# Out of equilibrium dynamics of the vortex glass

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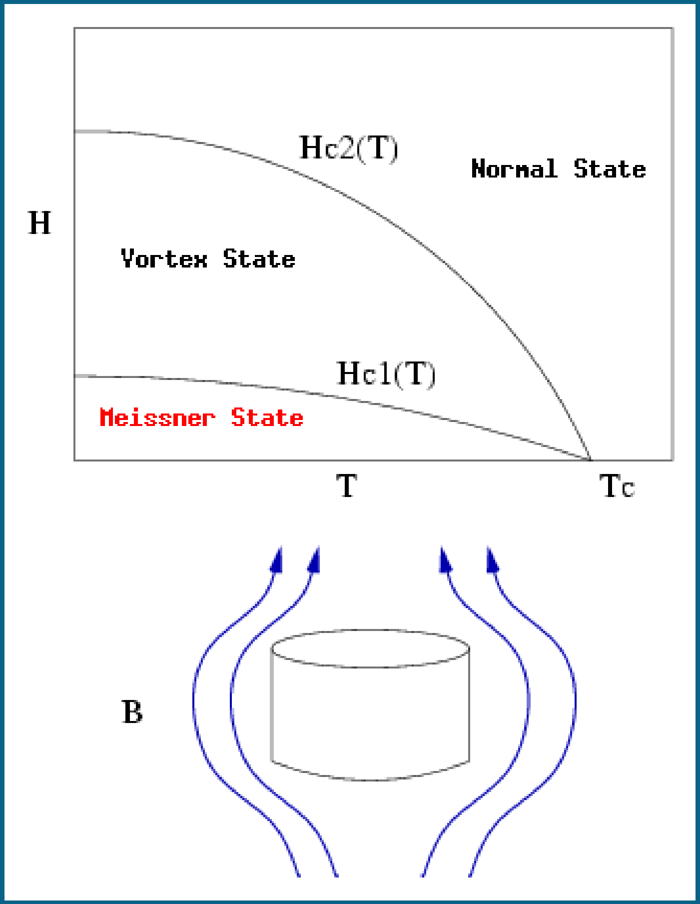
March 12, 2005

CONICET

# Outline

- Vortices in Superconductors
- Vortex dynamics model and vortex glass
- Single vortex line: Aging and FDT
- Vortex glass: Aging and FDT
- Conclusions

# Type II Superconductors:Vortices



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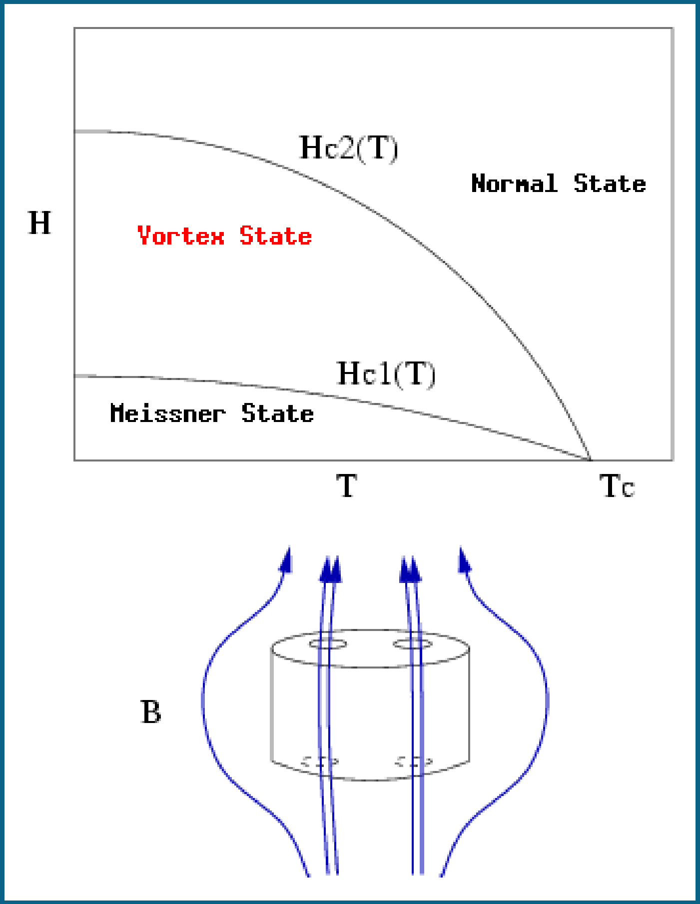

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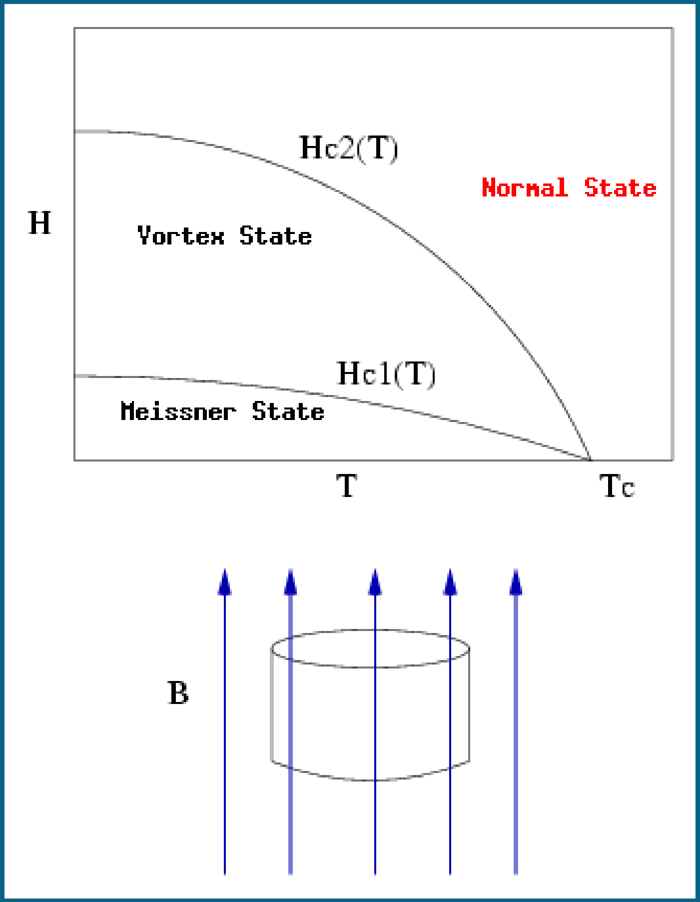

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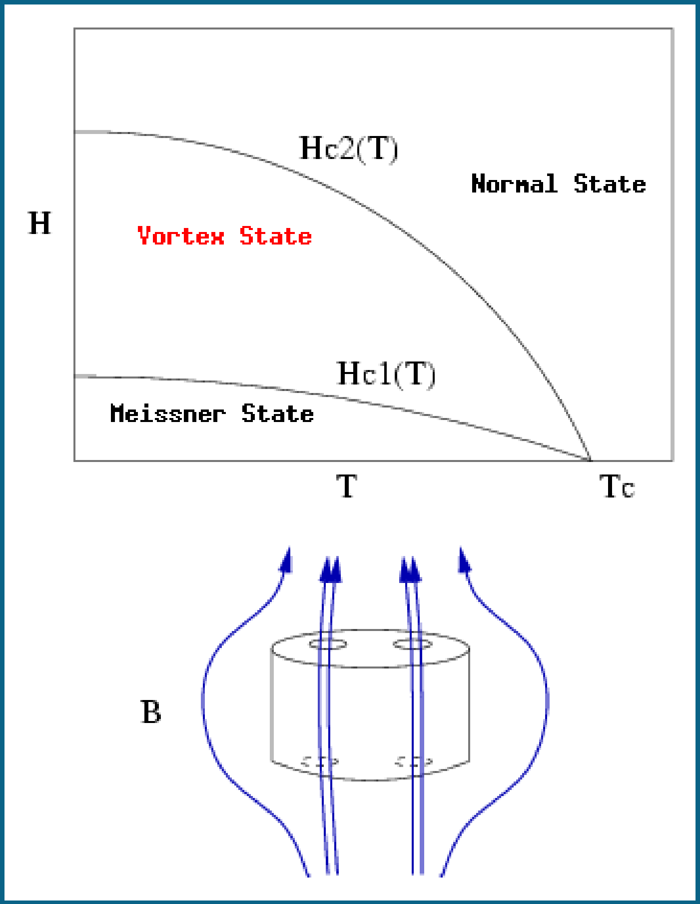


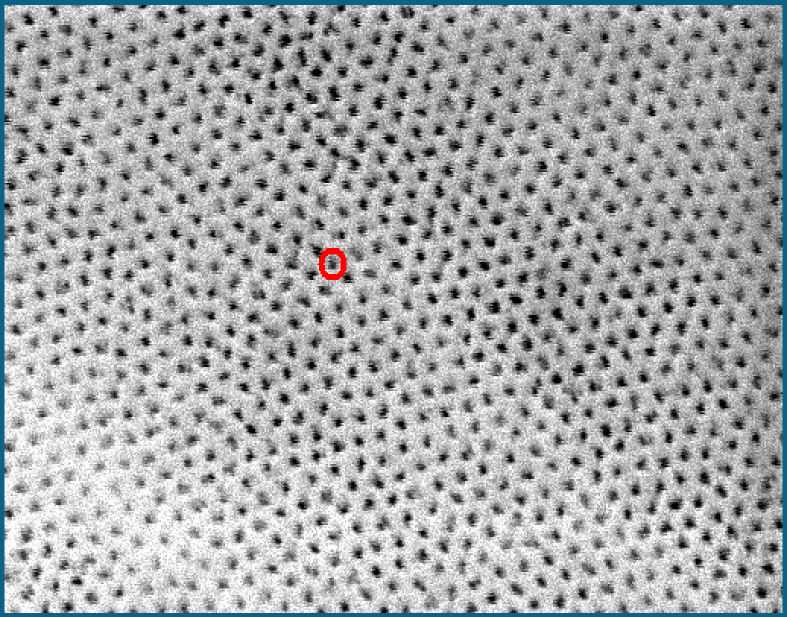

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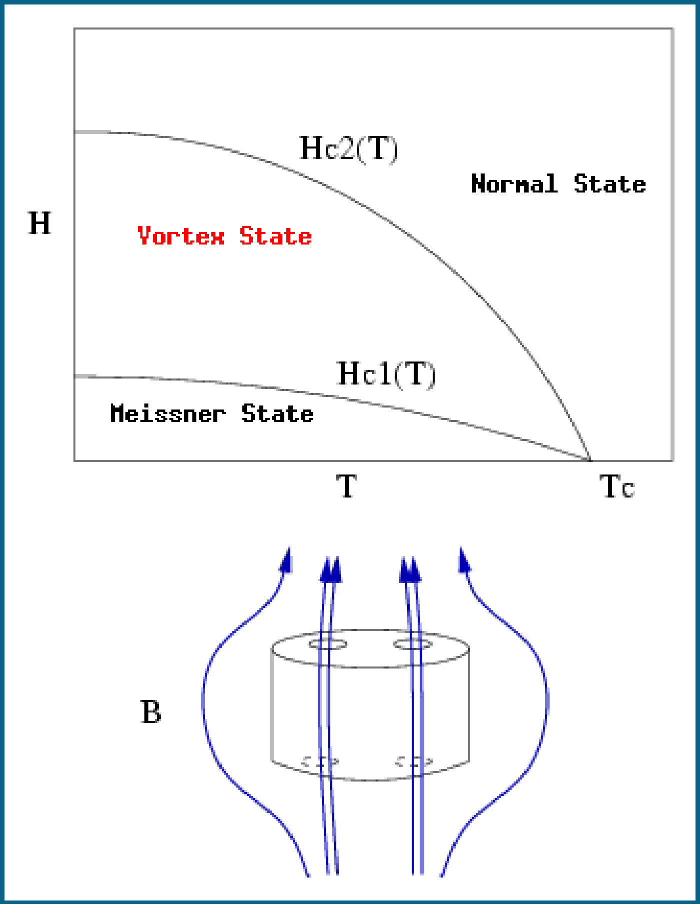


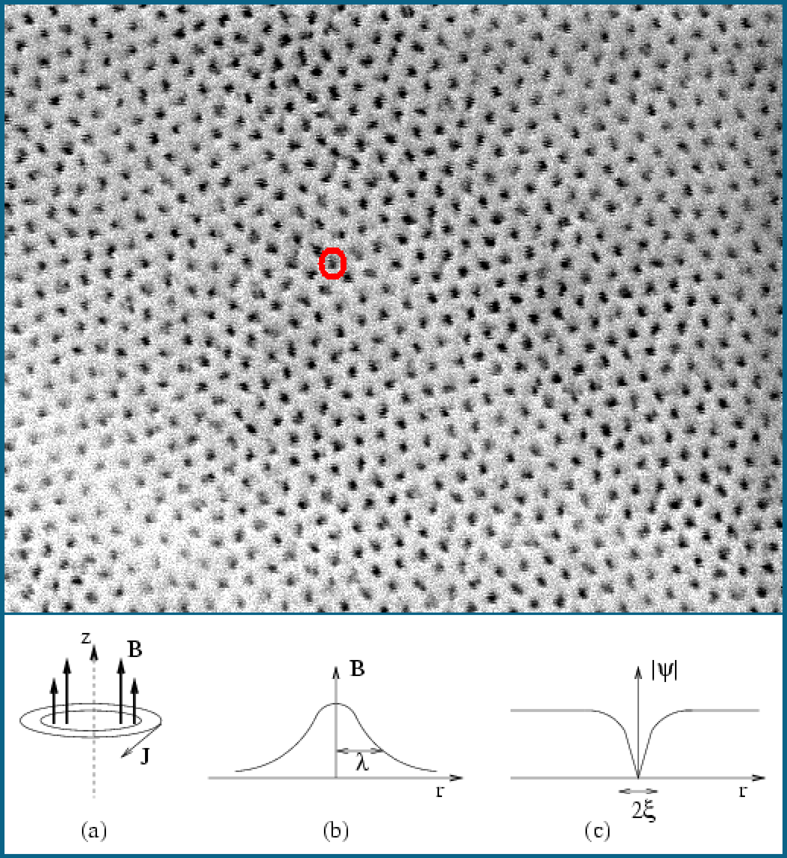


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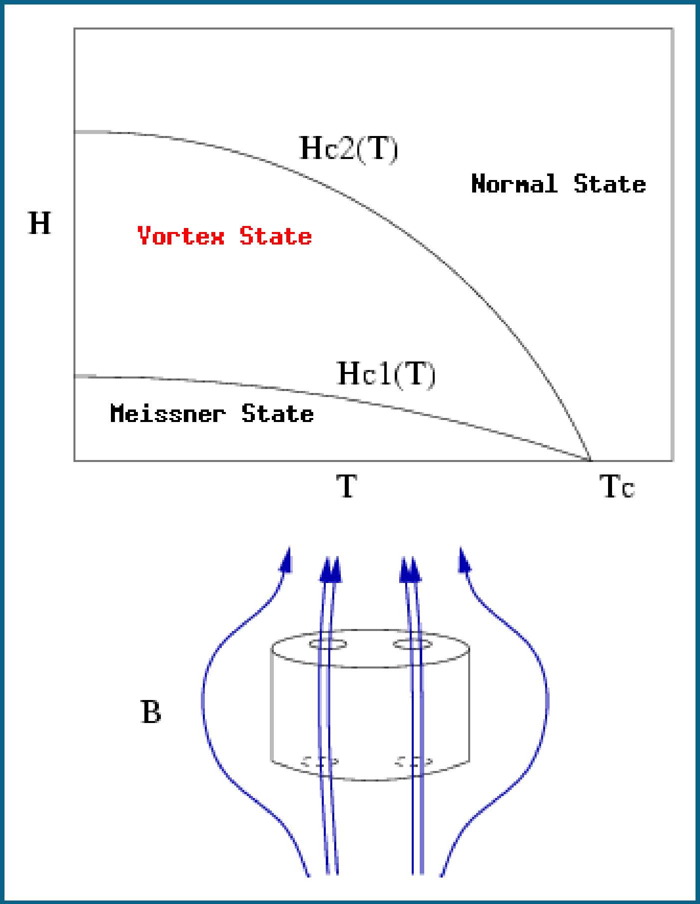


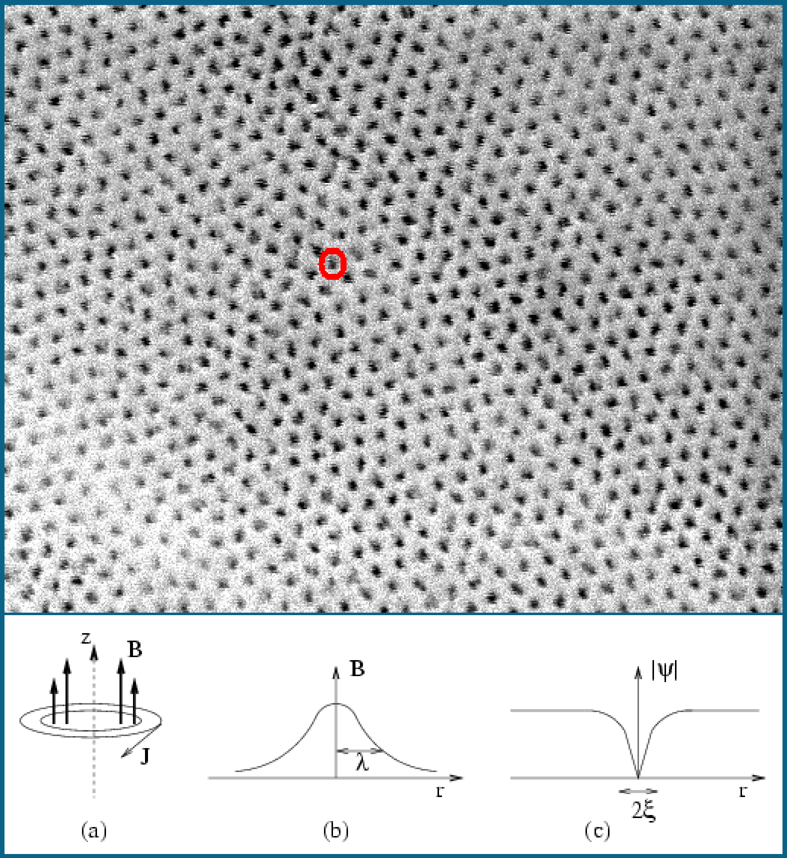


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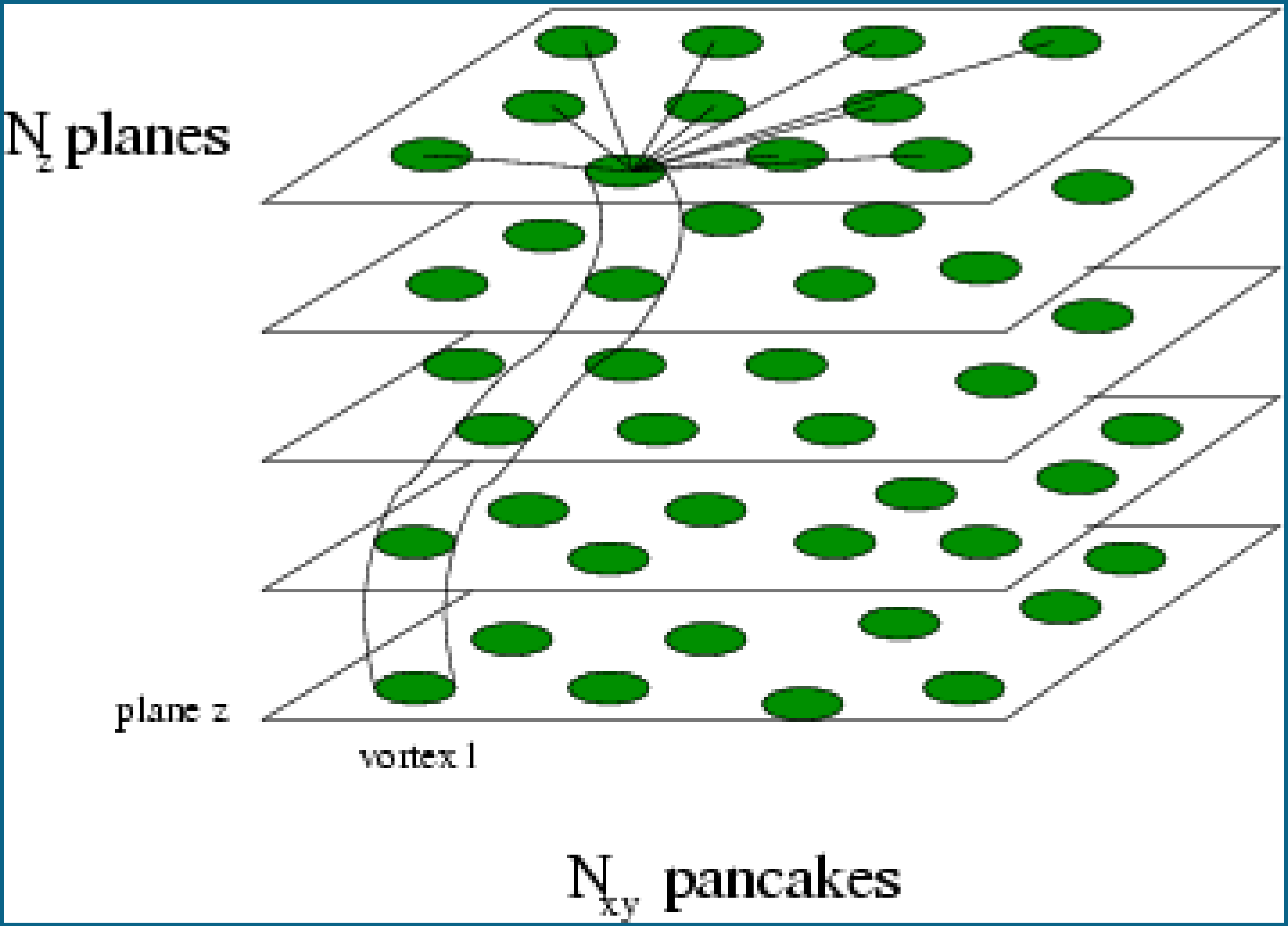
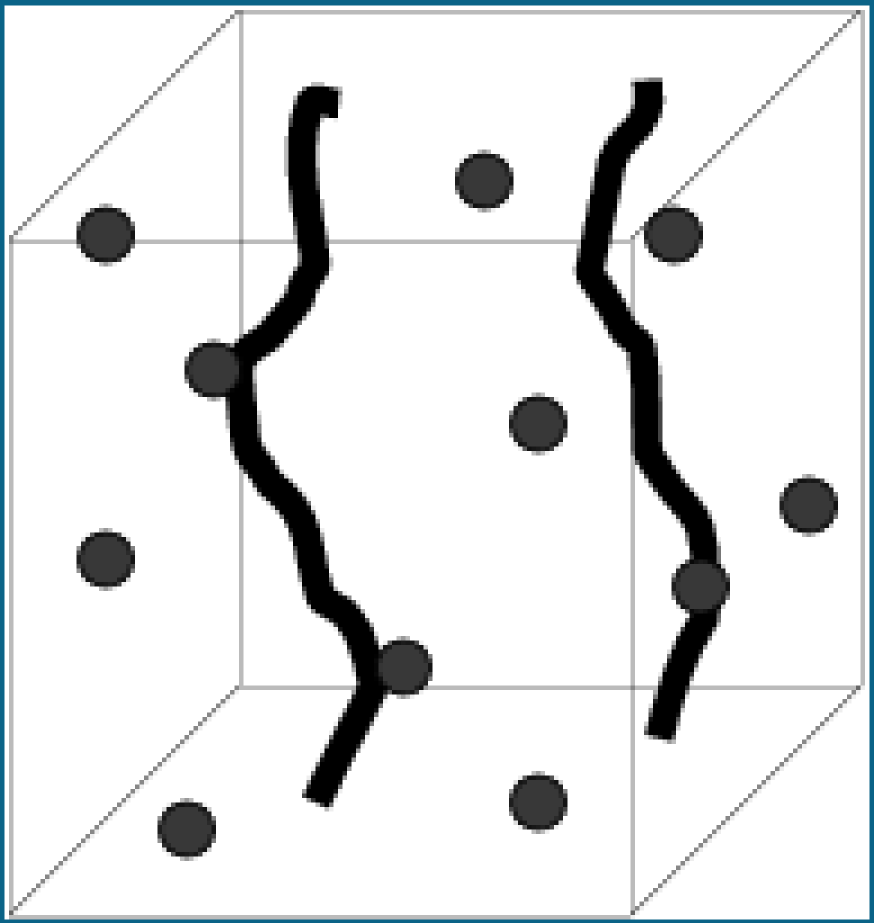
# Vortex Interactions



# Random Pinning



# Vortex dynamics model

$$\eta \overbrace{\frac{\partial \mathbf{r}_{lz}(t)}{\partial t}}^{friction} = \overbrace{\mathbf{f}_{lz}^{int}(t) + \mathbf{f}_{lz}^{elast}(t)}^{interactions} + \overbrace{\mathbf{f}_{lz}^{P}(t)}^{randompinning} + \overbrace{\mathbf{f}_{lz}^{T}(t)}^{thermalnoise} , \qquad (1)$$

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- $\mathbf{r}_{lz}$: 2D position of $l$-th vortex in $z$-th plane.
- Friction force: $\eta = \frac{\Phi_0 H_{c2} d}{c^2 \rho_n}$

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- $\mathbf{r}_{lz}$: 2D position of $l$-th vortex in $z$-th plane.
- Friction force: $\eta = \frac{\Phi_0 H_{c2} d}{c^2 \rho_n}$
- In-plane interaction force: $\mathbf{f}_{lz}^{int} = -\nabla_{\mathbf{r}_{lz}} U^p[\{\mathbf{r}_{lz}(t)\}]$ with

$$U^p[\{\mathbf{r}_{lz}(t)\}] = \sum_{l' \neq l} U\left(\frac{|\mathbf{r}_{lz'} - \mathbf{r}_{lz}|}{\lambda}\right), \quad (2)$$

with

$$U(x) = \frac{\Phi_0}{8\pi^2\lambda^2} K_0(x), \quad (3)$$

- Elastic (inter-plane) force: $\mathbf{f}_{lz}^{elast} = -\nabla_{\mathbf{r}_{lz}} U^e[\{\mathbf{r}_{lz}(t)\}]$ with

$$U^e[\{\mathbf{r}_{lz}(t)\}] = V\left(\frac{|\mathbf{r}_{(z+1)l} - \mathbf{r}_{lz}|}{2r_g}\right) + V\left(\frac{|\mathbf{r}_{(z-1)l} - \mathbf{r}_{lz}|}{2r_g}\right), \quad (4)$$

$$V(|\mathbf{x}|) = 2c_j(|\mathbf{x}| - 1) \text{ for } |\mathbf{x}| \geq 1, \quad c_j(|\mathbf{x}|^2 - 1) \text{ for } |\mathbf{x}| < 1, \quad (5)$$

with $\mathbf{x} = |\mathbf{r}_{z'l} - \mathbf{r}_{lz}|/2r_g$, $z' = z \pm 1$,
$r_g = d\xi_{ab}/\xi_c$, $d$ interplane distance, and

$$c_j = \frac{\epsilon_0 a}{\pi d}\left(1 + \ln\frac{\lambda}{a}\right)$$

- Random pinning force:

$$\mathbf{f}_{lz}^{P} = -\nabla_{\mathbf{r}_{lz}} \int d^2\mathbf{r}' u(\mathbf{r}', z) p(|\mathbf{r}_{lz} - \mathbf{r}'|), \qquad (6)$$

$$\langle u(\mathbf{r}, z) u(\mathbf{r}', z') \rangle = \gamma \delta(\mathbf{r} - \mathbf{r}') \delta_{zz'},$$

and

$$p(r) = \frac{2\xi^2}{r^2 + 2\xi^2}.$$

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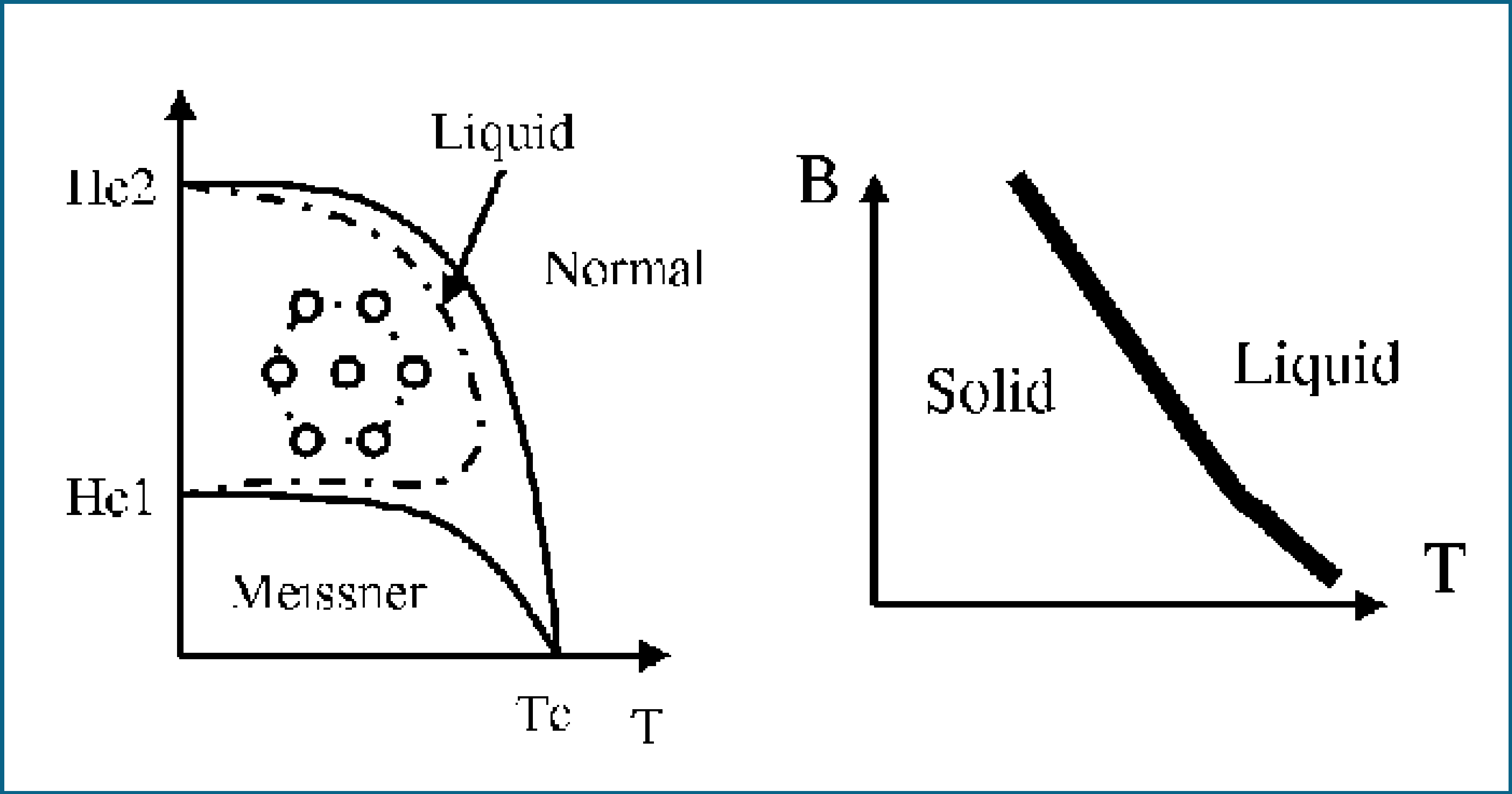
- Thermal (Langevin) force:

$$\langle f_{lz,\mu}^{T}(t) \rangle = 0,$$

$$\langle f_{lz,\mu}^{T}(t) f_{z'l',\mu}^{T}(t') \rangle = 2\eta k_B T \delta(t - t') \delta_{zz'} \delta_{ll'} \delta_{\mu\mu'},$$

$\mu, \mu' = x, y$

# Vortex Phase diagram without disorder



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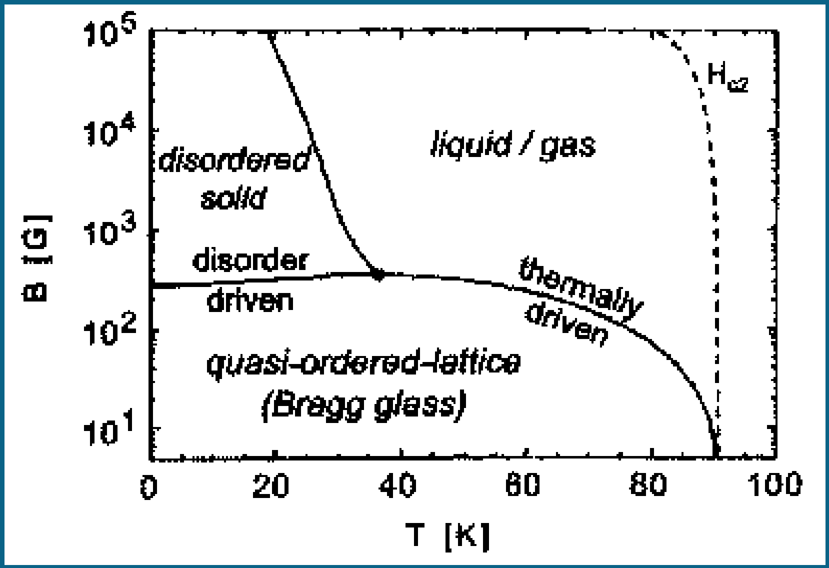
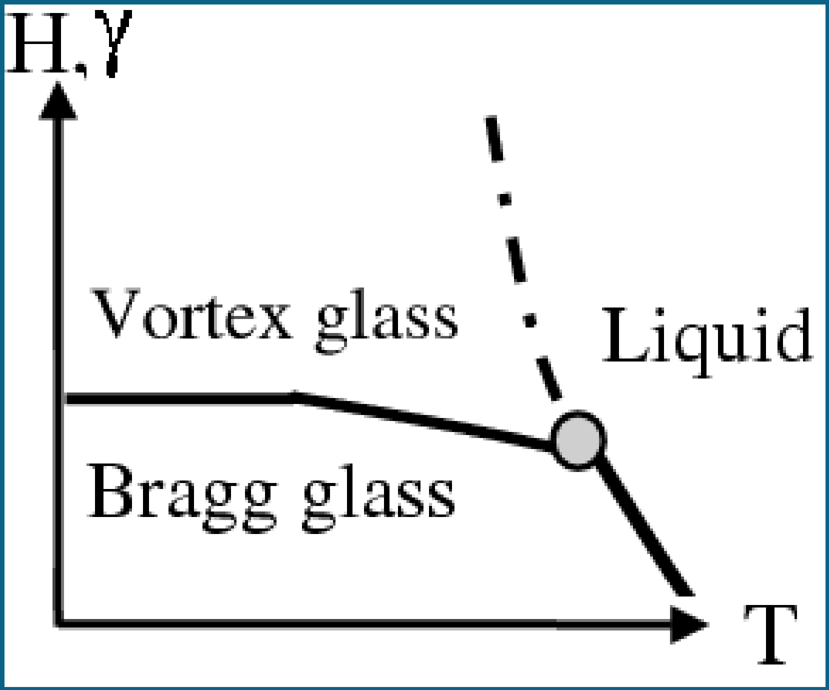
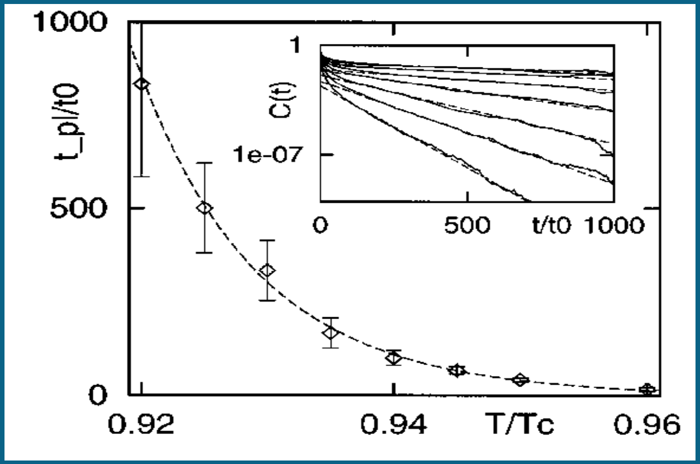



From experimental data in BiSrCaCuO

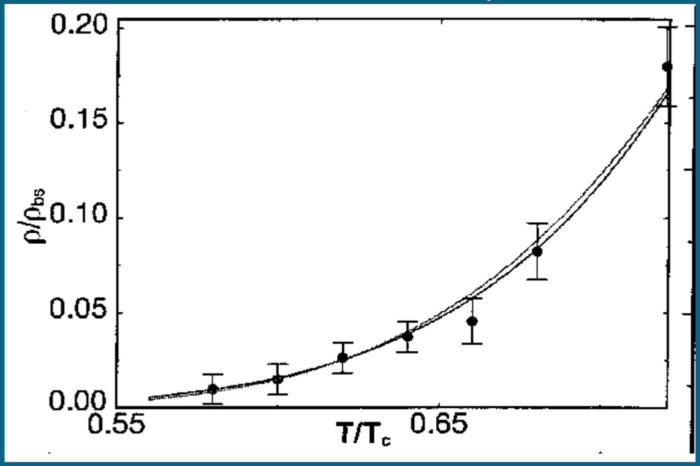



Schematic Phase Diagram

# Vortex Glass transition




van Otterlo et al, PRL 1998




Reichhardt et al, PRL 2000

- $C(t) = e^{-\langle [r_{lz}(t) - r_{lz}(0)]^2 \rangle / a^2} \sim e^{-t/t_{pl}}$

- Scaling theory : $\rho \sim |T - T_g|^{\nu(z-1)}$
  $\nu = 1.3 - 2$, $z = 3.1 - 6.5$

- Volger-Fulcher law : $\rho \sim e^{-1/(T - T_g)}$

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- Examples: *spin glasses, window glasses polymers, colloids, gels, dense liquides, granular matter, sheared fluids, etc,.....Vortex Matter?*

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*Energy flow, partial equilib ration, and effective temperatures in systems with slow dynamics*, Cugliandolo, Kurchan, Peliti, PRE, 1997.

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- Response: $\chi(t+t_w,t_w) = \lim_{\epsilon\to 0} \frac{\langle O(t)\rangle_\epsilon - \langle O(t)\rangle_0}{\epsilon}$, ($H_\epsilon = H - \epsilon(t)O$)

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- Equilibrium: *Fluctuation - dissipation theorem* ( $t_w \gg \tau_{eq}$)

$$\chi(t) = \frac{1}{k_B T}[C(0) - C(t)]$$

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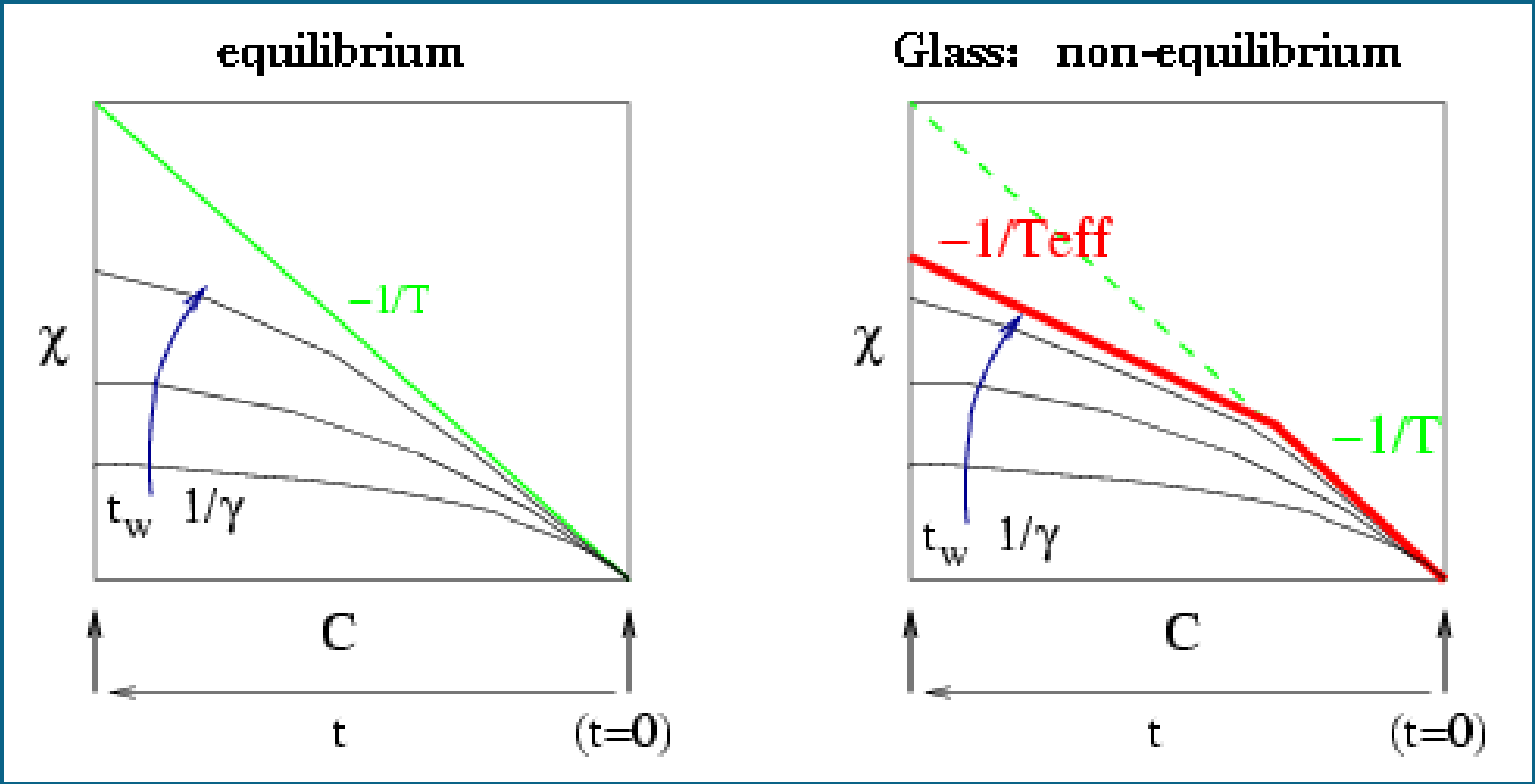
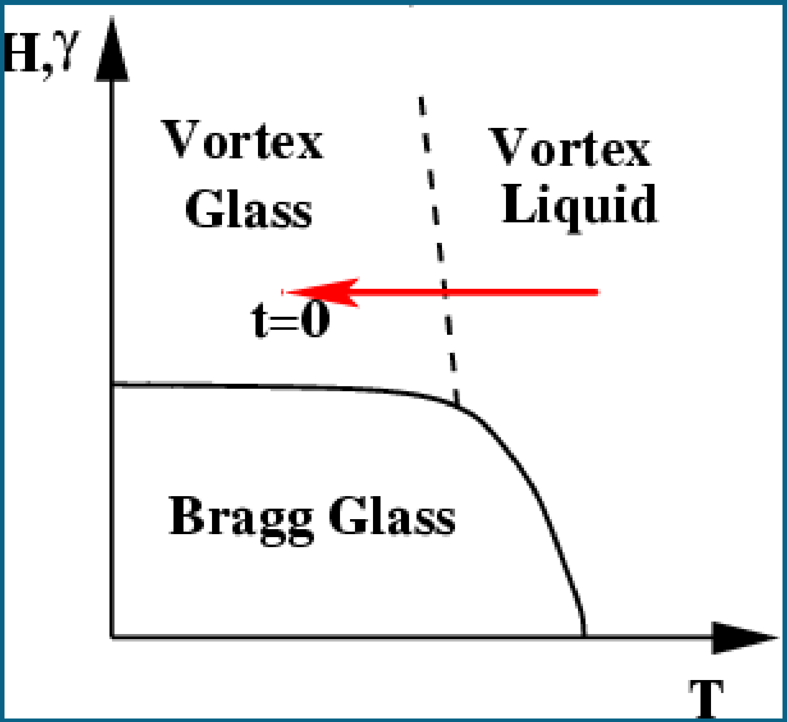
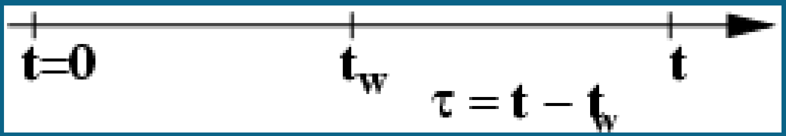
- Fluctuation - dissipation relations



# Simulations




- 56 pancakes per plane
- 50 planes
- Quench from $T_{start} = 0.3 > T_g$ to $T$



# Correlation functions measured

- Dynamic wandering

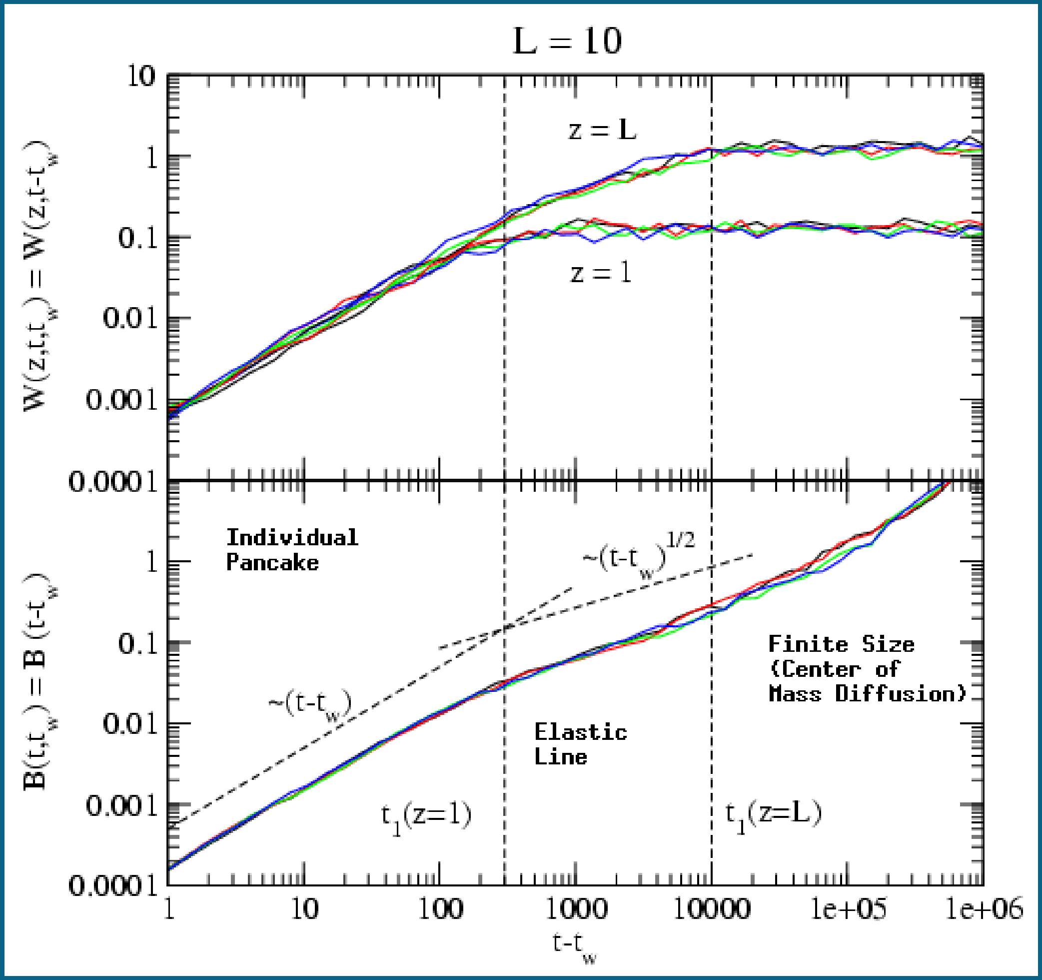
$$W(z,t,t_w) =$$

$$\langle \frac{1}{N_{xy}} \sum_l |[\mathbf{r}_{lz}(t) - \mathbf{r}_{l0}(t)] - [\mathbf{r}_{lz}(t_w) - \mathbf{r}_{l0}(t_w)]|^2 \rangle$$

- Mean square displacement (MSD)

$$B(t,t_w) = \langle \frac{1}{N} \sum_{lz} [x_{lz}(t) - x_{lz}(t)]^2 \rangle$$

# Single Vortex Line wihout disorder (Vortex Liquid)



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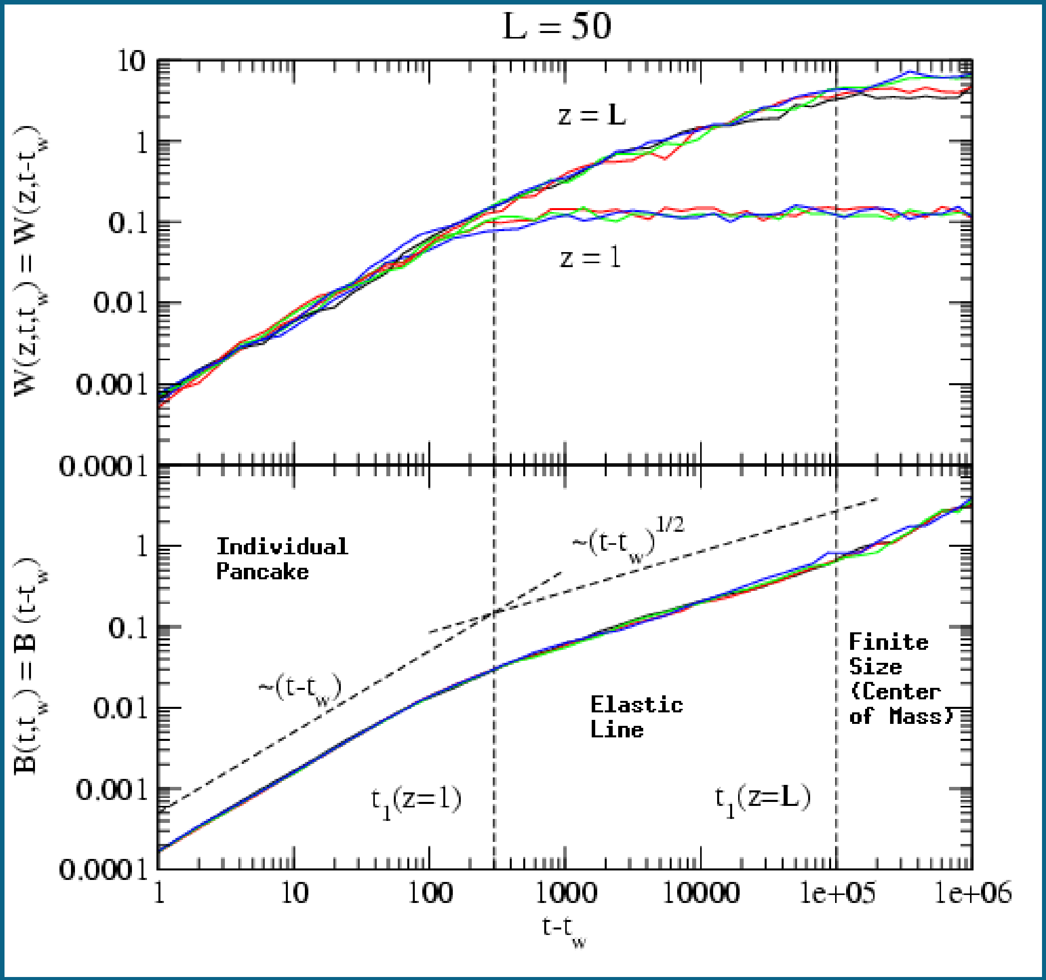

# Finite Size effect

$$\tau = t - t_w$$

$$W(z,\tau) \sim \begin{cases} \tau & \tau < t_{1,(z)} \\ z^{2\zeta_T} & \tau > t_{1,(z)} \end{cases}$$

$\zeta = \zeta_T = 1/2$ (thermal line wandering)

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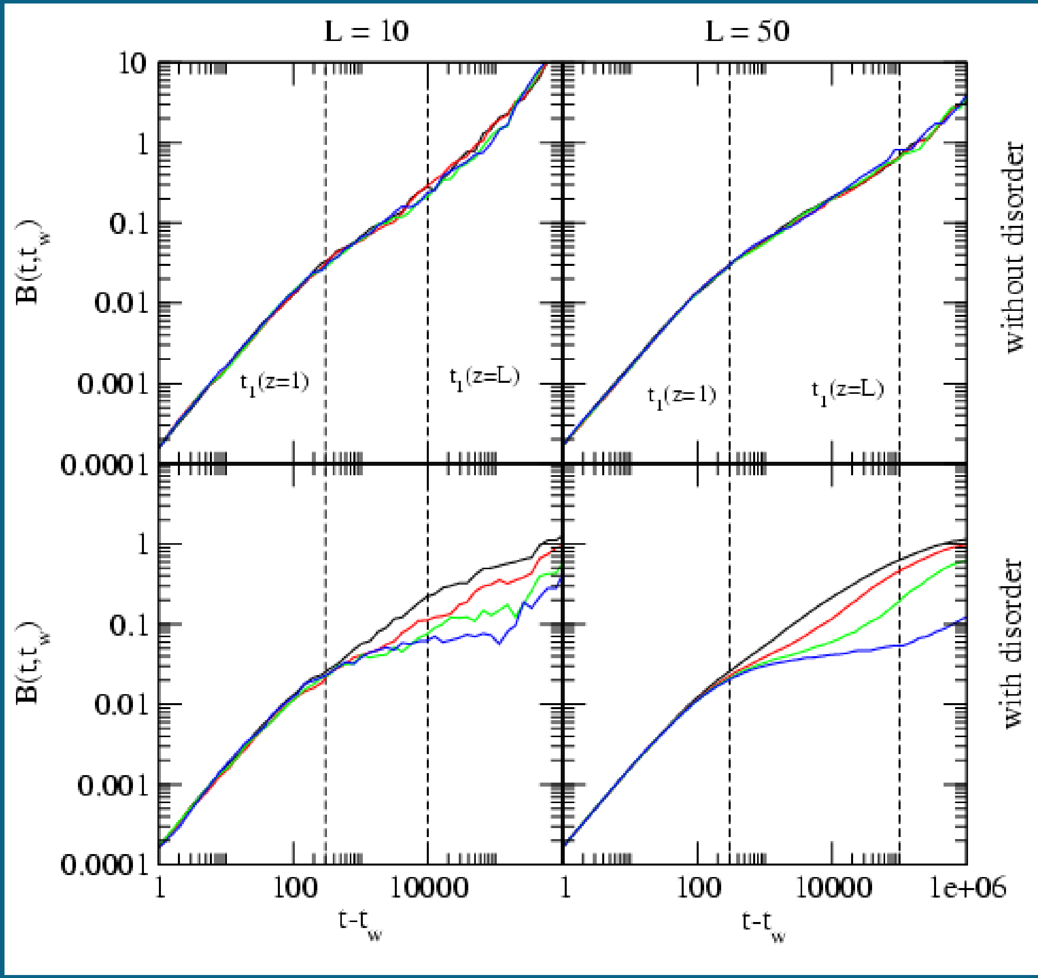
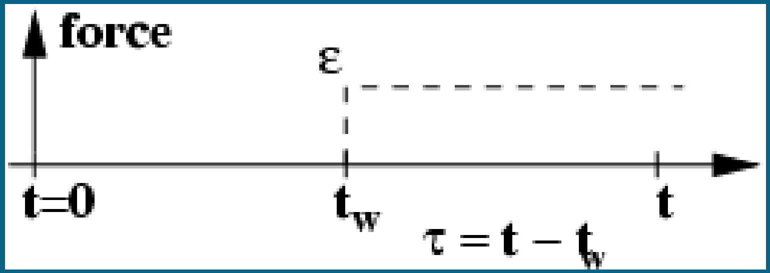
$$B(\tau) \sim \begin{cases} \tau & \tau < t_{1,(z=1)} & \text{single pancake} \\ \tau^{1/2} & t_{1,(z=1)} < \tau < t_{1,(z=N_z)} & \text{elastic line} \\ \tau & t_{1,(z=N_z)} < \tau & \text{center of mass} \end{cases}$$

# Single Vortex Line with Disorder: Aging



# Response Function

- Integrated Response



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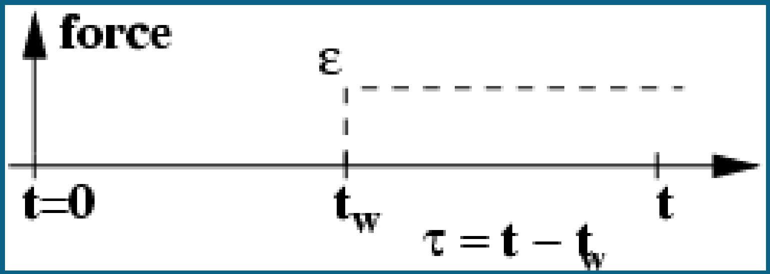
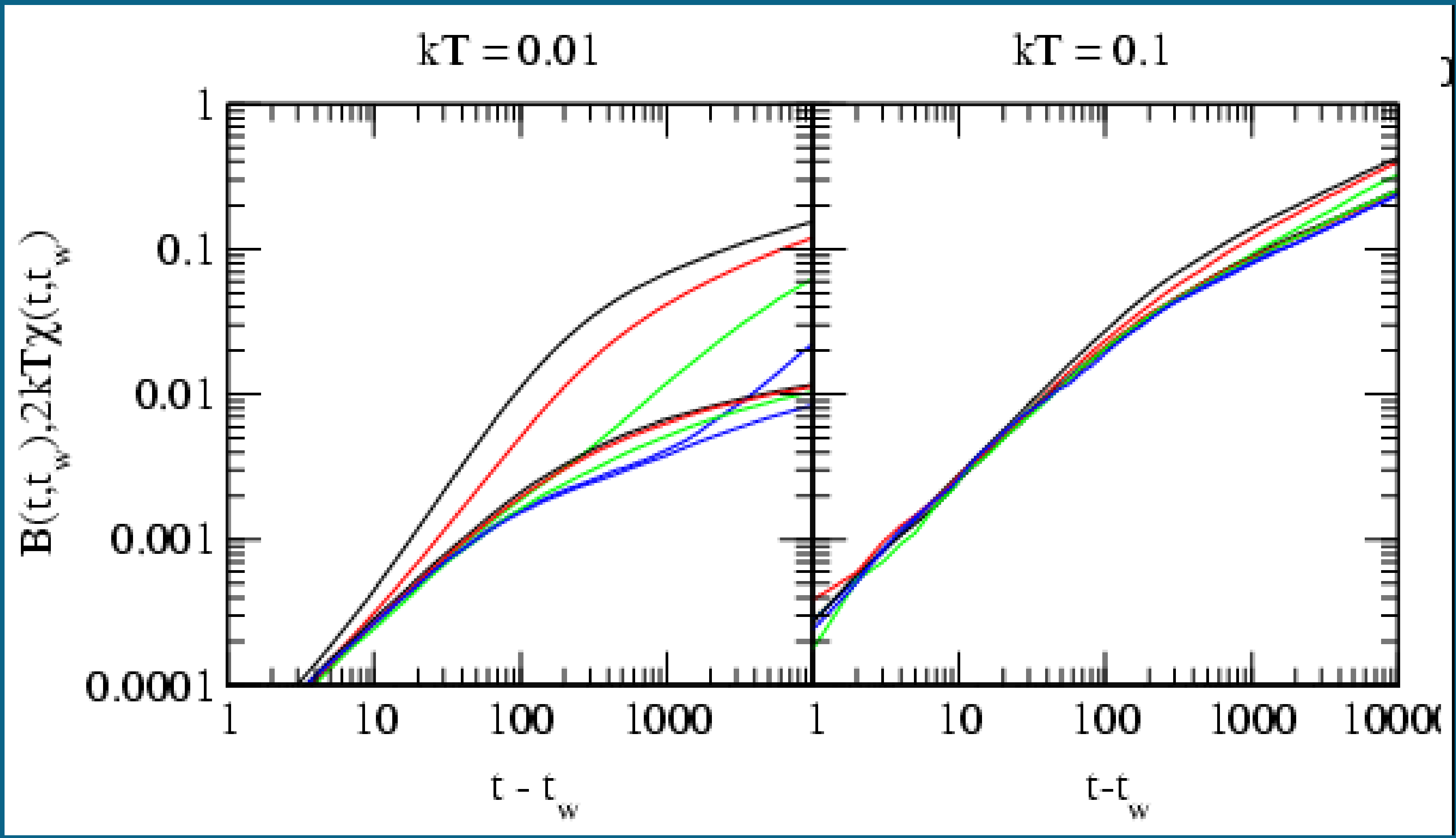



Random force for $t \geq t_w$: $\mathbf{f}_{lz} = \epsilon s_{lz} \hat{\mathbf{x}}$,
$\epsilon$ : perturbation strength, $s_{lz} = \pm 1$ , random

$$\chi(t, t_w) = \frac{\left\langle \frac{1}{N} \sum_{lz} s_{lz} x_{lz}(t) \right\rangle_\epsilon - \left\langle \frac{1}{N} \sum_{lz} x_{lz}(t) \right\rangle_{\epsilon=0}}{\epsilon}$$

$x_{lz}$, $x$-component of vortex position,

# Single Vortex Line with Disorder: Aging



# Multiplicative Scaling

Yoshino, PRL 1998, Directed Polimer in Random Media (1+1)

Define

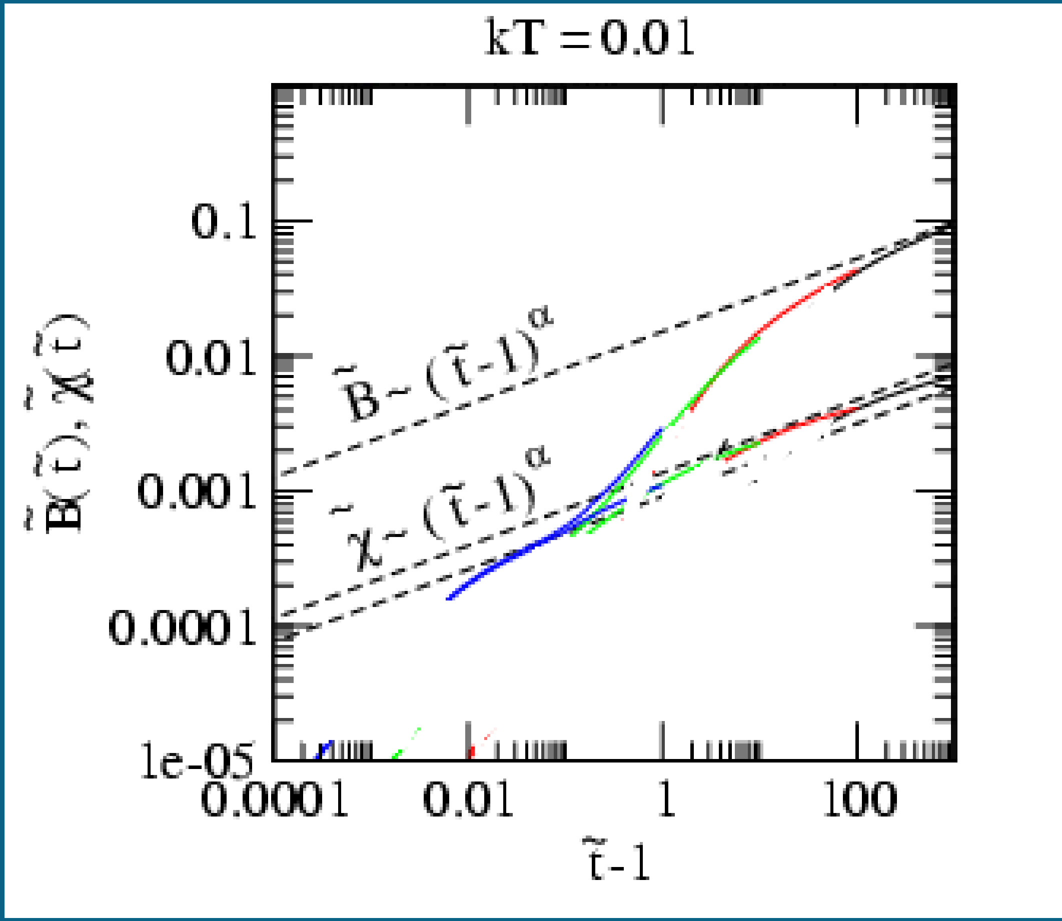
$$B(t,t_w) = \tilde{B}(\tilde{t})t^{-\alpha}$$
$$2kT\chi(t,t_w) = \tilde{\chi}(\tilde{t})t^{-\alpha}$$

with $\tilde{t} = t/t_w$ and

$$\tilde{B}(\tilde{t}) = \begin{cases} c_1(T)(\tilde{t}-1)^{-\alpha(T)} & \tilde{t} \ll 1 \\ c_2(T)(\tilde{t}-1)^{-\alpha(T)} & \tilde{t} \gg 1 \end{cases}$$

$$\tilde{\chi}(\tilde{t}) = \begin{cases} c_1(T)(\tilde{t}-1)^{-\alpha(T)} & \tilde{t} \ll 1 \\ y(T)c_2(T)(\tilde{t}-1)^{-\alpha(T)} & \tilde{t} \gg 1 \end{cases}$$

# Single Vortex Line with Disorder: Scaling



# Violation of FDT

$$\chi(t,t_w) = \frac{y(t,t_w)}{2T} B(t,t_w)$$

with $y(t,t_w) = \frac{T}{T_{\text{ef}}(t,t_w)}$

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- for $t < t_w$,

$$\tilde{\chi} = \tilde{B}$$

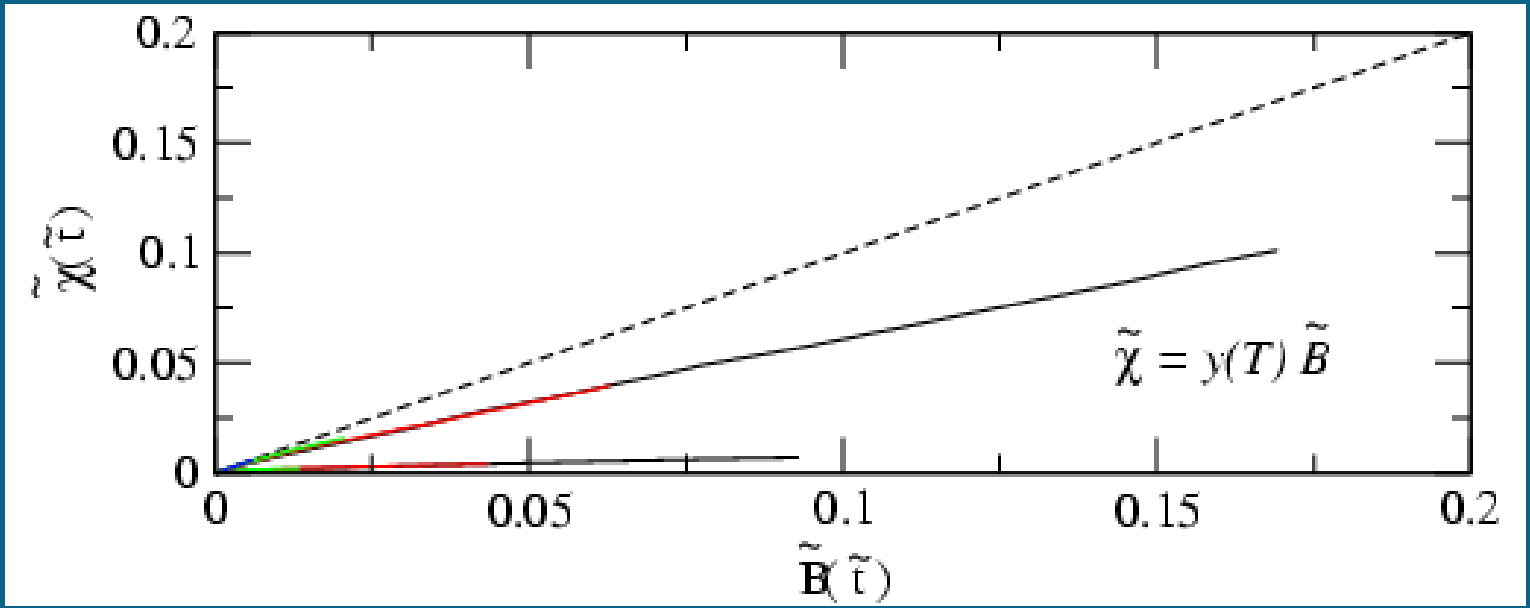
- for $t > t_w$

$$\tilde{\chi} = y(T)\tilde{B}$$

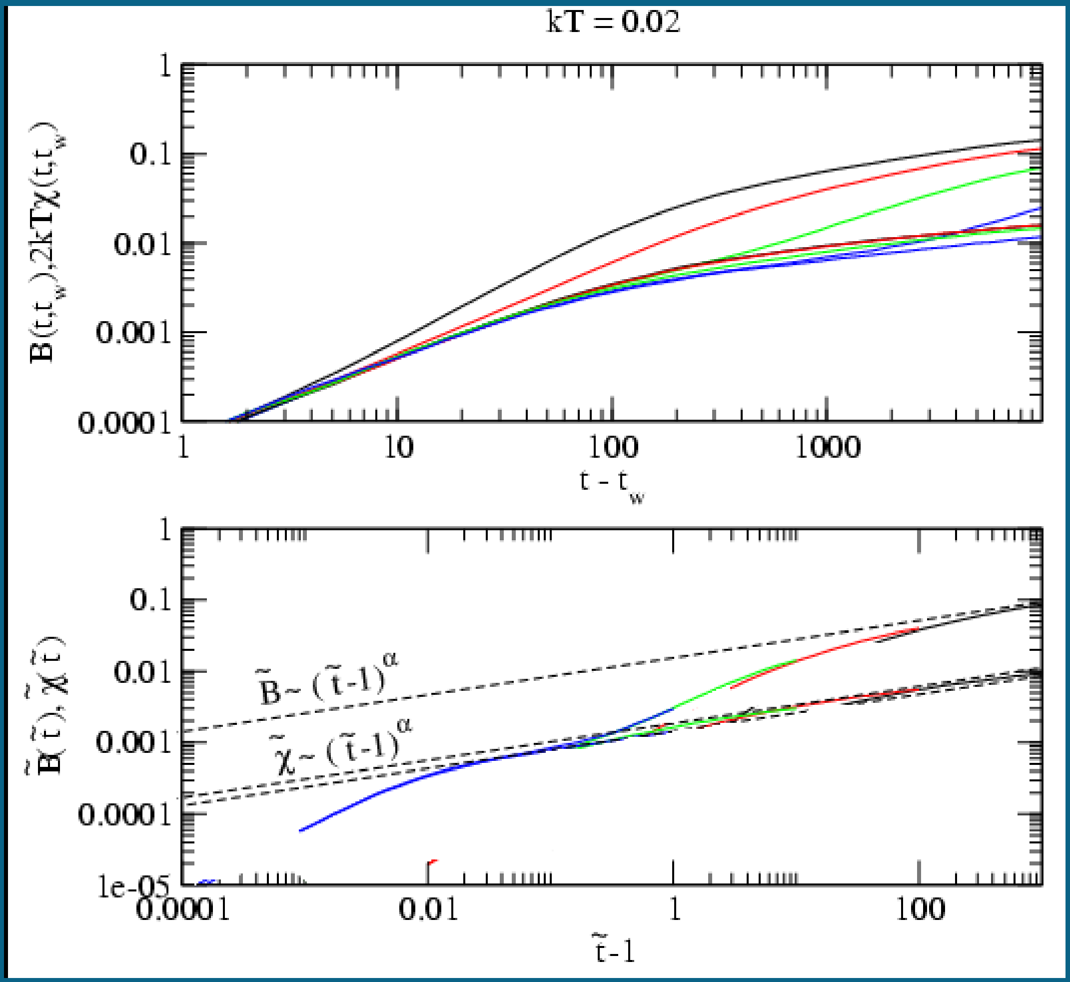
- Effective temperature:

$$y(T) = \frac{T}{T_{eff}}$$

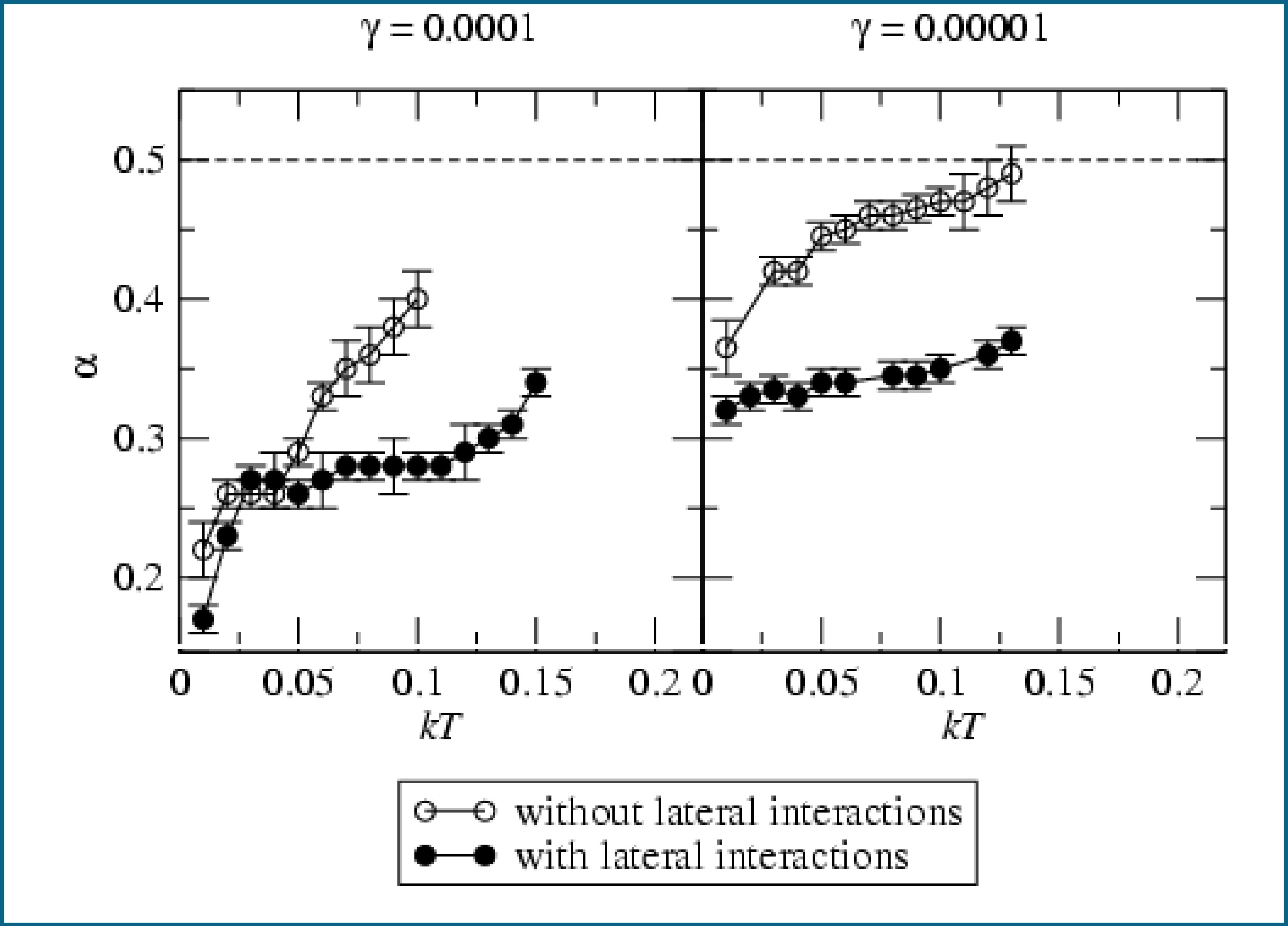
# Single Vortex Line with Disorder: Violation of FDT



# Vortex Glass: Aging and Scaling



# Scaling Exponents $\alpha(T)$



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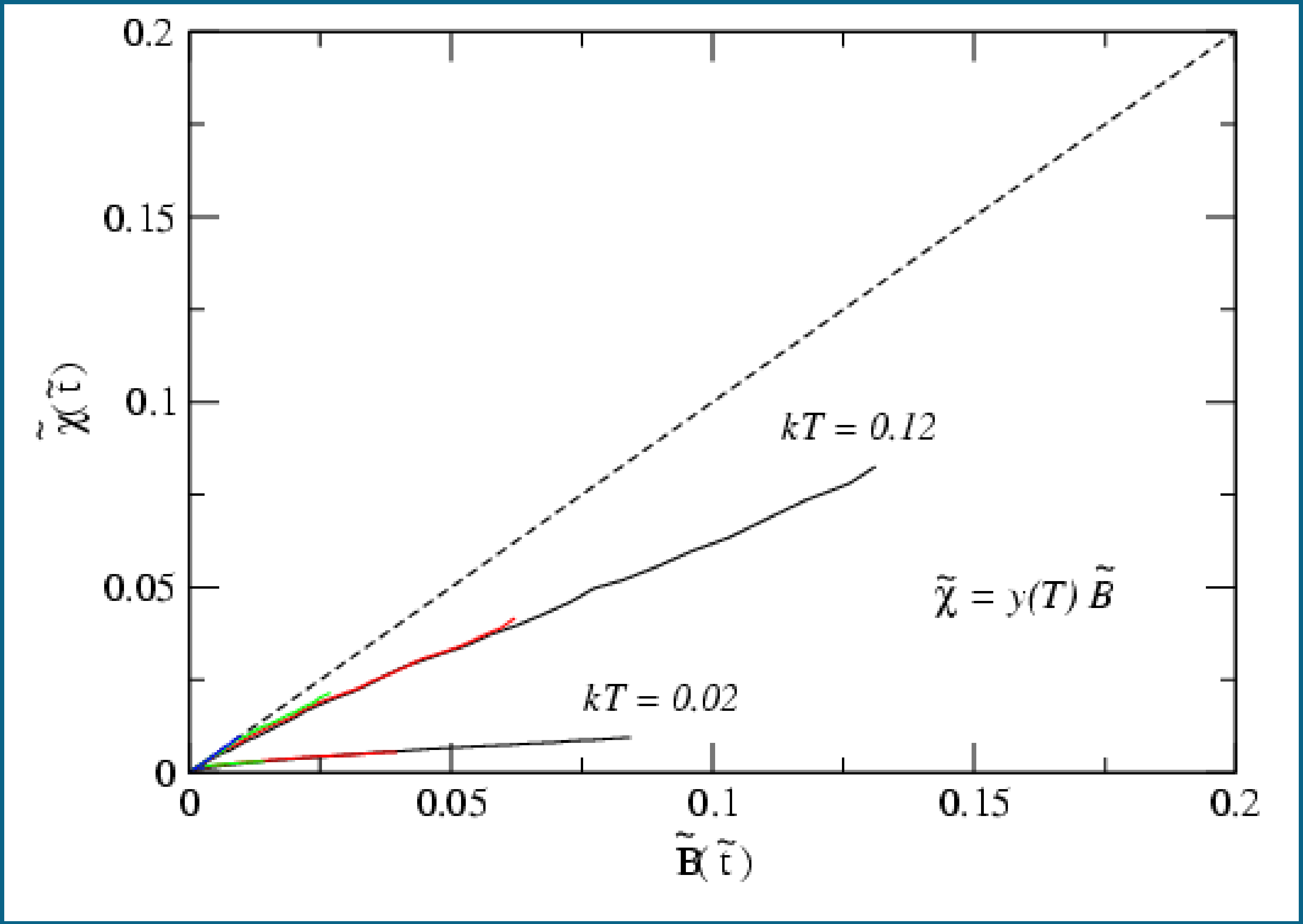
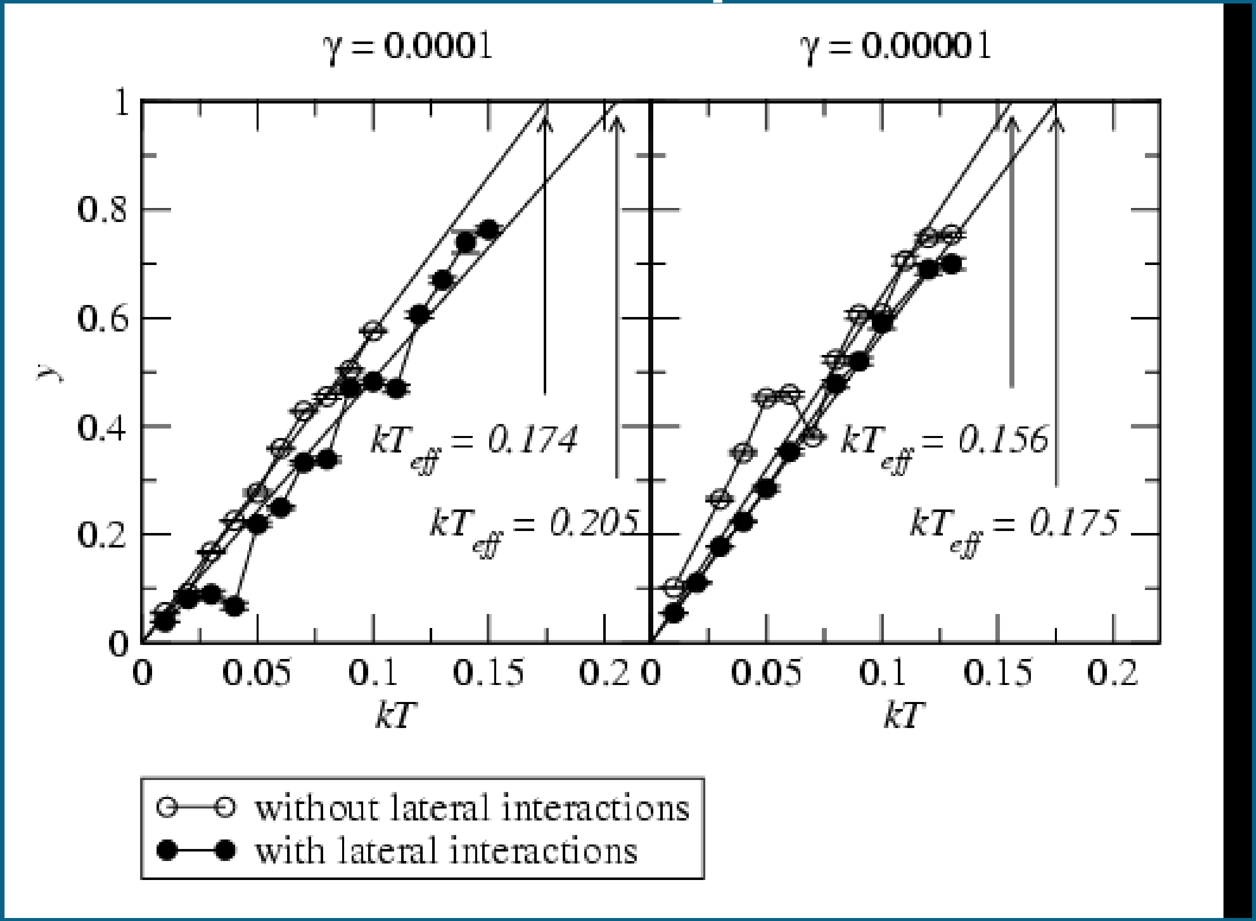


# Effective Temperature




$$y(T) = \frac{T}{T_{eff}(T)}$$

# Conclusions

- Slow dynamics dominated by elastic line relaxation
- Aging : multiplicative scaling (as polimers in random media)
- Violation of FDT after a quench to the vortex glass