PROGRAM FOR TODAY

0 – SLIDES WITH MOTIVATION 30-45 min

(THE NUMBERS FOLLOW THE # OF THE SECTIONS IN THE LECT. NOTES)

1-1 – BASIC NOTIONS 15 min

1.2 – THIS YEAR WE WILL NOT DISCUSS THE INEQUIVALENCE OF ENSEMBLES.

1-3 – BATHS & THEIR MODELLING.

15 min

$\sim 1h$

REFERENCES

DETAILS FROM SLIDES

- BOOKS

— ALTLAND - SIMONS CMFT
— GOLDENFELD PHASE TRANS RG
— PARISI STAT PHYS
— BERLINSKY - HARRIS INTRO STAT PHYS
— LESNE PHASE TRANS.
— CASTIGLIONE ET AL FORMALISM

THE COURSE WILL BE AN INTRODUCTION TO THEORET. METHODS IN STATISTICAL PHYSICS

CHAPTER 1 INTRODUCTION

1.1 BASIC NOTIONS

STATISTICAL PHYSICS is ABOUT GOING

FROM PARTICLES / AGENTS / ELEMENTS

TO

MACROSCOPIC BEHAVIOUR VIA
OBSERVABLES / ORDER PARAM

$N \gg 1$ ELEMENTS w/ MICROSC. DYNAMICS

$\Downarrow$ PROB TH & STAT. METH.

A FEW OBSERVABLES w/ NO (OR SIMPLE)
TIME DEPENDENCIES

Note that going from $N \gg 1$ to a few
may not need an "equilibrium" hypothesis
(to be made explicit below)

PROB THEORY & STATISTICAL METHODS
CAN BE APPLIED TO
DIFF MICROSC DYN. eg.

CLASSICAL OR QUANTUM
NON-RELAT OR RELAT.

$\Downarrow$

COLLECTIVE-EMERGENT PHENOMENA

EMPHASIS ON PHASE TRANSITIONS

START FROM CLASSICAL NON-RELAT

HERE, ALL IN EQUILIBRIUM (RECALL STAT PHYS LECT.)

BUT Comments on out of equilibrium

"STANDARD" METHODS BUT

Comments on new aspects too

OUT OF EQUILIBRIUM

(1) EXP. TIMES ARE TOO SHORT TO LET THE SYST EVOLVE TO EQUIL.

(2) THERE ARE EXT. DRIVING FORCES

(3) TOO MANY INT. OF MOTION (INTEGRABILITY)

MACROSC SYST OUT OF EQ. A FINE FIELD OF RESEARCH

RECALL : IN NOTES RECAP ON PASSAGE FROM MECHANICS $\longrightarrow$ STAT PHYS

(SHOULD BE KNOWN -)

MORE IN CASTIGLIONE ET AL, FOR EX

GOAL. UNDERSTAND THE MACROSCOPIC BEHAVIOUR AS A FUNCTION OF A FEW QUANTITIES CHARACT THE SYST "MACROSTATE"

- ENERGY, VOLUME, NUMBER PART CLOSED
  E V N $O(N)$

- TEMP., PRESSURE, CHEM POT OPEN
  T P μ $O(1)$

OBSERVABLES DERIVABLE FROM THERMODYN POTENTIALS

e.g. $F(\beta J, \beta h)$ FREE ENERGY

AS DERIVATIVES e.g. $m = \frac{\partial F}{\partial h}$

THE PROBLEM IS : HOW CAN ONE COMPUTE THE THERMOD. POTENTIALS

TO DO IT, ONE NEEDS TO IDENTIFY

a THE SYSTEM & ITS INTERNAL INTERACTIONS

b THE SITUATION : OPEN, CLOSED & EVENTUAL ROLE OF ENVIRONM.

AND FIGURE OUT HOW TO DEAL WITH THE MANY BODY SYST IN (STRONG) INTERACTION

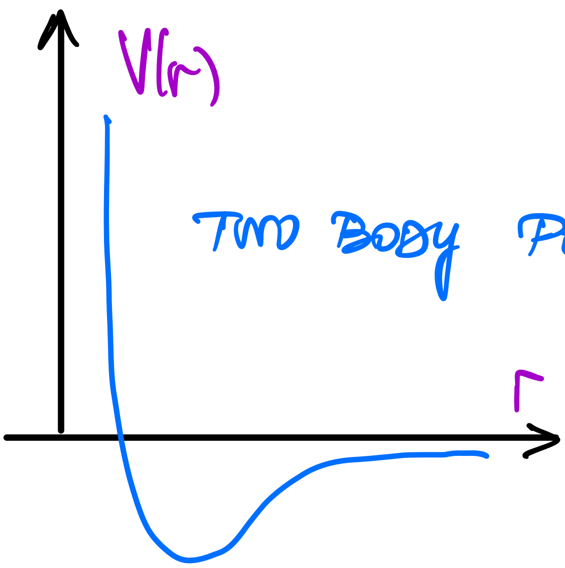
METHODS PERTURB THEORY, MEAN-FIELD, RENORM GROUP.

$\Rightarrow$ ALLOW ONE TO COMPUTE THERMODYN POTENTIAL & THEN ANALYZE IT

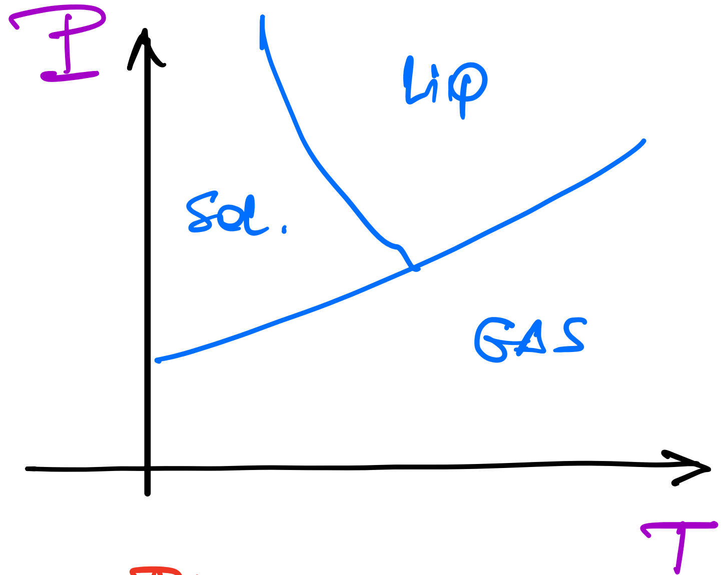
a MICROSCOPIC MODEL

PHYSICAL PARTICLE SYSTEMS FOLLOWING NEWTON CLASSICAL DYNAMICS

SOLID - LIQUID - GAS



MICROSCOPIC INTERACTIONS



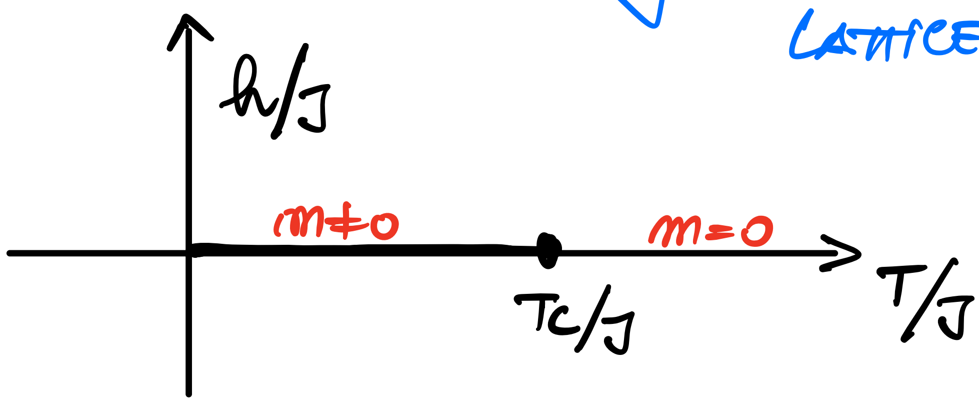
PHASE DIAGRAM (CONST V, N)

BUT DIFFICULT TO DEAL WITH ANALYTICALLY
BETTER FOCUS ON OTHER CASES WHICH ARE GOOD ENOUGH TO DISCUSS & UNDERST. PHASE TRANS.

SPIN MODELS

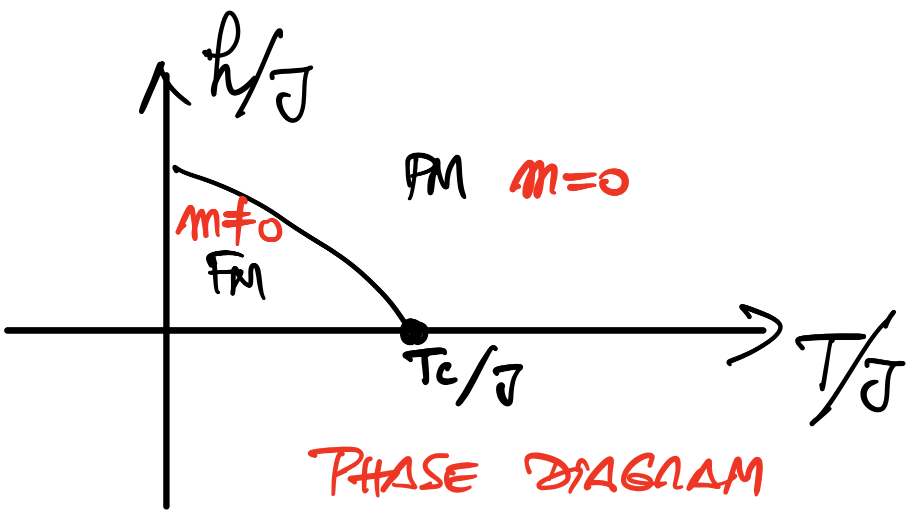
EASIER TO DEAL WITH ANALYTICALLY & NUMERICALLY

e.g. ISING MODEL ON A LATTICE



MAGNETISM

OR IN MEAN FIELD



PHASE DIAGRAM

MF: CURIE-WEISS

$$H = -\frac{J}{N}\sum_{i\neq j} S_i S_j$$

$$S_i = \pm 1$$

QUANTUM CASES

e.g. QUANTUM GASES / LIQUIDS / SOLIDS

NEW PHASES, SUCH AS SUPERFLUID ONES
(FLOW WITH NO DISSIPATION)

ALSO QUANTUM SPIN SYSTEMS

$\hat{H}[\{\hat{s}_i^a\}]$

OPERATORS

$1h + 30\,min = 1h\,1/2$

RECAP. FORMALISM — SEE LECTURES NOTES

N PART $\{\vec{Q}, \vec{P}\}$ PHASE SPACE

$\vec{Q} = \{\vec{q}_1, \dots, \vec{q}_N\}$

$\vec{P} = \{\vec{p}_1, \dots, \vec{p}_N\}$ $\quad \mathcal{X} = 2dN$-dim.

NEWTON DYN $\Longrightarrow$ TRAJECTORY FROM $\{\vec{Q}(0), \vec{P}(0)\}$ TO $\{\vec{Q}(t), \vec{P}(t)\}$

TAKE ENSEMBLE OF $\{\vec{Q}(0), \vec{P}(0)\}$ WITH

$$\rho_0(\vec{Q}_0, \vec{P}_0)$$

& EVOLVE THEM.

WHAT HAPPENS W/ $\rho(\vec{Q}(t), \vec{P}(t), t)$?

LIOUVILLE'S EQ. $\dfrac{d\rho}{dt} = \dfrac{\partial \rho}{\partial t} + \{H, \rho\}$

ERGODIC HYPOTHESIS

READ CASTIGLIONE, FALCIONI, LESNE VULPIANI
Phys Stat: Chaos et approx. multiéchelles

AFTER SOME TIME

$$\rho \rightarrow \rho(H)$$

e.g. NO EXPLICIT TIME DEP
ONLY THROUGH $H(\vec{q}, \vec{p})$

- OBSERVABLE CONSEQUENCES

THE TIME-AVER. OF NON-PATHOLOGICAL OBSERV Θ WHICH DEP ON TIME ONLY THROUGH $(\vec{q}(t), \vec{p}(t))$ REACHES A CONST.

$$\overline{\Theta} = \lim_{\tau \gg \tau_0} \frac{1}{\tau} \int_t^{t+\tau} dt' \, \Theta(\vec{q}(t'), \vec{p}(t'))$$

INDEP. OF t

AND COINCIDES WITH

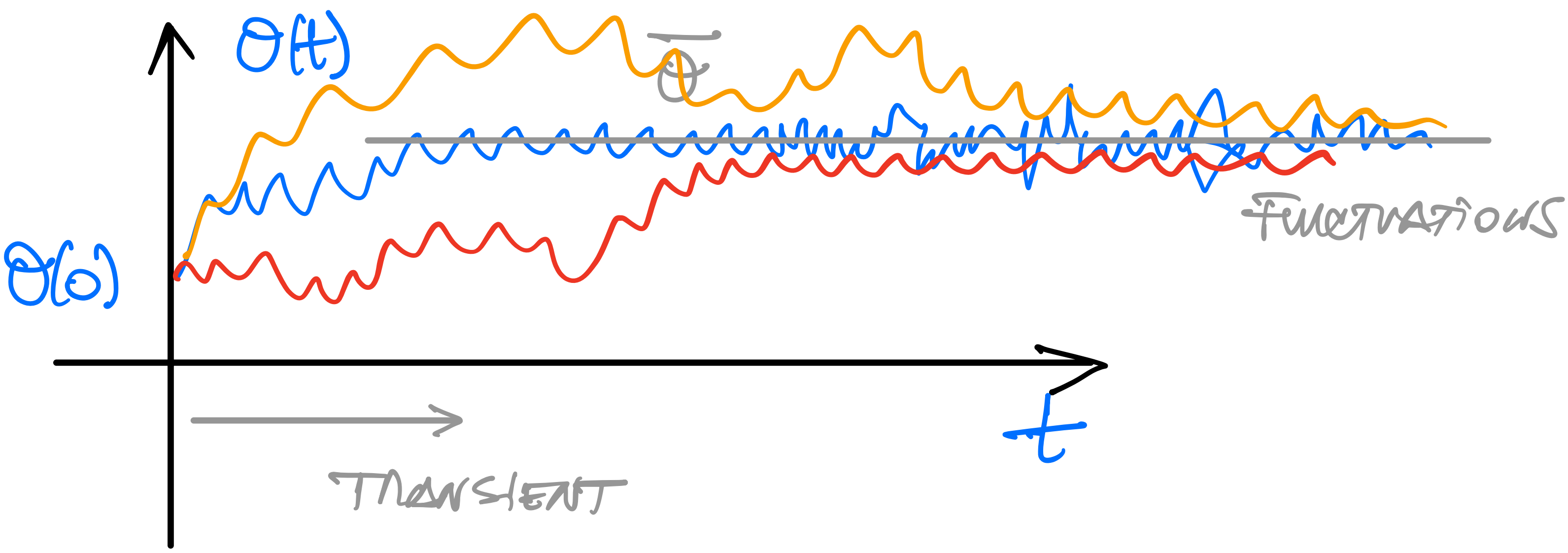
$$\langle \Theta \rangle = \int d\vec{P}\, d\vec{Q}\; \rho(H(\vec{P},\vec{Q}))\; \Theta(\vec{P},\vec{Q})$$

AN <u>ENSEMBLE</u> AVERAGE :

$$\boxed{\overline{\Theta} = \langle \Theta \rangle}$$

ASSUME FOR SIMPLICITY E IS THE ONLY CONST MOTION

MANY EX. IN PREVIOUS EXAMS & LECTURE NOTES.



WHILE $\quad \langle O \rangle = \dfrac{1}{N_{meas.}} \sum_{a=1}^{N_{meas.}} O_a$

AVERAGE OVER ≠ MEASURES, ALL AT THE SAME TIME AFTER PREPARATION (IN EXP. OR IN NUM. SIMULATION)

THIS SHOULD HOLD ∀ REASONABLE OBSERV. (SUFFICIENTLY "GLOBAL")

NB ONE CAN HAVE OUT OF EQ CASES IN WHICH, eg E_{kin} IS OK BUT E_{pot} IS NOT

MICRO-STATE VS. MACRO-STATE

e.g. A PRECISE POINT IN PHASE SPACE

$$\left(\vec{Q}(t), \vec{P}(t) \right)$$

OR A PRECISE SPIN CONF WILL BE CALLED A <u>MICRO-STATE</u>

INSTEAD AN ENSEMBLE OF MICRO STATES WITH THE SAME MACROSC PROPS WILL BE CALLED A <u>MACRO-STATE</u>

MICROCANONICAL ENSEMBLE

CLOSED SYSTEM $E = ct.$ $V = ct.$ $N = ct.$

HYPOTHESIS
$$\delta(H) = \begin{cases} \delta_0 & \forall \text{ conf w/ } E \\ 0 & \text{other confs.} \end{cases}$$

ALL CONFS ON THE CONST ENERGY $\mathcal{N}-1$ dim SURFACE IN PHASE SPACE ARE EQUALLY PROBABLE

DENSITY OF STATES

$$\boxed{g(E) = \int d\vec{p}\, d\vec{q}\;\; \delta\left(H(\vec{p},\vec{q}) - E\right)}$$

VOLUME OF PHASE SPACE WITH $H(\vec{p},\vec{q}) = E$

MICROCANON. ENTROPY

$$\boxed{S(E) = k_B \ln g(E)}$$

MICROCANONICAL TEMPERATURE

$$\boxed{\frac{1}{T(E)} = \frac{\partial S(E)}{\partial E}}$$

NB.

IF THERE ARE OTHER CONSERVED QUANTITIES, (eg. MOMENTUM $\vec{P}$ OR ANGULAR MOM. $\vec{L}$) RESTRICT FURTHER THE AVAILABLE PHASE SPACE, BUT STILL ASSUME $\rho(H, \vec{P}, \text{etc.}) = \rho_0$

FROM MICROCANONICAL TO CANONICAL

TRACE OUT A PART OF TOTAL SYST

ASSUME IT'S SHORT-RANGED COUPLED TO PART OF INTEREST

ASSUME "BATH" >> "SYSTEM" IN SIZE

FOLLOW NOTES

$$\rho(\{\vec{q}, \vec{p}\}) = \frac{e^{-\beta H(\{\vec{q}, \vec{p}\})}}{Z}$$

Z PARTITION FUNCTION

β $1/k_BT$ FIXED BY BATH PROP.

$-\beta F = \ln Z$ IT'S A FUNCT OF β, AND PARAM IN H

EQUILIBRIUM : MINIMIZE F

ALSO HOMEWORK

FLUCTUATION-DISSIPATION THEOREM

VARIATIONS WRT PARAM = FLUCTUATIONS

$$\underbrace{\frac{\partial \langle E \rangle}{\partial T}}_{\text{A SUSCEPTIBILITY}} = k_B \beta^2 \underbrace{\langle (E - \langle E \rangle)^2 \rangle}_{\text{A FLUCTUATION}}$$

- IT'S MODEL INDEPENDENT
- HOLDS IN THE CANONICAL ENSEMBLE
- IT HOLDS IN TOTAL GENERALITY, BEYOND ANY EXPANSION

PROOF

$$\langle E \rangle = -\frac{\partial \ln Z(\beta)}{\partial \beta}$$

TAKE THE DERIVATIVE EXPLICITLY $\Rightarrow$ DONE.

CONDENSATION OF ENERGY FLUCTUATIONS

$$\langle E \rangle = \Theta(N) \qquad \text{EXTENSIVITY}$$

$$\frac{\partial \langle E \rangle}{\partial T} = \Theta(N) \quad \text{AS WELL}$$

$$= \langle (E - \langle E \rangle)^2 \rangle$$

$$\Rightarrow \quad \frac{\text{DISPERSION}}{\text{AVERAGE}} = \frac{\Theta(N^{1/2})}{\Theta(N)} = \Theta(N^{-1/2})$$

$$\xrightarrow[N \to \infty]{} 0$$

NARROWER AND NARROWER $P(E)$

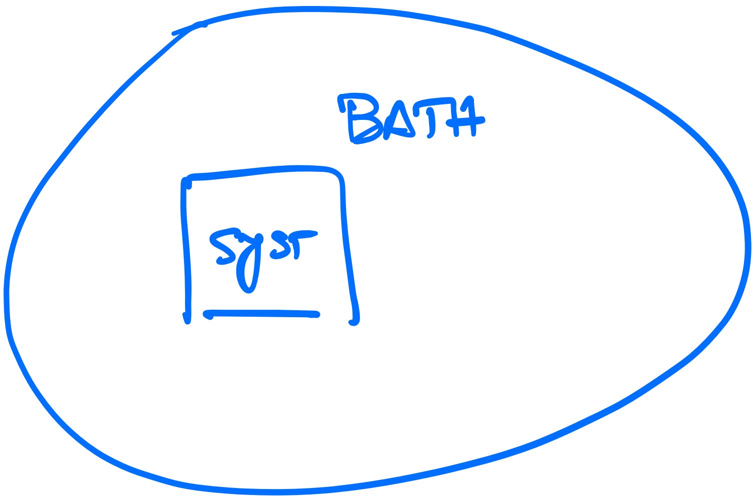
$$\Rightarrow \quad \langle E \rangle = E_{\text{microcanonical}}$$

$$\boxed{\text{JUSTIFICATION OF THE EQUIVALENCE OF ENSEMBLES}}$$

4.3 REDUCED SYSTEMS

SYSTEMS AND BATHS

- HOW TO MODEL A BATH ?
- HOW TO "REDUCE" THE SYSTEM ?



MEGA BATH.

$\downarrow$ FIXES β

$$Z = \sum_{\substack{\text{conf} \\ \text{bath}}} \sum_{\substack{\text{conf} \\ \text{syst}}} e^{-\beta H}$$

$$H = H_{syst} + H_{bath} + H_{int}$$

SAY THAT THE BATH IS MADE OF HARM OSC

$$H_{bath} = \sum_{a=1}^{N_{osc}} \left(\frac{\pi_a^2}{2m_a} + \frac{m_a \omega_a^2}{2} q_a^2 \right)$$

AND SYST JUST ONE PART

$$H = \frac{P^2}{2M} + V(x)$$

BILINEAR INT (SIMPLEST CHOICE)

$$H_{int} = x \sum_a c_a q_a$$

H IS QUADRATIC IN ALL VARIABLES

$\Rightarrow$ EASY

INTEGRATE OUT $\{\Pi_a, q_a\}$:

$$\int \prod_a dp_a \int \prod_a dq_a$$

SUM OVER CONFS.

$$Z_{RED} = \sum_{\substack{conf \\ syst}} e^{-\beta H_{RED}}$$

$$H_{RED} = H_{SYST} - \frac{1}{2} \sum_{a=1}^{N_{osc}} \frac{c_a^2}{m_a \omega_a^2} x^2$$

$$\boxed{H_{RED} \neq H_{SYST}}$$

- ARGUE $\quad c_a^2 = \sigma\left(1/N_{osc} \right)$

$\Rightarrow$ DROP CORRECTION

- OR NOT

AT THE BASIS OF

- LANGEVIN EQ FOR STOCH DYN
- ENVIRON. DRIVEN EFFECTS.

SHOW SLIDES WITH
EXAMPLES OF SYST + BATH

SPECIALLY IMPORTANT QUANTUM
MECHANICALLY !