Non potential forces

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Plan

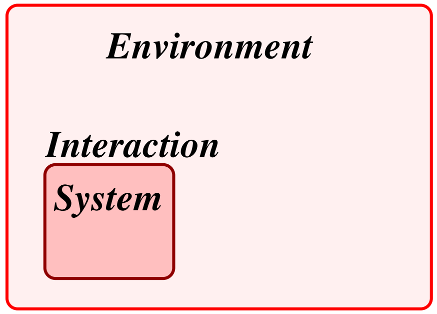
LFC, J. Kurchan, P. Le Doussal L. Peliti 98

Berthier, LFC, Iguain 00

Dynamics out of equilibrium

One condition for equilibrium explicitly broken

Take an open system coupled to an environment



Necessary :

— The bath should be in equilibrium

same origin of noise and friction.

— Deterministic force :
conservative time-independent forces only, $\vec{F} = -\vec{\nabla} V$.

— Either the initial condition is taken from the equilibrium pdf, or the latter should be reached after an equilibration time t_{eq} :

Models

Driven p-spin models

Hamiltonian (potential energy)

$$H_J[\{s_i\}] = -\sum_{i_1\dots i_p} J_{i_1\dots i_p} \underbrace{s_{i_1}\dots s_{i_p}}_{\text{symmetric}} + z\left(\sum_i s_i^2 - N\right)$$

under exchanges of any pair of indices $i_k \leftrightarrow i_j$

The random coupling exchanges taken from Gaussian pdf with zero mean and variance $[J^2_{i_1\dots i_p}] = p! J^2/N^{p-1}$ and they are also symmetric with respect to $i_k \leftrightarrow i_j$

Langevin dynamics

$$\mathrm{d}_t s_i(t) = +\sum_{i_2\dots i_p} J_{ii_2\dots i_p} s_{i_2}(t)\dots s_{i_p}(t) - z(t)s_i(t) + \xi_i(t)$$

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symmetric

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Langevin dynamics with no symmetric exchanges $J_{ii_2i_3\dots i_p} \neq J_{i_2ii_3\dots i_p}$

$$\mathrm{d}_t s_i(t) = +\sum_{i_2\dots i_p} J_{ii_2\dots i_p} s_{i_2}(t)\dots s_{i_p}(t) - z(t)s_i(t) + \xi_i(t)$$

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Exchanges

$$\boxed{J_{ii_2 i_3\dots i_p} = J^S_{ii_2 i_3\dots i_p} + \alpha J^A_{ii_2 i_3\dots i_p}}$$

Models

Driven p-spin models

Hamiltonian (potential energy)

$$H_J[\{s_i\}] = -\sum_{i_1\dots i_p} J_{i_1\dots i_p} \underbrace{s_{i_1}\dots s_{i_p}}_{\text{symmetric}} + z\left(\sum_i s_i^2 - N\right)$$

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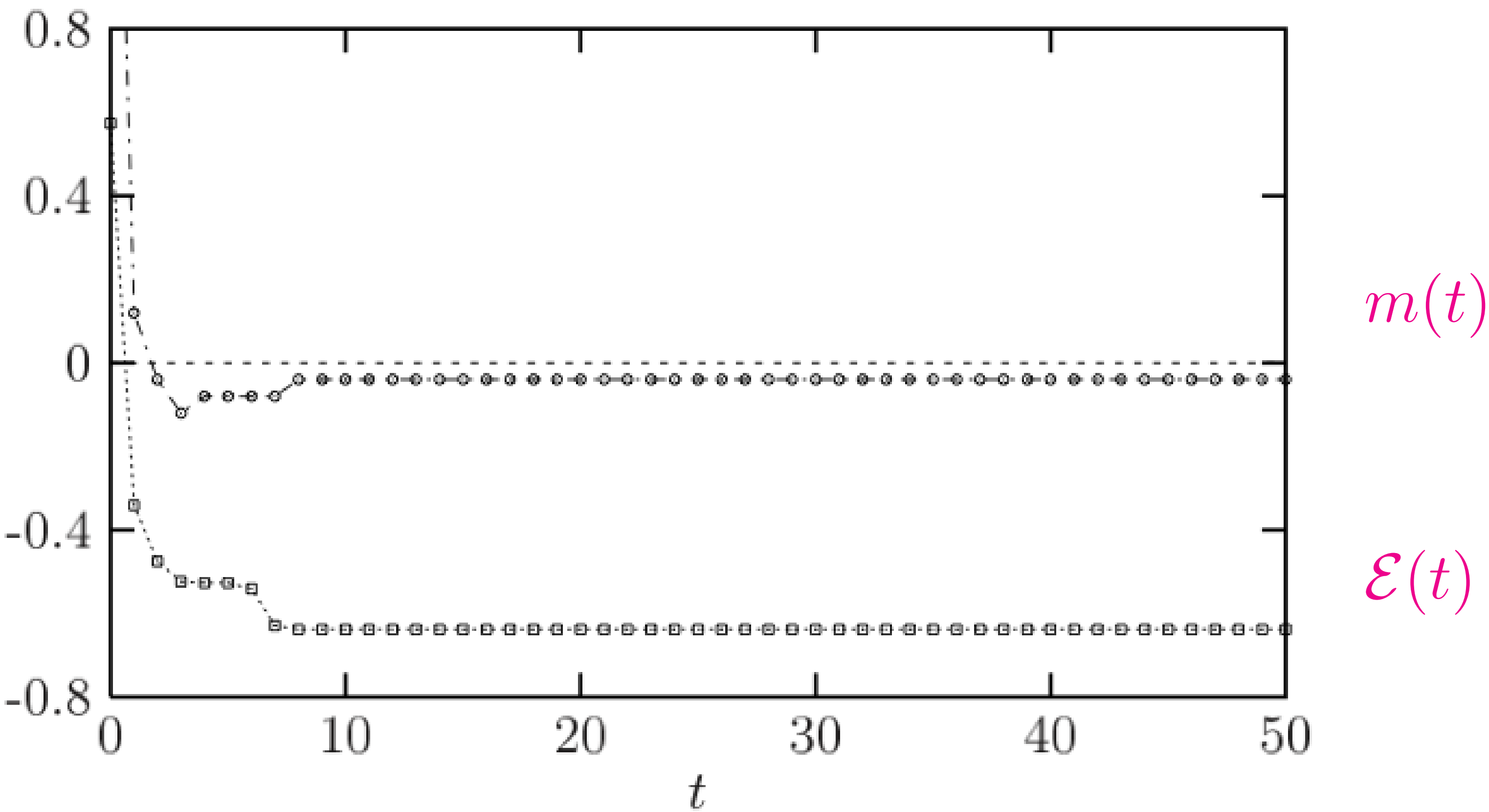
The random coupling exchanges taken from Gaussian pdf with zero mean and variance $[J^2_{i_1\dots i_p}] = p!J^2/N^{p-1}$ and they are also symmetric with respect to $i_k \leftrightarrow i_j$

Langevin dynamics with time-dependent forces

$$\mathrm{d}_t s_i(t) = \sum_{i_2\dots i_p} J^S_{ii_2\dots i_p} s_{i_2}(t)\dots s_{i_p}(t) - z(t)s_i(t) + h_i(\omega, t) + \xi_i(t)$$

Potential force

$p = 3$ **Ising spin model with** $N = 50$ **at** $T = 0.01$

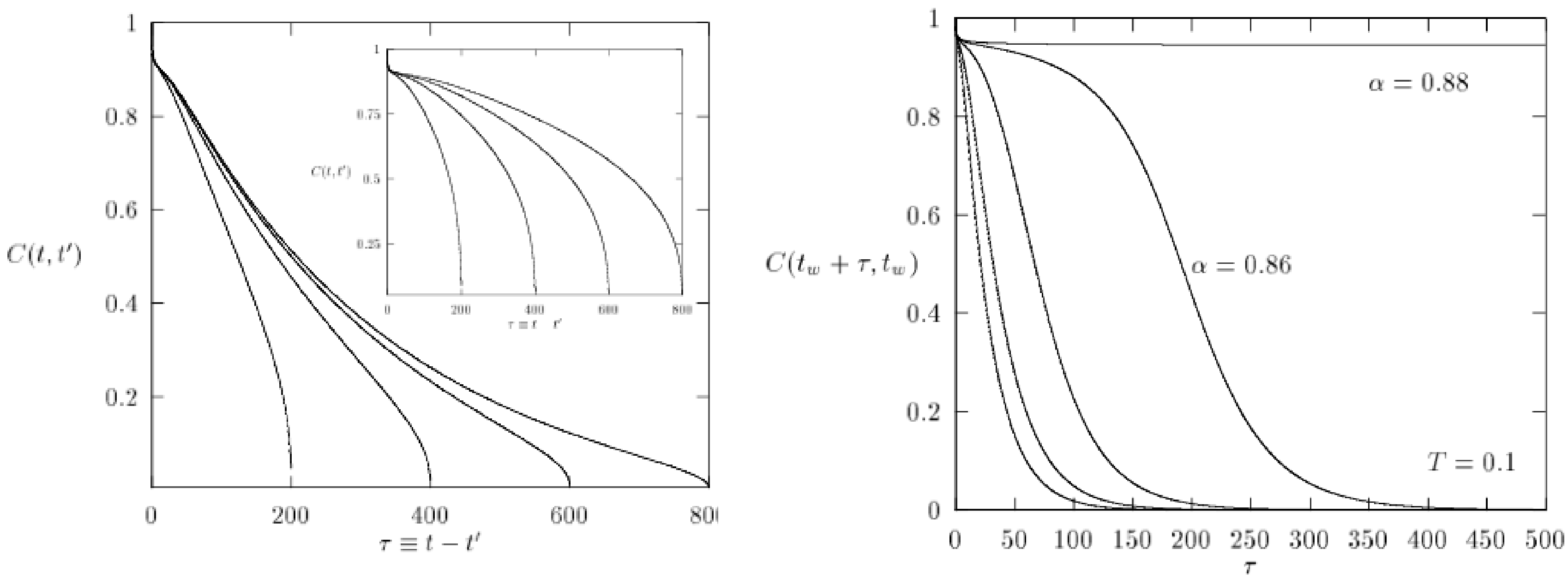


Magnetisation and energy density decay

Initial condition dependent metastable state reached $\mathcal{E}_\infty > \mathcal{E}_{\rm th} = -1.155\,J$

Non-potential force

Driven $p = 3$ Ising spin model with $N = 50$

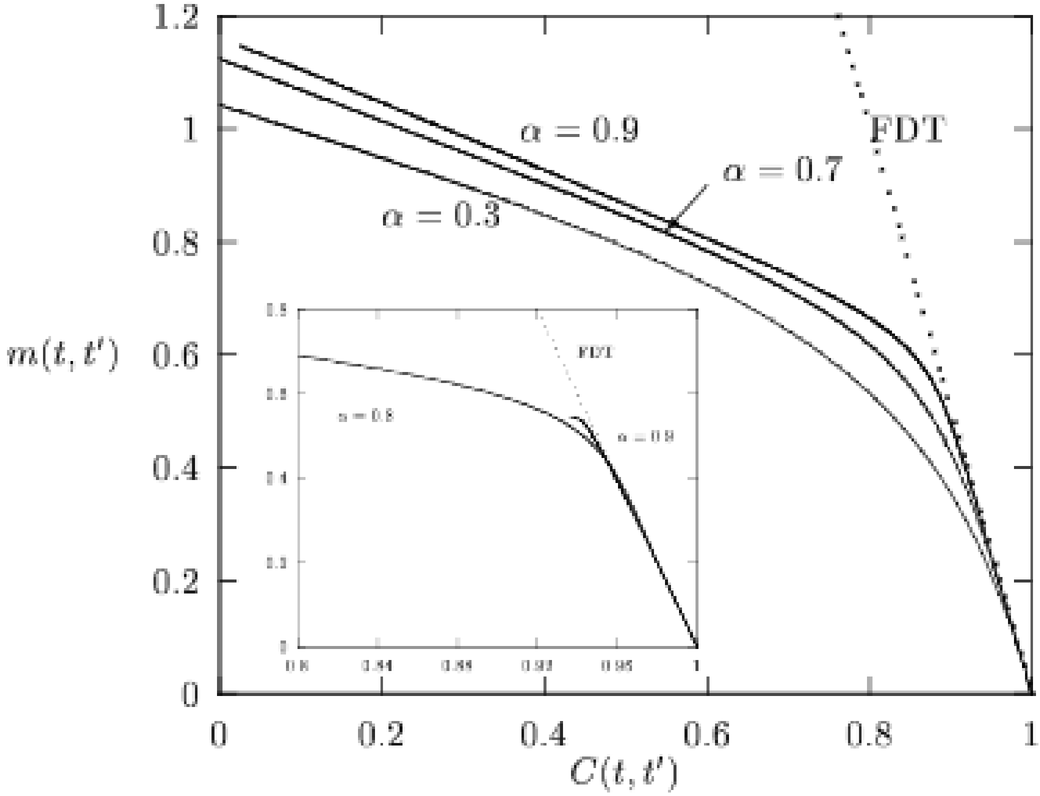


Waiting-time dependence (α fixed) and α dependence in steady state

$$J_{ii_2i_3\ldots i_p} = J^S_{ii_2i_3\ldots i_p} + \alpha J^A_{ii_2i_3\ldots i_p}$$

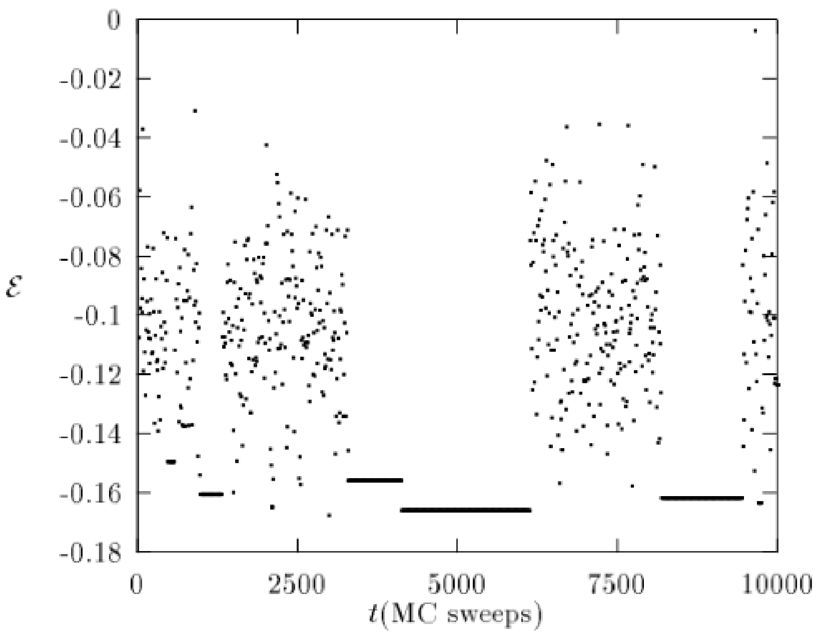
Non-potential force

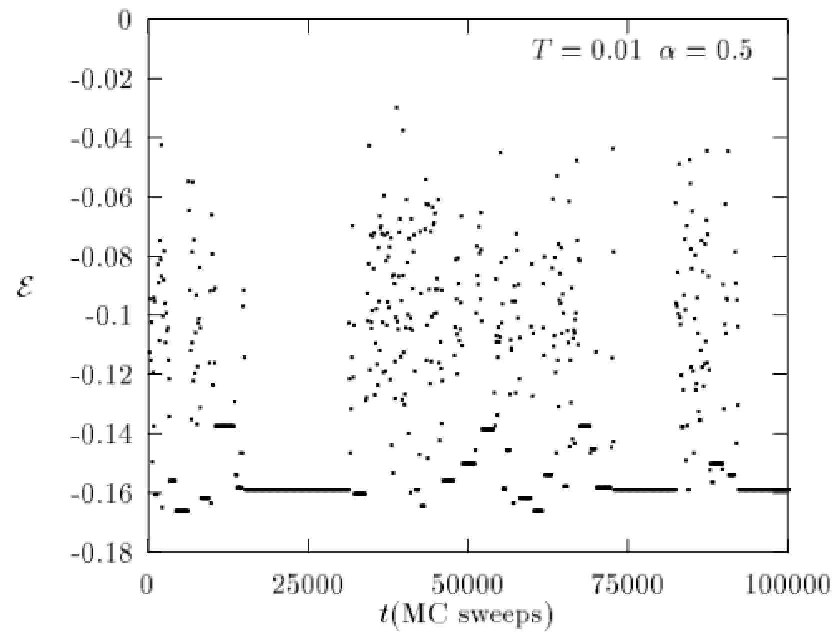
Driven $p = 3$ Ising spin model with $N \to \infty$



Non-potential force

Driven $p = 3$ Ising spin model with $N = 50$



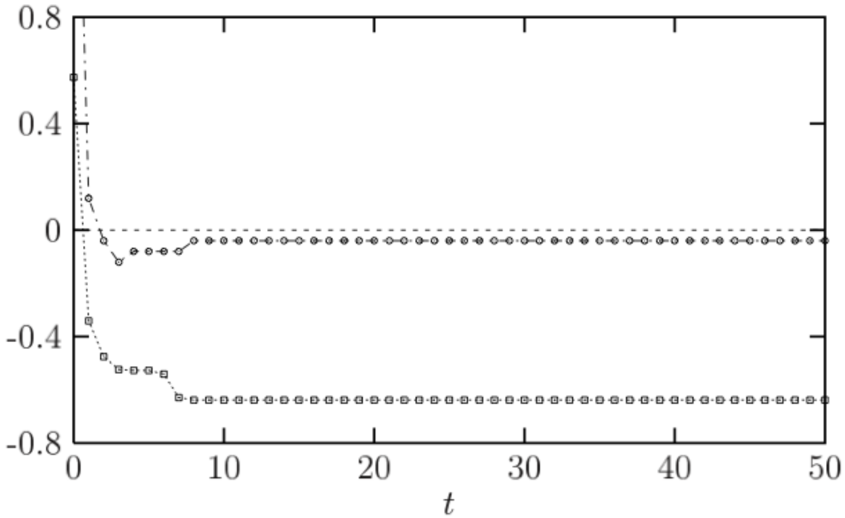


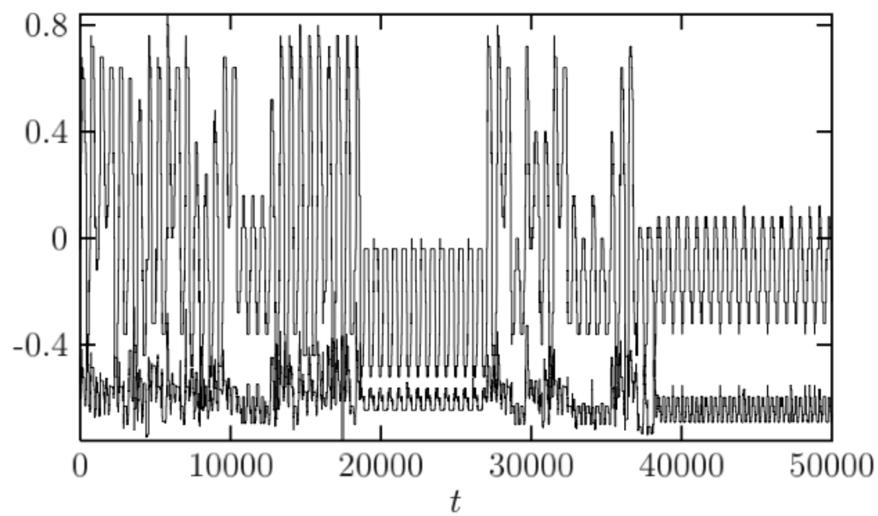
Time dependent energy density

$$J_{ii_2i_3\ldots i_p} = J^S_{ii_2i_3\ldots i_p} + \alpha J^A_{ii_2i_3\ldots i_p}$$

Time-dependent force

Driven $p = 3$ Ising spin model with $N = 50$





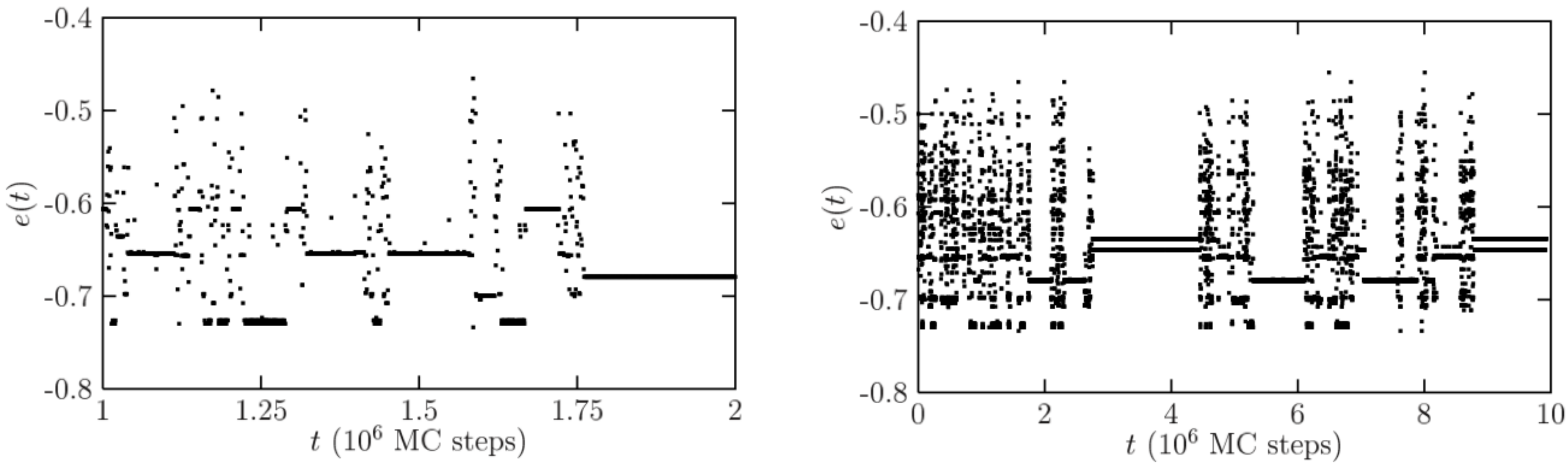
$h = 0$

$h(\omega, t) = h \cos(\omega t)$

Time dependent magnetisation and energy density

Time-dependent force

Driven $p = 3$ Ising spin model with $N = 50$



$h(\omega, t) = h \cos(\omega t)$ with $h = 2,\ \omega = 0.01$

Stroboscopic-time dependent magnetisation and energy density