Quantum (spin) glasses

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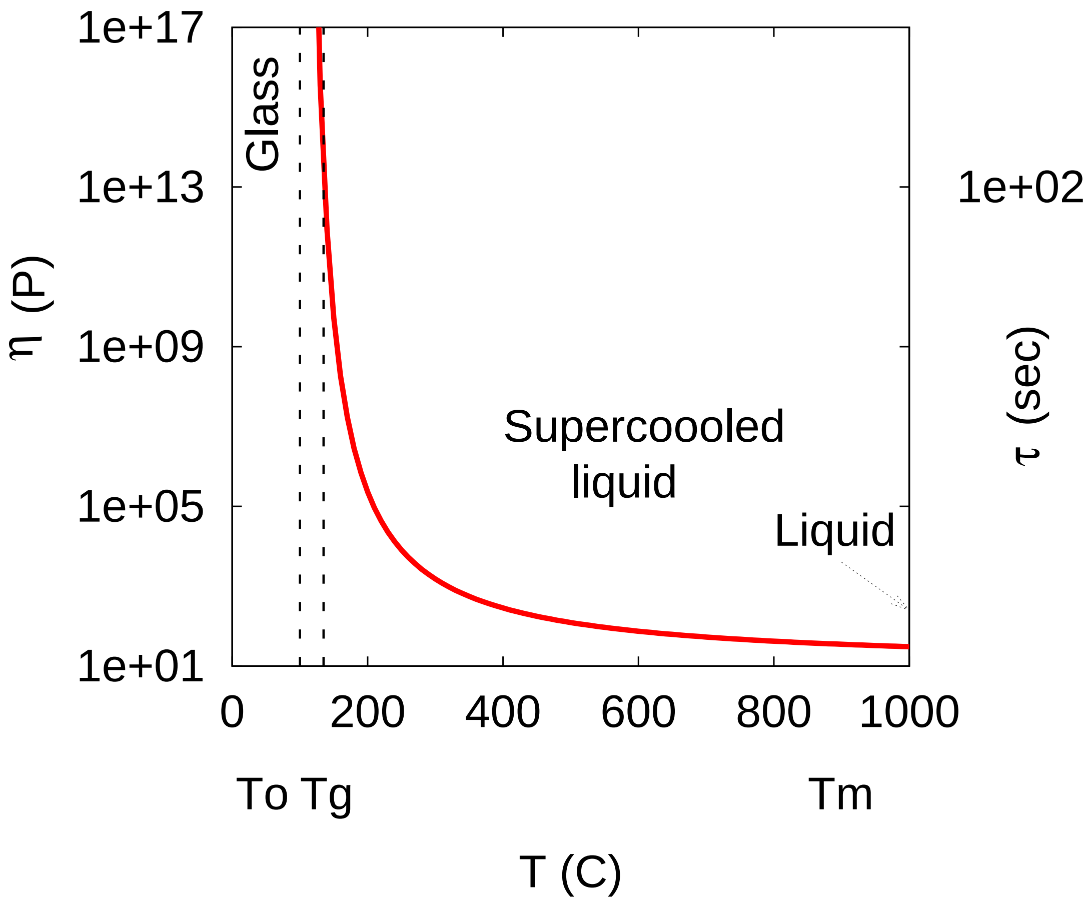
Classical glassy arrest

Observable (e.g. viscosity) *vs* control parameter (e.g. temperature)



T_g at $\tau = 10^2 s$; T_0 hypothetic static glass transition.

The glassy problem

- What is the mechanism for the slowing down observed in all types of samples whenever cooling is sufficiently fast ?

- Describe the dynamic crossover at T_g from

  'equilibrium dynamics' in the (metastable) super-cooled liquid

  to the 'non-equilibrium relaxation' in the glassy regime.

- Is there a static phase transition ? (Beyond experimental skills...)

Achievements

A highly non-trivial mean-field theory

(fully-connected disordered spin models $\equiv$

self-consistent approximations)

mimics T_g as a dynamic transition, T_o as a static transition

T. R. Kirkpatrick, D. Thirumalai & P. Wolynes (80s)

describes correctly the dynamics above W. Götze *et al* (80s)

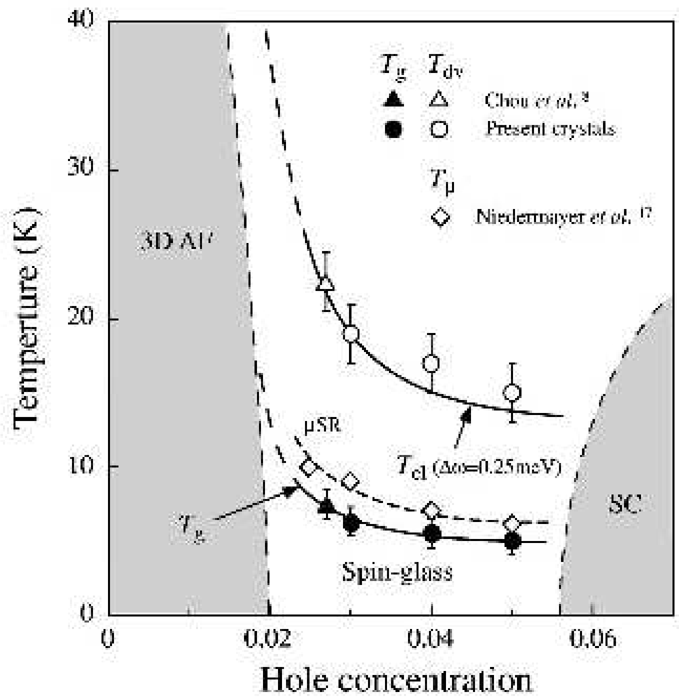
and below the cross-over region at T_g LFC & J. Kurchan (93)

Quantum fluctuations

Why should one include quantum fluctuations ?

SG phase

in High Tc SCs, $La_{1-x}Sr_2Cu_2O_4$



A. Aharony *et al*, PRL 60, 1330 (88). S. Wakimoto *et al*, PRB 62, 3547 (00).

Quantum fluctuations

What should one change from the mean-field approach to the glassy problem ?

Rôle of the environment

- Classically : ‘trivial’

  statics in the canonical ensemble,

  dynamics with an appropriate stochastic equation

  (Langevin, Glauber,...).

- Quantum mechanically : highly non-trivial.

  decoherence,

  localization transition in a dissipative two-level system

  A. J. Bray & M. A. Moore (84), S. Chakravarty (84). A. J. Leggett *et al* (80s).

Statics and dynamics of dissipative quantum glassy systems

- Real-time dynamics :
  glassy aspects, aging, effective temperatures ?

- Locus and order of the phase transition ?

- Decoherence, localization and interactions.

- Griffi ths phase ?
  Activated or conventional scaling ?

Quantum Ising disordered models

$$H_S = -\sum_{ij} J_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z + \sum_i \Gamma_i \hat{\sigma}_i^x + \sum_i h_i \hat{\sigma}_i^z \ .$$

$\hat{\sigma}_i$ Pauli matrices.

J_{ij} & Γ_i quenched random.

ij nearest neighbours on the lattice – finite d

or fully connected – mean-field

p-spin interactions

$$H_S = - \sum_{i_1 i_2 \ldots i_p} J_{i_1 i_2 \ldots i_p} \, \hat{\sigma}^z_{i_1} \hat{\sigma}^z_{i_2} \ldots \hat{\sigma}^z_{i_p} + \sum_i \Gamma_i \hat{\sigma}^x_i + \sum_i h_i \hat{\sigma}^z_i \ ,$$

$\hat{\sigma}_i$ Pauli matrices,

$J_{i_1 i_2 \ldots i_p}$ quenched random $\overline{J^2_{i_1 i_2 \ldots i_p}} = \dfrac{p! \tilde{J}^2}{2N^{p-1}}$

$i_1 i_2 \ldots i_p$ all possible p-uplets

‘Spherical’ model

A particle in a random potential

$$\hat{H}_S = H_J + \sum_i \frac{\hat{\Pi}_i^2}{2M}$$

Potential energy | Kinetic energy

$$[\hat{\Pi}_i, \hat{S}_j] = -i\hbar\delta_{ij}$$ Canonical commutation rules

$$\sum_i \langle \hat{S}_i^2 \rangle = N$$ Spherical constraint

$$\Gamma \equiv \hbar^2/(JM)$$ Strength of quantum fluctuations

Coupling to the environment

$$H_E = \sum_i \hat{\sigma}_i^z \sum_{a=1}^{\tilde{N}} c_{ia} \hat{x}_a + \sum_{a=1}^{\tilde{N}} \left(\frac{\hat{p}_a^2}{2m_a} + \frac{m_a \omega_a^2}{2} \hat{x}_a^2 \right)$$

+ counter-term.

$\tilde{N}$ indep. quantum harmonic oscillators per spin

Distribution of $c_{ia}, m_a, \omega_a \Rightarrow$ type of bath.

Spectral density $I(\omega) \propto \alpha \, \omega^s \quad \omega \ll \omega_c$

Dimensionless coupling constant α

Cut-off ω_c

$s = 1$ Ohmic $\quad s < (>) 1$ sub(super)-Ohmic

Reasons for the choice of model

Mean-fi eld limit : amenable to analytic calculations.

$p \geq 3$ spin interactions : connection to structural glasses.

Spherical case : real-time dynamics is easy to treat.

$p = 2$ spherical model : connection to domain growth

as described by the $O(N)$ model with $N \to \infty$ and $d = 3$

and the usual model for rotors. Slightly 'trivial'.

Oscillator bath : amenable to analytic calculations.

$1d$ model : Simulations of classical $2d$ system are feasible

Techniques to study the statics

Study of the free-energy density

Path-integral in imaginary time τ.

Gaussian integration over oscillator variables $\Rightarrow$

long-range ferromagnetic interactions in the τ direction.

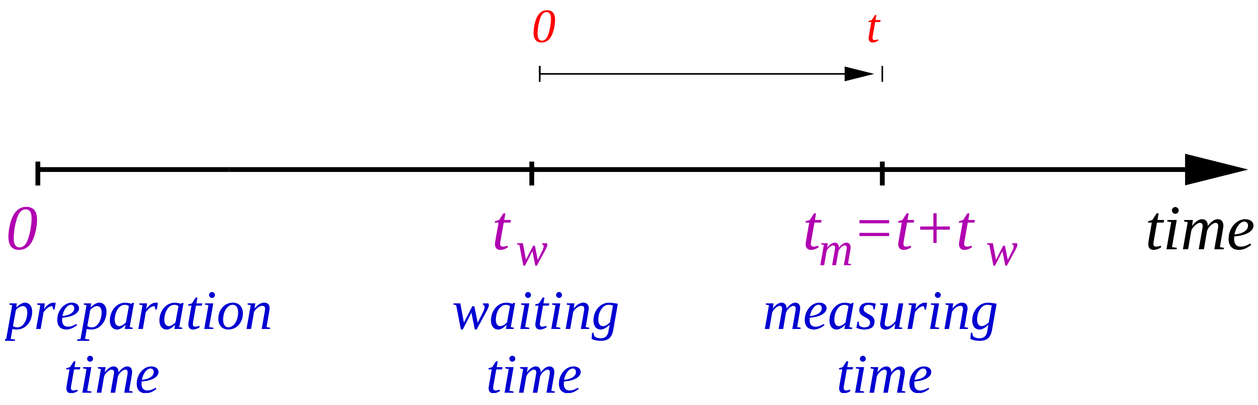
Replica trick to average over disorder

& replica symmetry breaking (mean-field).

Montecarlo simulations of $d+1$ classical model (finite d).

Real-time dynamics

Two-time dependence



Correlation

$$C(t+t_w,t_w)=\langle[\hat{O}(t+t_w),\hat{O}(t_w)]_+\rangle$$

Linear response

$$R(t+t_w,t_w)=\left.\frac{\delta\langle\hat{O}(t+t_w)\rangle}{\delta h(t_w)}\right|_{h=0}=\langle[\hat{O}(t+t_w),\hat{O}(t_w)]_-\rangle$$

Real-time dynamics

Study of the dynamic generating functional

Continuous models

Schwinger-Keldysh closed-time path-integral.

Gaussian integration over oscillator variables $\Rightarrow$

Two-time long-range interactions.

Typical initial conditions ($\hat{\rho}_{\mathsf{e}} \otimes \hat{\rho}_s$ and $\hat{\rho}_s$ ‘random’) :

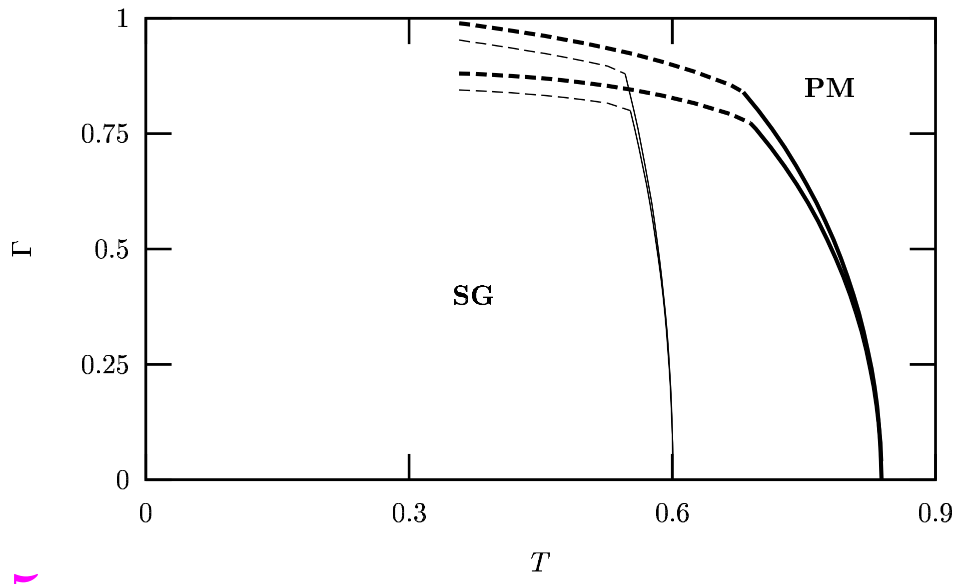
no need of replica trick to average over disorder.

Study of Dyson equations for correlations and responses.

Static & dynamic phase diagram

Dissipative Ising p-spin model with $p \geq 3$

dashed = 1st order, solid = 2nd order   thin = static, bold = dynamic



$\alpha = 0,\ 0.5$

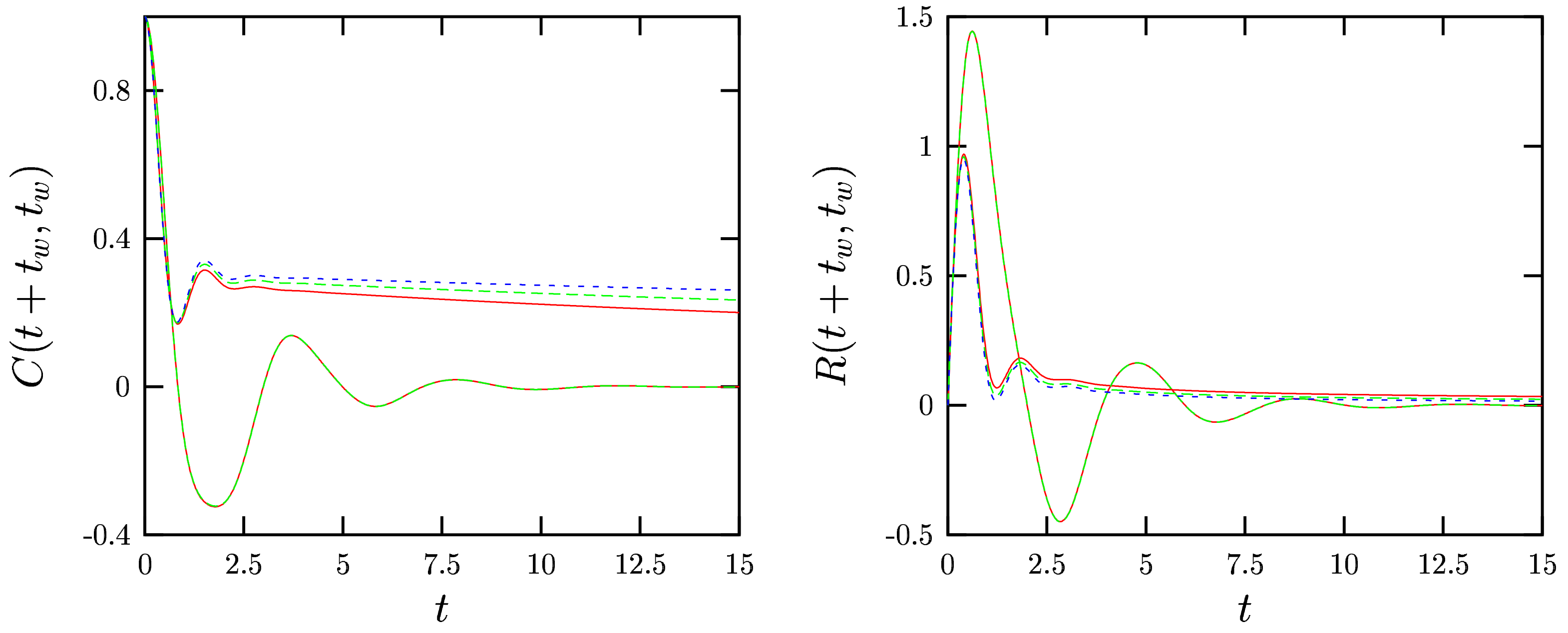
LFC, D. R. Grempel & C. A. da Silva Santos, PRL 85, 2589 (00)

LFC, D. R. Grempel, G. Lozano & H. Lozza, PRB 70, 024422 (04)

Real-time dynamics

Dissipative spherical p-spin model

Symmetric correlation Linear response

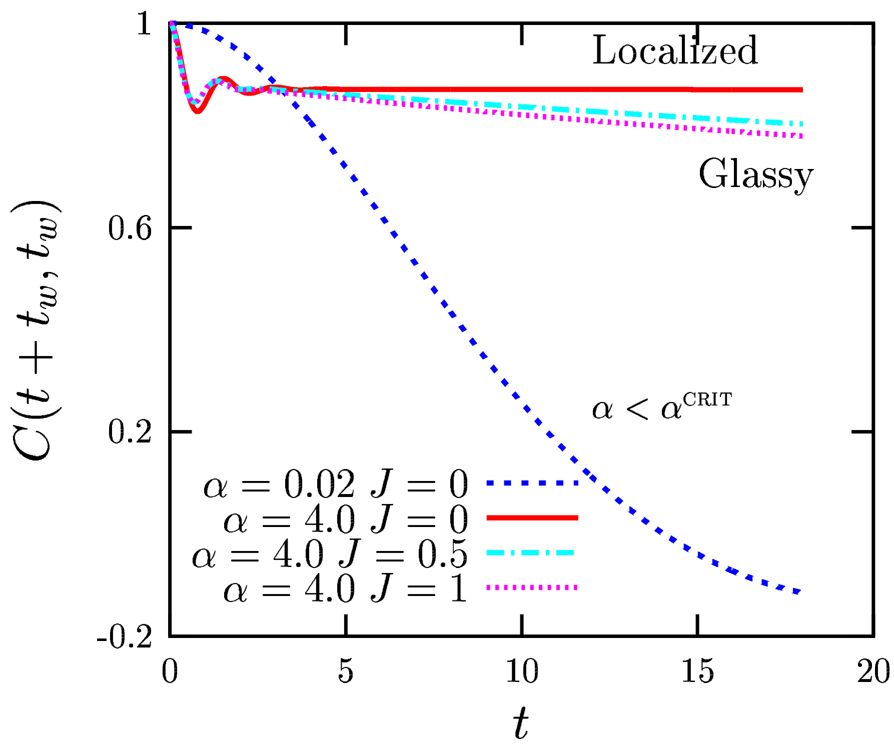


Comparison between $\alpha = 0.2$ (PM) and $\alpha = 1$ (SG)

LFC, D. R. Grempel, G. Lozano, H. Lozza & C. da Silva Santos, PRB 66, 014444 (02).

Interactions against lozalization

Dissipative spherical p-spin model



LFC, D. R. Grempel, G. Lozano, H. Lozza, C. A. da Silva Santos, PRB 66, 014444 (02).

FDT & effective temperatures

Classical glassy systems

In equilibrium $R(t) = -\frac{1}{T}\frac{dC(t)}{dt}$ with $t \geq 0$.

Glasses : breakdown of stationarity & FDT.

$$\chi(t+t_w, t_w) \equiv \int_{t_w}^{t+t_w} dt' \, R(t+t_w, t')$$

$$\to f\left(C(t+t_w, t_w)\right) \approx -\frac{1}{T_{\tt eff}} C(t+t_w, t_w)$$

in the long t_w and $t \gg t_w$ limit.

LFC, J. Kurchan & L. Peliti, PRE 55, 3898 (97)

FDT & effective temperatures

Dissipative quantum glassy models

The equilibrium FDT

$$R(t+t_w,t_w) \;=\; \int_{-\infty}^{\infty} \frac{id\omega}{\pi\hbar}\, e^{-i\omega t}\, \tanh\left(\frac{\beta\hbar\omega}{2}\right) C(\omega,t+t_w)$$

becomes

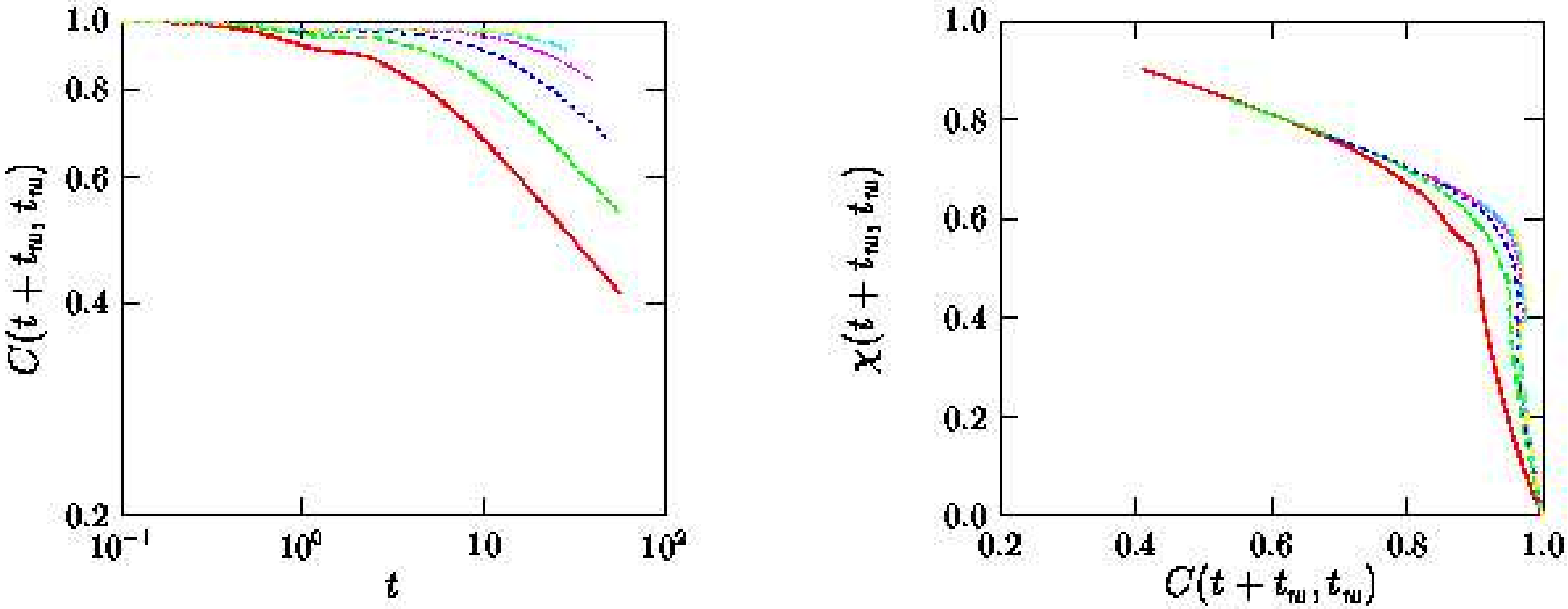
$$\chi(t+t_w,t_w) \;\approx\; -\frac{1}{T_{\tt eff}} C(t+t_w,t_w) \qquad\qquad t \gg t_w$$

if the integral is dominated by $\omega t \ll 1$ and $T \to T_{\tt eff}$ such that $\beta_{\tt eff}\hbar\omega \to 0$.

LFC & G. Lozano, PRL 80, 4979 (98)

Aging and effective temperatures

Dissipative p spin spherical model

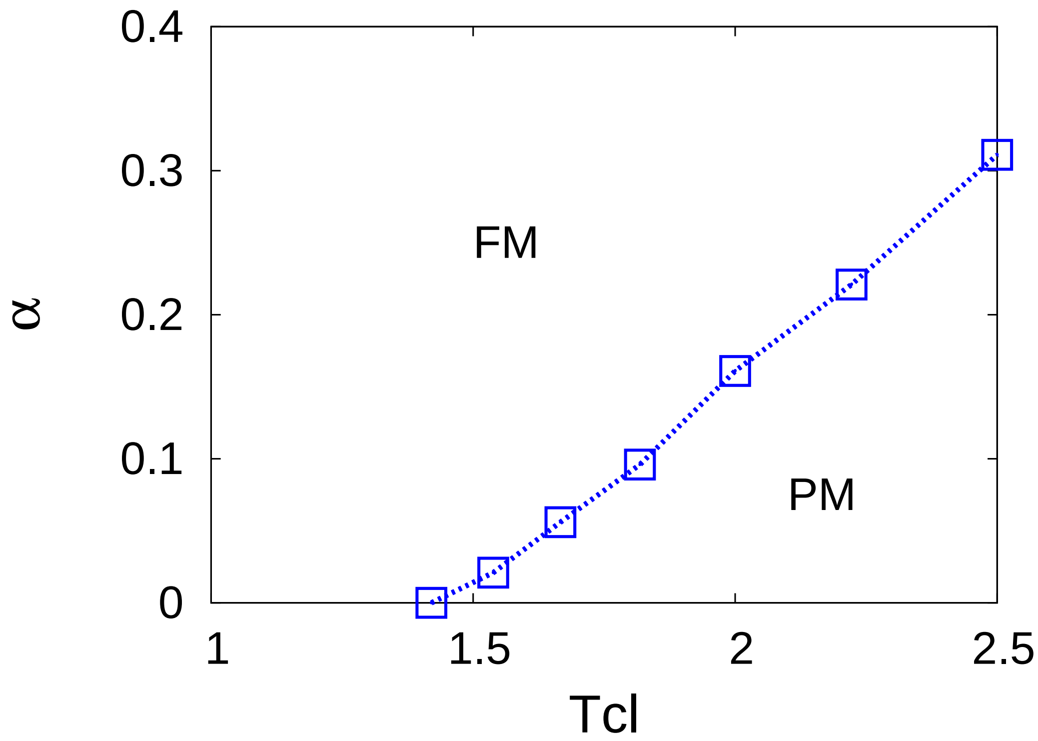


LFC & G. Lozano, PRL 80, 4979 (98) ; PRB 59, 915 (99)

Dissipative random Ising chain

Montecarlo simulations of the $2d$ classical counterpart

Static phase diagrams

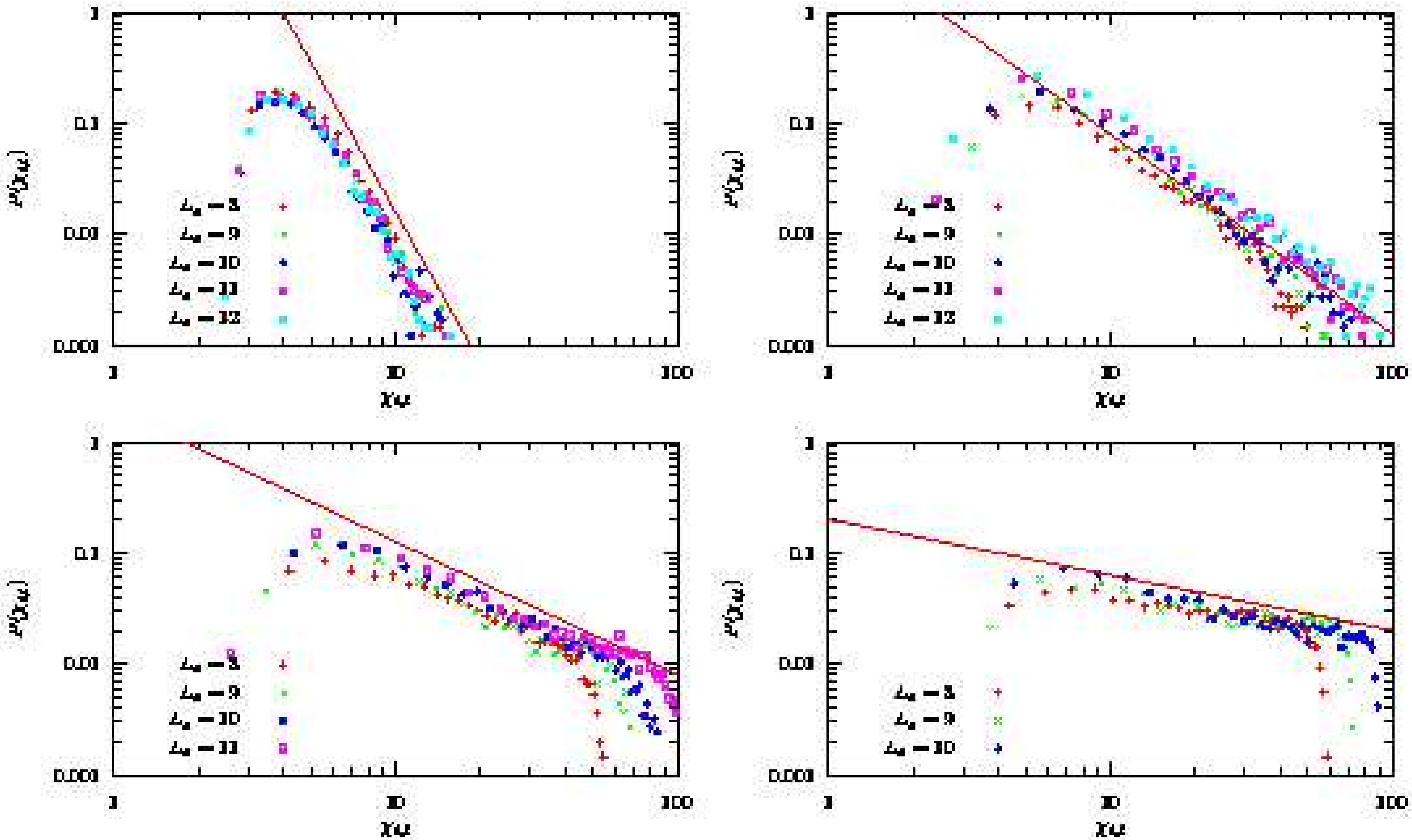


$T_{cl} \propto \Gamma$

LFC, G. S. Lozano & H. Lozza (04).

Dissipative random Ising chain

pdfs of local susceptibilities



$T_{cl} \propto \Gamma$

LFC, G. S. Lozano & H. Lozza (04).

Summary

- Dissipation favors the glassy phase.

- Real-time dynamics $\Rightarrow$ separation of time-scales, aging and effective temperature ($d = \infty$).

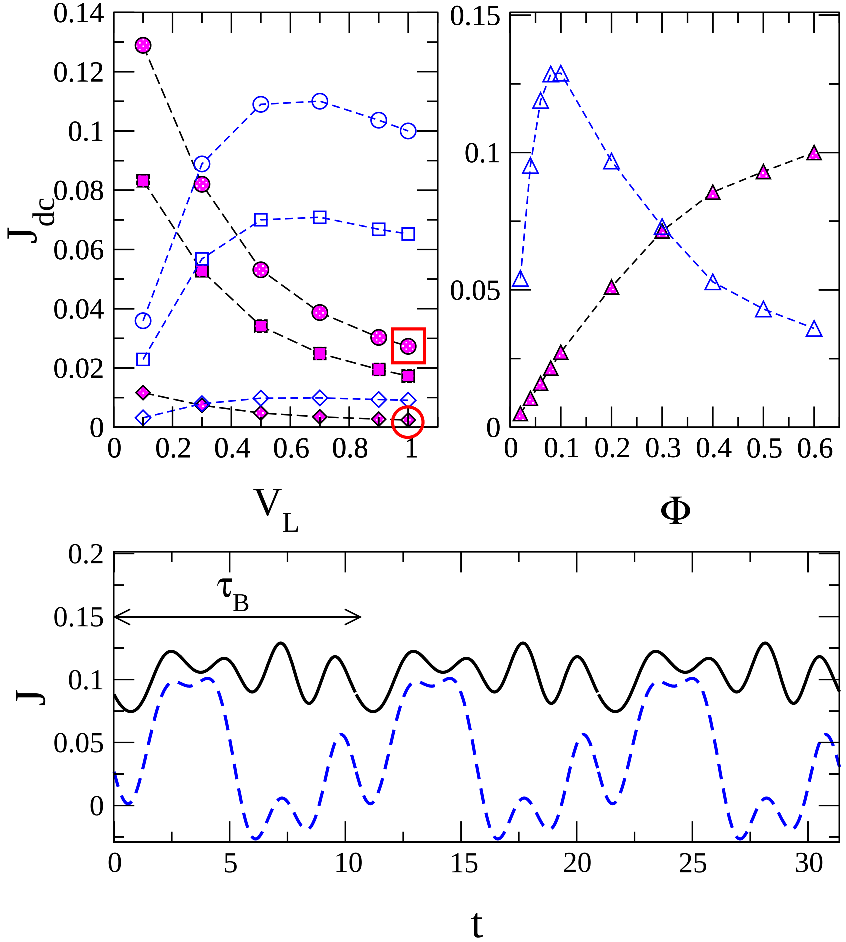
- First-order quantum phase transitions ($d = \infty$).

- Griffiths phenomena still present in $d = 1$ (at least for moderate couplings to the environment).

- $T_{\tt eff}$ relevant to other quantum non-equilibrium systems.

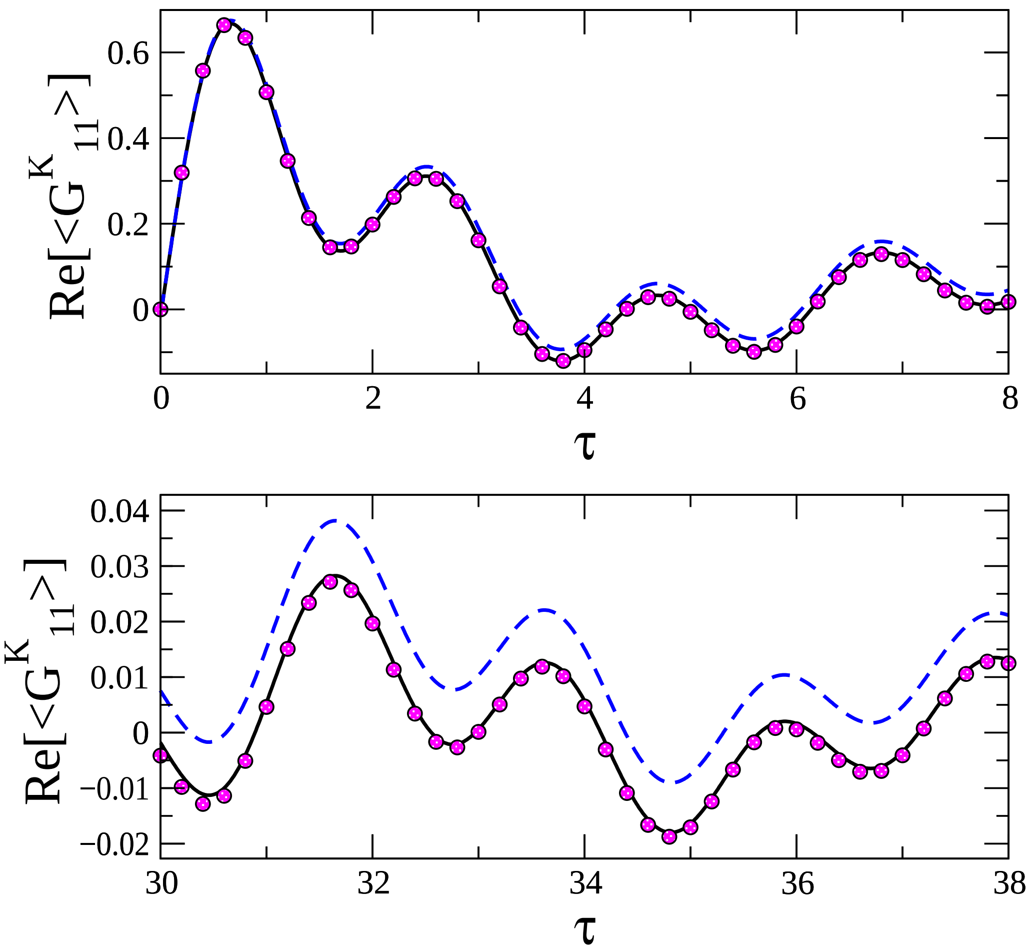
e.g., mesoscopic dissipative conducting ring driven by a time-dependent magnetic field. L. Arrachea & LFC, cond-mat/0407427

Current in driven conducting ring



L. Arrachea & LFC, cond-mat/0407427

$T_{\texttt{eff}}$ in conducting ring



L. Arrachea & LFC, cond-mat/0407427