

# Advanced Statistical Physics

## Exam 2

January 2018

Surname :

Name :

Master :

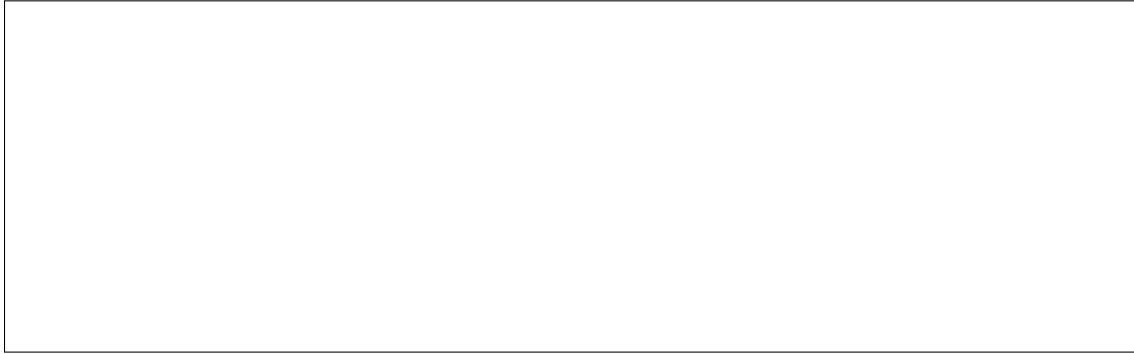
1. Consider a system made of  $N$  point-like particles confined in a volume  $V = L^d$  in  $d$  dimensions.

- Is the potential  $V(r) = e^{-r/\xi}$ , with  $r$  the distance between two point-like particles in the system, long-range or short-range? Justify the answer.

- Give a physical example of a long-range two-body potential energy. Justify the answer.

- Which condition on the parameters  $(N, V)$  is needed to have a phase transition?

- Present a possible phase diagram for such a system.



2. Consider a system of  $N$  Ising spins placed on the vertices of a  $d$  dimensional lattice, and potential energy given by the sum of a two-body interaction over nearest-neighbour spins on the lattice,  $-Js_i s_j$  and  $J > 0$ .

- In which cases do you expect a phase transition at finite temperature?



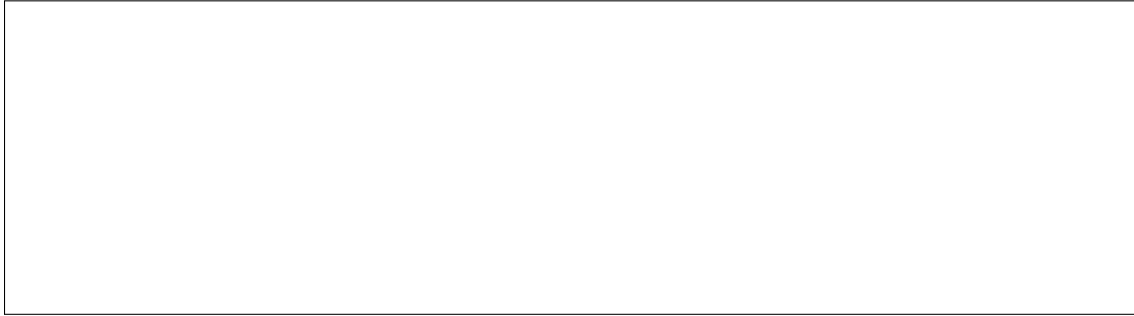
- Consider a triangular lattice in two dimensions. Which phases do you expect?



- In cases with a finite temperature transition, explain the mechanism for it.



- What is a pinning field? Explain its role.

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- Explain the Peierls argument. What is its purpose?

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3. Take the Mattis model  $H = -\sum_{i \neq j} \xi_i \xi_j s_i s_j$  with  $s_i = \pm 1$  Ising spins and  $\xi_i$  taken from a bimodal probability distribution with zero mean and variance  $\sigma_\xi^2$ .

- Is this model disordered?

- Is it frustrated?

- Justify the answer to the previous question mathematically.

4. Take a random matrix with elements drawn from a Gaussian probability measure with zero average and variance  $\sigma^2$ .

Which is the probability density of the matrix eigenvalues?

5. Take an Ising antiferromagnetic model with nearest neighbour interactions on a triangular lattice. The interaction strength between pairs of spins is  $J < 0$ .
- Give an approximate expression for the entropy of the ground state. Explain your answer.

6. What are the Griffiths argument and the Griffith phase? Explain.