FINITE DIMENSIONAL SYSTEMS

QUESTIONS

- DOES DISORDER CHANGE THE LOW T PHASE WITH RESPECT TO THE PURE MODEL ?

  J_{ij} WITH FRUST. $\Rightarrow$ EXPECTED
  J_{ij} WITHOUT FRUST $\Rightarrow$ NO

  e.g. SPIN-GLASSES VS DISORDERED MAGNETS

- THE PROPS OF THE PHASES SIGNIFICANTLY ?

  GRIFFITHS PHASE IN NON-FRUST CASES

- THE ORDER OF PHASE TRANS ?

  ROUNDING $\Rightarrow$ 1st $\rightarrow$ 2nd ORDER

- IN CONTINUOUS PHASE TRANS, THE CRITICAL EXP. ?

  HARRIS CRITERIUM

- SOME EXACT RESULTS

  GAUGE INVARIANCE & NISHIMORI LINES

HARRIS CRITERIUM

TAKE A PURE SYST WITH A 2nd ORDER PHASE TRANS

$$\xi_{PURE} \sim |T - T_c^{PURE}|^{-\nu_{PURE}}$$

MODIFY IT BY "ADDING" DISORDER

e.g. $-J\, s_i s_j \to -J_{ij}\, s_i s_j$

ASSUME STILL A 2nd ORDER PHASE TRANS.

$$\xi_{DIS} \sim |T - T_c^{DIS}|^{-\nu_{DIS}}$$

CLAIM

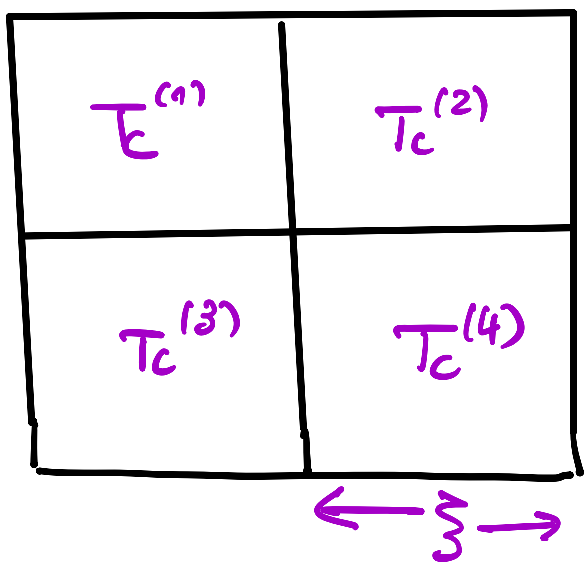
$\nu_{PURE} > \frac{2}{d} \Longrightarrow$ CRITICAL EXP UNCHANGED

$\nu_{PURE} < \frac{2}{d} \Longrightarrow$ CHANGE IN CRIT EXP. ν

PROOF

BREAK SYST IN BLOCKS OF SIZE

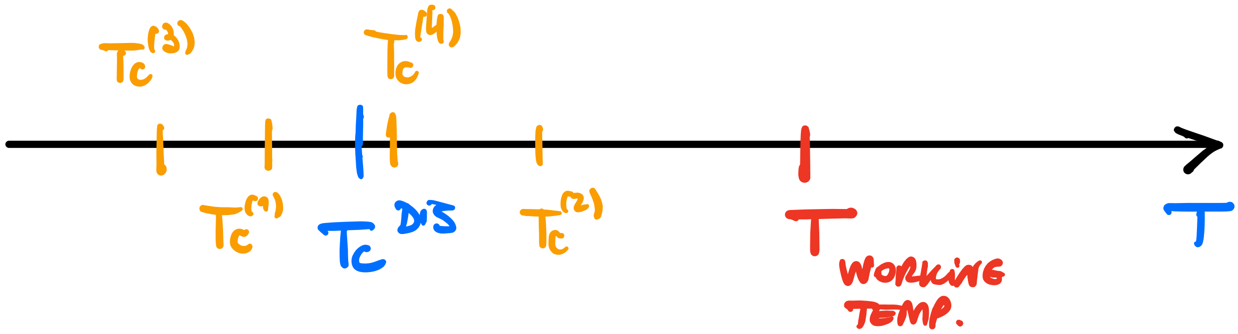
$$\xi_{DIS} = \xi$$



BECAUSE OF FLUCT IN J_{ij} THE BLOCK'S CRIT TEMP. CAN BE DIFF. FROM BLOCK TO BLOCK

THINK INTER. COULD BE VERY STRONG SOMEWHERE $\Longrightarrow T_c \uparrow$

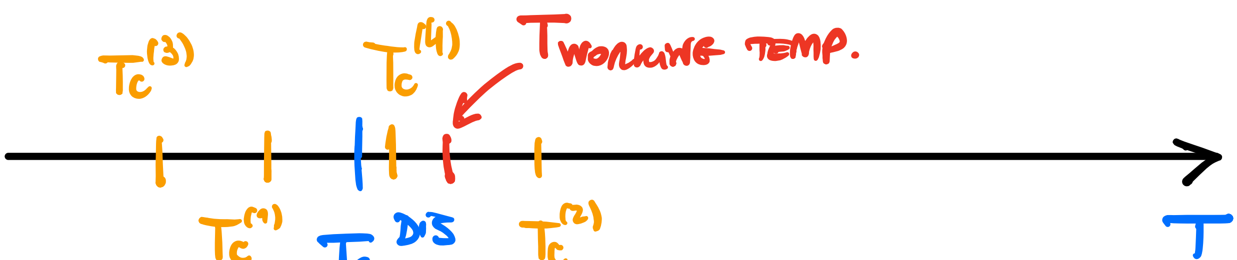
WHILE WEAK ELSEWHERE $\Longrightarrow T_c \searrow$



TAKE THE WORKING T TO BE ABOVE ALL $T_c^{(k)}$

$T > T_c^{(k)}$ (ALSO T_c^{DIS})

$\Rightarrow$ THE SYST IS DISORDERED,
IN PM PHASE



SOME BLOCKS HAVE $T_c^{(k)} > T$

THEY ARE IN THE
ORDERED PHASE

DIFFERENT CASE

ASK $T > T_c^{(k)} \quad \forall k$

TO NOT MODIFY THE CRITICAL BEHAVIOUR

EQUIVALENT TO

$$T - T_c^{Dis} > T_c^{(k)} - T_c^{Dis}$$

$$\Delta T > \Delta T_c^{(k)}$$

WE NOTE THAT $\Delta T_c^{(k)}$ WILL ALSO DEPEND ON TEMP. WHEN WE DECREASE $\Delta T \to 0$ & APPROACH THE CRITICAL POINT.

QUESTION: HOW DO THEY BOTH DEP. ON T ?

HOW DO THEY COMPARE?

START FROM

$$T - T_c^{DIS} > T_c^{(k)} - T_c^{DIS}$$

$$\Delta T > \Delta T^{(k)}$$

NOW, WE RELATE ΔT TO THE CORR LENGTH

$$\xi_{DIS} \sim |\Delta T|^{-\nu_{DIS}} \Longrightarrow |\Delta T| \sim \xi_{DIS}^{-1/\nu_{DIS}}$$

WE CALLED $\xi_{DIS} = \xi \Longrightarrow$

$$|\Delta T| \sim \xi^{-1/\nu_{DIS}}$$

BUT, IF THERE'S NO CHANGE IN CRIT EXPONENT

$$\nu_{DIS} = \nu_{PURE}$$

$$\xi_{DIS} \sim |\Delta T|^{-\nu_{DIS}} = |\Delta T|^{-\nu_{PURE}}$$

$$\boxed{|\Delta T| \sim \xi^{-1/\nu_{PURE}}}$$

WHAT IS $\Delta T^{(k)}$?

$$\Delta T^{(k)} = T^{(k)} - T_c^{DIS}$$

IT'S CLEAR THAT IF THE SIZE OF THE BOX DIVERGES $\Rightarrow$ $T_c^{(k)} - T_c^{DIS} = 0$

SO, THE VARIATION SHOULD DECREASE WITH THE BOX SIZE. HOW?

THE CRITICAL TEMP. OF A FM MODEL GROWS WITH J

WE EXPECT THE SAME IN A DISORDER MODEL

SO EACH BOX WILL HAVE

$$T_c^{(k)} \propto J^{(k)}$$

WHAT IS $J^{(k)}$?

THE INTERACTIONS ARE DISORDERED. RANDOM BOND ISING MODEL

ONE CAN GUESS IT'S THE AVERAGED J_{ij} ON THE BOX

THE J_{ij} ARE QUENCHED RANDOM VARIABLES WITH AVERAGE

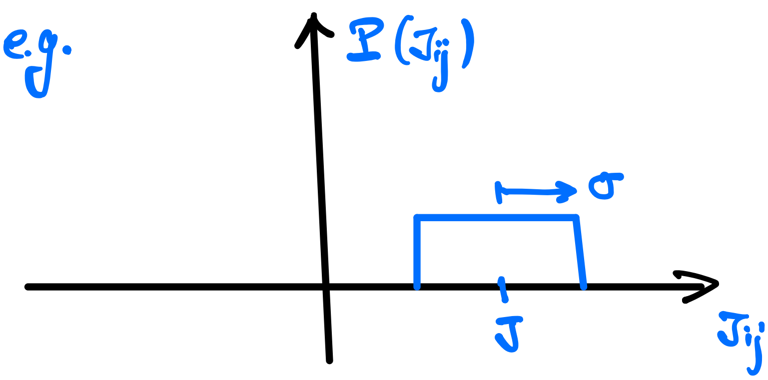
$$[J_{ij}] = J$$

$$[J_{ij}] = J$$

AND VARIANCE

$$[J_{ij}^2] - [J_{ij}]^2 = \sigma_J^2$$

e.g.



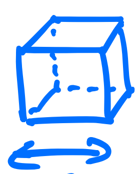
FINITE AVERAGE AND DISPERSION

THERE ARE AS MANY J_{ij} TO SUM OVER AS SITES IN THE BOX SINCE N.N. INTERACTIONS

$$\#\text{ INT.} = \underbrace{(\#\text{ SITES})}_{\text{LARGE}}\ \underbrace{(\text{COORD. LATTICE})}_{\text{FINITE}}$$

HOW MANY SITES IN THE BOX ?

ξ_{PURE}^{d}



ξ_{PURE}

SINCE

$$\xi_{PURE} = \xi_{DIS}$$

CENTRAL LIMIT THEOREM

$$J^{(k)} \sim \frac{1}{\xi^d} \sum_{ij \in Box(k)} J_{ij}$$

$J^{(k)}$ IS GAUSSIAN DISTRIBUTED WITH

AVERAGE $[J^{(k)}] = [J_{ij}]$

AND VARIANCE $\sigma_{J^{(k)}} \sim \sigma_J \cdot \xi^{-d/2}$

THEREFORE

$$\Delta J^{(k)} \sim \sigma_{J^k} \sim \xi^{-d/2}$$

WHICH CORRECTLY VANISHES FOR $\xi \to \infty$

THIS IS THE REDUCTION FACTOR

$$|\Delta J^{(k)}| \sim \xi^{-d/2}$$

$$\Rightarrow \quad \xi^{-1/\nu_{PURE}} > \xi^{-d/2}$$

$$\Longrightarrow \qquad \frac{1}{\nu_{PURE}} < \frac{d}{2}$$

$$\boxed{\nu_{PURE}\, d > 2}$$

THIS IS THE HARRIS CRITERIUM

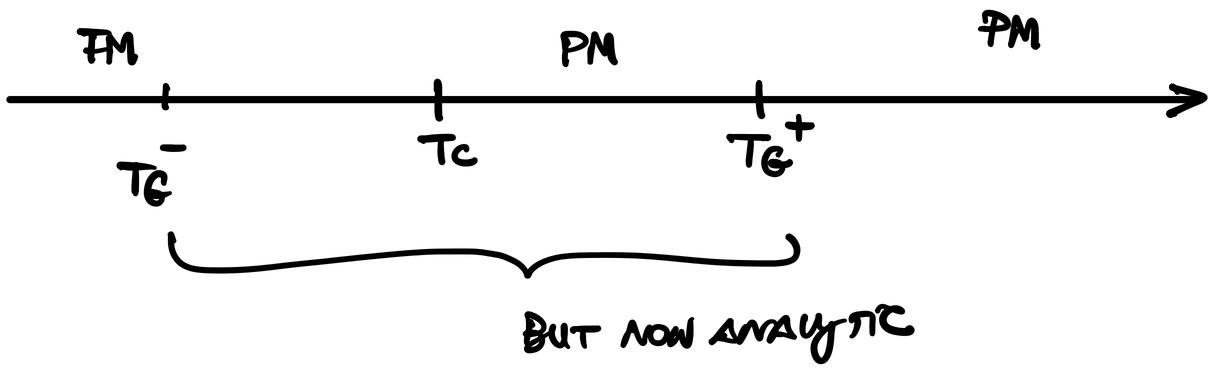
SOMETIMES WRITTEN IN TERMS OF THE SPECIFIC HEAT EXPONENT

$$\alpha = 2 - d\nu$$

$$\Longrightarrow \qquad \alpha < 0 \qquad \Longrightarrow \qquad \text{NO CHANGE}$$

Griffiths Phase

Griffiths showed that $-\beta f$ is non-analytic in a range of temps. above T_c



Dilute FM

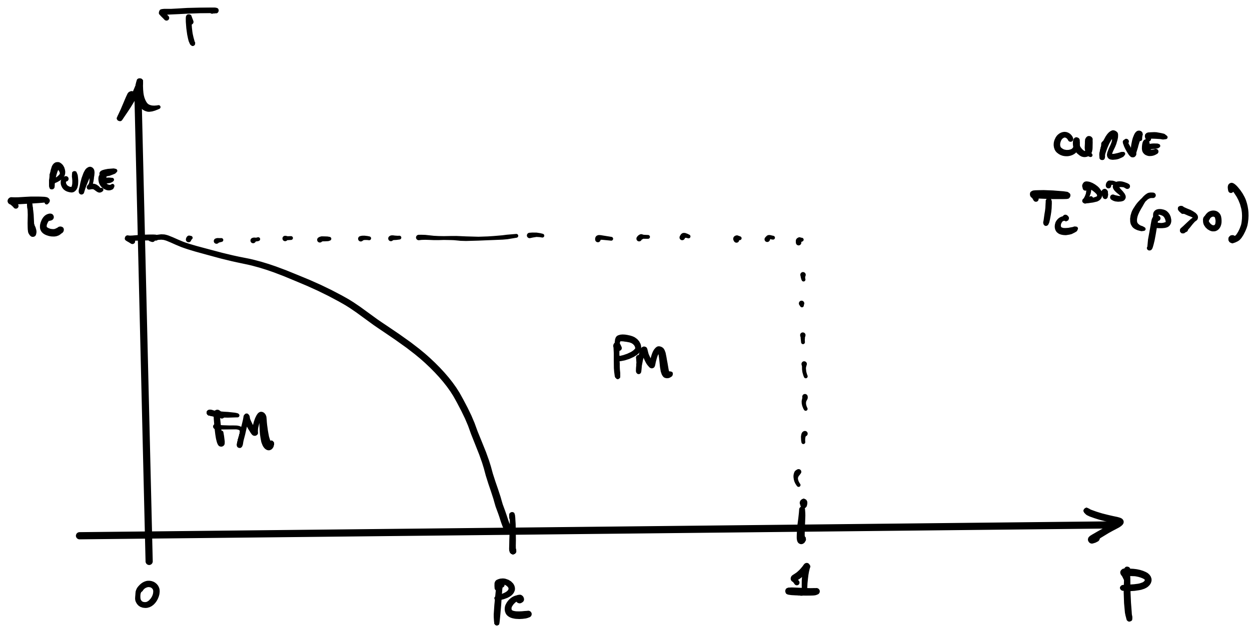
Site occupied w/ prob $1-p$

empty p

From percolation we know there's a

p_c above which $\nexists$ perc cluster

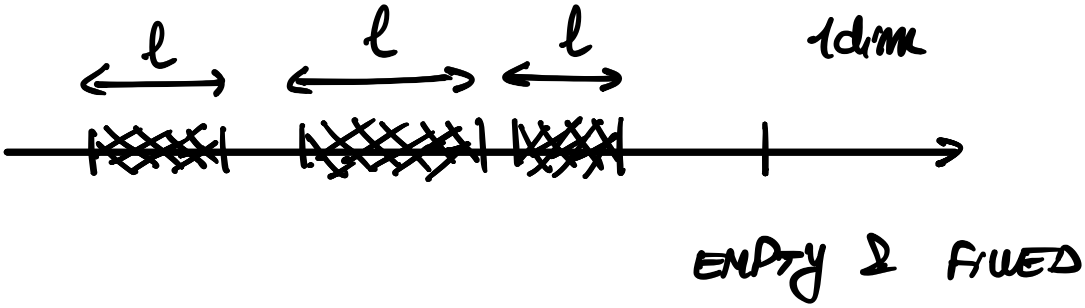
below $\exists$ perc cluster



T_c CANNOT BE DIFF FROM ZERO ABOVE p_c

SHOULD BE THE KNOWN VALUE OF THE "FULL" SYSTEM AT $p=0$

PERCOLATION



SEGMENT OF OCCUPIED SITE OF LENGTH ℓ

AND TWO EMPTY SITES AT THE BORDERS

CLUSTER LINEAR SIZE PROBABILITY

$$\mathcal{P}(\ell) = (1-p)^{\ell} \; p^2 \qquad d=1$$

$$= p^2 \; e^{\ell \ln (1-p)}$$

$$= p^2 e^{-c(p)\ell} \qquad c(p) \equiv -\ln(1-p) > 0$$

d DIMENSIONAL CASE

$$\mathcal{P}(\ell) \sim (1-p)^{\ell^d} \; p^{\ell^{d-1}}$$

ℓ^d VOLUME $\quad \ell^{d-1}$ SURFACE OF CLUSTER

$$\mathcal{P}(\ell) \sim e^{\ell^d \ln(1-p) + \ell^{d-1} \ln p}$$

$$\sim e^{\ell^d \ln(1-p)} = e^{-c(p)\ell^d} \qquad d \text{ GENERAL}$$

THE MAGN. DENSITY

$$m(T,h) = \frac{1}{N}\sum_i \langle s_i \rangle = \sum_\ell P(\ell)\, m(\ell;T,h)$$

A SUM OVER CLUSTERS OF SIZE ℓ
EACH WITH ITS OWN PROBABILITY

EACH CLUSTER OF FINITE SIZE, AT TEMP IN BTW

$$T_c^{DIS} < T < T_c^{PURE}$$

IS COHERENT, AND MAGNETIZES

WE ESTIMATED $P(\ell)$, WE NOW NEED $m(\ell;T,h)$

$$m(\ell;T,h) \sim \mu \cdot \ell^d$$

$$\Delta E(T,h) = E_{ALIGNED}(\ell) - E_{ANTIALIGNED}(\ell)$$

$$= -2h\mu\,\ell^d$$ DEPENDS ON h

IMAGINE THAT SMALL CLUSTERS CAN BE EASILY FLIPPED
AT THE WORKING T WHILE LARGE ONES ARE FROZEN

THE SEPARATION is CONTROLLED BY $k_B T$

$$\Delta E > k_B T \qquad \text{FROZEN}$$

$$\Delta E < k_B T \qquad \text{FLIPPABLE}$$

$$m_{\text{FROZEN}}(T,h) \sim \sum_{\Delta E > k_B T} P(\ell)\, m(\ell_i; T, h)$$

$$\sim \int_{\ell_c}^{\infty} d\ell \; e^{-\ell^d c(p)} \mu \ell^d$$

ℓ_c SUCH THAT $\Delta E_c \sim k_B T$

$$2h\mu\ell_c \sim k_B T$$

$$\ell_c \sim \frac{k_B T}{\mu h}$$

$$m_{\text{FROZEN}} \sim \int_{\frac{k_B T}{\mu h}}^{\infty} d\ell \; e^{-\ell^d c(p) + \ln(\mu \ell^d)}$$

SADDLE-POINT EVALUATION OF THE INTEGRAL

$$\text{"ACTION"}(\ell) = -\ell^d c(p) + \underbrace{\ln(\mu \ell^d)}$$

first term $\gg$ ↓ Drop

⇓

BORDER OF INT. DOMINATES

$$m_{FROZEN} \sim e^{-c(p)\,\ell_c^d} \sim e^{-c(p)\frac{k_B T}{\mu h}}$$

$$\boxed{m_{FROZEN} \sim e^{-c(p)\frac{k_B T}{\mu h}}}$$

ESSENTIAL SINGULARITY FOR $h \to 0$

CONTRIBUTION OF RARE REGIONS WHICH EXIST WITH PROB

$$e^{-c(p)L^d}$$

VERY LOW PROBABILITY

ESSENTIAL SINGULARITY

$$f(x) = e^{-\frac{a}{x}} \xrightarrow[x\to 0]{} 0$$

$$f'(x) = +a \frac{1}{x^2} e^{-a/x} \xrightarrow[x\to 0]{} 0$$

ALL DERIVATIVES WILL DIVERGE, PROBLEMS TO TAYLOR EXP. AROUND $x=0$

ESTIMATE χ FROM m^2, VANISHING CONTRIBUTION OF THE RARE REGIONS SINCE

$$\chi_{\text{FROZEN}} \xrightarrow[h\to 0]{} 0$$

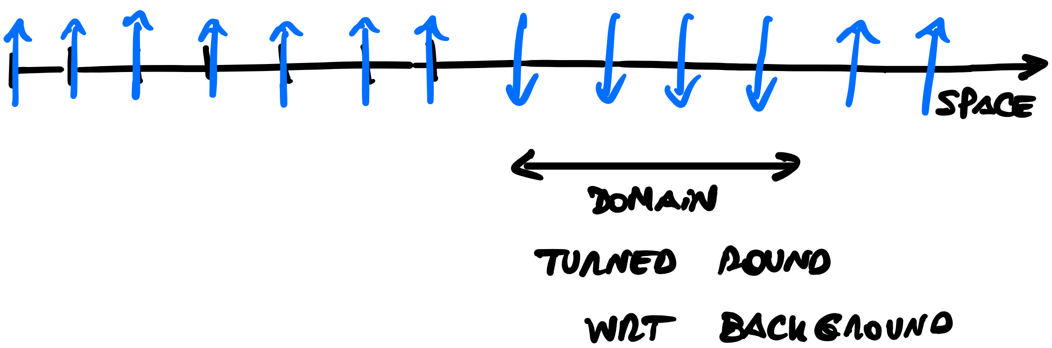
COMMENT

IN OTHER PROBLEMS $\chi(\ell)$ COULD DEPEND DIFFERENTLY ON ℓ AND COMPENSATE THE EXP DECAY WITH VOLUME OF $P(\ell)$

IMRY - MA ARGUMENT

WHICH IS THE LOWER CRITICAL DIMENSION OF THE RANDOM FIELD ISING MODEL ?

- RECALL PEIERLS' ARGUMENT FOR $T_c = 0$ IN 1d IM



ARGUMENT COMPARE FREE-ENERGY WITH DOMAIN
" " WITHOUT "

GROUND STATES : TWO ALL UP OR ALL DOWN
FOCUS ON ONE OF THEM

$E_0 = -JN$ $\qquad$ $S_0 = k_B \ln 1 = 0$

$F_0 = -JN$

1st EXCITED STATE : ONE DOMAIN & TWO DOMAIN WALLS

FOCUS ON A DOMAIN WITH A FIXED LENGTH ℓ

$$E_1 = -JN + 2 \cdot 2J \qquad S_1 = k_B \ln N$$
$$\sim -JN$$

ONE CAN PLACE THE DOMAIN AT EACH SITE

$$F_1 = -JN + k_B T \ln N$$

NOW COMPARE
$$\Delta F = F_1 - F_0$$
$$= -\cancel{JN} - k_B T \ln N + \cancel{JN}$$

$$\Delta F = -k_B T \ln N \xrightarrow[N\to\infty]{} \infty$$

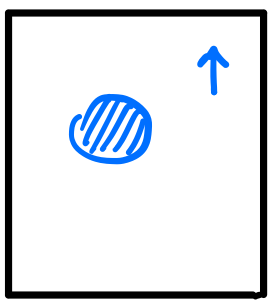
SO, IT'S FAVOURABLE TO CREATE A DOMAIN IF $T>0$

$\Rightarrow$ NO FINITE T TRANSITION IN THE 1d IM

WE'LL USE A SIMILAR ARGUMENT IN THE RFIM

- STABILITY OF AN ORDERED FM PHASE AT $T=0$

UNDER A CHANGE OF $h \to -h$



$\uparrow h \to \downarrow h$

FATE OF $\uparrow$ STATE WHEN h IS TURNED FROM $\uparrow$ TO $\downarrow$?

GAIN IN ENERGY DUE TO ALIGNMENT WITH FIELD

$$\Delta E = -2h\,v \propto h\,\ell^{d}$$

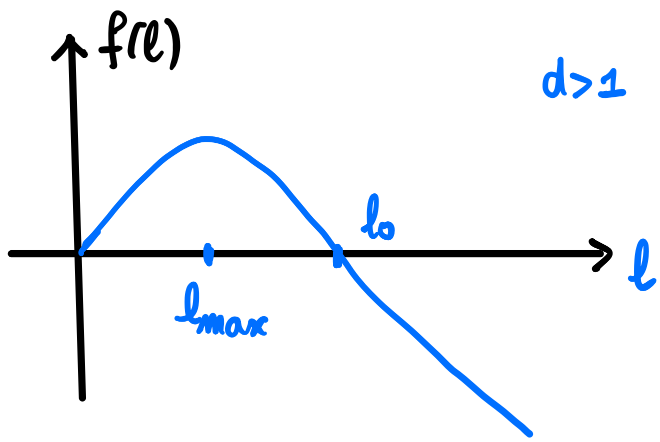
LOSS DUE TO INTERFACIAL ENERGY COST

$$\Delta E = +J S \propto J\,\ell^{d-1}$$

NEGLECTING ANGULAR PREFACTORS LINKED TO THE FORM OF THE BUBBLE

$$\Delta E_{TOT} = J\ell^{d-1} - h\,\ell^{d} = f(\ell)$$

$$f(0)=0 \qquad f(\ell\to\infty) = -\infty$$



$$f'(\ell) = J(d-1)\,\ell^{d-2} - h d\,\ell^{d-1} = 0$$

FIXES ℓ_{max}

$$\Rightarrow \quad \ell_{max} = \frac{J(d-1)}{h d}$$

WE SEE THAT $d=1$ IS SPECIAL

$$\ell_{max} \propto \frac{J}{h}$$

THE HEIGHT OF THE "BARRIER" GOES AS

$$\Delta E_{TOT}(\ell_{max}) = J\ell_{max}^{d-1} - h\,\ell_{max}^{d}$$

$$\propto J\left(\frac{J}{h}\right)^{d-1} = \frac{J^d}{h^{d-1}}$$

WHAT ABOUT ℓ_0 ? $\quad f(\ell_0)=0$

$$f(l_0) = J\, l_0^{d-1} - h\, l_0^{d} = 0 \quad \Longrightarrow$$

$$l_0 = \frac{J}{h} \qquad \text{or } l_0 = 0 \quad (d>1)$$

BOTH l_{max} AND l_0 HAVE THE SAME DEPENDENCE ON $\frac{J}{h}$

THIS IS "NATURAL".

THE STRONGER THE J, THE HARDER IT IS TO MAKE A SUCCESSFUL BUBBLE.

THE STRONGER THE h, THE EASIER TO TURN ROUND THE BUBBLE.

- BUBBLES WITH $l < l_{max}, l_0$ $\Longrightarrow$ NOT SUCCESSFUL
- " " $l > l_{max}, l_0$ $\Longrightarrow$ TURN ROUND & CONQUER THE SAMPLE SINCE

$$\Delta E_{TOT} < 0$$

ZERO TEMP. ARGUMENT.

• THE RFIM $$H = -J \sum_{\langle ij \rangle} s_i s_j - \sum_i h_i s_i$$

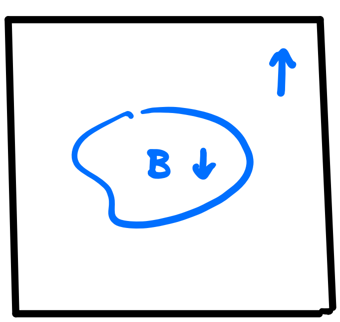
UNIFORM FM COUPLING OF NN ON LATTICE

$P(h_i)$ s.t. $[h_i] = 0$ $[h_i^2] = h^2 < +\infty$

LOCAL RANDOM FIELDS

ASSUME IT'S FM ORDERED AND THAT WE TURN ROUND A BUBBLE.
WORK AT $T = 0$

AND SEE WHAT HAPPENS AS A FUNCTION OF h



IMAGINE THAT THE BULK IS ORIENTED ↑ AND WE REVERSE A BUBBLE B WITH TYPICAL SIZE

ℓ^d = VOLUME

ℓ^{d-1} = SURFACE

WE ASSUME IT'S REGULAR, NO FRACTALITY

ENERGY ESTIMATE $\{ S_i = +1, \ \forall i \}$

$$E_0 = - J z N - \sum_i h_i$$

WITH THE BUBBLE THERE COULD HAVE BEEN A VOLUME GAIN (BETTER ALIGNEMENT W/ LOCAL FIELDS) BUT A SURFACE COST (INTERFACE ENERGY)

NOW FOCUS ON SPINS IN $\mathcal{B}$ ONLY SINCE THE REST HASN'T CHANGED

$$\Delta E_{\text{VOLUME}} \propto - \sum_i h_i \propto - h \, \ell^{d/2}$$

CENTRAL LIMIT THEOREM

$$\Delta E_{\text{SURFACE}} \propto J \ell^{d-1}$$

$$\Delta E_{\text{TOTAL}}(\ell) \propto - h \, \ell^{d/2} + J \ell^{d-1} = f(\ell)$$

WE REPEAT THE ANALYSIS OF THE FM UNDER A FIELD.

$\Delta E_{TOT}(\ell = 0) = 0$ $\quad \lim_{\ell \to \infty} \Delta E_{TOT}(\ell)$ DEPENDS ON d

WHICH POWER WINS ?

$$\frac{d}{2} = d-1 \implies d=2$$ BREAK POINT

if $d<2$ e.g. $d=1 \implies \frac{d}{2}=\frac{1}{2}$ & $d-1=0$

$$\frac{d}{2} > d-1$$

if $d>2$ e.g. $d=3 \implies \frac{d}{2}=\frac{3}{2}$ & $d-1=2$

$$\frac{d}{2} < d-1$$

INTEREST IN $d>2 \implies$

THE SECOND TERM DOMINATES IN THE $\ell \to \infty$ LIMIT

$$\lim_{\ell\to\infty} -h\,\ell^{d/2} + J\,\ell^{d-1} \longrightarrow \infty$$

$$J>0$$

$$f'(\ell) \propto -h\,\frac{d}{2}\,\ell^{d/2-1} + J(d-1)\,\ell^{d-2}$$

$$\propto -hd\,\cancel{\ell^{\frac{d-2}{2}}} + J(d-1)\,\ell^{\cancel{d-2}\ \frac{d-2}{2}} = 0$$

if $d>2$

$$\boxed{\ell_{ext} \propto \left(\frac{h}{J}\right)^{\frac{2}{d-2}}}$$

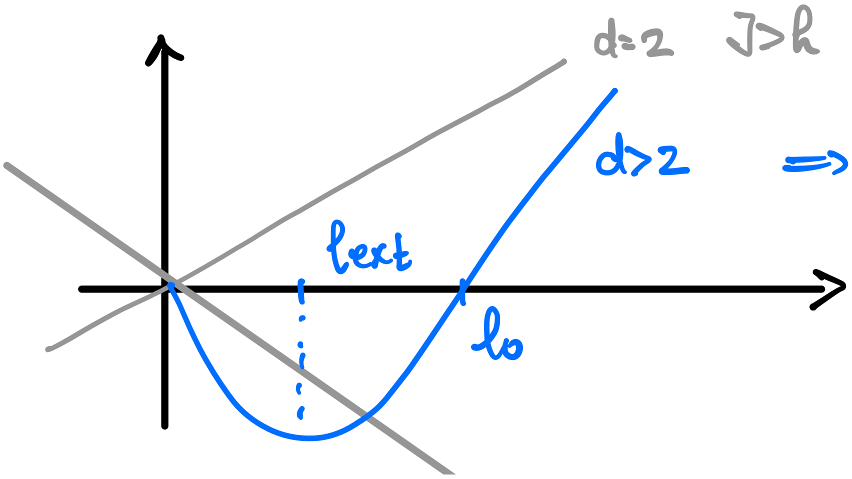
WHAT ABOUT ℓ_0 ?

$$f(\ell_0)=0 = -h\,\ell_0^{d/2} + J\,\ell_0^{d-1} = 0$$

$$J\,\ell_0^{d/2-1} = h$$

$$\ell_0 = \left(\frac{h}{J}\right)^{\frac{2}{d-2}}$$

SAME BEHAVIOUR AS THE ONE OF ℓ_{max}



$\Longrightarrow$ NOT GOOD TO MAKE DOMAINS

$\Downarrow$

NO ORDER

$J < h$

LOCAL GAUGE INVARIANCE
NISHIMORI

IDEA: USE SYMMETRIES TO PROVE GENERAL PROPS ON CERTAIN OBSERVABLES

e.g. $$\int_{-a}^{a} dx \; x \, f(x) = 0 \qquad \text{if } f(x) = f(-x)$$

PROOF: CHANGE VARIABLES IN INTEGRAL

$$y = -x \quad \Rightarrow \quad x = -y$$

$$\int_{a}^{-a} (-dy)\,(-y)\, f(-y) =$$

$$= -\int_{-a}^{a} dy \; y \, f(y)$$

SINCE $f(y) = f(-y)$

RENAME $x = y$

$$= -\int_{-a}^{a} dx \; x \, f(x)$$

$\Longrightarrow$ INTEGRAL $=0$

ANOTHER EXAMPLE

$$\langle S_i \rangle = \sum_{\{S_i = \pm 1\}} S_i \, e^{+\beta \sum_{\langle ij \rangle} J_{ij} S_i S_j}$$

CHANGE VARIABLES $\sigma_i = -S_i$

$$= \sum_{\{\sigma_i = \pm 1\}} (-\sigma_i) \, e^{\beta \sum_{\langle ij \rangle} J_{ij} \sigma_i \sigma_j}$$

CHANGE NAME $\sigma_i = S_i$

$$= - \sum_{\{S_i = \pm 1\}} S_i \, e^{\beta \sum_{\langle ij \rangle} J_{ij} S_i S_j}$$

$\Longrightarrow$ $\langle S_i \rangle = 0$ $\quad \forall \{J_{ij}\}$

JUST BECAUSE OF

$$H(\{S_i\}) = H(-\{S_i\})$$

GLOBAL SYMMETRY $S_i \longrightarrow -S_i \quad \forall i$

WHAT ABOUT A LOCAL SYMMETRY ?

SIMILARITY WITH GAUGE INVARIANCE
CHANGE $\{S_i\}$ BUT ALSO $\{J_{ij}\}$ WHICH PLAY
THE ROLE OF GAUGE FIELDS

NISHIMORI

eg. WORK WITH ISING VARIABLE

$S_i = \pm 1$

$\sigma_i = \eta_i S_i$ ALSO ISING $\eta_i = \pm 1$ FIXED

$\eta_i \sigma_i = S_i$

NOTE THAT η_i NEED
NOT BE ALL THE SAME
eg $\{1, -1, 1, 1, -1, -1, -1 \ldots\}$

$$H_J[\{S_i\}] = - \sum_{\langle ij \rangle} J_{ij} S_i S_j$$

$$-$$

$$= -\sum_{\langle ij \rangle} J_{ij}\, \eta_i \eta_j\, \sigma_i \sigma_j$$

$$= -\sum_{\langle ij \rangle} \bar{J}_{ij}\, \eta_i \eta_j\, \eta_i \eta_j\, \sigma_i \sigma_j$$

$$= -\sum_{\langle ij \rangle} \bar{J}_{ij}\, \sigma_i \sigma_j$$

THE SAME HAMILTONIAN

$J_{ij} = \bar{J}_{ij}\, \eta_i \eta_j$ CHANGE IN COUPLING STRENGTHS

if J_{ij} WAS BIMODAL $= \pm 1$

η_i ALSO BIMODAL $= \pm 1$

$\Rightarrow$ $\bar{J}_{ij}$ BIMODAL

LET'S USE THESE IDEAS TO STUDY THE ISING SPIN-GLASS

$J_{ij} = \pm J$ WITH

$$\mathcal{P}(J_{ij}) = p\,\delta_{J_{ij},J} + (1-p)\,\delta_{J_{ij},-J}$$

$$H = -\sum_{\langle ij\rangle} J_{ij}\, s_i s_j$$

WE REWRITE THE BIMODAL PROBABILITY

$$\mathcal{P}(J_{ij}) = \frac{e^{k_p J_{ij}/J}}{2\,\mathrm{ch}\, k_p}\left[\delta_{J_{ij},J} + \delta_{J_{ij},-J}\right]$$

$$e^{2k_p} = \frac{p}{1-p}$$

CHECK

$$\mathcal{P}(J) = \frac{e^{k_p}}{2\mathrm{ch}k_p} = \frac{\sqrt{\frac{p}{1-p}}}{\sqrt{\frac{p}{1-p}} + \sqrt{\frac{1-p}{p}}}$$

$$\sqrt{\cancel{p}}$$

$$= \frac{\cancel{\sqrt{}}\;1-p}{\cancel{\sqrt{\frac{p}{1-p}}}\left[1+\sqrt{\frac{(1-p)^2}{p^2}}\right]}$$

$$= \frac{1}{1+\frac{1-p}{p}} = p$$

$$\mathbb{P}(-J) = \frac{e^{-kp}}{2\operatorname{ch} kp}$$

$$= \frac{\cancel{\sqrt{\frac{1-p}{p}}}}{\cancel{\sqrt{\frac{1-p}{p}}}\left[1+\sqrt{\frac{p^2}{(1-p)^2}}\right]}$$

$$= 1-p$$

TRANSF OF THE DISTRIBUTION

$\bar{J}_{ij} = J_{ij}\,\eta_i\,\eta_j \qquad \eta_i = \pm 1$ FIXED

JUST MULT BY
± 1 WITH prob $1/2$

$$P(J_{ij}) = \frac{e^{kp J_{ij}/J}}{2\,\mathrm{ch}\, kp}\left[\delta_{J_{ij},J} + \delta_{J_{ij},-J}\right]$$

$$= \frac{e^{kp \overline{J}_{ij}\eta_i\eta_j/J}}{2\,\mathrm{ch}\, kp}\left[\delta_{\overline{J}_{ij}\eta_i\eta_j,J} + \delta_{\overline{J}_{ij}\eta_i\eta_j,-J}\right]$$

STILL WITH $e^{2kp} = \frac{p}{1-p}$

THE AVERAGED ENERGY is

$$[\langle H_J \rangle]_J =$$

$$= \sum_{\{J_{ij} = \pm J\}} \frac{e^{kp \sum_{\langle ij \rangle} J_{ij}/J}}{(2\,\mathrm{ch}\, kp)^{N_B}}$$

$$\frac{\sum_{\{s_i\}}\left(-\sum_{\langle ij\rangle} J_{ij}\, s_i s_j\right) e^{\beta \sum_{\langle ij\rangle} J_{ij}\, s_i s_j}}{\sum_{\{s_i\}} e^{\beta \sum_{\langle ij\rangle} J_{ij}\, s_i s_j}}$$

$$= \sum_{\{\bar{J}_{ij}\}} \frac{e^{K_p \sum_{\langle ij\rangle} \bar{J}_{ij}\, \eta_i \eta_j / J}}{(2\,\mathrm{ch}\, K_p)^{N_B}} \; \frac{\sum_{\{\bar{s}_i\}}\left(-\sum_{\langle ij\rangle} \bar{J}_{ij}\, \bar{s}_i \bar{s}_j\right) e^{\beta \sum_{\langle ij\rangle} \bar{J}_{ij}\, \bar{s}_i \bar{s}_j}}{\sum_{\{\bar{s}_i\}} e^{\beta \sum_{\langle ij\rangle} \bar{J}_{ij}\, \bar{s}_i \bar{s}_j}}$$

CAN NOW RENAME $\quad \bar{J}_{ij} \to J_{ij}$

$\bar{s}_i \to s_i$

THE ONLY DIFF ARE THE $\eta_i \eta_j$ IN THE pdf WEIGHT.

SUM OVER THEM $\frac{1}{2^N}\sum_{\{\eta_i=\pm1\}}$ SINCE THE CALC. DID NOT DEPEND ON THEM

NISHIMORI CHOICE OF PARAM $K_p=\beta J$

NUM. & DEN. FACTORS CANCEL

$$= \sum_{\{\bar{J}_{ij}\}} \frac{1}{2^N} \cancel{\sum_{\{\eta_i\}}} \frac{\cancel{e^{K_p\sum_{\langle ij\rangle}\bar{J}_{ij}\eta_i\eta_j/J}}}{(2\cosh K_p)^{N_B}} \; \frac{\sum_{\{\bar{S}_i\}}\left(-\sum_{\langle ij\rangle}\bar{J}_{ij}\bar{S}_i\bar{S}_j\right)e^{\beta\sum_{\langle ij\rangle}\bar{J}_{ij}\bar{S}_i\bar{S}_j}}{\cancel{\sum_{\{\bar{S}_i\}} e^{\beta\sum_{\langle ij\rangle}\bar{J}_{ij}\bar{S}_i\bar{S}_j}}}$$

NOW, IGNORE THE BARS, CHANGE NAMES BACK TO J_{ij}, S_i

$$= \frac{1}{2^N} \frac{1}{(2\mathrm{ch}\, K_p)^{N_B}} \sum_{\{J_{ij}\}} \sum_{\{S_i\}} \left(- \sum_{\langle ij \rangle} J_{ij} S_i S_j \right) e^{\beta \sum_{\langle ij \rangle} J_{ij} S_i S_j}$$

$$= \frac{1}{2^N} \frac{1}{(2\mathrm{ch}\, K_p)^{N_B}} \left(-\frac{\partial}{\partial \beta} \right) \sum_{\{J_{ij}\}} \sum_{\{S_i\}} e^{\beta \sum_{\langle ij \rangle} J_{ij} S_i S_j}$$

$$= \frac{1}{2^N} \frac{1}{(2\mathrm{ch}\, K_p)^{N_B}} \left(-\frac{\partial}{\partial \beta} \right) \sum_{\{J_{ij}\}} \sum_{\{S_i\}} \prod_{\langle ij \rangle} e^{\beta \underbrace{J_{ij} S_i S_j}_{\tau_{ij}}}$$

$$= \frac{1}{2^N} \frac{1}{(2\mathrm{ch}\, K_p)^{N_B}} \left(-\frac{\partial}{\partial \beta} \right) \sum_{\{S_i\}} \prod_{\langle ij \rangle} \sum_{\{\tau_{ij} = \pm J\}} e^{\beta \tau_{ij}}$$

$$= \frac{1}{\cancel{2^N}} \frac{1}{(\cancel{2}\mathrm{ch}\, K_p)^{N_B}} \left(-\frac{\partial}{\partial \beta} \right) \cancel{2^N} (\cancel{2}\mathrm{ch}\, K_p)^{N_B}$$

$$= \frac{1}{(\mathrm{ch}\, K_p)^{N_B}} N_B \, (\mathrm{sh}\, K_p)^{N_B} \, J$$

$$\boxed{[\langle H_J \rangle] = N_B J (\mathrm{th}\, K_p)^{N_B}}$$