Advanced Statistical Physics

Exam

January 2018

Surname :
Name :
Master :

1. Write the equation that states the *ergodic hypothesis*

Explain the meaning of the two members of this equation

2. Consider a system of N particles confined inside a volume V, and potential energy given by the sum, over pairs of the elementary constituents, of a two-body translationally invariant potential. Illustrate, with one equation, the *extensivity* property of the potential.

Illustrate, with one equation, the *additivity* property of the potential.

Explain why the violation of these properties may affect the equivalence of ensembles.

3. Mention a model with a topological phase transition

Which are the topological defects in this model? Give one example.

Explain, with words, the phase transition mechanism.

4. Take an energy spectrum $E_1 < \ldots < E_n < \ldots$
Which would be the distribution of spacings between consecutive levels if these were just independent random numbers?

Give the main characteristics of the distribution of level spacings found, for example, in heavy nuclei?

Which is the property of the level spacings needed to capture the distribution just described?

5. Define *frustration*

Give an example of a frustrated model.

Compute the canonical average of $\langle \sin s_i^3 \rangle$ in a model with Hamiltonian $H_J = -J \sum_{\langle ij \rangle} s_i s_j$ where the sum runs over nearest neighbours on a triangular lattice and $J > 0$. Which is the result of the average in the case $J < 0$? Explain the reasoning.

An Ising spin model with $N = 5$ spins and two body interactions is defined on a random graph. The exchanges

$$
\begin{array}{lllllllll}
J_{12} & = & -1.00 & J_{25} & = & 0.4 & J_{23} & = & -0.1 \\
J_{35} & = & 0.81 & J_{34} & = & -0.7 & J_{45} & = & 0.03
\end{array}
$$

couple the spins labeled with i and j and all other are zero. Is this model frustrated? Justify the answer.

6. Give an example of a Hamiltonian with quenched randomness and explain what kind of physical problem it describes.

Which are the main effects of quenched randomness on a phase transition?

Explain the difference between "annealed" and "quenched" disorder.

What is the main difference between "weak" and "strong" quenched disorder?

Take the Ising model

$$H = -J \sum_{\langle ij \rangle} s_i s_j - \sum_{i=1}^{N} h_i s_i \tag{1}$$

with $J > 0$ and h_i taken from a probability distribution $P(h_i)$ with zero mean and variance σ_h^2. The first sum runs over nearest neighbours on a cubic lattice in dimension d.

Is its free-energy density self-averaging? Justify your answer.

Which kind of phases do you expect?

What is(are) the order parameter(s) in this problem?

Explain the Imry-Ma droplet argument that allows one to estimate in which dimension this model can sustain an ordered phase.

Draw an schematic phase diagram.

Write the mean-field equation(s) for the order parameter(s). Explain briefly how these equations are obtained.

Take now the case in which the model (1) is defined on a full complete graph.
Explain the order of magnitude expected for each term in the Hamiltonian.

What choice would you make for J?

Name a technique that is apt to deduce an expression for the disorder averaged free-energy density of this problem.