Consignes

Le rapport sera écrit en **5 pages maximum**.

Il doit avoir la structure d'un article scientifique; il avoir :

– un titre,
– un auteur,
– un résumé (6 lignes maximum),
– un corps qui peut être séparé en sections,
– une conclusion,
– une bibliographie.

Bien évidement, le rapport ne doit pas être une traduction de l'article choisi. Le texte doit montrer que l'étudiant a fait un travail de réflexion pour situer le problème étudié dans le contexte du cours et dans une problématique générale. Il doit montrer également que étudiant s'est posé et a donné réponse à la question "pour quoi on étudie ce problème", autrement dit, "pour quoi ce problème est intéressant".

En ce qui concerne les développements analytiques présentés ou mentionnés dans les articles, il n'est pas nécessaire de les reproduire dans le rapport ; il faut, pourtant, donner les idées générales qui sont derrière les résultats obtenus.

Il est conseillé d'approfondir l'étude en consultant les références cités dans l'article choisi où bien en cherchant des articles reliés.

Papers

1. Extreme value statistics and fluctuating interfaces

Eric Bertin (DTP GENEVA), Maxime Clusel (ILL) *Generalised extreme value statistics and sum of correlated variables*, cond-mat/0601189

We show that generalised extreme value statistics -the statistics of the k-th largest value among a large set of random variables- can be mapped onto a problem of random sums. This allows us to identify classes of non-identical and (generally) correlated random variables with a sum distributed according to one of the three (k-dependent) asymptotic distributions of extreme value statistics, namely the Gumbel, Frechet and Weibull distributions. These classes, as well as the limit distributions, are naturally extended to real values of k, thus providing a clear interpretation to the onset of Gumbel distributions with non-integer index k in the statistics of global observables. This is one of the very few known generalisations of the central limit theorem to non-independent random variables. Finally, in the context of a simple physical model, we relate the index k to the ratio of the correlation length to the system size, which remains finite in strongly correlated systems.

Satya N. Majumdar, Alain Comtet *Airy Distribution Function: From the Area Under a Brownian Excursion to the Maximal Height of Fluctuating Interfaces*, J. Stat. Phys, 119 (2005) 777

The Airy distribution function describes the probability distribution of the area under a Brownian excursion over a unit interval. Surprisingly, this function has appeared in a number of seemingly unrelated problems, mostly in computer science and graph theory. In this paper, we show that this distribution also appears

in a rather well studied physical system, namely the fluctuating interfaces. We present an exact solution for the distribution $P(h_m, L)$ of the maximal height h_m (measured with respect to the average spatial height) in the steady state of a fluctuating interface in a one dimensional system of size L with both periodic and free boundary conditions. For the periodic case, we show that $P(h_m, L) = L^{-1/2} f(h_m L^{-1/2})$ for all L where the function $f(x)$ is the Airy distribution function. This result is valid for both the Edwards-Wilkinson and the Kardar-Parisi-Zhang interfaces. For the free boundary case, the same scaling holds $P(h_m, L) = L^{-1/2} F(h_m L^{-1/2})$, but the scaling function

$F(x)$ is different from that of the periodic case. We compute this scaling function explicitly for the Edwards-Wilkinson interface and call it the F-Airy distribution function. Numerical simulations are in excellent agreement with our analytical results. Our results provide a rather rare exactly solvable case for the distribution of extremum of a set of strongly correlated random variables. Some of these results were announced in a recent Letter [S.N. Majumdar and A. Comtet, Phys. Rev. Lett., 92, 225501 (2004)].

2. Equilibrium phase transitions

G. Fletcher, *A mechanical analog of first- and second-order phase transitions*, Am. J. Phys. **65**, 74 (1997).

A mechanical model that exhibits first- and second-order phase transitions is analyzed. The possible configurations are found first by using Newtonian mechanics and second by determining the minimum of the effective potential energy taken from the Lagrangian. A comparison is made between the effective potential energy method and the Landau theory of phase transitions. Phase diagrams are obtained for the mechanical system and are compared with those of a ferromagnet.

3. Inequivalence of ensembles in systems with long range interactions

Julien Barré, David Mukamel, Stefano Ruffo, *Inequivalence of ensembles in a system with long range interactions*, cond-mat/0102036, Phys. Rev. Lett. **87**, 030601 (2001).

We study the global phase diagram of the infinite range Blume-Emery-Griffiths model both in the canonical and in the microcanonical ensembles. The canonical phase diagram is known to exhibit first order and continuous transition lines separated by a tricritical point. We find that below the tricritical point, when the canonical transition is first order, the phase diagrams of the two ensembles disagree. In this region the microcanonical ensemble exhibits energy ranges with negative specific heat and temperature jumps at transition energies. These results can be extended to weakly decaying nonintegrable interactions.

D. Mukamel, S. Ruffo, N. Schreiber, *Breaking of ergodicity and long relaxation times in systems with long-range interactions*,

The thermodynamic and dynamical properties of an Ising model with both short range and long range, mean field like, interactions are studied within the microcanonical ensemble. It is found that the relaxation time of thermodynamically unstable states diverges logarithmically with system size. This is in contrast with the case of short range interactions where this time is finite. Moreover, at sufficiently low energies, gaps in the magnetization interval may develop to which no microscopic configuration corresponds. As a result, in local microcanonical dynamics the system cannot move across the gap, leading to breaking of ergodicity even in finite systems. These are general features of systems with long range interactions and are expected to be valid even when the interaction is slowly decaying with distance.

4. Scaling and interfaces

A.L. Barabasi and H.E. Stanley, *Fractal Concepts in Surface Growth*, (Cambridge University Press, Cambridge, 1995).

5. Stochastic resonance – just definition and discussion

L. Gammaitoni, P. Hänggi, P. Jung, F. Marchesoni, *Stochastic resonance*, Rev. Mod. Phys. **70**, 223 (1998).

6. Random walks and applications

M. O. Magnasco, *Forced thermal ratchets*, Phys. Rev. Lett. **71**, 1477 (1993).

We consider a Brownian particle in a periodic potential under heavy damping. The second law forbids it from displaying any net drift speed, even if the symmetry of the potential is broken. But if the particle is subject to an external force having time correlations, detailed balance is lost and the particle can exhibit a nonzero net drift speed. Thus, broken symmetry and time correlations are sufficient ingredients for transport.

E. P. Raposo, S. M. de Oliveira, A. M. Nemirovsky, and M. D. Coutinho-Filho, *Random walks: a pedestrian approach to polymers, critical phenomena and field theory*, Am. J. Phys. **59**, 633 (1991).

Main concepts in critical phenomena such as universality, critical dimension, analogies with the statistics of polymers and scaling theory, together with renormalization group ideas and rudimentary methods in field theory are presented through the simple and intuitive properties of random walks.

This work, which requires little previous knowledge of mathematics besides elementary calculus, has served as the basis of a program to introduce undergraduate students to research in theoretical physics.

7. Domain growth

J. M. Kim, A. J. Bray, M. A. Moore, *Domain growth, directed polymer and self-organized criticality*, Phys. Rev. A **45**, 8546 (1992).

The nature of the roughness of the growing surface modeled by the Kardar-Parisi-Zhang (KPZ) equation has been further studied by defining a 'domain' structure for it. This was done by mapping the height $h(x,t)$ of the surface onto a 'spin' $S(x,t) = sgn[h(x,t) - \langle h(t) \rangle]$, where $\langle h(t) \rangle$ is the mean height of the growing surface at time (t). It was then found that the growth of the surface has useful analogies with the domain-coarsening process in nonequilibrium systems quenched into an ordered phase. Thus, in $d = 1 + 1$, the average size $\langle l \rangle$ of spin domains grows as $t^{1/3}$ and the domain-size distribution $P(l,t)$ is consistent with $l^{-3/2} f(l/t^{2/3})$. We find that the autocorrelation function $A(t) = \langle S(x,t)S(x,0) \rangle$ decays as $t^{-}\alpha$, with $\alpha(d = 1+1) = 1.00.08$ and $\alpha(d = 2+1) = 1.50.1$. The form of $P(l,t)$ in higher dimensions and the connection between the spin autocorrelation function and the height autocorrelation function are discussed. For computational convenience, the KPZ equation was studied by transforming it to the problem of directed polymers in a random potential.

B. Derrida, C. Godrèche, I. Yekutieli, *Scale invariant regimes in one dimensional models of growing and coalescent droplets*, Phys. Rev. A **44**, 6241 (1991).

We consider several simplified models of breath figures in one dimension. For all these models, the combined effects of growth and of coalescence of droplets lead to a scale-invariant regime with a stable distribution of the distances between droplets. We show that at the mean-field level there exist one-parameter families of such stable distributions, each distribution being characterized by its decay at infinity. We explain how the mean-field theory can be improved by taking into account the effect of pair or higher correlations. For some models one can check that the pair and higher correlations are factorized, meaning that correlations are absent and that therefore the mean-field theory is exact. Finally, we show that a very simple model of domain growth related to spinodal decomposition, the one-dimensional Potts model in the limit of an infinite number of states, also possesses a one-parameter family of stable distributions analogous to what we obtained for breath figures.

8. Quenched disorder

R. da Silveira, *An introduction to breakdown phenomena in disordered systems*, Am. J. Phys. 67, 1177 (1999).

H. Yoshino, *Off equilibrium dynamics of a (1+1)-dimensional directed polymer in random media*, J. Phys. A **29**, 1421 (1996).

O. Perkovič, K. Dhamen, and J. P. Sethna, *Avalanches, Barkhausen noise, and plain old criticality*, Phys. Rev. Lett. **75**, 4528 (1995).

J. P. Sethna, *Hysteresis, avalanches, and Barkhausen noise*, http://xxx.lanl.gov/cond-mat/9704059; Lecture 10 in Les Houches.

T. Natterman, Y. Shapir, I. Vilfan, *Interface pinning and dynamics in random systems*, Phys. Rev. B **42**, 8577 (1990).

9. The spherical model: replica method and dynamics

J. M. Kosterlitz, D. J. Thouless, and Raymund C. Jones *Spherical Model of a Spin-Glass*, Phys. Rev. Lett. **36**, 1217 (1976).

A spherical model of a spin-glass is solved in the limit of infinite-ranged interactions with a Gaussian probability distribution. We use the known properties of a large random matrix, and show that the results are identical to those obtained by the $n \to 0$ trick. We believe that the solution is exact.

L. F. Cugliandolo, D. S. Dean *Full dynamical solution for a spherical spin-glass model* J. Phys. A **28**, 4213 (1995).

We present a detailed analysis for the Langevin dynamics of a spherical spin-glass model (the spherical Sherrington-Kirkpatrick model). The effects of initial conditions on the ultimate dynamical behaviour are closely examined. In addition, the effects of temperature variations in the model are studied. Somewhat surprisingly, this simple model captures some of the effects seen in laboratory spin-glasses.

10. Phase space geometry and slow dynamics

Jorge Kurchan and Laurent Laloux, *Phase space geometry and slow dynamics* J. Phys. A **29**, 1929 (1996).

We describe a non-Arrhenius mechanism for the slowing down of dynamics that is inherent to the high dimensionality of the phase space. We show that such a mechanism is at work both in a family of mean-field spin-glass models without any domain structure and in the case of ferromagnetic domain growth. The marginality of spin-glass dynamics, as well as the existence of a 'quasi-equilibrium regime' can be understood within this scenario. We discuss the question of ergodicity in an out-of equilibrium situation.

11. Random matrices and replica theory

S F Edwards and R C Jones *The eigenvalue spectrum of a large symmetric random matrix*, J. Phys. A: Math. Gen. 9 1595-1603 (1976).

A new and straightforward method is presented for calculating the eigenvalue spectrum of a large symmetric square matrix each of whose upper triangular elements is described by a Gaussian probability density function with the same mean and variance. Using the n to 0 method, the authors derive the semicircular eigenvalue spectrum when the mean of each element is zero and show that there is a critical finite mean value above which a single eigenvalue splits off from the semicircular continuum of eigenvalues.

12. The random energy model

B. Derrida, *Random-Energy Model: Limit of a Family of Disordered Models*, Phys. Rev. Lett. **45**, 79 (1980),

In this Letter, a simple model of disordered systems 'the random-energy model' is introduced and solved. This model is the limit of a family of disordered models, when the correlations between the energy levels become negligible. The properties are qualitatively the same as those of the Sherrington-Kirkpatrick model. Moreover, this random-energy model looks like a simple approximation to any spin-glass model.

B. Derrida, *Random-energy model: An exactly solvable model of disordered systems*, Phys. Rev. B 24, 2613 (1981)

A simple model of disordered systems 'the random-energy model' is introduced and solved. This model is the limit of a family of disordered models, when the correlations between the energy levels become negligible. The model exhibits a phase transition and the low-temperature phase is completely frozen. The corrections to the thermodynamic limit are discussed in detail. The magnetic properties are studied, and a constant susceptibility is found at low temperature. The phase diagram in the presence of ferromagnetic pair interactions is described. Many results are qualitatively the same as those of the Sherrington-Kirkpatrick model. The problem of using the replica method is analyzed. Lastly, this random-energy model provides lower bounds for the ground-state energy of a large class of spin-glass models.

13. Trap model

Giulio Biroli, *A Crash Course on Aging*, cond-mat/0504681, Lecture Notes (22 pages) given at "Unifying Concepts in Glass Physics III", Bangalore (2004); to be published in JSTAT.

In these lecture notes I describe some of the main theoretical ideas emerged to explain the aging dynamics. This is meant to be a very short introduction to aging dynamics and no previous knowledge is assumed. I will go through simple examples that allow one to grasp the main results and predictions.

and references therein.

14. Temperatures effectives

Leticia F. Cugliandolo *Temperatures effectives*, le chapitre correspondant des notes des Houches, 2002

Patrick Ilg, Jean-Louis Barrat *Effective temperatures in a simple model of non-equilibrium, non-Markovian dynamics*, cond-mat/0601618

The concept of effective temperatures in nonequilibrium systems is studied within an exactly solvable model of non-Markovian diffusion. The system is coupled to two heat baths which are kept at different temperatures: one ('fast') bath associated with an uncorrelated Gaussian noise and a second ('slow') bath with an exponential memory kernel. Various definitions of effective temperatures proposed in the literature are evaluated and compared. The range of validity of these definitions is discussed. It is shown in particular, that the effective temperature defined from the fluctuation-dissipation relation mirrors the temperature of the slow bath in parameter regions corresponding to a separation of time scales. On the contrary, quasi-static and thermodynamic definitions of an effective temperature are found to display the temperature of the fast bath in most parameter regions.

Antonio Prados, J. Javier Brey *Effective dynamics and steady state of an Ising model submitted to tapping processes* Physical Review E 66, 041308 (2002)

A one-dimensional Ising model with nearest neighbour interactions is applied to study compaction processes in granular media. An equivalent particle-hole picture is introduced, with the holes being associated to the domain walls of the Ising model. Trying to mimic the experiments, a series of taps separated by large enough waiting times, for which the system freely relaxes, is considered. The free relaxation of the system corresponds to a T=0 dynamics which can be analytically solved. There is an extensive number of metastable states, characterized by all the holes being isolated. In the limit of weak tapping, an effective dynamics connecting the metastable states is obtained. The steady state of this dynamics is analyzed, and the probability distribution function is shown to have the canonical form. Then, the stationary state is described by Edwards thermodynamic granular theory. Spatial correlation functions in the steady state are also studied.

15. Optimisation problems

O.C. Martin, R. Monasson, R. Zecchina, *Statistical mechanics methods and phase transitions in optimization problems*, Theoretical Computer Science 265 (1-2) (2001) 3-67, cond-mat/0104428.

Recently, it has been recognized that phase transitions play an important role in the probabilistic analysis of combinatorial optimization problems. However, there are in fact many other relations that lead to close ties between computer science and statistical physics. This review aims at presenting the tools and concepts designed by physicists to deal with optimization or decision problems in an accessible language for computer scientists and mathematicians, with no prerequisites in physics. We first introduce some elementary methods of statistical mechanics and then progressively cover the tools appropriate for disordered systems. In each case, we apply these methods to study the phase transitions or the statistical properties of the optimal solutions in various combinatorial problems. We cover in detail the Random Graph, the Satisfiability, and the Traveling Salesman problems. References to the physics literature on optimization are provided. We also give our perspective regarding the interdisciplinary contribution of physics to computer science.