Advanced Statistical Physics

Exam - 2nd Session

January 2019

Surname :
Name :
Master :

Analysis of a disordered model

Take a particle with mass m in a one dimensional harmonic potential

$$V(x) = \frac{1}{2} m\omega^2 x^2 \tag{1}$$

with the real frequency ω taken from a probability distribution $q(\omega)$. The position of the particle is given by the real variable x.

1. Compute the free-energy at fixed ω and inverse temperature $\beta = 1/(k_B T)$, with k_B Boltzmann's constant.

2. Find an expression for the probability distribution of F_ω for a generic $q(\omega)$.

3. Are the fluctuations Gaussian?

4. Express the disorder averaged free-energy $[F_\omega]$ for a generic distribution of ω.

5. Calculate the disorder averaged free-energy $[F_\omega]$ for a Gaussian distribution of ω with zero mean and variance σ^2.

6. Explain how the typical value of the free-energy, F_ω^{typ}, should be obtained, again for a generic $q(\omega)$.

7. Determine F_ω^{typ} for a Gaussian distribution of ω with zero mean and variance σ^2.

8. Explain the self-averaging property and the conditions under which we proved it in the lectures.

9. Is this model self-averaging? Discuss the result found for the simple harmonic oscillator clearly and justify your answer.

10. Do you expect a phase transition? Why?

A useful integral is $\int_0^\infty dy\, e^{-ay^2} \ln y = -\frac{1}{4}\left(C + \ln 4a\right)\sqrt{\frac{\pi}{a}}$ where C is a constant given by $-C = \int_0^1 dx \ln \ln 1/x$.

Study of a phase diagram

Figure 1 shows the phase diagram of the model
Take an Ising model defined on a ring and with Hamiltonian

$$H = -\frac{K}{2}\sum_i (s_i s_{i+1} - 1) + \frac{J}{2}\left(\sum_i s_i\right)^2 . \tag{2}$$

The coupling constants are $K > 0$ and $J > 0$.

1. How should the coupling constant scale with system size to have a reasonable competition between the two terms in H?

2. How would you reduce the calculation of the partition function of this problem to the one of an Ising model with nearest-interactions on the same ring?

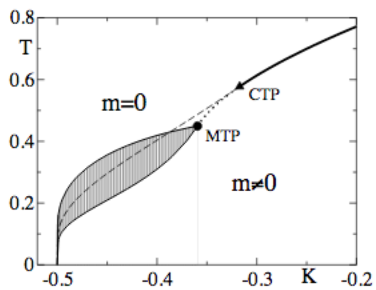

Figure 1: The phase diagram of the model defined in Eq. (2).

3. The phase diagram in the canonical and microcanonical ensemble are given in Fig. 1. What do you think the various line represent? Discuss the phase diagram.