From Brownian motion to impurity dynamics in $1d$ quantum liquids

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Quantum : J. Bonart & LFC, Phys. Rev. A **86**, 023636 (2012) & EPL **101**, 16003 (2013).

Classical : J. Bonart, LFC & A. Gambassi, J. Stat. Mech. P01014 (2012) ;

work in progress with G. Gonnella, G. Laghezza, A. Lamura and A. Sarracino

Taipei, July 2013

Preamble

- Interest is the dynamics of complex out of equilibrium classical and quantum matter (glasses, active matter, spin-ice, *etc.*)
- One way to study some of these systems is to use tracers and follow their evolution to characterise the medium.
- Here : revert the problem, back to Brownian motion for relatively simple baths and interactions.

Plan

- **Classical dissipative dynamics**

  Langevin's approach to Brownian motion.

  White/coloured noise.

  General Langevin equations with additive noise.

  Examples : protein dynamics, tracers in active matter.

  Microscopic modelling : equilibrium unchanged, dynamics modified.

- **Quantum systems**

  Quantum Brownian motion and quenches

  Experimental realisation and theoretical description

  Bath and interaction modelling

  Polaron effect and potential renormalisation

  Consequences & conclusions

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Langevin's approach

A Brownian particle immersed in a liquid in thermal equilibrium

The effect of the collisions with the liquid particles is mimicked by two forces :

a viscous drag that tends to slow down the tracer, $\vec{f}(\vec{v}) = -\eta_o \vec{v} + \dots$

the relevant parameter is the friction coefficient η_o.

a random noise $\vec{\xi}$ that mimics thermal agitation

the relevant parameter is temperature T.

If the collision time τ_c satisfies $\tau_c \ll t_0$ (a particle's 'microscopic' time),

the statistics of the noise is Gaussian (central limit theorem).

If $\tau_c \ll t$ (the observation time)

the noise is white and the dynamics are Markovian (no memory).

Langevin's approach

Stochastic Markov dynamics

From Newton's equation $\vec{F} = m\vec{a} = m\dot{\vec{v}}$ and $\vec{v} = \dot{\vec{r}}$

$$\boxed{m\dot{v}_a = -\eta_o v_a + \xi_a} \qquad\qquad (\vec{F}_{\rm ext} = \vec{0})$$

with $a = 1, \dots, d$ (the dimension of space),

m the particle mass,

η_o the friction coefficient,

and $\vec{\xi}$ the time-dependent thermal noise with Gaussian statistics,

zero average $\langle \xi_a(t) \rangle = 0$ at all times t,

and delta-correlations $\langle \xi_a(t)\xi_b(t') \rangle = 2\,\eta_o k_B T\,\delta_{ab}\delta(t-t')$.

Dissipation : for $\eta_o > 0$ the averaged energy is not conserved,

$$2\langle E(t) \rangle = m\langle v^2(t) \rangle \neq \text{ct.}$$

The relation between friction coefficient & noise amplitude ensures

equipartition $\boxed{m\langle v^2(t) \rangle \to k_B T}$ for $t \gg t_r^v \equiv \frac{m}{\eta_o}$

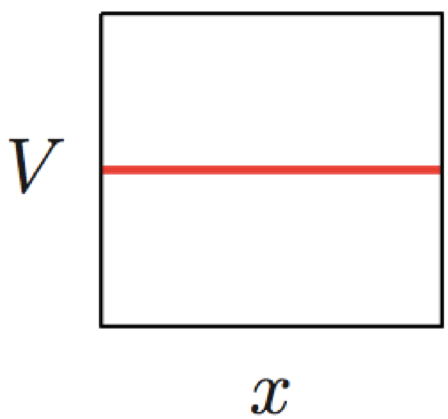
Langevin's approach

Markov normal diffusion

For simplicity : take a one dimensional system, $d = 1$.

$$\boxed{\langle x^2(t) \rangle \to 2D\, t^{2H}}$$

(for $t \gg t_r^v = m/\eta_o$)



with D the diffusion coefficient,

$\boxed{D = k_B T/\eta_o}$ the Einstein relation,

H the Hurst exponent and

$\boxed{H = 1/2}$ i.e. normal diffusion

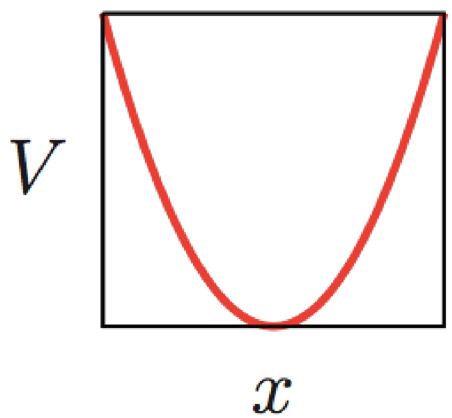
for a white bath (no correlations).

Langevin's approach

Markov $1\mathrm{d}$ relaxation in a harmonic potential

$$F = -\frac{dV(x)}{dx} \quad \text{with} \quad V(x) = \frac{1}{2}\kappa x^2$$

Underdamped dynamics for parameters such that $\boxed{\kappa > \eta_0^2/m}$



$$\boxed{\langle x^2(t)\rangle \to \frac{k_B T}{\kappa} + \text{ct} \ \sin \Omega t \ e^{-t/t_r^x}}$$

with $\boxed{\Omega \propto \sqrt{\frac{\kappa}{m} - \frac{\eta_o^2}{m^2}} \underset{\eta_o^2 \ll m\kappa}{\longrightarrow} \sqrt{\frac{\kappa}{m}}}$

Damped ($\eta_o \neq 0$ but small) oscillations in a confining harmonic potential

Asymptotic equilibrium (equipartition)

Langevin's approach

Markov $1\mathrm{d}$ relaxation in a harmonic potential

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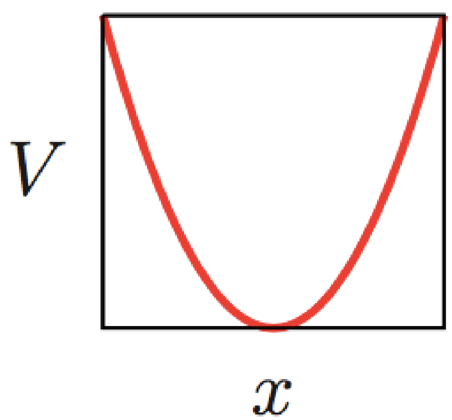


$$\boxed{\langle x^2(t)\rangle \to \frac{k_B T}{\kappa} + \text{ct}\, e^{-t/t_r^x}}$$

for $t \gg t_r^v = m/\eta_o$,

and $t_r^x \propto \eta_o/\kappa$,

the position-relaxation time.

Exponential relaxation (η_o large) in a confining potential

Asymptotic equilibrium (equipartition)

General Langevin equation

The system, $\{r_a\}$, with $a = 1, \dots, d$, coupled to a **general environment in equilibrium** evolves according to the **non-Markov eq.**

$$\boxed{\underbrace{m\dot{v}_a(t)}_{\text{Inertia}} + \underbrace{\int_{t_0}^{t} dt' \, \Sigma_{\mathbf{B}}^{\mathbf{K}}(t-t') v_a(t')}_{\text{friction}} = \underbrace{-\frac{\delta V(\{\vec{r}\})}{\delta r_a(t)}}_{\text{deterministic force}} + \underbrace{\xi_a(t)}_{\text{noise}}}$$

Coloured noise with correlation $\langle \, \xi_a(t)\xi_b(t') \, \rangle = k_B T \, \delta_{ab} \, \Sigma_{\mathbf{B}}^{\mathbf{K}}(t-t')$ and zero mean.

T is the **temperature** of the bath and k_B the Boltzmann constant.

The **friction kernel** is $\Sigma_{\mathbf{B}}^{\mathbf{K}}(t-t')$ [generalizes $2\eta_o \delta(t-t')$].

Colored noise

Equilibrium unaltered, dynamics altered

Generic : Most of the exact **fluctuation-dissipation relations** in and out of equilibrium remain unaltered for generic $\Sigma_{\rm B}^{\rm K}$, *e.g.* the fluctuation-dissipation theorem, fluctuation theorems, *etc.* Equilibrium at $V(\vec{r})$ is reached.

Aron, Biroli, LFC 10

Particular : The observables' **time-dependent functional form** depends on the noise, *i.e.* on $\Sigma_{\rm B}^{\rm K}$ or its Fourier transform $\mathrm{S}(\nu)/\nu$

with the spectral density $$\boxed{\mathrm{S}(\nu) = \eta_{\rm o} \left(\frac{\nu}{\omega_o}\right)^\alpha e^{-\nu/\omega_c}}$$

for $\alpha \leq 1$ the high-frequency cut-off ω_c can be set to infinity and

$$\boxed{\Sigma_{\rm B}^{\rm K}(t) = \frac{\eta_{\rm o}}{\omega_o^{\alpha-1}\Gamma_{\rm E}(1-\alpha)}\, t^{-\alpha} \qquad \text{with} \qquad 0 < \alpha \leq 1}$$

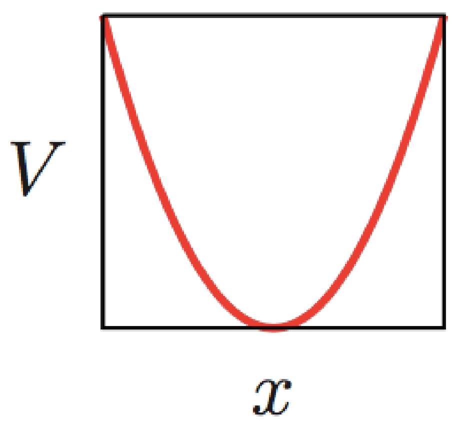
with $\eta_{\rm o}$ the 'friction coefficient' and $\Gamma_{\rm E}$ the Euler-function.

for $\alpha > 1$ a $\omega_c < +\infty$ is needed and $\boxed{\Sigma_{\rm B}^{\rm K}(t) = t^{-\alpha} g\left(\omega_c t\right)}$

Langevin's approach

sub-Ohmic $1\mathrm{d}$ over-damped relaxation in a harmonic potential

$F = -\frac{dV}{dx}$ with $V(x) = \frac{1}{2}\kappa x^2$ & bath with $\alpha \leq 1$



$$\boxed{\langle x^2(t)\rangle \to \frac{k_B T}{\kappa} + \text{ct } E_{\alpha,1}\left(\frac{t}{t_r^x}\right)}$$

for $t \gg t_r^v = m/\eta_o$,

and $t_r^x \propto \eta_o/\kappa$

the position-relaxation time.

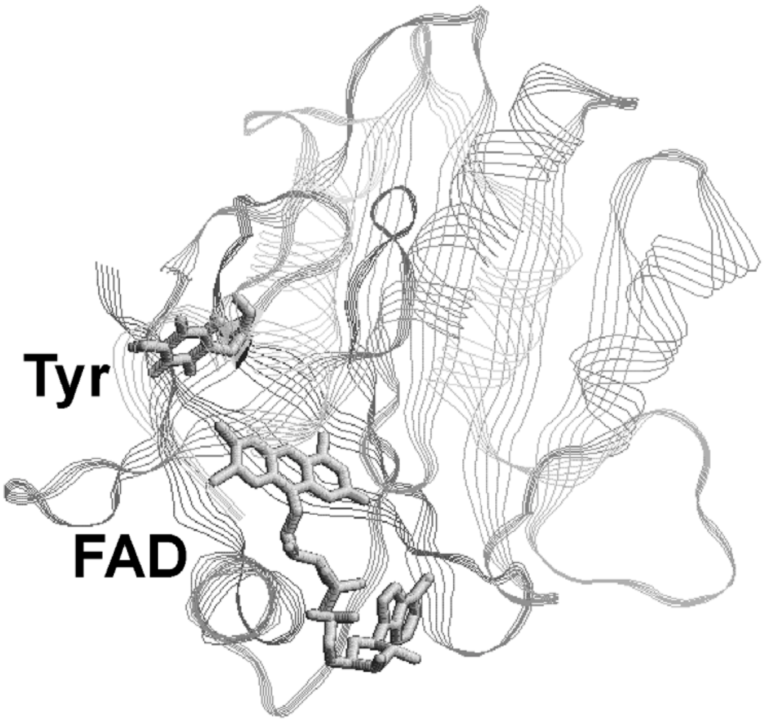
with $E_{\alpha,1}(z) = \sum_{k=0} \frac{z^k}{\Gamma_E(\alpha k+1)}$ the Mittag-Leffler function.

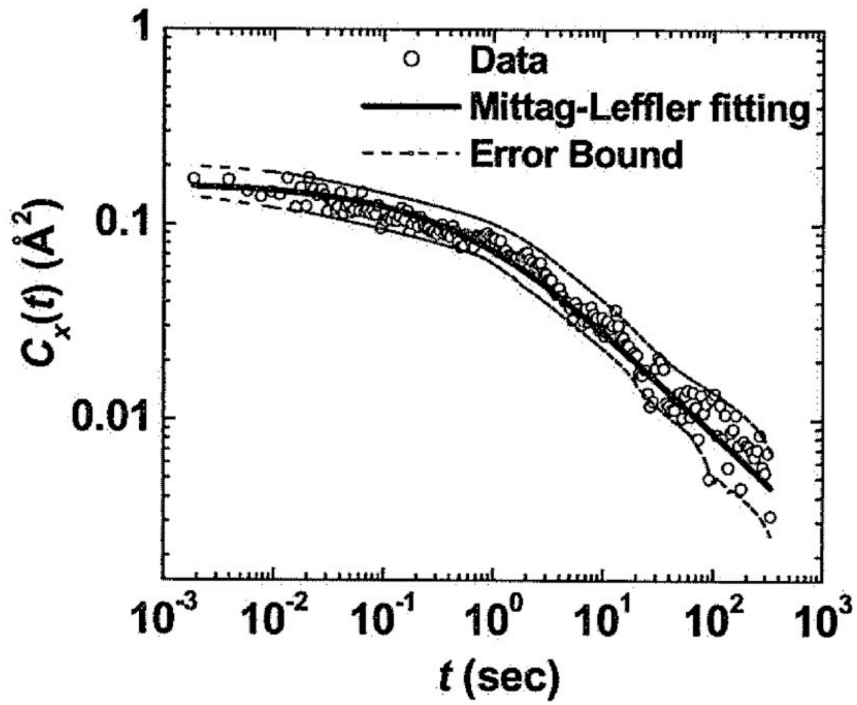
Only for an Ohmic bath $\alpha = 1$ the relaxation is exponential $E_{1,1}(z) = e^z$

sub-Ohmic bath $\alpha < 1$: $E_{\alpha,1}(z) \to z^{-1}$ for $z \to -\infty$ **algebraic decay.**

Protein dynamics

Questions : what are the potential and the bath ?





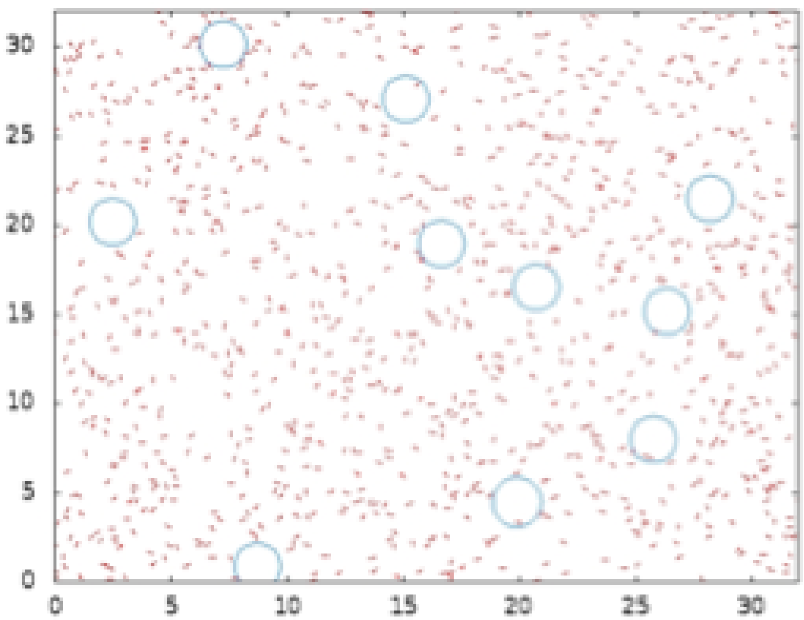
$x(t)$ distance between Tyr and FAD $\boxed{\alpha = 0.51 \pm 0.07}$

Yang *et al* 03 ; Min, Luo, Cherayil, Kou & Xie 05

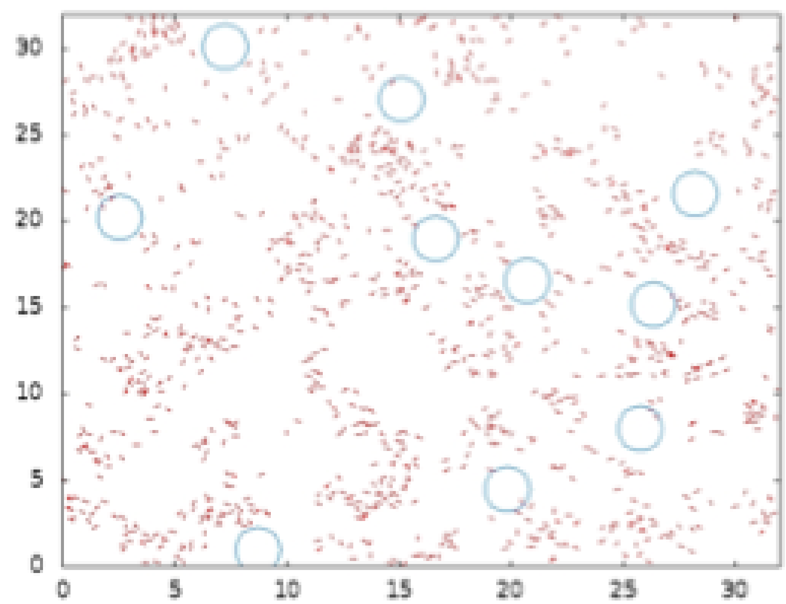
Active matter

Monitoring tracers' diffusion to characterise the environment

Liquid-like regime



Clustering regime



Langevin description ?

White/colored, additive/multiplicative out of equilibrium noise ?

Work in progress w/ G. Gonnella, G. Laghezza, A. Lamura (Bari) &

A. Sarracino (Roma/Paris)

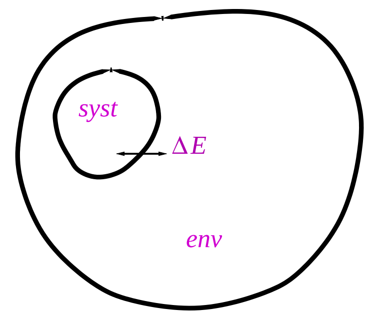
Microscopic modelling

From deterministic to stochastic

In order to derive an effective Langevin equation for a **classical** system coupled to a **classical** bath one writes down :

the Hamiltonian of the ensemble

$$\boxed{\mathcal{H} = \mathcal{H}_{syst} + \mathcal{H}_{env} + \mathcal{H}_{int}}$$



The dynamics of all variables are given by Newton rules.

One has to give the initial $\{\vec{q}_n(0), \vec{\pi}_n(0); \vec{r}(0), \vec{p}(0)\}$.

Dissipative case : if $\mathcal{H}_{int} \neq 0$ the total energy is conserved, $E =$ ct, but each contribution is not ; in particular, $E_{syst} \neq$ ct and we'll take $\boxed{E_{syst} \ll E_{env}}$

Microscopic modelling

From deterministic to stochastic

E.g., an ensemble of $1d$ harmonic oscillators and coupling $\sum_n c_n q_n f(x)$

$$\boxed{\mathcal{H}_{env} + \mathcal{H}_{int} = \sum_{n=1}^{\mathcal{N}} \left[\frac{\pi_n^2}{2m_n} + \frac{m_n \omega_n^2}{2} \left(\frac{c_n}{m_n \omega_n^2} f(x) - q_n \right)^2 \right]}$$

Classically, one can <u>solve</u> Newton's equations for the oscillator variables.

Assuming that the initial conditions are taken from a p.d.f. $\varrho(t_0)$, that the **environment** is coupled to the sample at t_0,

$$\boxed{\varrho(t_0) = \varrho_{syst}(t_0)\varrho_{env}(t_0)}$$

and that its variables are characterized by a **Gibbs-Boltzmann distribution** at inverse temperature β,

$$\boxed{\varrho_{env}(t_0) \propto e^{-\beta(\mathcal{H}_{env}+\mathcal{H}_{int})}}$$

one finds :

a **Langevin equation with multiplicative/additive colored-noise** (e.o.m.)

or a **reduced dynamic generating functional** $\mathcal{Z}_{red}$ (functional formalism).

General Langevin equation

Recall

The system, $\{r_a\}$, with $a = 1, \dots, d$, coupled to a **general environment in equilibrium** evolves according to the **non-Markov eq.**

$$\boxed{\underbrace{m\dot{v}_a(t)}_{\text{Inertia}} + \underbrace{\int_{t_0}^{t} dt'\, \Sigma_{\mathrm{B}}^{\mathrm{K}}(t-t') v_a(t')}_{\text{friction}} = \underbrace{-\frac{\delta V(\{\vec{r}\})}{\delta r_a(t)}}_{\text{deterministic force}} + \underbrace{\xi_a(t)}_{\text{noise}}}$$

Coloured noise with correlation $\langle\, \xi_a(t)\xi_b(t')\,\rangle = k_B T\, \delta_{ab}\, \Sigma_{\mathrm{B}}^{\mathrm{K}}(t-t')$ and zero mean.

The **friction kernel** is $\Sigma_{\mathrm{B}}^{\mathrm{K}}(t-t')$ [generalizes $2\eta_o\delta(t-t')$].

One can derive Langevin eqs. with multiplicative coloured noise as well.

Correlated initial conditions can be treated in the functional formalism.

Functional formalism

Martin Siggia Rose formalism

Janssen 76, de Dominicis 78

Obtain the generating functional $\boxed{\mathcal{Z}_{red}[\zeta] = \int \mathcal{D}\text{variables}\ e^{-S[\zeta]}}$

with the action given by $\boxed{S = S_{\rm det} + S_{\rm init} + S_{\rm diss} + S_{\rm sour}[\zeta]}$

where $S_{\rm det}$ characterises the deterministic evolution, $S_{\rm init}$ the initial distribution, $S_{\rm diss}$ the dissipative and fluctuating effects due to the bath, and $S_{\rm sour}$ the terms containing the sources ζ.

Correlations between the particle and the bath at the initial time $t_0 = 0$ are taken into account *via* $\varrho(t_0)$ and then $S_{\rm init}$.

Once written in this way, the usual field-theoretical tools can be used. In particular, the minimal action path contains all information on the dynamics of quadratic theories.

Plan

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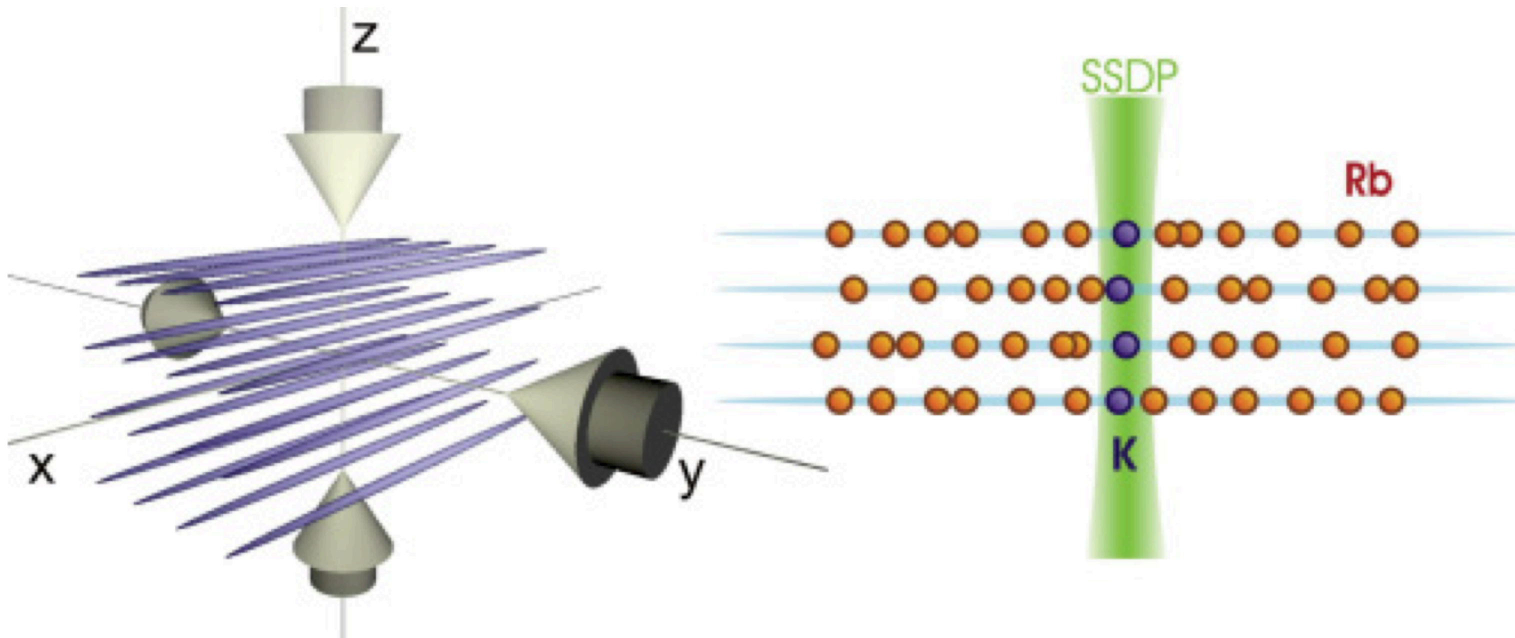
A quantum impurity

in a one dimensional harmonic trap



K atom : the impurity (1.4 on average per tube)

Rb atoms : the bath (180 on average per tube)

all confined in one dimensional tubes

$T \simeq 350$ nK

$\hbar\beta\sqrt{\kappa_0/m} \simeq 0.1$

Catani ***et al.*** **12 (Firenze)**

A quantum impurity

in a one dimensional harmonic trap

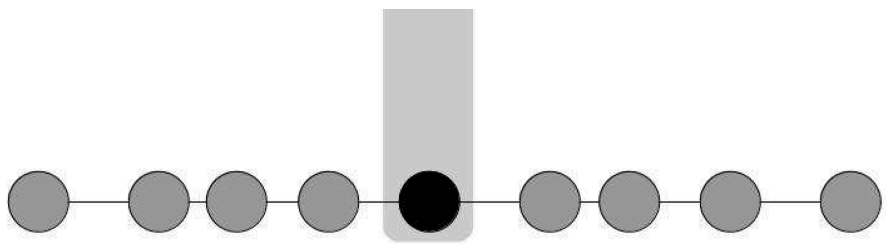
One atom trapped by a laser beam

$$\hat{\mathcal{H}}^0_{syst} = \frac{1}{2m}\,\hat{p}^2 + \frac{1}{2}\kappa_0\,\hat{x}^2$$

in contact with a bath made by a different species $\hat{\mathcal{H}}_{env}$.

Hamiltonian of the coupled system includes an interaction term

$$\hat{\mathcal{H}}_0 = \hat{\mathcal{H}}^0_{syst} + \hat{\mathcal{H}}_{env} + \hat{\mathcal{H}}_{int}$$



Catani *et al* **12**

All atoms are within a wider (κ small) one-dimensional harmonic trap (not shown).

Experimental protocol

A quench of the system

Initial equilibrium of the coupled system : $\boxed{\hat{\varrho}(t_0) \propto e^{-\beta \hat{\mathcal{H}}_0}}$

with $$\hat{\mathcal{H}}_0 = \hat{\mathcal{H}}^0_{syst} + \hat{\mathcal{H}}_{env} + \hat{\mathcal{H}}_{int}$$

and $$\hat{\mathcal{H}}^0_{syst} = \frac{1}{2m}\,\hat{p}^2 + \frac{1}{2}\kappa_0\,\hat{x}^2$$

At time $t_0 = 0$ the impurity is released, the laser blade is switched-off and the atom only feels the *wide* confining harmonic potential $\boxed{\kappa_0 \to \kappa}$ subsequently, as well as the bath made by the other species.

Question : what are the subsequent dynamics of the particle ?

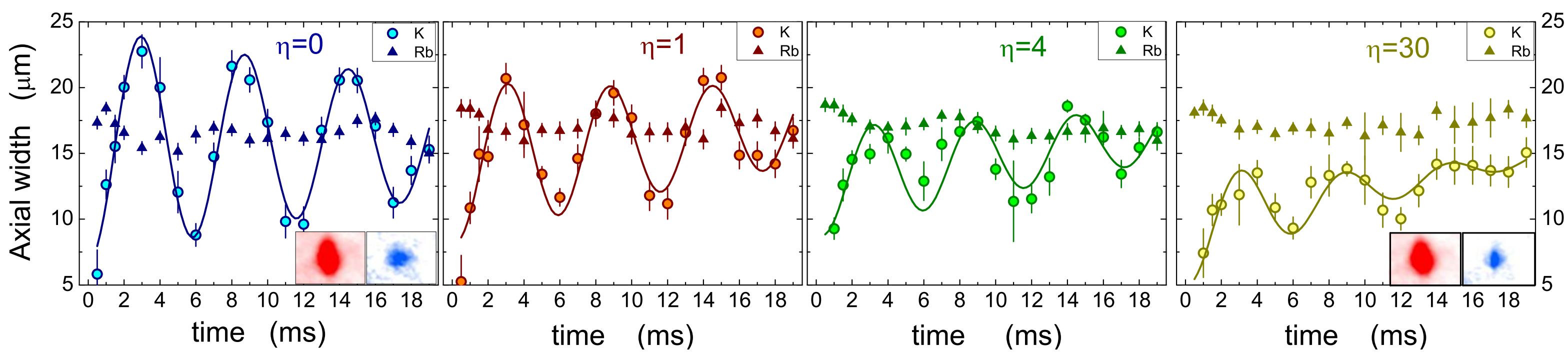
Catani *et al.* **12**

Equal-times correlation

Experiment : breathing mode

$$\boxed{\sigma^2(t) = C_x(t,t) \equiv \langle \hat{x}^2(t) \rangle}$$

Under damped oscillations



For four values of the coupling to the bath, $\eta \propto w/\omega_L$

(interaction strength impurity-bath / interaction strength bath-bath).

Catani ***et al.*** **12**

Reduced system

Model the environment and the interaction

E.g., an ensemble of quantum harmonic oscillators and $\sum_n c_n \hat{q}_n f(\hat{x})$

$$\boxed{\hat{\mathcal{H}}_{env} + \hat{\mathcal{H}}_{int} = \sum_{n=1}^{\mathcal{N}} \left[\frac{\hat{\pi}_n^2}{2m_n} + \frac{m_n \omega_n^2}{2} \left(\frac{c_n}{m_n \omega_n^2} f(\hat{x}) - \hat{q}_n \right)^2 \right]}$$

Quantum mechanically, one can <u>solve</u> Heisenberg's equations for the oscillator operators.

an **operator Langevin equation with a force that depends on the oscillator's initial values and is an operator**
c-valued approximations are wrong ; it is not obvious how to handle generic $\hat{\varrho}(t_0)$ with this approach.

A **reduced dynamic generating functional $\mathcal{Z}_{red}$**
is a much more powerful technique.

Functional formalism

Influence functional

Feynman-Vernon 63, Caldeira-Leggett 84

Obtain the generating functional $$\boxed{\mathcal{Z}_{red}[\zeta] = \int \mathcal{D}\text{variables } e^{\frac{i}{\hbar}S[\zeta]}}$$

with the action given by $$\boxed{S = S_{\rm det} + S_{\rm init} + S_{\rm diss} + S_{\rm sour}[\zeta]}$$

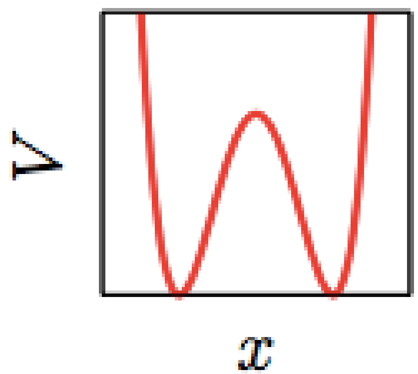
where $S_{\rm det}$ characterises the deterministic evolution, $S_{\rm init}$ the initial density matrix, $S_{\rm diss}$ the dissipative and fluctuating effects due to the bath, and $S_{\rm sour}$ the terms containing the sources ζ.

Correlations between the particle and the bath at the initial time $t_0 = 0$ are taken into account *via* $\hat{\varrho}(t_0)$ and then $S_{\rm init}$.

Once written in this way, the usual field-theoretical tools can be used. In particular, the minimal action path contains all information on the dynamics of quadratic theories.

Quantum dynamics

Some non-trivial effects under quantum Ohmic dissipation ($\alpha = 1$)



$P_{tunn} \to 0$

Suppression of tunnelling or

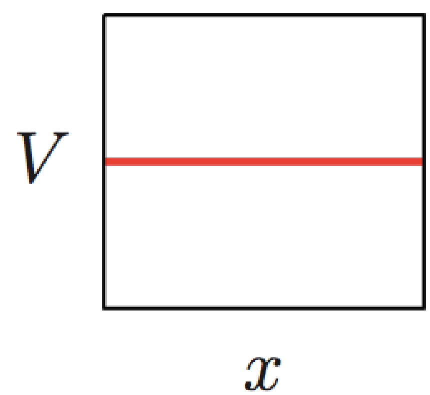
Localisation in a double well potential

at $k_B T = 0$ for $\tilde{\eta}_o > 1$

Bray & Moore 82, Leggett et al 87

Slowed-down diffusion

$$\langle x^2(t) \rangle \to \begin{cases} \dfrac{2 k_B T}{\eta_o}\, t & \text{Classical } k_B T \neq 0 \\ \dfrac{\hbar}{\pi \eta_o} \ln t & \text{Quantum } k_B T = 0 \end{cases}$$



Schramm-Grabert 87

The model

The bath in the experiment

The environment is made of interacting bosons in one dimension that we model as a Luttinger liquid.

The local density operator is $\hat{\rho}(x) = \rho_0 - \frac{1}{\pi}\frac{d}{dx}\hat{\phi}(x)$.
A canonical conjugate momentum-like operator $\hat{\Pi}(x)$ is identified.

One argues

$$\hat{\mathcal{H}}_{env} = \frac{\hbar}{2\pi}\int dx \left[\frac{u}{K}\left(\frac{d\hat{\phi}(x)}{dx}\right)^2 + \frac{uK\pi^2}{\hbar^2}\hat{\Pi}^2(x)\right]$$

The sound velocity u and LL parameter K are determined by the microscopic parameters in the theory. For, e.g., the Lieb-Liniger model of bosons with contact potential $\hbar\omega_L \sum_{i<j} \delta(\hat{x}_i - \hat{x}_j)$, one finds $u(\gamma)K(\gamma) = h\pi\rho_0/m_b$ and an expression for $K(\gamma)$ with $\gamma = m_b\omega_L/(\hbar\rho_0)$. $\boxed{\gamma_{\text{exp}} \simeq 1 \textbf{ Catani et al. 12}}$

t-DMRG of Bose-Hubbard model confirmation for $\hbar w$ small and $\hbar\omega_L$ large

S. Peotta, M. Polini, D. Rossini, F. Minardi & R. Fazio 13

The model

The interaction in the experiment

- The interaction is $\hat{\mathcal{H}}_{\rm int} = \int dr dr' \, U(|r - r'|) \, \delta(\hat{x} - r') \, \hat{\rho}(r)$ with $\tilde{U}(p) = \hbar w e^{-p/p_c}$, quantized wave-vectors $p \to p_n = \pi\hbar n/L$, and L the 'length' of the tube.
- After a transformation to ladder operators $\hat{b}^\dagger_n, \; \hat{b}_n$ for the bath, the coupling $\hat{\mathcal{H}}_{\rm int}$ becomes $\hat{\mathcal{H}}_{\rm int} \propto \sum_{p_n} i p_n \tilde{U}(p_n) e^{-\frac{i p_n \hat{x}}{\hbar}} \hat{b}_{p_n} + \text{h.c.}$
- One constructs the Schwinger-Keldysh path-integral for this problem.
- Low-energy expansion : $e^{\frac{i p_n x_\pm}{\hbar}}$ to quadratic order, the action becomes the one of a particle coupled to a bath of harmonic oscillators with coupling constants determined by p_n. The spectral density $\mathbf{S}(\nu)/\nu$ is fixed. *A further approximation, $L \to \infty$, is to be lifted later.*

The model

Schwinger-Keldysh generating functional

The effective action has delayed quadratic interactions (both dissipative and noise effects) mediated by

$$\Sigma^{\mathbf{K}}_{\mathbf{B}}(t-t') = 2\int_0^\infty d\nu \, \frac{\mathbf{S}(\nu)}{\nu} \, \cos[\nu(t-t')]$$

with the (Abraham-Lorentz) spectral density ($\hbar = 1$)

$$\mathbf{S}(\nu) \; = \; \frac{\pi}{2L} \sum_{p_n} \frac{K}{2\pi} \, |p_n|^3 \, |\tilde{U}(p_n)|^2 \; \delta(\nu - u|p_n|)$$

$$\to \quad \boxed{\eta_{\rm o} \left(\frac{\nu}{\omega_c}\right)^3 e^{-\nu/\omega_c}} \qquad \text{continuum limit for} \quad L \to \infty$$

$\eta_{\rm o} = K w^2 \omega_c^3/u^4$ with $\omega_c = u p_c$ **Super-Ohmic diss.** $\boxed{\alpha = 3}$

K LL parameter, u LL sound velocity, $\hbar w$ strength of coupling to bath, ω_c high-freq. cut-off

The model

Schwinger-Keldysh generating functional

The action is quadratic in all the impurity variables.

The generating functional of all expectation values and correlation functions can be computed by the stationary phase method (exact in this case) as explained in, e.g.,

Grabert & Ingold's review

with some extra features : rôle of initial condition, quench in harmonic trap, non-Ohmic spectral density, possible interest in many-time correlation functions.

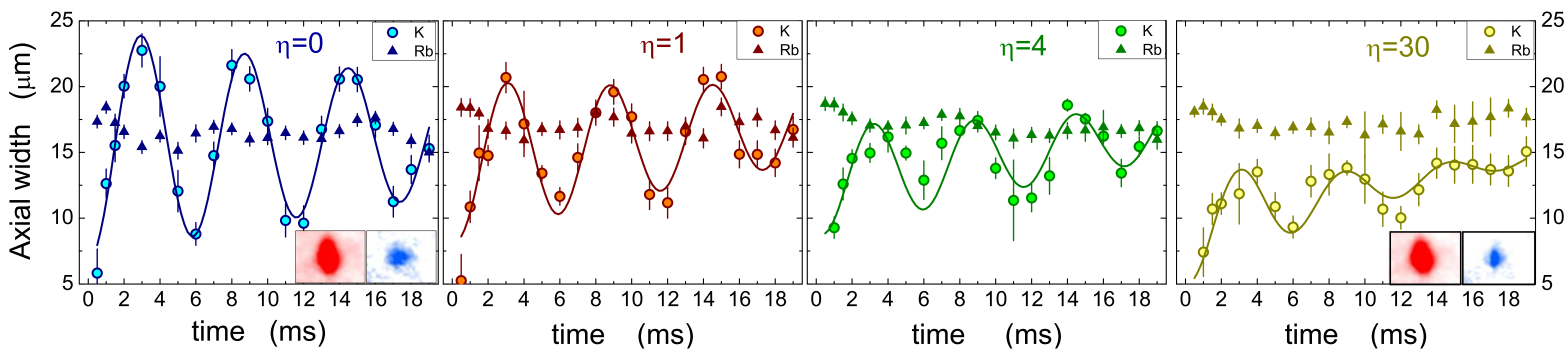
The equal-times correlation $\boxed{C_x(t,t) = \langle \hat{x}^2(t) \rangle}$ is thus calculated, ignoring for the moment the polaron effect (mass renormalisation) and the potential renormalisation due to the fact that the bath itself is confined.

Equal-times correlation

Experiment : breathing mode

$$\boxed{\sigma^2(t) = C_x(t,t) \equiv \langle \hat{x}^2(t) \rangle}$$

Damped oscillations for four values of the coupling to the bath



around the same asymptotic value, that is independent of $\eta = 0, 1, 4$.
(For $\eta > 4$ the system is no longer $1d$)

Catani *et al.* **12**

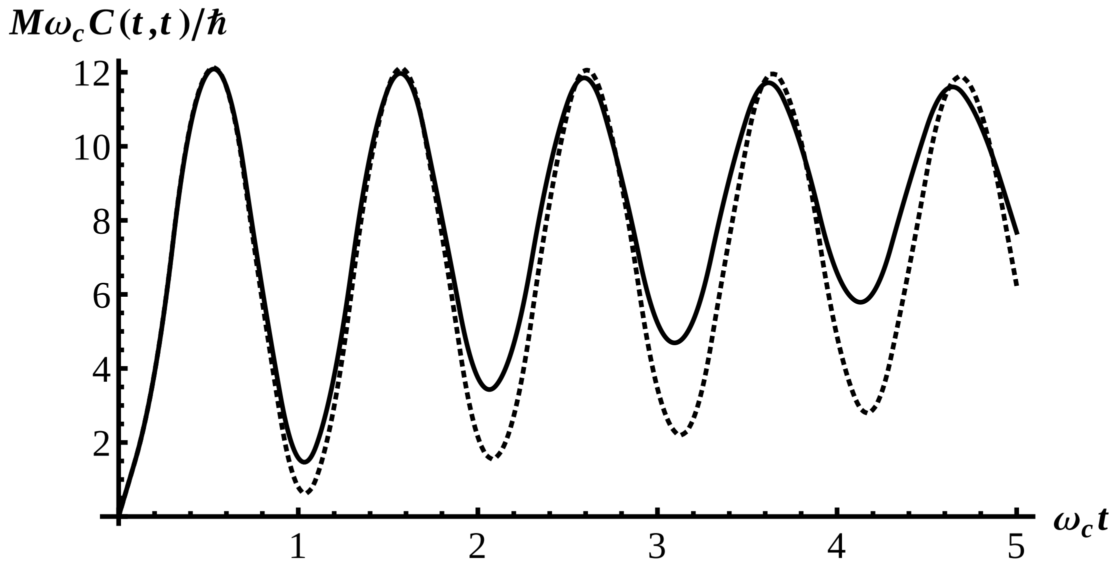
NB independence of the width of breathing mode, $\lim_{t\to\infty} \sigma^2(t)$ on η for relatively small η. Also seen in lattice numerics **Peotta** *et al.* **13**.

Equal-times correlation

Theory

$$\boxed{\sigma^2(t) = C_x(t,t) \equiv \langle \hat{x}^2(t) \rangle}$$

Damped oscillations

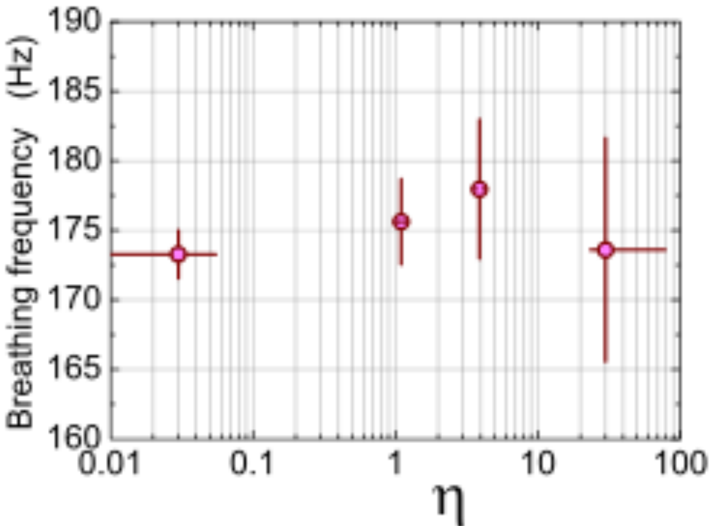


For two values of the coupling to the bath (to be made precise below).

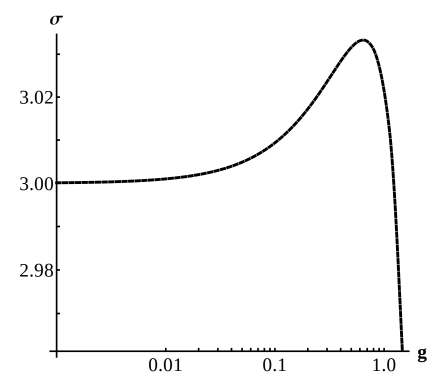
Bonart & LFC 12

Oscillating frequency

Theory vs. experiment



Ω/ω_c for $\boxed{\sqrt{\kappa/m}\,\omega_c^{-1} = 3}$



η_o

The frequency Ω increases with the coupling to the bath η_o for sufficiently narrow (large $\sqrt{\kappa/m}\,\omega_c^{-1}$)) harmonic traps.

The super-Ohmic $\mathrm{S}(\nu)$ is responsible for this 'classical' feature.

The height of the peak depends on $\sqrt{\kappa/m}\,\omega_c^{-1}$ with ω_c the cut-off of the bath spectral function. The order of magnitude of this tiny effect (1%) is similar to the one measured, but the errorbars are larger (5%).

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Beyond

Polaron & potential renormalisation

Another point of view : $\boxed{\Omega \simeq \sqrt{\kappa/m} \text{ for all } \eta \leq 4}$

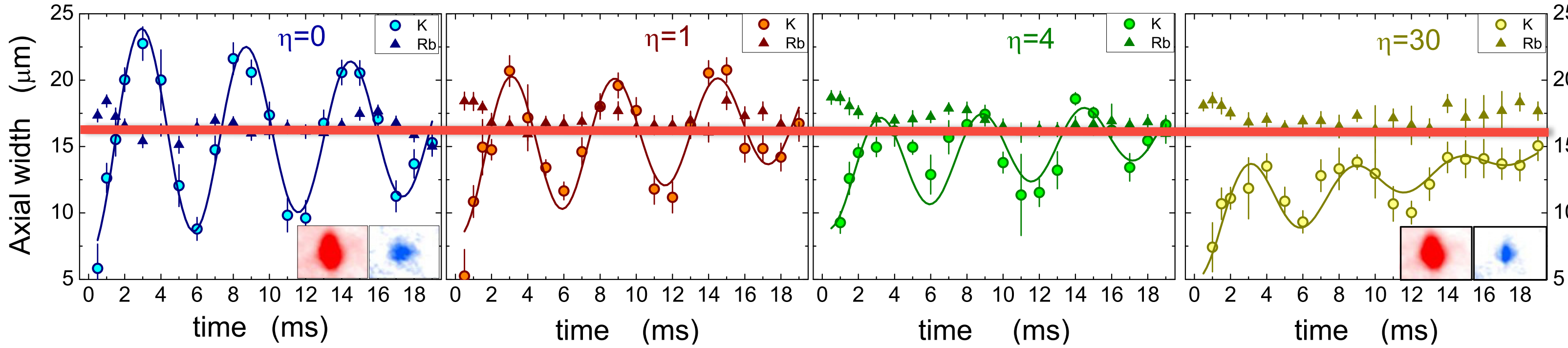
- The coupled Hamiltonian resembles strongly Frölich Hamiltonian and an impurity mass renormalisation $m \to m^*(\eta)$ should be expected.
- However, the constancy of Ω with η suggests that the harmonic potential spring constant should also be renormalised $\kappa \to \kappa^*(\eta)$ to counteract this change and keep the natural frequency $\sqrt{\frac{\kappa^*}{m^*}}$ (roughly) constant.

$$\boxed{\text{However, this is inconsistent with } \langle \hat{x}^2(t) \rangle \to \frac{k_B T}{\kappa^*(\eta)}}$$

since one does not observe an η-dependence of the asymptotic cloud width (for $\eta \leq 4$).

Equal-times correlation

Experiment : breathing mode



around the same asymptotic value, that is independent of $\eta = 0, 1, 4$.
(For $\eta > 4$ the system is no longer $1d$)

Catani *et al.* **12**

Independence of the width of breathing mode, $\lim_{t\to\infty} \sigma^2(t)$ on η for relatively small η. Also seen in lattice numerics **Peotta** *et al.* **13.**

A way out

- Polaron. The bath density profile follows the impurity creating a dressed impurity with renormalised mass, $m^* = (1+\mu(v))m$,

  with $\mu(v)$ estimated from the kinetic energy gained by the impurity after rapid acceleration from v_o to v due to injection of energy that goes partially into a wave excitation, dynamic analysis of e.o.m.

- Potential renormalisation. The impurity needs more energy to create a bath excitation (e.g., bosons have to climb the potential). The potential felt by the impurity gets renormalised $\kappa^* = (1+\tilde{\mu}(v))\kappa$,

  with $\tilde{\mu}(v)$ estimated from the force acting on the impurity & density cloud.

- Experimentally : the dynamics feel these two effects but the asymptotic statics does not : $\lim_{t\to\infty} \sigma^2(t) \to k_B T/\kappa$.

- Repeat Schwinger-Keldysh formalism & Gaussian approximation analysis with m^* and κ^* for the dynamics.

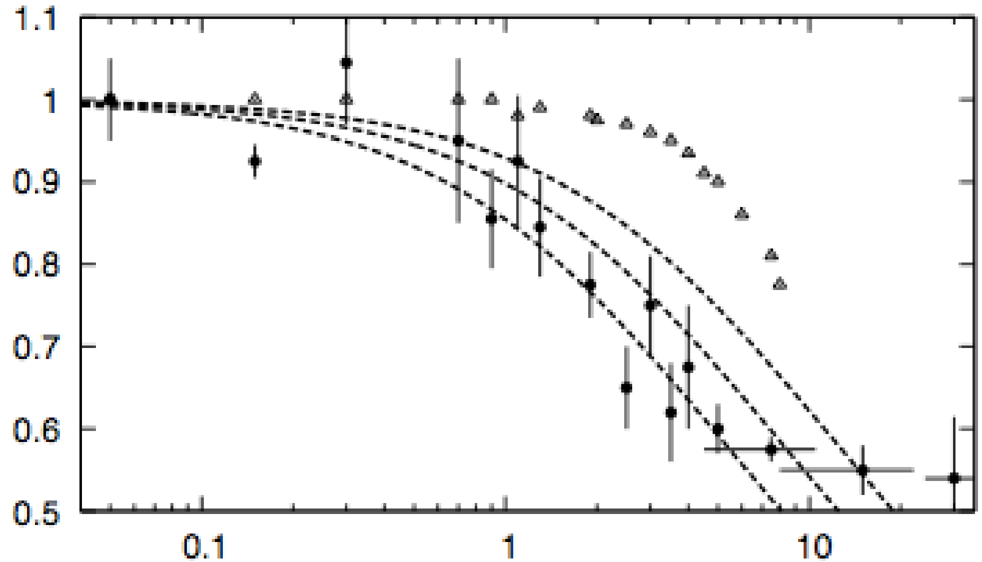
- Asymptotic behaviour cannot be described with the Gaussian approximation ; an interpolation is proposed.

Bonart & LFC EPL 13

Potential renormalisation

Theory vs. experiment

$$\frac{\sigma_p(\eta \neq 0)}{\sigma_p(\eta = 0)} \simeq \sqrt{\kappa/\kappa^*}$$



$$2E_0 \simeq k_B T \simeq \kappa^* \sigma_p^2(\eta) \simeq \kappa \sigma_p^2(\eta = 0)$$

$$\eta \propto w/\omega_L$$

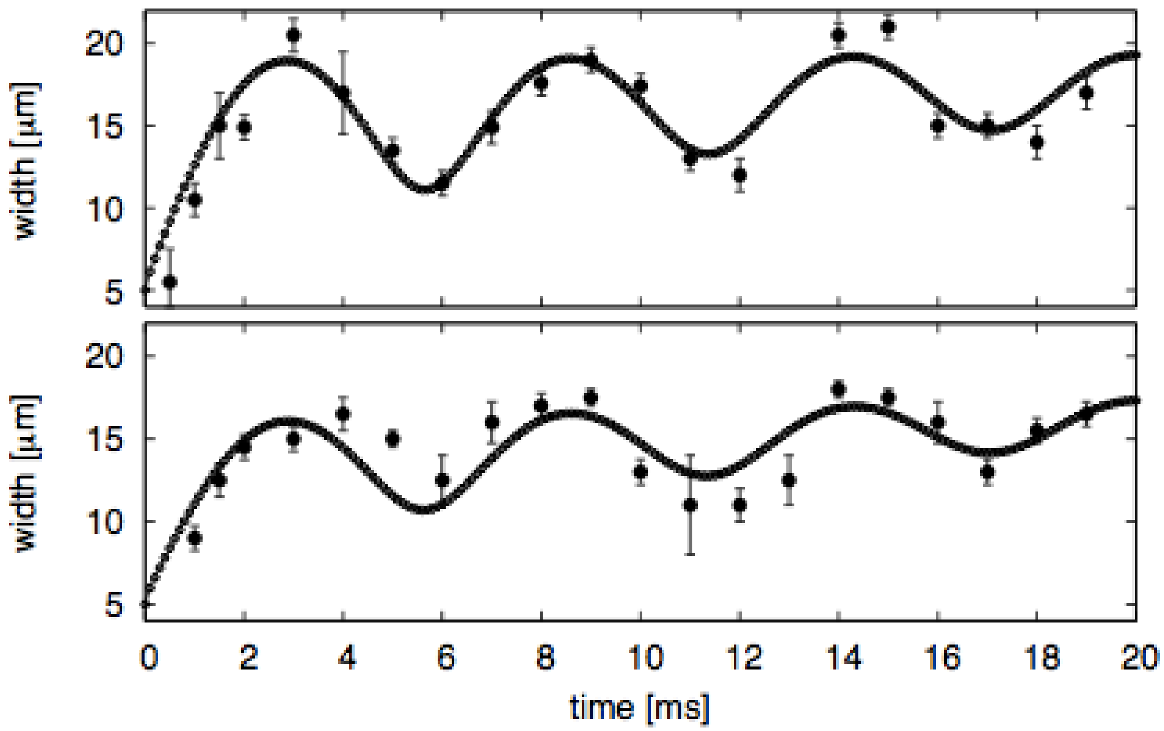
Experimental data points estimated from amplitude of the first oscillation $\sigma_p(\eta \neq 0)/\sigma_p(\eta = 0)$ **Catani et al. 12**

Triangles : equilibrium variational theory for $\sqrt{m/m^*}$ **Catani et al. 12**

Dotted lines : $\sqrt{\kappa/\kappa^*}$ for $\gamma = 0.2,\ 0.35,\ 0.5$ **Bonart & LFC EPL 13**

Breathing mode

Theory vs. experiment



$w/\omega_L = 1$

$w/\omega_L = 4$

Dynamics with m^* and κ^*, interpolation to $\lim_{t\to\infty}\sigma^2(t) \to k_BT/\kappa$:

$$\boxed{\sigma^2(t) = \frac{\hbar^2\kappa_0}{4k_BT}\mathcal{R}(t) - \frac{\kappa^*}{k_BT}\mathcal{C}^2_{eq}(t) + \frac{k_BT}{\kappa^*} + \left(1 - e^{-\Gamma t}\right)\left(\frac{k_BT}{\kappa} - \frac{k_BT}{\kappa^*}\right)}$$

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Summary

- **Classical and quantum dynamics**

  technically very similar once in the path-integral formalism.

- **Classical systems**

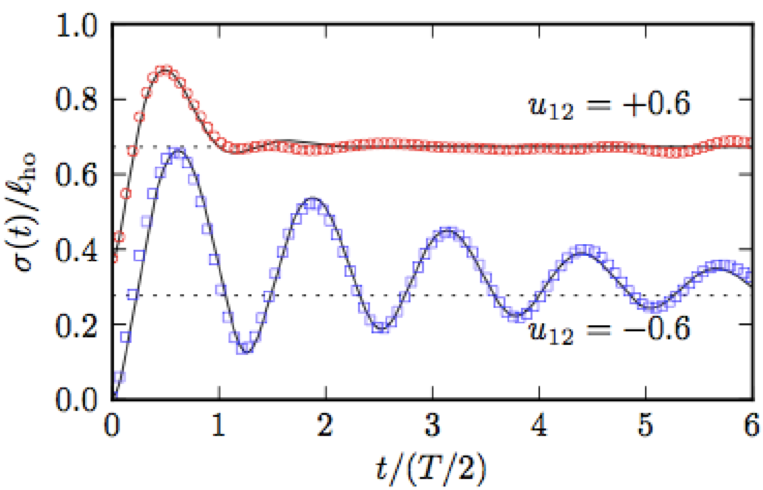
  Single particle : non-Markovian environments are very popular in bio-physics ; they are usually the 'unknown'

- **Quantum systems**

  Quantum Brownian motion and quenches : a rather simple problem with non-trivial consequences of the coupling to the bath.

Breathing mode w/TDMRG

Bose-Hubbard model for the bath & interaction



Flat trap with length $L = 250$

$N_b = 22$ bosons

$n_i = \langle \hat{b}_i^\dagger \hat{b}_i \rangle \simeq$ ct $\lesssim 0.1$ in $2L/3$

Model $\simeq$ Lieb-Liniger

Coupling $\mathcal{H}_{\rm int} = u_{\rm int} \sum_i \hat{n}_i \hat{N}_i$

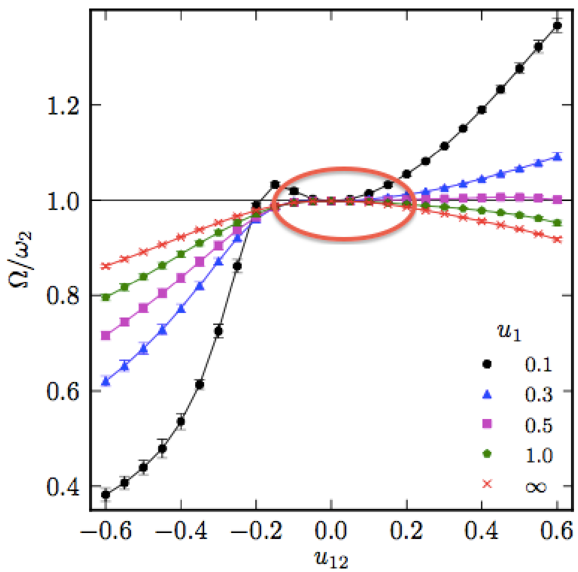
Mass difference mimicked by $J_2/J_1 = 2$.

Study of Ω yields approximate independence of $u_{\rm int}$ for $u_b \gtrsim 0.5$ (Tonks-Girardeau limit).

S. Peotta, D. Rossini, M. Polini, F. Minardi, R. Fazio 13

Breathing mode w/TDMRG

Bose-Hubbard model for the bath & interaction



$u_{\rm int}$

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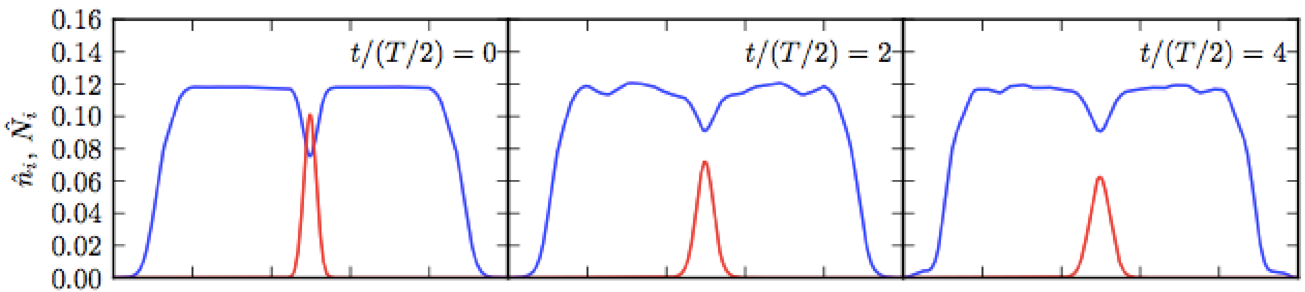
Mean occupation numbers

TDRG data



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A way out : details

- Polaron. Dressed impurity with renormalised mass $m^* = (1+\mu(v))m$,

  with $\mu \propto \omega_c w^2 K/(mu^4) f(v_o, v)$, estimated from the kinetic energy gained by the impurity after rapid acceleration from v_o to v due to injection of energy that goes partially into a wave excitation.

- Potential renormalisation. The potential felt by the impurity gets renormalised $\kappa^* = (1 + \tilde{\mu}(v))\kappa$.

  with $\tilde{\mu}(v) = Kwug(v, u)$ estimated from sum of the force felt by the impurity $-\kappa\hat{q} \to -\kappa$ plus the one felt by the cloud around it $-\kappa \int_{-L/2}^{L/2} dx\, x\, \hat{\rho}(x) \to -\kappa\tilde{\mu}(v)$

- Dynamics with m^* and κ^*, interpolation to $\lim_{t\to\infty} \sigma^2(t) \to k_BT/\kappa$:

$$\boxed{\sigma^2(t) = \frac{\hbar^2 \kappa_0}{4k_BT}\mathcal{R}(t) - \frac{\kappa^*}{k_BT}\mathcal{C}_{eq}^2(t) + \frac{k_BT}{\kappa^*} + (1 - e^{-\Gamma t})\left(\frac{k_BT}{\kappa} - \frac{k_BT}{\kappa^*}\right)}$$

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Breathing mode

Amplitude of first oscillation σ_p

- The impurity is initially in equilibrium with the bath at a ‘high’ temperature T (the thermal energy is order 10 times the potential one).

  Its mean energy per d.o.f. is $2E_0 \simeq k_B T$.

- At the first peak the amplitude can be estimated as $\kappa^* \sigma_p^2(\eta) \approx k_B T$ (dissipation only affects κ^*). Therefore,

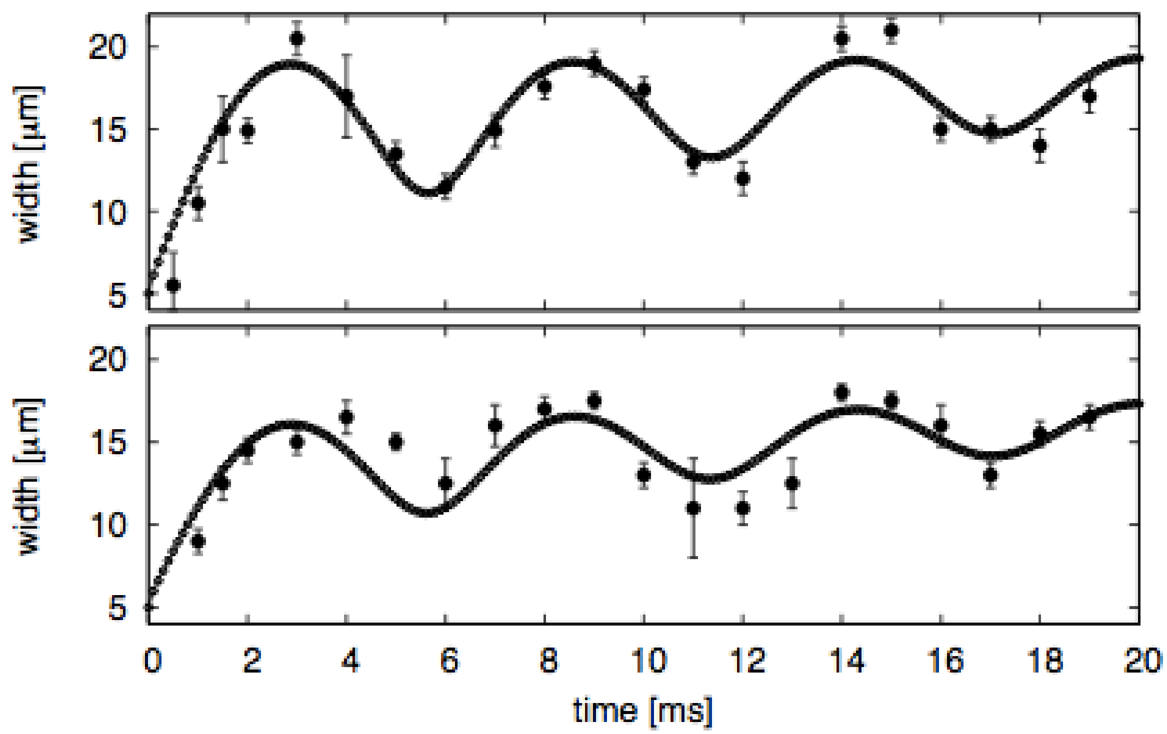
$$\boxed{\frac{\sigma_p(\eta \neq 0)}{\sigma_p(\eta = 0)} = \sqrt{\frac{\kappa}{\kappa^*}}}$$

- As the breathing frequency is approximately η-independent $\Omega \simeq \Omega^*$:

$$\sqrt{\kappa/\kappa^*} \simeq \sqrt{m/m^*}$$

Breathing mode

Theory



$$\sigma^2(t) = C_x(t,t)(m^*,\kappa^*) + \left(1 - e^{-\Gamma t}\right)\left(\frac{k_B T}{\kappa} - \frac{k_B T}{\kappa^*}\right)$$

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