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ANTINUCLEONS
AND
HIGH ENERGY NUCLEON-NUCLEON SCATTERING

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HIGH ENERGY PHYSICS

I plan to discuss some of the methods, especially the new ones, for isolating one component in a beam containing many kinds of particles.
Some of these methods have been used extensively in studying antiprotons in the presence of many other particles. Results of experiments on antiprotons and antineutrons will be discussed. As time allows we may go on to nucleon-nucleon scattering.

In the use of high-energy accelerators it is frequently necessary to select, from among all the particles emerging from a target, a particular component. For instance, in studying antiprotons and K particles it is frequently necessary to use a selector that is sensitive only to particles of some chosen mass.

We will review here some of the methods used to isolate a particular component, with some emphasis on the newer methods. Special attention will be given to counter arrangements which select particles of a particular mass, arrangements that are being used a great deal at Berkeley and Brookhaven.

On many occasions we will need particle selectors that are capable of selecting from a beam a rare component.
For example, if we may use the statistical theory of Fermi as a guide, we expect that even when using very high energy accelerators antiprotons will be rather rare reaction products. See for instance the following results of Fermi.

$\overline{W}$ (Rel.en in c.m.system of 2 N)	W (Rel.en.in lab.syst.of one N)	Fraction of collision resulting in antinucleons
4 +	7 + 4	0.I
4.5	9.I	0.00I5 (X)
5.0	II.5	0.0036
5.5	I4.3	0.0047
6.0	I6.9	0.0054

(X) $0.I(0.5)^{7/2}=0.0088$ = overestimate by factor 6

This table is from E.Fermi, Prog.Theor.Phys. 5, 570 (1950) Note that except for the energy region very near threshold the rise in antiproton production with energy is very slow , and that antiprotons are a rare product even at high energies, according to this theory.

In describing mass selectors let us restrict our attention to singly charged particles. They may be either positive or negative but we assume the sign of this charge is known, usually from their direction of bending in a magnetic field.

We can determine the mass of a particle by measuring any two of the quantities in the following list.

Possible measurable quantities.

1. Momentum - (by measuring magnetic rigidity , where B = magnetic flux density and = radius of orbit curvature. Current practice allows arranging magnets to give 1% spread in momentum and an uncertainty of 2% in the value of mean momentum. Better accuracy could be obtained if it were quite important to do so.
2. Velocity -
   (a)By time-of-flight measurement over a known distance. Current practice allows flight-time measurements to 5×10^{-9} sec by purely electronic means, perhaps 3×10^{-9} sec if unusually good technique is used in electronics, and to 1×10^{-9} sec by photographing the pulses on an oscilloscope screen.
   (b)By velocity-selecting Cherenkov counters. Current practice allows determining velocity to about 3%, as will be discussed in detail.

c) By grain-density measurement in emulsion.

3-Range

4- $P\beta$ (momentum times velocity)

a) by measuring multiple coulomb scattering.

b) by measuring rigidity against deflection in an electrostatic field.

5-Energy (applicable in special cases)

In special cases limits may be places on the mass of a particle by measurement of either :

6 Energy of annihilation or decay

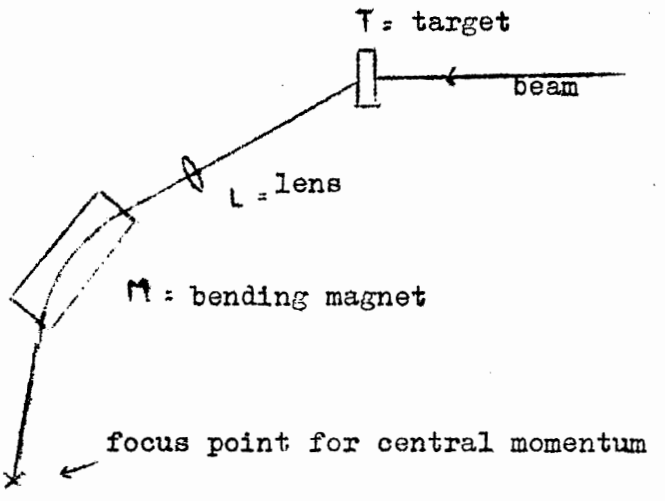
7 Rate of emission of bremsstrahlung

Basic momentum selector.

Many experimental arrangements used at present include some kind of selection apparatus to select certain charged particles out of many that emerge from some target that has been struck by the beam of an accelerator. The simplest of these is a mementum selector using magnetic fields.

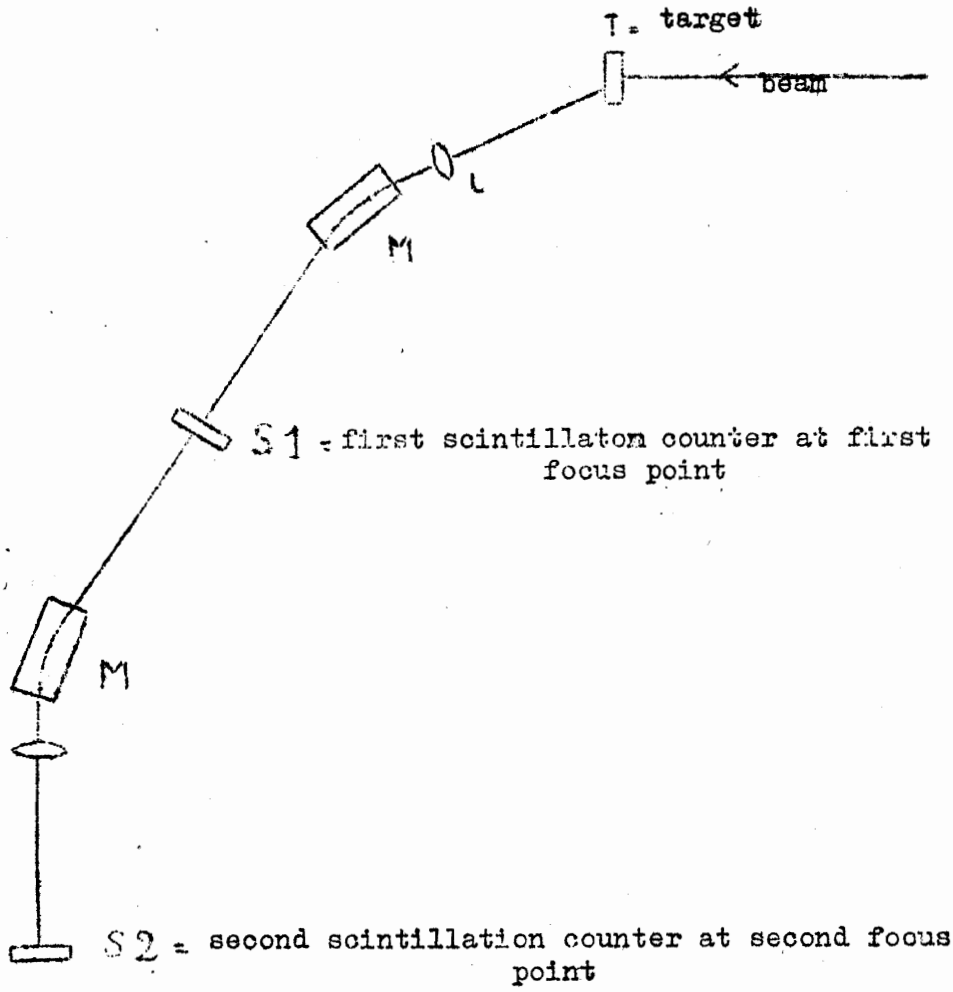
At the present time the most elementary arrangement usually includes magnetic focussing.

For instance:



In some cases bending and focussing magnets can be combined.

If the particles to be selected are long lived a double magnetic spectograph is frequently advisable. Such an arrangement is shown in the following sketch.



See Fig.I of Chamberlain, Segrè, Wiegand and Ypsilantis, Phys. Rev. 100, 947 (1955).

Two important advantages are to be had.

I°) The momentum dispersion present at the focus of the first analyses (position of SI) can be compensated at the position S2, the second focus. This means that all particles which pass through the system come to a much better focus at S.2 than at S.I.

2°) The first counter(in this case SI) counts only those particles in or near the selected momentum. In particular the first counter need not be filled with counts from particles irrelevant to the experiment. General background is usually reduced.

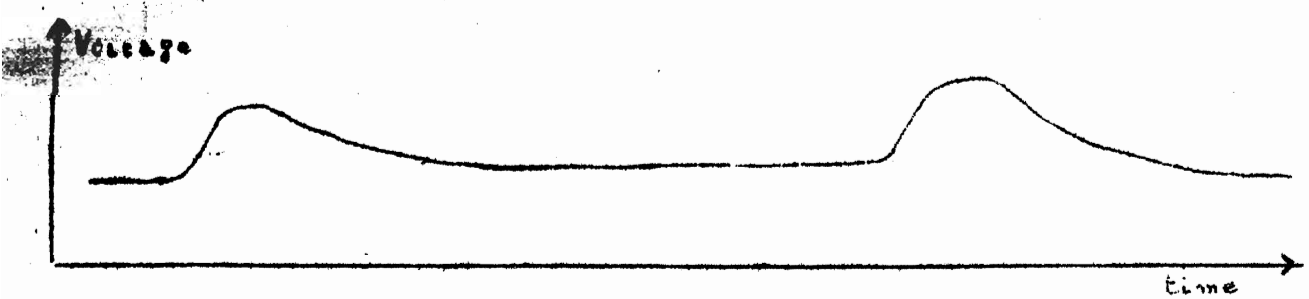
It is still true in this arrangement that when SI and S2 (scintillation counters) count in coincidence then one has an excellent assurance that a particle of the desired momentum passed through the system.

The system may be further improved by putting another magnetic lens near SI - a so called field lens. Such a lens can be used to focus those particles coming out of lens Q I onto the entrance aperture of Q 2, thus eliminating some loss of intensity in the double magnetic analysis.

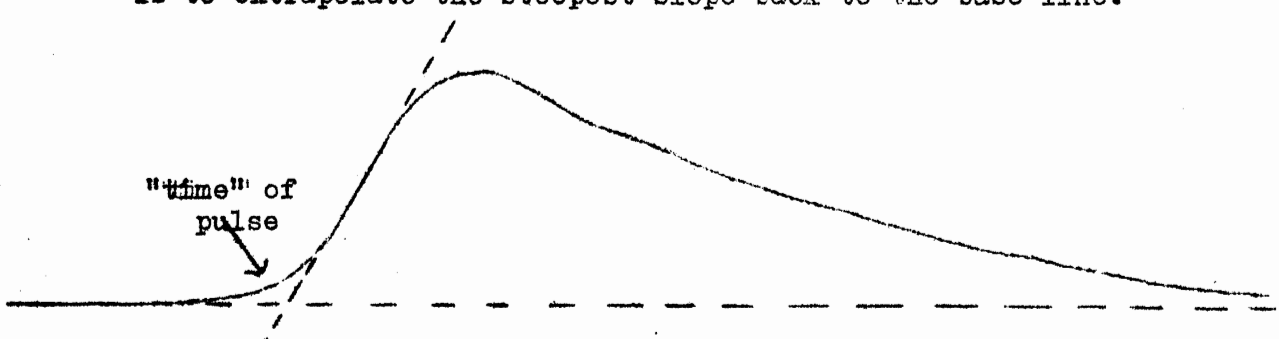
Available magnetic lenses with 4" diameter aperture have focal lengths of 2 - 3 meters for particles of about I000 Mev/c, momentum. Such magnetic lenses would require 40 or 20 kilowatts of electrical power, and cost from 5000 to I0.000 dollars.

Time-of-flight measurement by photographing pulses.

If the outputs of two counters are combined electrically they may be displayed on the same oscilloscope sweep and the time between the two pulses determined more accurately than by purely electronic means.



A good method of determining the time of an event from the pulse is to extrapolate the steepest slope back to the base line.

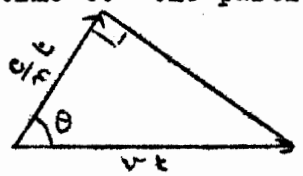


This method of assigning the time to each pulse has advantage that it is not severly affected if there is some saturation in the electronic system such as would distort the highest part of the pulse. The observed r.m.s. time error using the system has been 1×10^{-9} sec even when three Hewlett-Packard (Type 460A) amplifiers were used to supply the pulse voltage to the deflecting plates of the oscilloscope.

Velocity determination by Cherenkov velocity - selecting counter.

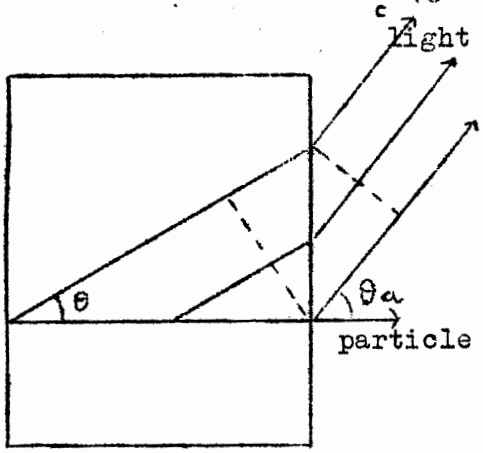
Cherenkov radiation is emitted when a particle travels faster than the velocity of light in that medium.

In time t: the particle travels distance v t; the electromagnetic disturbance travels distance (c/n)t, A Huyghens' construction gives the expression relating v and θ.



θ = angle between light direction and particle direction.

$$\cos\theta = \frac{\frac{c}{n}}{v} = \frac{1}{\frac{v}{c}n} = \frac{1}{\beta n} \qquad \sin\theta = \sqrt{1 - \frac{1}{\beta^2 n^2}}$$



θ_a = angle of light in air

$$\frac{\sin\theta_a}{\sin\theta} = n \qquad \text{We assume } n_{air} = 1$$

$$\sin\theta_a = n\sqrt{1 - \frac{1}{\beta^2 n^2}}$$

The intensity of Cherenkov light is given by

$$I = \text{photons emitted per cm} = \frac{4\pi^2 e^2 Z^2}{h c^2}\Delta\nu\left(1 - \frac{1}{\beta^2 n^2}\right)$$

$$= \frac{2\pi Z^2 \Delta\nu}{137 c}\sin^2\theta$$

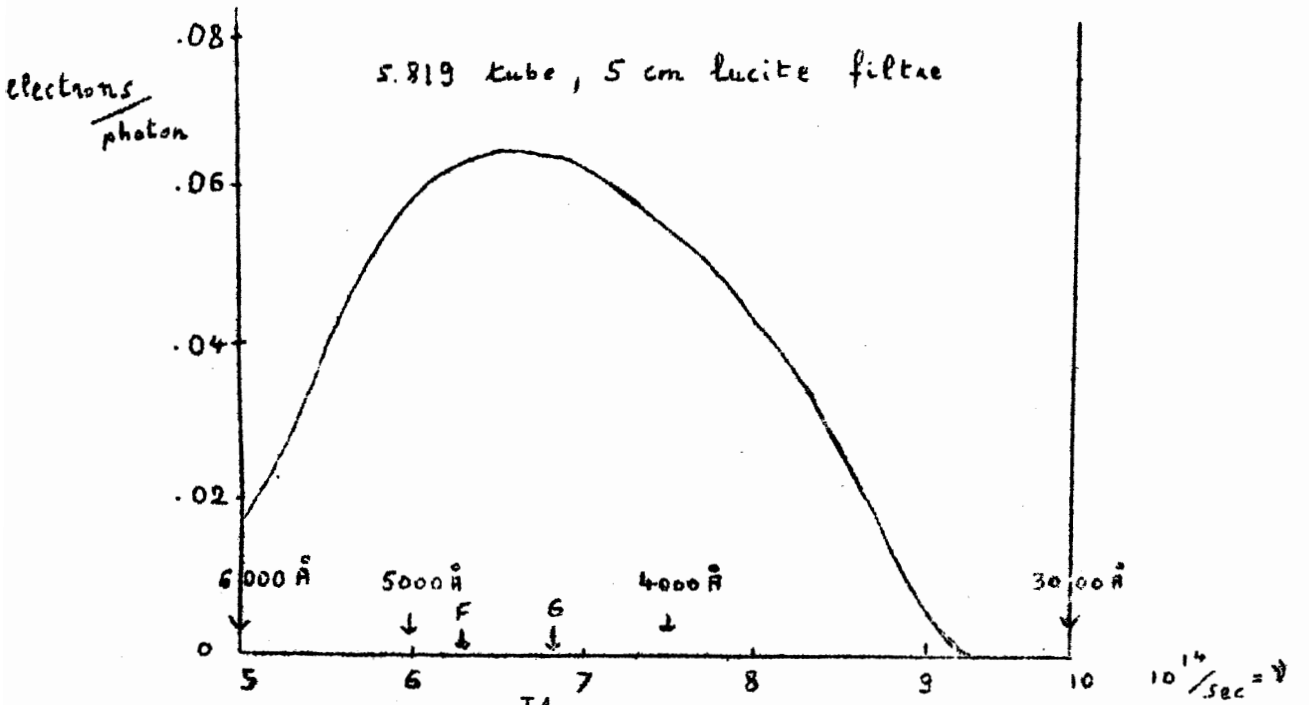
Z = charge of particle, usually 1 or - 1 in our application.

ν = frequency of light emitted.

Ist report: Cherenkov, P.A., Compt.rendu Académie Sciences U.S.S.R. 2 45I (I934) - Explanation: I Frank and I. Tamm, Compte rendu Académie Sciences U.S.S.R. I4, I09 (I937) -

The Cherenkov light, to be useful in the usual way, must impinge on a conventional photomultiplier tube, such as the RCA58I9 or RCA68I0. The sensitivity of one such photomultiplier tube, as a function of frequency is shown in the following plot, taken from John Marshall, Annual Rev.Nuclear Sciences. 4 , I4I (I954). -

The area under the curve determines the yield of photoelectrons from the photocathode of the tube.



area = .06 X 2.9 X 10^{14}

Yield of photoelectrons = .06 440 X $\left(1 - \frac{1}{\beta^2 n^2}\right)$ per cm of path

= 26 $\left(1 - \frac{1}{\beta^2 n^2}\right)$ per cm path length.

= 26 $\sin^2 \theta$ per cm.

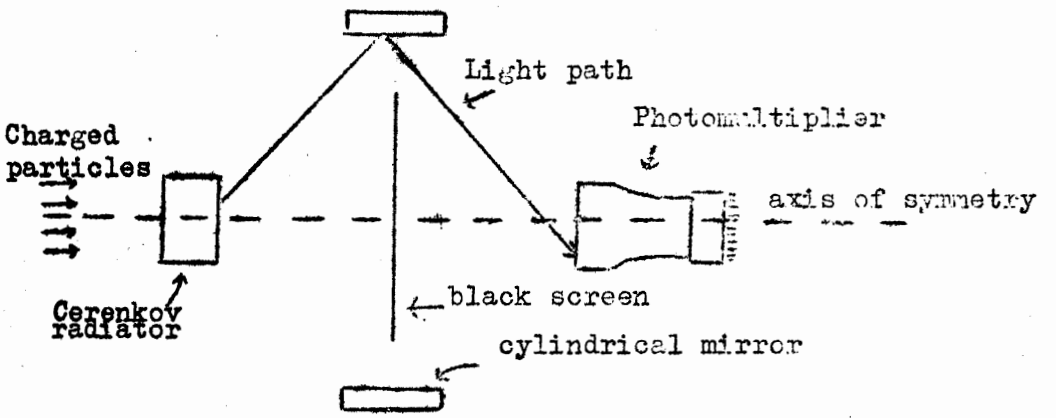
We assume 26 photoelectrons are required in practice less some loss due to poor optics: often divided between several photomultiplier tubes.

ℓ = length of radiator = $\frac{1}{\sin^2\theta} = \frac{1}{1 - \frac{1}{\beta^2 n^2}} = \frac{\beta^2 n^2}{\beta^2 n^2 - 1}$

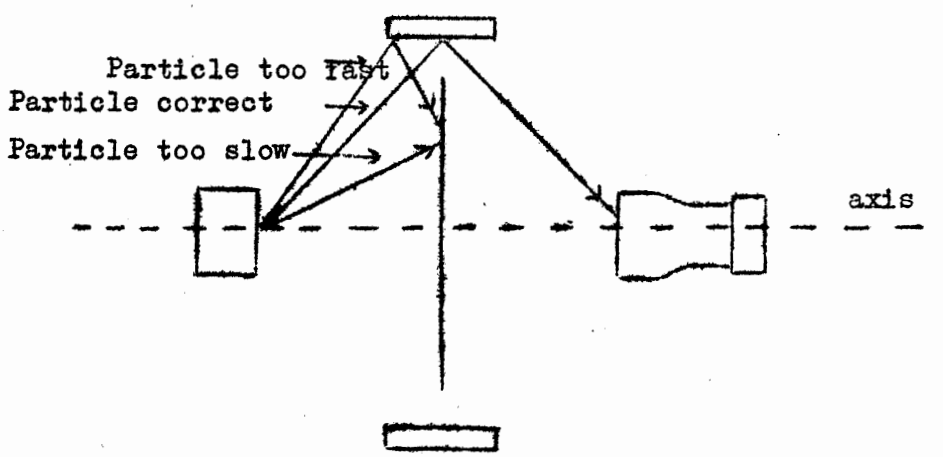
βn	ℓ
I.05	I0.8 cm.
I.I0	5.8
I.I5	4.I
I.20	3.3
I.25	2.8
I.30	2.5
I.40	2.0
I.50	I.8

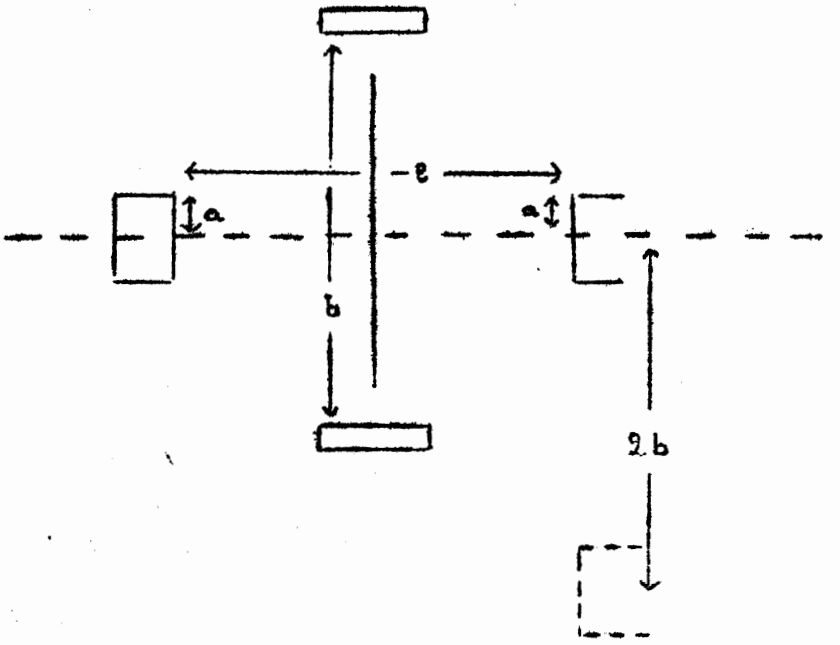
There are many possible forms of velocity-selecting Cherenkov counter. We shall discuss mainly the one used in the antiproton selection apparatus of Chamberlain, Segrè, Wiegand and Ypsilantis. Phys.Rev.100, 947 (1955). It employs a cylindrical (aluminized) mirror surface with diameter much greater than the Cherenkov-radiator diameter to produce an image of the radiator on the photomultiplier tube as shown in the figure below.

Velocity-selecting counter



The action of the counter in selecting particles of a chosen velocity is shown in the next figure.





We have previously shown that θ_a, the angle of the Cherenkov light in air is given by

$$\sin\theta_a = \sqrt{1 - \frac{1}{\beta^2 n^2}}$$

and in order that the light reach the photomultiplier we should have

$$\tan\theta_a = \frac{2b}{e}$$

By inspection of the geometry of the instrument and by straight-forward differentiation we may determine the effect of several factors on the uncertainty in velocity resolution. We express the uncertainty in terms of $\beta = v/c$. The uncertainty inherent in the geometry of the instrument (finite size of radiator and photomultiplier tube) is

$$(\Delta\beta)_{geom} = 4\beta^3 b e^2 a / (e^2 + 4b^2)^2$$

Another uncertainty arises from the optical dispersion of the radiator material. If the index of refraction n changes by an amount Δn over the useful range of light wave lengths used this gives rise to an uncertainty in β of $(\Delta\beta)_{\text{optical dispersion}} = n\beta^3 \Delta n$

Another limitation of the velocity resolution comes from the change in velocity of the particle as it passes through the radiator. Taking the length of the Cherenkov radiator as ℓ (the dimension parallel to the axis of the instrument) we have

$$(\Delta\beta)_{\text{energy loss}} = \frac{1}{2}\,\frac{d\beta}{dT}\,\frac{dT}{dx}\,\ell$$

where T is the kinetic energy of the particle being counted and dT/dx is the energy loss per unit distance in the radiator material.

The last source of uncertainty in β is that due to multiple scattering in the radiator material. We may express the angle of multiple scattering as

$$\frac{21\ \text{Mev}}{p\,v}\sqrt{\frac{\ell}{L}}$$

where p and v are momentum and velocity of the particle and L is the radiation length (in cm) of the radiator material. Using this expression the uncertainty in β is

$$(\Delta\beta)_{\text{mult.scatt.}} = \frac{\sin\theta}{n\cos^2\theta}\,\frac{21\ \text{Mev}}{M c^2 \gamma \beta^2}\sqrt{\frac{\ell}{L}}$$

where M is the mass of the particle being counted and $\gamma = 1/\sqrt{1-\beta^2}$

As an example, consider the velocity-selecting Cherenkov counter used for selecting antiprotons (Chamberlain, Segrè, Wiegand and Ypsilantis, Phys. Rev. I00, 947 (I955)).

a = radiator radius = 3 cm.

b = mirror radius = I9 cm.

n = index of refraction, fused quartz = I.47

$\beta = v/c = 0.75$ $\gamma = I.5I$ (480 - Mev protons)

ℓ = length of radiator = 6 cm.

$\cos\theta = I/\beta n = 0.9I$

$\theta = 24.5^\circ$

$\sin\theta = 0.4I5$

$\sin\theta_a = 0.6I0$

$\theta_a = 37.6^\circ$

$\cos\theta_a = 0.793$

$\tan \theta_a = 0.769$

e = distance from radiator to photomultiplier = 49.5 cm.

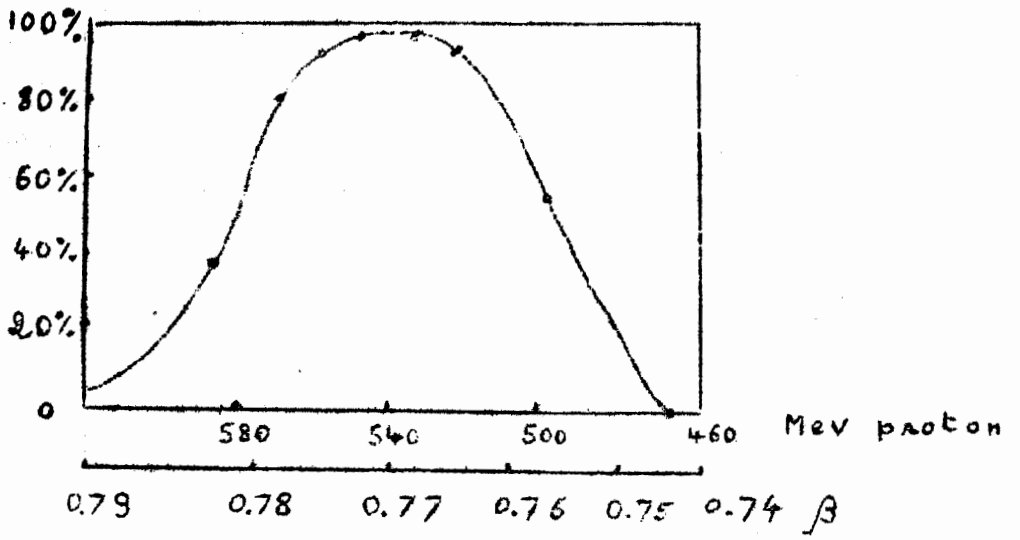
$(\Delta\beta)$(geometry) = I.55 X 10^{-2}

$(\Delta\beta)$(optical dispersion) = 0.62 X 10^{-2}

$(\Delta\beta)$(energy loss) = 0.77 X 10^{-2}

$(\Delta\beta)$(mult. scatt.) = 0.47 X 10^{-2}

The calculated uncertainties compare well with the actual performance as determined experimentally using a pure beam of positive protons, obtained by magnetic selection at the Bevatron. The following figure shows the experimentally determined efficiency as a function of proton energy and proton velocity.



At the chosen velocity the efficiency reaches 95%, while for very large velocities the efficiency drops to 3%.

As actually used, the instrument contains an equilateral triangle of 3 plane mirrors that divides the light between 3 photomultiplier tubes. This division of light is necessary only to avoid having the particles (to be counted) pass through the photomultiplier tube. This refinement is not essential to an understanding of the mechanism of the counter.

When the light is divided between 3 photomultipliers the pulses are very small. The efficiency curve shown just above was obtained using a coincidence circuit that accepted events when any 2 of the 3 photomultipliers showed pulses.

Main advantages of this type of counter.

I- Counter ignores (to a large extent) particles of the wrong velocity, hence number of unwanted counts reaching electronic circuits is drastically reduced .

2- Gives velocity selection without long paths involved in time-of-flight velocity selection.

3- Counter ignores particles going in completely wrong direction.

Disadvantage.

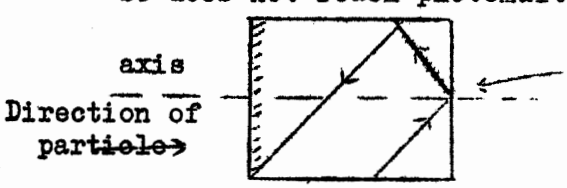
I- Radiator must be fairly thick so nuclear collisions can result in fast products that cause counter to count.(3% of relativistic pions count falsely)

Fitch counter.

Another, simpler, variety is the Fitch counter,designed for velocity band pass.

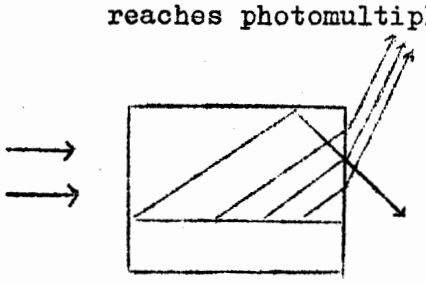
Uses cylindrical Cherenkov radiator.

Radiation from very fast particles is totally internally reflected, so does not reach photomultiplier and particle does not count.

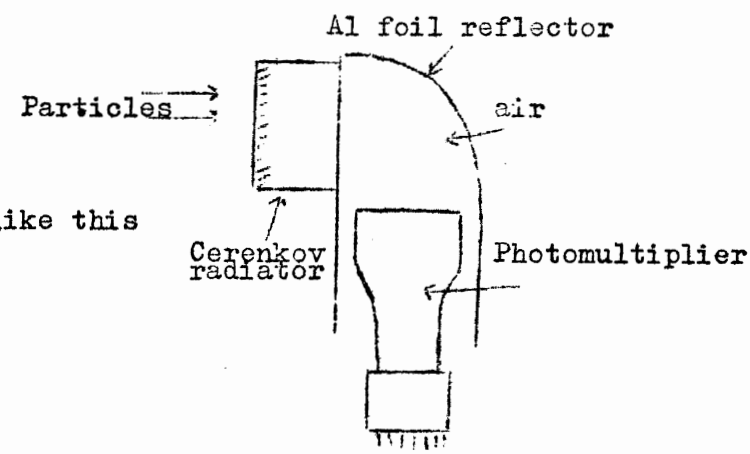


Internal reflection keeps particle from counting if $\beta > \frac{1}{\sqrt{n^2-1}}$

If particle is within acceptable range of velocity then radiation reaches photomultiplier and particle is counted.



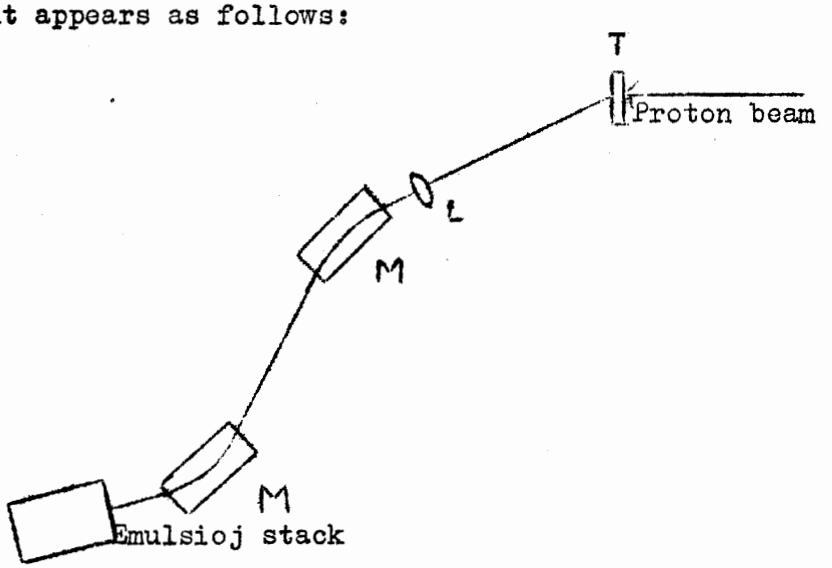
Actual counter may look like this (see left)



Obviously index of refraction n must be $> 1.41 = \sqrt{2}$ in order to get any rejection by internal total reflection. If n = I.62, counter will not count unless $0.62 < \beta < 0.79$ providing particles traverse radiator parallel to axis. Also in Fitch counters, nuclear collisions of relativistic particles may cause particles to count which should be rejected. This background is 3% to I0% of relativistic mesons.

Selecting antiprotons using Emulsions.

The magnet arrangement used to observe antiprotons in emulsions has been very similar to systems already discussed, with the exception that an additional bending magnet must be placed directly before the emulsion stock to cause positive particles, especially positive protons, to enter the stack in a completely different direction than that of the negative particles. The arrangement appears as follows:

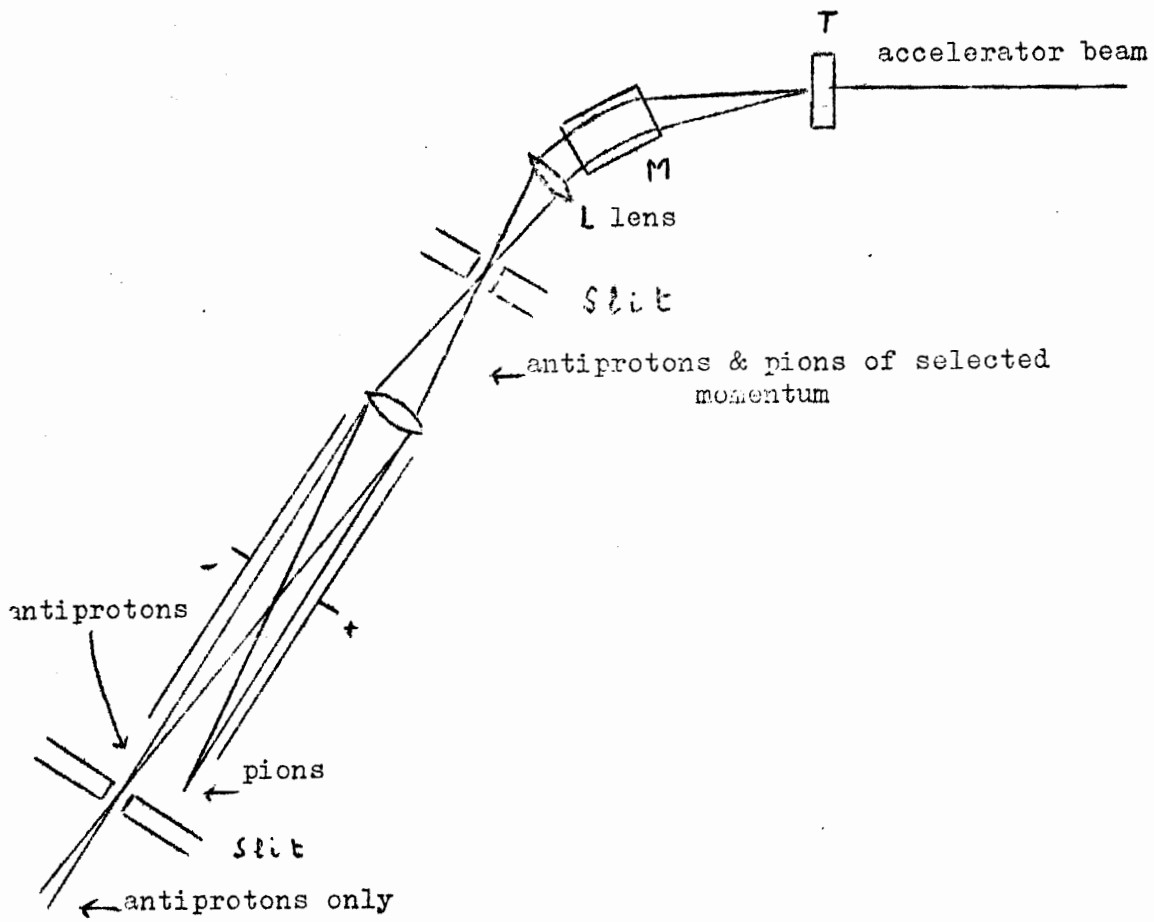


In order to find the antiprotons without a very great scanning effort it is necessary to use a reduced momentum, about 700 Mev/c, corresponding to an antiproton kinetic energy of 200 Mev. At this energy the antiproton have about twice minimum ionization and can be recognized by eye by experienced scanners even though they are a very small fraction of the magnetically selected negative beam. Since we are looking here for lower energy antiprotons they must

have been omitted in the c.m. system with a higher energy (in the backward direction) so the antiprotons are even more rare than in the cases already discussed. There is one antiproton for every 500.000 minimum-ionizing particles. In spite of this large ratio of background to effect the antiprotons can be recognized by their heavier ionization when the tracks are inspected near their entrance in the emulsion stack. The antiproton tracks can then be followed either to the end of their range or until they interact in flight in the emulsion.

Selecting a beam of particles by electrostatic and magnetic deflection.

Many experiments, such as the antiproton experiments, are severely handicapped because there are so many undesired particles in the beam (30.000 π^- to 1 antiproton). It should be possible to form a beam of known momentum using magnets, and then sort out this beam the desired particles by electrostatic deflection. In a simple form such a system might look like this:



Using cgs units and es units, the lateral displacement of a particle due to the electrostatic field $\mathcal{E}$ acting for a distance d along the path of the particle will be:

$$s = \frac{1}{2} a t^2 = \frac{1}{2} a \left(\frac{d}{v}\right)^2 = \frac{1}{2} \frac{e \mathcal{E} d^2}{p v}$$

where e is charge, p momentum, v velocity, a acceleration.

As an example we may consider 480-Mev antiprotons (p = I060Mev/c) for which $p v = m c^2 \gamma \beta^2$ = 790 Mev = I.27 X I0 $^{-3}$ergs. We take $\mathcal{E}$ to be 30.000 v/cm corresponding to 300000 volts across I0 cm. spacing between plates, or $\mathcal{E}$ = I00 esu. Then

$$s = 1.89 \times 10^{-5} d^2 \quad (d \text{ and } s \text{ in cm})$$

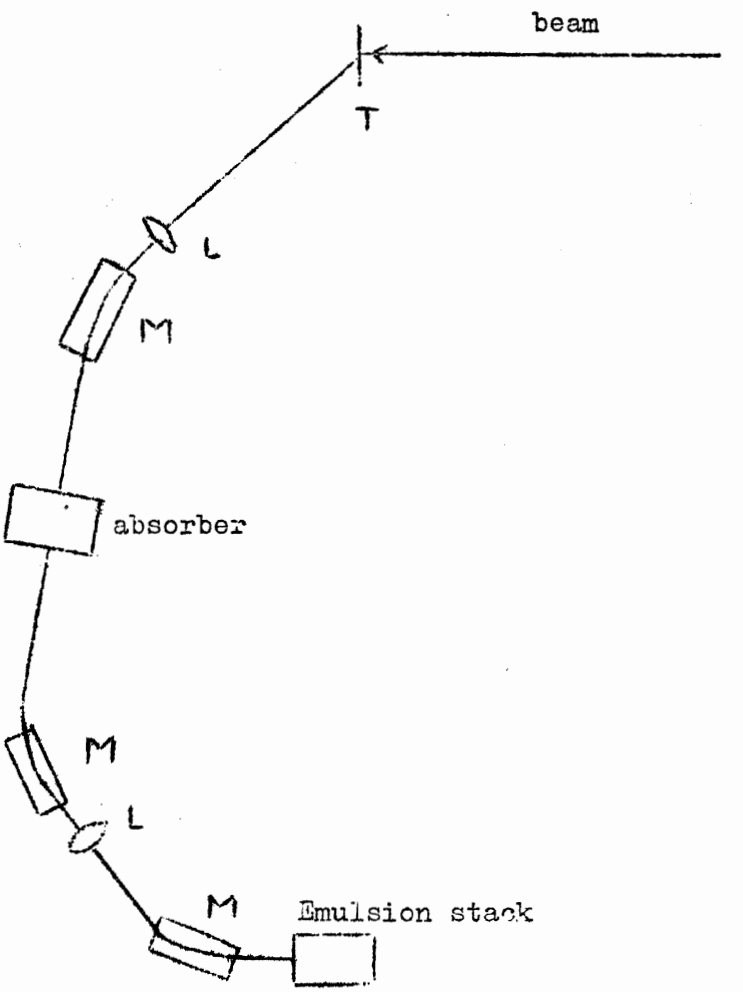
Taking as a reasonable length for the electric field d = 800 cm. we find s = I2.I cm. Relativistic particles of the same momentum would be deflected 9.I cm. so a separation of 3.0 cm. between p^- and π^- could be obtained. This separation would probably not be too useful unless present magnetic lenses were somewhat refined by a careful study to eliminate their aberrations. However a much more optimistic example can be chosen if one starts with lower momentum particles.

An electrostatic deflection system of the general variety described here has been started in construction by Wenzel and Lambertson at the Radiation Laboratory, U niversity of California. It is a promising method, although a number of problems may prove somewhat troublesome. One of these is due to $\pi - \mu$ decay. μ mesons may appear as a significant contaminant of the final beam because the orbit is slightly altered when the $\pi - \mu$ decay occurs.

Selecting a beam of particles by momentum analysis and differential absorption.

Some measure of success has already been achieved in making a separated antiproton beam by using a momentum selector,

followed by an absorber, and this followed by another momentum selector, Chamberlain, Goldhaber, Jauneau, Segrè, Steiner et al. In the absorber particles of different masses lose momentum at different rates so that the second momentum selector brings different particles to different focus points.
The apparatus looked as follows:

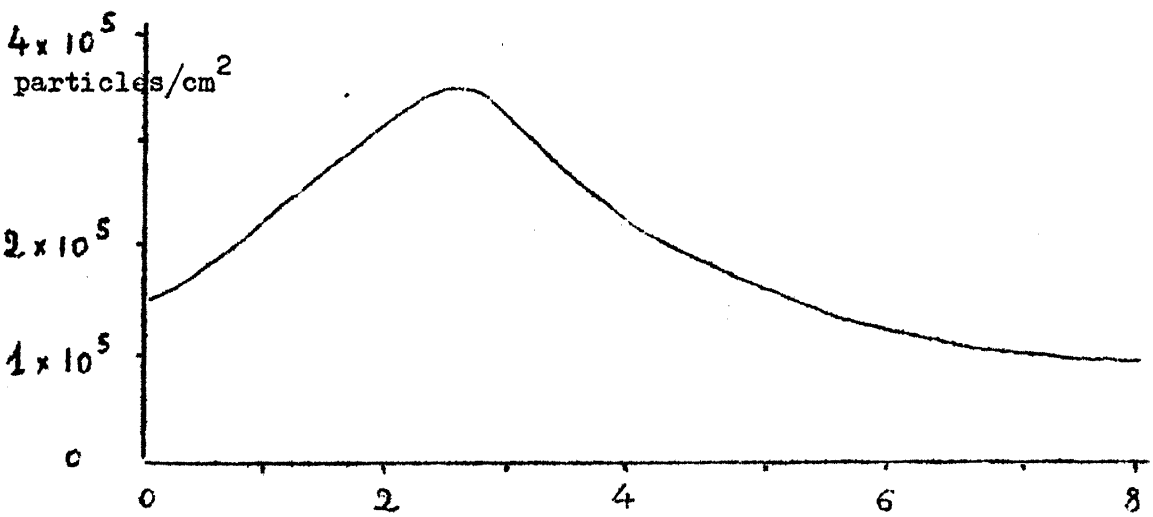


The magnets between the target and the absorber were adjusted to momentum 819 Mev/c. The absorber consisted of 20 g/cm^2 of lithium hydride. Pions leaving this absorber had an average momentum of 776/Mev/c, while antiprotons at the same point had 700 Mev/c. At the second focus (at the emulsion stack) the separation of antiproton image from pion image was I5 cm.

The detector in this case was an emulsion stack, approximately I7cm x I0 cm x 7 cm. At the momentum (700 Mev/c) at which the

antiprotons entered the stack the usual relativistic particle-to-antiproton ratio is 500000 (as observed without mass separation). In this exposure the relativistic particle-to-antiproton ratio was 30000, indicating an improvement by more than a factor of I0. Instead of being limited, by background, to some 20 antiprotons per stack we obtained in this stack about 200 antiprotons in an exposure equivalent to about 2 hours of full beam at the Bevatron.

By looking carefully at the stack it is possible to estimate the composition of the background. The number of pions in the background can be estimated from the density of stars in the emulsion. The number of electrons can be estimated by observing the shower curve - measuring the number of minimum tracks per square centimeter at various depths within the stack. In this case the curve looked as follows:

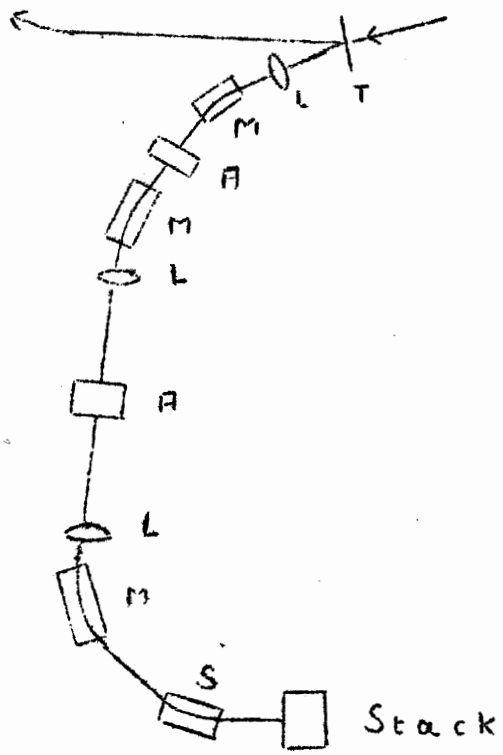


The component not pions or electrons is assumed to be muons. On the basis of this analysis the background is estimated to be 4% pions, 40% electrone, and 56% muons.

In order to further reduce background we plan to make an exposure of emulsions with a double separation in an apparatus as follows:

A = absorber

S = sweeping magnet



It is hoped and expected that the resulting antiproton beam will be at least as pure as 1 antiproton in 3000 background particles. Such a beam may be sufficiently rich in antiprotons to warrant trying to use a hydrogen bubble chamber as detector.

Spectrum of baryons.

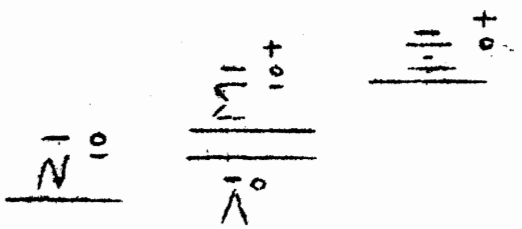
The spectrum of baryons, particles with the nucleon number N = 1, is believed to be as follows:

$\bar{\Xi}^{\bar{0}}$ $\quad$ $\bar{\Sigma}^{\bar{0}}$ $\quad$ $\bar{\Lambda}^{\bar{0}}$ $\quad$ N^{+}

S = -2 $\quad$ S = -1 $\quad$ S = 0

The strangeness is indicated as S, according to the Gell-Mann scheme. All the indicated particles have been observed except the neutral cascade particle $\bar{\Xi}^0$.

Presumable each of these should have an opposite, its anti - particle, with the same mass, but opposite strangeness S and opposite charge. The spectrum of antibaryons $N = -1$ should then look as follows:



$S = 0 \qquad S = 1 \qquad S = 2$

Of these, only the two forms of antinucleon have been observed as yet. However, in time, the others are expected to be produced and observed. In any reaction, nucleon number N and charge number Q should be rigidly conserved. In order to be a strong reaction strangeness S and isotopic spin I should also be conserved.

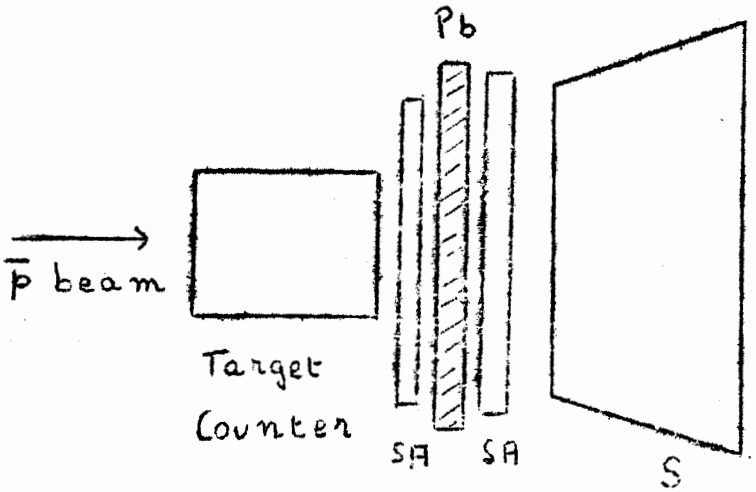
If there is invariance under the transformation $T\ C\ P$ (T = time reversal, C = charge conjugation, P = space reflection), as there is according to the Lüders theorem, then the mass and lifetime of each antiparticle should be the same as for the corresponding particle. Thus we expect the antiproton is stable in vacuum, and the antineutron should have the same lifetime as the neutron.

We have already discussed the methods of detecting antiprotons. The results indicate the mass of the experimentally observed anti - proton is the same as that of the proton, within 2 percent.

The antineutron is difficult to detect among the directly produced products from the Bevatron target because it may too easily be confused with energetic neutrons, however, it has been observed in a charge-exchange reaction. The elementary reaction is believed to be

$$\bar{p} + p \rightarrow \bar{n} + n$$

The production of antineutrons was first observed by Cork,Lamberton,Piccioni and Wenzel, in an apparatus constructed approximately as follows:



The target counter is a container holding liquid scintillator solution, approximate chemical composition C.H. The Charge exchange reactions to be detected took place in the scintillator solution either on protons within carbon or on free protons. The purpose of making this material be also a counter was to allow rejection of events in which antiprotons annihilated within the target counter. Such annihilation events would be accompanied by large pulses from the target counter and could be rejected on that basis.

If an event was to be interpreted as an antineutron then it should show no pulses in either of the counters labeled SA (scintillator, anticoincidence) because the neutral antineutron should pass through these counters without making any interaction. The antineutrons were to be detected by their large energy release in counter S (large pulse height in S). The counter S consisted of a large vat of liquid scintillator, or, in part of their experiment, counter S was replaced by a large lead-glass Cherenkov counter.

In either case an antineutron was said to be detected if: (a) an antiproton entered the target counter making only a small pulse, (b) showed nu pulse in either of counters SA, (C) showed a large pulse in counter S, commensurate with over I.8 Bev.energy release at annihilation. The lead sheet Pb served to convert gamma rays and thus keep them from being confused with antineutrons.

Production cross sections for antiprotons.

From the work of Chamberlain, Segrè, Wiegand and Ypsilantis a production cross section for antiprotons could be estimated. The method of computation requires a little thought because the apparatus includes two momentum spectrographs, one feeding into the other. This means that the momentum is over-determined and there are many stops or diaphragms through the apparatus.

The result obtained is the cross section per nucleon in copper for production of antiprotons at 0° (forward direction) with antiproton momentum I.I9 Bev/c. The copper target was bombarded by protons of kinetic energy 6.2 Bev. The result:

$$\frac{d^2\sigma}{d\Omega\, dp} = 1 \times 10^{-30} \text{ cm}^2/\text{Sterad.} - \text{Bev}/c$$

The observed cross section has not been compared with a statistical theory calculation done for just these circumstances, but the observed cross section is believed to be comparable or somewhat larger than that predicted from statistical theory. The result given above should replace an erroneous result given in the Cern symposium report of I956.

When Cork, Lamberton, Piccioni and Wenzel set up their antiproton selector based on momentum and time-of-flight measurement, they obtained some approximate yield numbers for antiprotons at various antiproton momenta. If their numbers are re-expressed to be proportional to $d^2\sigma/d\Omega\, dp$ they are:

Momentum	Yield
0.75 Bev/c	0.1
0.90 Bev/c	0.3
I.I5 Bev/c	1.
I.4I	3

In the latest run of Agnew, Chamberlain, Segrè, Sternier, Wiegand, Ypsilantis et al, targets of CH_2 and C were compared for their antiproton production, under circumstances that did not permit an absolute cross section to be determined. The result may be given in terms of hydrogen and carbon as follows:

$$\frac{\sigma_{pp \to \bar{p}}}{\sigma_{pC \to \bar{p}}} = 0.11 \pm 0.06$$

when the cross sections of the two nuclei are compared. If the yields per nucleon are compared then we obtain

$$\frac{\text{Yield per nucleon in H}}{\text{Yield per nucleon in C}} = 1.3 \pm 0.7$$

The calculated ratio of total yields of antiprotons (according to statistical theory) from hydrogen and carbon(per nucleon)is no larger than 0.13 (compare with 1.3 above).
The reason for the theoretical prediction of less yield per nucleon from hydrogen than carbon is that the internal momentum of nucleons in carbon should aid considerably the antiproton production at the Bevatron energy (close to threshold).

While the observed production of antiprotons in hydrogen seems at first sight surprisingly large, we believe it may well be accounted for by two factors:

(a) The production in hydrogen is concentrated to smaller angles than the production from carbon, hence detection in the forward direction favors hydrogen over carbon (and the theoretical calculation has only been made for total yield).

(b) There may be a great deal of reabsorption (annihilation) of antiprotons in carbon, not taken into account in the simple theory.

Very little information is as yet available on the excitation function for antiproton production - the production cross section as a function of Bevatron proton beam energy. An approximate excitation function was determined in the original antiproton work of Chamberlain, Segrè, Wiegand and Ypsilantis, Phys.Rev. 100, 1947 (1955),

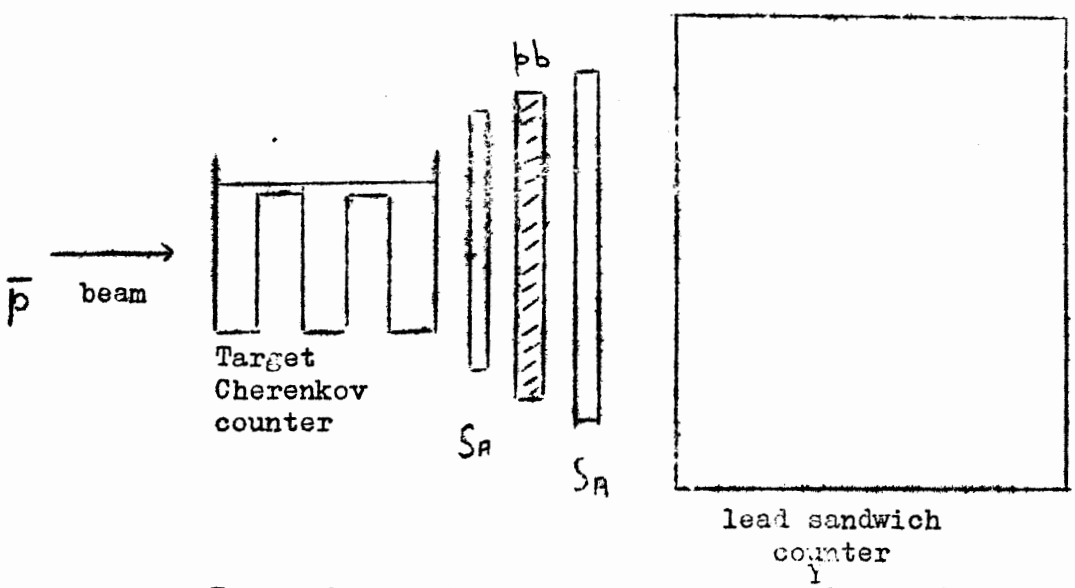
giving yields as follows:

Bevatron proton beam energy	Antiprotons per 10^5 negative pions
4.2 Bev.	0 ± 0.2
5.0	0.8 ± 0.5
6.2	2.3 ± 0.3

Note that only the ratio to mesons was measured, not a strict production cross section.

Production cross sections for Antineutrons by Charge-Exchange.

Chamberlain, Elioff, Keller, Segrè, Steiner, Weingart, Wiegand and Ypsilentis have detected the charge exchange reaction in a few elements using an apparatus as follows:



The charge exchange reactions to be detected were to take place in the target Cherenkov counter or material placed in the slots within that counter, and an annihilation in that vicinity was detected by a pulse in the Cherenkov counter(containing water or methyl alcohol). Since the antiprotons were too slow to give Cherenkov radiation in water or methyl alcohol, a pulse in the target Cherenkov counter would be grounds for rejecting an event.

The antineutron, to be useful, had to pass through the counters S A and the lead sheet without giving any pulse and cause a large (annihilation) pulse in the lead sandwich counter. The lead sandwich counter consisted of a 60 cm. cube filled with alternate sheets of 0.2 cm lead and 0.6cm plastic scintillator. The counter has an average density of 3.8 g/cm^3 and a radiation length of about 3 cm, so most of the annihilation energy could be contained within the counter in many cases.

After the charge-exchange reaction had been studied in CH_2, by placing that material in the slots of the target Cherenkov counter, it could be ascertained that the information from the target Cherenkov counter was not essential to the antineutron detection, hence it was removed and replaced by carbon (graphite) for one run and lead for another run.

The results obtained may be expressed in terms of the total cross section for charge exchange of an antiproton to yield an antineutron in the forward direction at angles up to I7 degrees from the forward direction as follows:

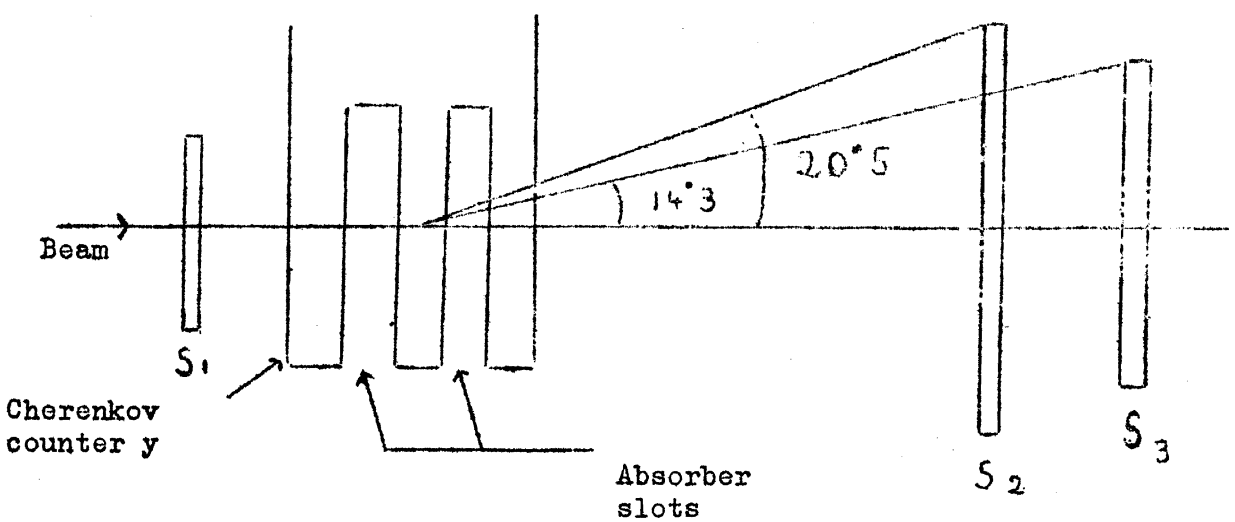
$\sigma_{\bar{p}, Pb \rightarrow \bar{n}}$	$= 3.8^{+4.0}_{-2.5}$ mb	(I event)
$\sigma_{\bar{p}, C \rightarrow \bar{n}}$	$= 4.0 \pm$ I.5mb	(6 events)
$\sigma_{\bar{p}, CH_2 \rightarrow \bar{n}}$	$= 10.0 \pm 2.8$	(I3 events)
$\sigma_{\bar{p}, P \rightarrow \bar{n}}$	$= 13.0 \pm$ I.6	

Note that the nuclei hydrogen, carbon, and lead all have comparable charge exchange cross sections. No detailed explanation is yet made, but it seems clear that in heavy elements, and even in carbon, the annihilation events cause most antinucleons to be lost at collision. Only the outside regions of these nuclei are available for the charge exchange reaction.

Interaction Cross sections of antiprotons.

Some of the earliest measurements on antiprotons were determinations of annihilation and total cross sections in beryllium and copper. The results indicated cross sections about twice as large as those for ordinary protons (Chamberlain, Keller, Segrè, Steiner, Wiegand and Ypsilantis, Phys.Rev.I02, I637 (I956).

More recent measurements by the same group have been performed using an apparatus sketched below



The material whose cross section was to be measured was placed in the slots of the water Cherenkov counter y. Antiprotons of this beam were too slow to produce Cherenkov radiation in counter y, but annihilation processes produced fast charged mesons and neutral mesons that could be detected in counter y, which thus acted as a detector of annihilation events. The counters S 2 and S 3 acted to determine the transmission of the sample (absorber) with cut-off angles of I4.3° and 20.5° as indicated in the diagram.

Because of uncertainty in the proper gain setting of counter y when the experiment was begun it was necessary to photograph the pulses from that counter and to determine from the resulting pulse height spectra the correct interpretation of pulses from the counter. The method is very laborious but shoudl give reliable results.

The results are quoted I4° (and 20°) cross sections, meaning the cross section for annihilation plus the cross section for scattering to angles wider than I4°(and 20°). Furthermore, the annihilation cross sections are also given.

For comparison and to check the operation of the apparatus the cross sections for protons were also measured. The results are (cross sections in mb.):

Substance and Energy(Mev)	I4°		20°		Annih.
	$\bar{p}$	p	$\bar{p}$	p	$\bar{p}$
Oxygen (457 Mev)	559 ±10	292 ±2	5I9 ±30	246 ± 2	456 ±9
Copper (411 Mev)	I240 ±82	7I9 ±5	I220 ±83	650 ±4	I040 ±6I
Silver (43I Mev)	I630 ±I70	I052 ±6	I640 ±I83	924 ±6	I500 ± I57
Lead (436 mev)	----	----	(2680) ±354	(I46I) ±I0	20I0 ±I82

Only statistical errors are shown. Some of the lead results are doubtful because an excessive thickness of lead absorber was used.

The data for oxygen of the previous table were obtained in a slightly different apparatus using a counter y without slots and slightly thicker in the beam direction. With this alteration measurements were made using liquid oxygen, water, and heavy water. Substraction of cross sections gave the following results in mb at 457 Mev.

Substance	14°		20°		Annih.
	$\bar{p}$	p	$\bar{p}$	p	$\bar{p}$
H	104 ±8	25.1 ±1.2	104 ±8	24.4 ±1.3	89 ±7
d - H	70 ±8	28.7 ±1.1	70 ±8	---	46 ±8
n	112	31.6	112	---	74

The errors are again statistical errors. The proton cross sections are within 7 mb of known proton cross sections in all cases. The difference (d-H) between deuteron and proton cross sections is not equal to the neutron cross section because there is considerable hiding by one nucleon behind the other. This effect has been studied by Glauber, and the calculation extended to apply to the present case by J.S.Blair, Using Blair's results the cross sections for (d-H) have been corrected to yield the cross sections for the neutron (n). Since the corrections are very large it is not possible to claim high accuracy.

The conclusions are that the elementary $\bar{p}\,p$ and $\bar{p}\,n$ cross sections are very large (about 4 times as large as $p\,p$ and $p\,n$ cross sections) and that they are mainly (70 per cent to 80 per cent) annihilation in each case.

The total cross section for $\bar{p}\,p$ has been determined by Cork, Lambertson, Piccioni and Wenzel at a number of anti-proton energies. Interpolating between their results we find at the same energy (99 ± 7) mb. Since their cross section was determined in a very good geometry (cut-off angle about 4°) there should be little doubt that their cross section is a real total cross section including diffraction scattering.

Optical model calculations, using the size of the annihilation cross-section for $\bar{p}p$ to determine the size of an appropriate disc, indicated that most(90 %) of the diffraction scattering should fall within I4° and practically all within 20°.

This means that the cross sections determined with cut-off angles of I4° and 20° do not include diffraction scattering. Comparing the various results we conclude that the forward scattering, including most of the diffraction scattering, is $-(104 \pm 8) + (99 \pm 7) = (-5 \pm 11)$ mb. A negative result is absurd, but the errors allow some leaway.

It is still somewhat puzzling that the diffraction scattering should be as small as about I0 mb. If the proton were thought of as representing a black disc for the antiproton then we would expect a diffraction cross section equal to the absorption (annihilation) cross section, namely 89 mb. Fortunately we can say that as soon as we allow even a small bit of transparency the diffraction scattering drops very materially. Nevertheless the small forward scattering is a surprising result and it makes a simple theoretical interpretation somewhat difficult.

Theorem.

I would like to mention a theorem I have found that is based on a charge independence. The charge-exchange differential cross section may be thought of as due to a difference between isotopic spin $I = 0$ scattering and $I = 1$ scattering.
Writing the wave function for $\bar{p}p$ as

$$\bar{p}p = 2^{-\frac{1}{2}}\left(-u^{\circ}_{1} + u^{\circ}_{0}\right)$$

and for $\bar{n}n$ as

$$\bar{n}n = 2^{-\frac{1}{2}}\left(u^{\circ}_{1} + u^{\circ}_{0}\right)$$

we may show that after scattering the wave function should be

$$(-a_1 + a_0)\frac{\bar{n}n}{2} + (a_1 + a_0)\frac{\bar{p}p}{2}$$

showing that the amplitude for the state $\bar{n}n$ is proportional to the difference between scattering amplitude a_0 for $I = 0$ state and the scattering amplitude a_1 for $I = 1$ state.

The difference between $I = 1$ and $I = 0$ scattering may also be represented in another way. The total $\bar{p}p$ cross section may be written as

$$\sigma_{tot}\ \bar{p}p = \frac{\sigma_1}{2} + \frac{\sigma_0}{2}$$

Similarly,

$$\sigma_{tot}\ \bar{p}n = \sigma_1$$

The difference is

$$\sigma_{tot}\ \bar{p}n - \sigma_{tot}\ \bar{p}p = \frac{\sigma_1}{2} - \frac{\sigma_0}{2}$$

$$= \frac{2\pi}{k}\ Im\left[a_1(0^\circ) - a_0(0^\circ)\right]$$

where in the last step we have applied the optical theorem. Expressing all of the amplitudes and differential cross sections at 0°, we have

$$\frac{d\sigma}{d\omega}\ \bar{p}p \to \bar{n}n\ (0^\circ) = \left|\frac{-a_1 + a_0}{2}\right|^2 \geq \left[\frac{Im\,(a_1 - a_0)}{2}\right]^2$$

$$= \left(\frac{k}{4\pi}\right)^2 \left(\sigma_{tot}\ \bar{p}n - \sigma_{tot}\ \bar{p}p\right)^2$$

The theorem is then stated

$$\frac{d\sigma}{d\omega}\ \bar{p}p \to \bar{n}n\,(0^\circ) \geq \left|\frac{k}{4\pi}\right|^2 \left(\sigma_{tot}\ \bar{p}n - \sigma_{tot}\ \bar{p}p\right)^2$$

In practice the theorem is not terribly restrictive. If we use the charge-exchange pn collisions (for which detailed cross sections are known) as a measure of the forward peaking of the charge-exchange process we may estimate that the left-hand side of the inequality is no larger than 40mb/sterad. Taking k (lab.system) as $5 \times 10^{13}\,cm^{-1}$ we find that the difference between the two total cross sections cannot be larger than 50 mb. The experimental results satisfy the inequality very safely.

Another theorem.

We have shown that the scattering amplitude at angle θ for the reaction $\bar{p}p \to \bar{p}p$ is

$$\frac{(a_1 + a_0)}{2}$$

and hence the differential cross section is

$$\sigma_{\bar{p}p \to \bar{p}p} = \frac{|a_1 + a_0|^2}{4}$$

Similarly we obtain the differential cross section

$$\sigma_{\bar{p}p \to \bar{n}n} = \frac{|-a_1 + a_0|^2}{4}$$

Noting that the differential cross section at the same angle θ for $\bar{p}n \to \bar{p}n$ is

$$\sigma_{\bar{p}n \to \bar{p}n} = |a_1|^2$$

we obtain the inequality

$$\sigma_{\bar{p}p \to \bar{p}p}(\theta) + \sigma_{\bar{p}p \to \bar{n}n}(\theta) \geq \frac{1}{2}\ \sigma_{\bar{p}n \to \bar{p}n}\ (\theta)$$

which has been given by Malenka and Primakoff, Phys.Rev.105, 338 (1957).

This theorem is very similar to one known for nucleon - nucleon scattering, namely

$$\sigma_{np\to np}(\theta) + \sigma_{np\to pn}(\theta) \geq \frac{1}{2}\,\sigma_{pp\to pp}(\theta)$$

$$\sigma_{np\to np}(\pi-\theta)$$

Optical model calculations.

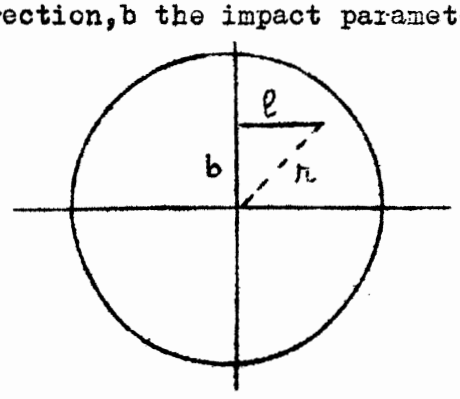
It is perhaps worth while to try to explain the cross sections of antiprotons on complex nuclei in terms of the elementary nucleon-antinucleon cross sections. At the present time we look at these calculations to see if everything is self-consistent. Originally they were regarded as a method of getting some estimate of the elementary interaction cross section, before direct measurements on hydrogen or deuterium had been made.

One of the first of these calculations for antiprotons was done by Gerson Goldhaber .

He used the following expression:

$$\sigma_{reaction,\,z} = 2\pi \int_0^\infty \left[1 - e^{-\int \sigma_0 \rho\, d\ell}\right] b\, d\,b$$

where is the elementaty nucleon-antinucleon total cross section (average of $\bar{p}n$ and $\bar{p}p$), ρ is the density of nucleons at a distance r from the center of the nucleons, ℓ is a coordinate parallel to the beam direction, b the impact parameter, $b^2 + \ell^2 = r^2$



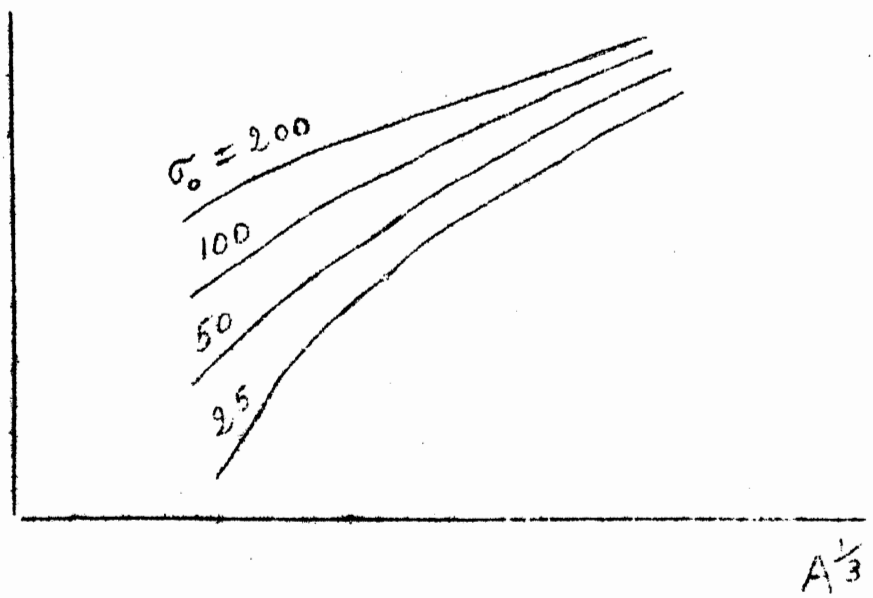
The factor

$$e^{-\int \sigma_0 \rho \, dl}$$

is then the transparency of the nucleus at impact parameter b. For the density function ρ Goldhaber took the Saxon potential, developed largely in connection with the scattering of protons

$$\rho = \rho_c \frac{1}{1 + e^{(r-R)/a}}$$

where $R = 1.33\, A^{1/3}$, with R and r expressed in units 10^{-13}cm. Goldhaber obtained calculated curves for antiproton cross sections as a function of atomic weight A for various values of the elementary cross section.



Comparison of his calculated curves with observed cross sections in complex nuclei indicated the elementary cross-section was between 100 and 200 mb. However, the radius used in the Saxon potential is considerably larger than that found by Hofstadter electron scattering. Use of the Hofstadter density distribution in the same calculation does not give agreement with experiment.

Apparently the difficulty can be overcome by taking into account the finite radius of interaction. Another calculation has been performed by Donald Keller in which the nucleon density ρ is replaced by the average density in a small spherical region about the point in question. The radius of the small spherical region may be taken from the elementary annihilation cross section (now known) or by adjusting it to give the correct cross section for hydrogen (for which $\rho(\vec{r}) = \delta(\vec{r})$). In either case good agreement can then be found using Hofstadter's nucleon density distribution

RESULTS FROM PHOTOGRAPHIC EMULSIONS.

The original intent of the emulsion work was to obtain as soon as possible an event that gave a proof of the annihilation property of the so-called antiproton that had been observed in counter experiments. The proof would involve finding an antiproton star in which the visible energy release in the star was definitely greater than the rest energy(plus kinetic energy, if any) of the incoming particle. When this was found it would be proof that this particle really had the property not just of changing its own mass-energy into visible energy but of annihilating a particle from the nucleus as well. This proof was supplied in the work of Chamberlain, Chupp, Fainpong, Goldhaber, Lofgren, Segrè, Wiegand, Amaldi, Baroni, Castagnoli, Franzinetti and Manfredini, Phys.Rev.102, 921(1956).

The work then shifted to studies of properties of antiprotons that are not direct consequences of the antinucleonic nature of particle.

At the present time the best summary of this work is contained in a collaborative paper of Barkas, Birge ,Chupp,Ekspong, Goldhaber,Heckman,Perkins,Sandweiss, Segrè,Smith,Stork, Van Rossum, Amaldi, Baroni,Castagnoli, Franzinetti and Manfredini, Phys.Rev.105, I037 (I957) . The paper describes 35 antiproton stars.

From the numbers of antiproton annihilations occurring at rest and the number occurring in flight it has been possible to estimate nuclear cross sections for emulsion for comparison with computed geometrical cross sections or with known meson cross sections. The results substantiate qualitatively the measurements with counters, at a lower energy ($\sim$ I40 Mev.) and with perhaps somewhat limited statistical accuracy.(Some results from a separated antiproton beam exposure were given at the Rochester Conference, I957, indicating a similar cross section based on 233 antiprotons.)

The annihilation stars have been analyzed to obtain information about the elementary (nucleon-antinucleon) annihilation process. The stars at rest show, on the average, the energy divided as follows:

(I005 $\pm$ I70) Mev in charged pions
(265 $\pm$ 20) Mev in nuclear excitation
(I50 $\pm$ I20) Mev in K particles
(448 $\pm$ 200) Mev in neutral particles (by substraction)

I868 Mev, known energy available.

The results above make it seem very probable that most annihilation processes lead predominantly to pions, and occasionally K-mesons, and that the observed nuclear excitation is due to collisions of mesons with the remainder of the nucleus, In fact, so : little energy appears as nuclear excitation that it seems that the annihilation processes must be taking place for the most part in the outer-most region of the nucleus where the density of nucleons is quite small.

If this were not true, one would expect more mesons to collide with the nucleus, yielding greater than the observed nuclear excitation. The stars in flight show a greater proportion of their e nergy appearing as nuclear excitation, but the fraction is still small enough to suggest that even in flight the average annihilation is taking place near the outside of the nucleus.

The observed charged-pion multiplicity (average number of observed charged pions per annihilation) is 2.6 ± 0.3. When corrected for an estimated loss in scanning this number becomes 2.9. By assuming charge independence - (total number of pions equals 3/2 the number of charged mesons) and correcting for an average I.0 mesons absorbed to give nuclear excitation, it is possible to obtain the estimate 5.4 ± 0.8 for the elementary pion multiplicity. An independent estimate of the multiplicity has been obtained by dividing the total available energy for pions by the (corrected) average energy of pions. This result is $5.2. \pm 0.5$, in excellent agreement of the number above. The combined result is given as $5.3. \pm 0.4$ pions per elementary annihilation.

In the same work seceral methods have ben used to measure the antiproton mass, on antiprotons. mostly that come to rest before annihilating. The combined result indicates a mass equal to the proton mass, with an accuracy of 2.5 percent.

From the 35 annihilation stars, 4 K-particles were found in all. Since none of the K particles came to rest in the emulsion stack their identity had to be established by ionization and multiple-scattering measurements.(In a later exposure, however, one K-meson from annihilation did stop in the stack and was positively identified). If we assume half of the K mesons are charged, and hence detectable, we would say the ratio of K mesons to pions is 8/I85, or one K meson for each 23 pions.

The Fermi statistical theory does not seem capable of explaining the observed pion multiplicity in annihilation except with the unreasonable choice of interaction volume of $12 \left(4\pi/3\right) \left(\hbar/\mu c\right)^3$, where μ is the pion mass.

Theory of the large elementary annihilation cross sections

Koba and Takeda have tried to explain the large cross section by utilizing a wave mechanical effect that allows the absorption cross section of an absorbing region to be slightly larger than the geometrical cross section (depending on the particle wave length). By using an absorbing region of radius $(2/3) \left(\hbar / \mu c\right)$ they have calculated results of 61 mb annihilation cross section and 33 mb scattering. In spite of their efforts the calculated annihilation cross section is too small and scattering cross section too large to give agreement with presently available experimental results.

Chew has suggested that since antinucleon-nucleon forces should be closely related to nucleon-nucleon forces (especially for large separation of the particles) it should be possible to use information about nucleon-nucleon forces in explaining annihilation. His hope is that one may be able to obtain an effective potential for antinucleon-nucleon interaction with a strongly absorbing region only at very small distances which would give agreement with experiment. As yet there are no numerical results to bear out such a hope.

Determination of charged-particle trajectories by current-carrying wires.

In all of the experiments at Berkeley that have required magnetic lenses, the action of the lenses has been checked by using a current-carrying wire under tension to imitate the particle trajectories in the magnetic field. A brief argument may be used to show the equivalence, at least for the simplest case in which the magnetic field is perpendicular to the plane of the orbit . The relation between $B\rho$ =(magnetic field) times (radius of curvature) and momentum p is

$$B\rho = \frac{pc}{e}$$

The curvature is then

$$\frac{1}{\rho} = \frac{eB}{pc}$$

for the particle trajectory. On the other hand, a current carrying wire under tension T must have the following equal and opposite forces on a length ds of the wire:

$$J\,ds\,B = T\frac{d\theta}{ds}\,ds$$

if it is in equilibrium. This gives the wire curvature as:

$$\frac{1}{\rho} = \frac{d\theta}{ds} = \frac{JB}{T}$$

where J = wire current in e.m.u., T = tension in c.g.s.units. This curvature is again proportional to B , so the wire will imitate the particle trajectory at all points in the field

(no matter how non-uniform the field) if

$$J = \frac{T\,e}{p\,c}$$

typical values might be $p = 1\ Bev/c$ (or $pc = 1 Bev = 1.6 \times 10^{-3}$ erg)

$J = 0.2$ e.m.u $= 2$ amps, $T = 6.7 \times 10^{5}$ cgsu $= 690$ grams

The current in the wire should be opposite in direction to the beam current being represented. Also the wire is unstable if it extends from one focus to another in the magnetic system. The wire may start at one focus, but then it must be terminated at a fixed point that falls short of a focus. By making lateral position measurements at a pair of points along the wire, one obtains the data from which an extrapolation can be made to the position of the focus.

Difficulty of electrostatic separation of short-lived particles.

The separation of one mass particle from another by electrostatic deflection is not practicable unless the particles to be selected are reasonably long lived. Since the separation, if it is to be performed, must best be done with particles that are not too relatisvistic, we may use a non-relativistic treatment to determine the time a particle must spend in the electrostatic field in order that it suffer a reasonable deflection. Letting s be the lateral deflection, t the time, a the acceleration, E the electric field in e.s.u., e the electronic charge we have

$$s = \frac{a t^2}{2} \qquad t = \sqrt{2s/a}$$

$$a = \frac{e E}{M} = 3 \times 10^{16}\ cm/sec^2$$

if the electric field is 30.000 v/cm or I00 e.s.u, and M is taken as the proton mass for this example. If we require a deflection S of I0 cm we find

$$t = 3 \times 10^{-8} \text{ sec}$$

This clearly shows that electrostatic separation is not useful for particles with lifetimes much less than $I0^{-8}$ sec.

Proton-Proton differential scattering cross-section.

Only a very brief account of proton-proton scattering will be included here, as far as measurements without polarization are concerned. Details are here taken from Chamberlain, Segrè, and Wiegand, although many others have made similar experiments at a great variety of proton energies.

The cyclotron external proton beam was obtained by multiple coulomb scattering at very small angles on a target within the cyclotron vacuum tank. (Measurements have since shown the beam unpolarized). In the process of getting the beam away from the cyclotron magnetic field a magnetic analysis was performed. Range curves, obtained using two ionization chambers with a variable absorber between them, showed the beam to be of only one component which was therefore assumed to be pure protons. The beam was collimated by a one-meter long brass collimeter which could be accurately aligned to be parallel to the proton beam. The proton energy was about 340Mev.

Argon-filled parallel-plate ionization chambers in connection with integrating electrometers were used as a secondary standard of integrated beam. The primary standard was a Faraday-cup consisting of a 15cmx15cm cylindrical brass block supported in vacuum. See Phys.Rev.83 -923(1951)

The targets consisted of CH_2 and C, or liquid hydrogen, in various phases of the work.

The counters have most recently been scintillation counters (with plastic scintillators) and the solid angle within which particles were counted was determined by the geometrical size of one of the counters. In some cases, both scattered proton and recoil proton were counted in coincidence, in which case one of the counters was small, and defined the solid angle, and the other counter was quite large, so it would be sure to count all the companion protons of protons that reached the small counter.

Only a very qualitative description of the proton-proton interaction can be obtained unless the scattering measurements are augmented by polarization experiments.

Polarized Proton Beams.

With the discovery by Oxly, Cartwright, Rouvina, Baskir, Klein, Ring and Skillman (Phys.Rev. 91, 419 (1953), see also Phys.Rev.93, 806 (1954) by the first three authors) that protons could be highly polarized by scattering from targets such as beryllium and carbon, it became possible in a number of laboratories to use highly polarized-proton beams. See Phys.Rev.102, 1659, (1956) for references to work at Berkeley, Chicago, Carnegie Tech. Harvard and Harwell.

To obtain the polarized beam, protons are scattered, let us say to the left, at a first target (perhaps carbon) which is usually within the cyclotron vacuum tank. To demonstrate the polarization of the once-scattered protons they are scattered a second time, perhaps on a target of the same material. If they show a preference for scattering to the left (rather than at an equal angle to the right) at the second target then the polarization has been demonstrated.

For scattering of protons (spin I/2) on a spin-zero target the scattering matrix must be of the form

$$M = g(\theta) + h(\theta)\,\vec{\sigma}\cdot\vec{n}$$

in which

$$\vec{n} = \frac{\vec{k}\times\vec{k}'}{|\vec{k}\times\vec{k}'|}$$

and $\vec{k}$ and $\vec{k}'$ are proton momenta before and after scattering. $\vec{n}$ is a unit normal to the scattering plane. See Michel's notes, Ecole d'Eté de Physique Théorique, 1957, for mathematicalmethods.

We use the following relationship: the beam polarization is:

$$P_{beam} = \frac{N_{up} - N_{down}}{N_{up} + N_{down}}$$

where N_{up} and N_{down} are the numbers of beam particles with spins directed up and down along some chosen direction - the direction on which a polarization component is desired to be known. Also

$$\vec{P}_{beam} = \langle \vec{\sigma} \rangle_{beam}$$

The polarization induced by a given scattering process (as scattering on carbon at 310 Mev at 13° lab.) (when an unpolarized beam is used) will be called P. It may be related to other parameters as follows: For an unpolarized beam the density matrix is $\rho = 1/2$. The beam after scattering may be represented by the density matrix

$$\rho' = M \rho M^* = (g + h\vec{\sigma}\cdot\vec{n})(1/2)(g^* + h^*\vec{\sigma}\cdot\vec{n})$$

and the expectation value or average value of the spin (times the intensity) in the scattered beam is

$$I\langle\vec{\sigma}\rangle = Tr(\rho'\vec{\sigma}) = Tr(M\rho M^*\vec{\sigma})$$

$$= Tr\left[(g + h\vec{\sigma}\cdot\vec{n})(1/2)(g^* + h^*\vec{\sigma}\cdot\vec{n})\vec{\sigma}\right]$$

$$= (g h^* + g^* h)\vec{n}$$

while the intensity is

$$I = Tr\left(M \rho M^*\right) = |g|^2 + |h|^2$$

so the polarization is

$$\langle \vec{\sigma} \rangle = \frac{(g h^* + g^* h)}{|g|^2 + |h|^2}$$

Therefore after one scattering we have

$$P_{beam} = P_1 = \frac{(g h^* + g^* h)}{|g|^2 + |h|^2}$$

This beam may be represented by the density matrix

$$\rho = \left(\frac{1}{2}\right)\left(1 + \vec{P}_1 \cdot \vec{\sigma}\right)$$

At a second scattering, the scattering matrix may again be given as $M = g + h\,\vec{\sigma}\,\vec{n}$ and the intensity for scattering in a given direction will be

$$I = Tr\left(M \rho M^*\right) = \left(\frac{1}{2}\right) Tr\left[(g + h\vec{\sigma}\cdot\vec{n})(1 + \vec{P}_1\vec{\sigma})(g^* + h^*\vec{\sigma}\cdot\vec{n})\right]$$

$$= \left(|g|^2 + |h|^2\right) + \left(g h^* + g^* h\right)\vec{P}_1 \cdot \vec{n}$$

Thus the scattered intensity depends on scattering angle θ through g and h and also on the azimuthal scattering angle through the product $\vec{P}_1 \cdot \vec{n}$ of the polarization vector with the unit normal $\vec{n}$ to the scattering plane.

We may also write $I_0 = |g|^2 + |h|^2$

$$\vec{P}_2 = \frac{gh^* + g^*h}{|g|^2 + |h|^2}\,\vec{n}$$

$$I = I_0\left(1 + \vec{P}_1 \cdot \vec{P}_2\right) = I_0\left(1 + P_1 P_2 \cos\Phi\right)$$

Using $\Phi = 0^\circ$ and 180° and letting the asymmetry be indicated as e we have

$$e = \left(I_0 - I_{180}\right) / \left(I_0 + I_{180}\right)$$

$$= \left(1 + P_1P_2 - 1 + P_1P_2\right) / \left(1 + P_1P_2 + 1 - P_1P_2\right) = P_1P_2$$

If, in particular, the conditions at both targets (I and 2) are the same then we may say

$$P_1 = P_2 = P$$

$$e = P^2$$

This means measurement of e determines P except for sign. (Its direction could be guaranteed from theory (with parity conservation) to be in the same direction $\vec{n}$ as indicated above.)

The sign of P is positive ($\vec{P}$ parallel to $\vec{n}$) for scattering at angle of 5° to 20° from light elements at about 300 Mev. : as determined by Marshall and Marshall (Phys.Rev.98 I398 (I955)) by degrading a high-energy polarized beam down to about 5 Mev. and scattering from helium under circumstances governed by a complete phase-shift analysis that gave a known polarization versus energy. The sign of the polarization is the same as that predicted from the sign of spin-orbit coupling used in shell-model theory.

Thus far we have discussed only scattering of protons by a spin-zero nucleus. Wolfenstein has shown that exactly the same considerations apply to the scattering by unpolarized nuclei of finite spin providing that the collisions are time-reversible. This means that the collisions must be elastic. Thus some care must be taken to ensure elastic collisions at both targets to ensure the relation

$$e = P^2$$

In part, absorbers or magnetic analysis have been used to guarantee that only the highest energy particles leaving the targets were utilized. Also, the scattering angle is chosen small enough so that diffraction scattering predominates over inelastic scattering.

The polarized proton beam most used in Berkeley has a polarization $P = 0.76$, at 315 Mev. It was made by scattering from beryllium at 13°.

Triple Scattering Experiments.

By restricting our attention to interactions satisfying parity conservation we may see which components of polarization of particles after scattering can depend on the various components of polarization before the scattering. We are presuming here that a first scattering would be used to produce the polarized beam, a second scattering (the one under investigation) would change the polarization in some way to be determined, and a third scattering would be used to measure the polarization of the scattered particles.

In the beam, we could say that the quantity $\langle \vec{\sigma} \rangle_1 \cdot \vec{n}$ is a scalar quantity, being the product of two pseudovectors, while the quantities $\langle \vec{\sigma} \rangle_1 \cdot \vec{k}$ and $\langle \vec{\sigma} \rangle_1 \cdot \vec{n} \times \vec{k}$ are pseudo scalars.

The subscript 1 means "after the first scattering" and would therefore refer to the beam used at the second target. The subscript 2 will indicate "after the 2nd scattering" Using the same considerations for the components of spin after scattering, we would allow the final spin components to depend on the initial spin components only to the following extent:

$$I \langle \vec{\sigma} \rangle_2 \cdot \vec{n} = I_0 \left(P + D \langle \vec{\sigma} \rangle_1 \cdot \vec{n} \right)$$

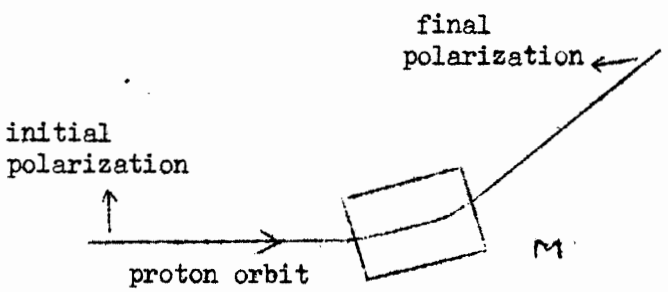
$$I \langle \vec{\sigma} \rangle_2 \cdot \vec{s} = I_0 \left(A \langle \vec{\sigma} \rangle_1 \cdot \vec{k} + R \langle \vec{\sigma} \rangle_1 \cdot \vec{n} \times \vec{k} \right)$$

$$I \langle \vec{\sigma} \rangle_2 \cdot \vec{k}'_2 = I_0 \left[A' \langle \vec{\sigma} \rangle_1 \cdot \vec{k} + R' \langle \vec{\sigma} \rangle_1 \cdot \vec{n} \times \vec{k} \right],$$

where I is the scattered intensity at the given angle and I_0 the scattered intensity at the same angle for an unpolarized beam, $\vec{s}$ is a unit vector $\vec{s} = \vec{n} \times \vec{k}'$, with $\vec{k}'$ the unit vector in the direction of momentum after scattering, and the coefficents D, A, R, A', R', are functions

of scattering angle θ , as defined by Wolfenstein, Phys.Rev. 96, I654 (I954). P is polarization, as defined previously. Clearly P is the polarization that would be induced by the scattering at the (second) target if an unpolarized beam were incident there, P is determined by a double scattering experiment but the other coefficients could be determined by triple scattering experiments, at least in principle.

Only the quantities D and R can be measured in the simplest triple scattering experiments because only they can be treated with a transversely polarized beam and an analyzer for transverse polarization. Some work has been done to measure the quantity A, by bending a (transversely) polarized proton beam in a magnetic field. Due to the large anomalous magnetic moment of the proton a longitudinal polarization can be induced by a reasonable bending in a magnetic field:



In principle R' could me measured using the same technique but this has not yet been done, and would be experimentally more difficult because the beam of particles after the second scattering is usually much more divergent than the beam hitting the second target.

For complex nuclei (carbon and aluminium) a few measurements of R and D have been made; see Phys.Rev.I02, I659 (I956). The measurements of R are useful to determine more fully the scattering matrix. Consider the scattering matrix M for scattering from spin-zero nucleus.

M may be written as

$$M = g + h\,\vec{\sigma}.\vec{n} \quad ,$$

as mentioned previously. Measurements of I_o and P can be taken into account by writing

$$g + h = \left[I_o(1+P)\right]^{1/2} e^{i\alpha}$$

$$g - h = \left[I_o(1-P)\right]^{1/2} e^{i(\alpha+\beta)}$$

As far as I_o and P are concerned these are just identities, in that these expressions automatically give the indicated I_o and P. The phase factor α is an over-all phase which is not available to experimental measurement directly, but the phase β does not affect the quantity R. Wolfenstein has shown that R is given by

$$R = (1 - P)^{1/2} \cos(\Theta - \beta)$$

in which Θ is the angle of scattering.
Comparisons of R from experiment and from calculations based on potential interactions to represent a complex nucleus have not given very good agreement. This presumably indicates that in this case we do not have a satisfactory potential model to represent the scattering process.

From spin-zero targets the parameter D should always be 1.- It has been measured for carbon, mainly to check the technique of measuring D under especially difficult conditions.

Experimental Conditions in a Triple-Scattering Experiment.

The details of a triple scattering experiment can perhaps best be given in connection with the scattering from hydrogen

(second target). The first target (beryllium) was within the cyclotron vacuum tank. The incident beam was of the order of 10^{-6} amperes, The protons scattered to the left at 13° were formed into a beam and magnetically analyzed to exclude all except the highest energy component, which was presumed elastically scattered. A collimator 5 cm in diameter allowed a beam of about 10^{6} protons per second outside the cyclotron shielding wall, and the beam was reasonably parallel. Its energy was 315 Mev, and the polarization 0.76.

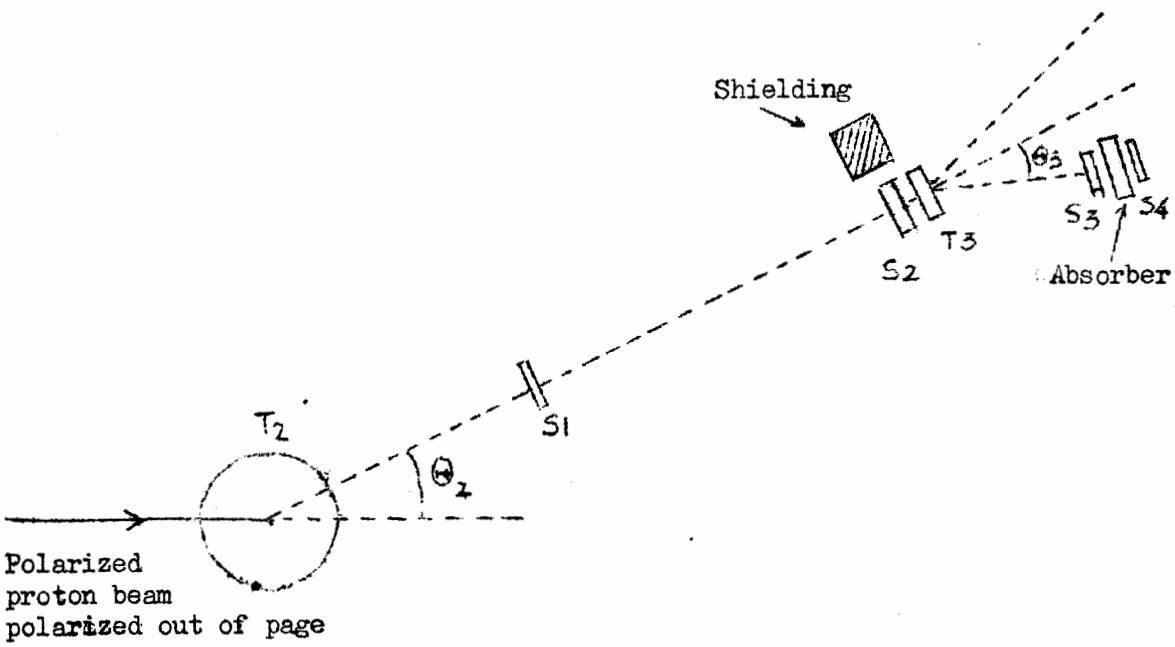
The polarized beam impinged on the liquid hydrogen target (16cm thick). The twice-scattered beam was formed, not by a collinator, but by two scintillation counters in coincidence. The counters were 8 cm square, and were respectively 50 cm and 100 cm away from the liquid hydrogen target in a typical case. The intensity of the beam so formed was about 30 protons per second.

The third scattering target was of carbon or beryllium, placed directly behind the second of the beam-forming counters mentioned above, and the three-times scattered protons were counted in a counter telescope either to "left" or to "right" at about 13° scattering angle. Absorbers were used in the counter telescope to limit the counted protons to reasonable energies. The counting rate was typically 0.03 per second. A sketch of the apparatus at the 2nd. and 3rd. targets looks as follows, as it has been used to measure D. For that measurement all scatterings occur in a horizontal plane.

The targets are indicated as T2 (hydrogen) and T3 (carbon or beryllium). Scintillation counters are indicated S1, S2, etc. Scattering angles are indicated Θ_2, and Θ_3.

The only shielding used in this part of the apparatus consisted of two iron blocks, one on each side of counter S2(only one of which is shown in the sketch). Great care was used to be sure particles could not scatter off these shielding blocks and at the same time register in all counters S1, S2,S3,S4.

This means a little space was left between counter S2 and the shielding block. The shielding blocks served only to reduce the S3 and S4 counting rates.



The triple scattering experiments on hydrogen could not be extended to very large angles θ_2 (even though the whole range 0° to 90° lab. would be of theoretical interest) because the analyzing power of the third-target arrangement drops rapidly for proton energies at the third-target of less than 150 Mev. Since statistics are limited and the scattering angle at the third target rather poorly defined, good analyzing power was somewhat essential to a convincing result.

Although absorber was used to limit the counted protons to those that had scattered nearly elastically at both the 2nd.

and the 3rd.targets, some inelastic scattering was certainly included. This meant that the 3rd.target analyzing power had to be checked by using a beam of known polarization and observing how much left-right asymmetry the known beam produced. When this calibration was done great care was taken to ensure that the beam incident on the last target was as similar as possible to that used in the actual experiment. Target 2 was replaced by a piece of uranium or lead to give (by multiple scattering) a rather divergent beam leaving target 2. The angle Θ_2 was reduced to zero so the 3rd.target was in the polarized beam. Absorbers of graphite were placed near the target 2 , position to allow the same average energy of proton to reach target 3 as reached that point when the experiment on hydrogen was performed. Through the relation

$$P_3 = \frac{e_3}{P_{beam}} = \frac{e_3}{0.76}$$

The analyzing power P_3 of the 3rd target was determined.

The quantity D was measured at several angles between 23° and 80° c.m., and R over the same range of angles. The quantity A was measured at 3 angles only, ranging from 25° to 76° c.m. See Phys.Rev.105, 288 (1957)

Theory of Proton-Proton Scattering Analysis.

Wolfenstein, Ashkin, Dalitz, and perhaps others have shown that for the proton-proton system the scattering matrix can always be written as:

$$M = B\,S + C\,(\sigma_{1N} + \sigma_{2N}) + (1/2)\,G\,(\sigma_{1K}\sigma_{2K} + \sigma_{1P}\sigma_{2P})\,T + (1/2)\,H\,(\sigma_{1K}\sigma_{2K} - \sigma_{1P}\sigma_{2P})\,T + N\,\sigma_{1N}\sigma_{2N}\,T$$

where B, C, G, H, and N are arbitrary functions of the scattering angle (indicated θ for c.m.angle), S is the singlet projection operator, T is the triplet projection operator, I and 2 respectively indicate incident proton and target proton, σ_{IN} indicates the spin component of incident proton in the direction of the unit normal $\vec{n}$ ($\sigma_{IN} = \vec{\sigma}_1 \cdot \vec{n}$) and the directions of unit vectors $\vec{K}$ and $\vec{P}$ are given by difference and sum of final and initial momenta $\vec{K} = \vec{k}' - \vec{k}$ and $\vec{P} = \vec{k}' + \vec{k}$ (except $\vec{K}$ and $\vec{P}$ are <u>unit</u> vectors in those directions). The form of M is dictated by these requirements:

I°) The terms should be linearly independent.

2°) They should be symmetrical in indices 1 and 2.

3°) The terms must retain their sign under time reversal.

In applying the last condition notice that $\vec{\sigma}$ and $\vec{n}$ both change sign under time reversal.

In order to avoid the necessity of antisymmetrizing the initial states it is convenient to place requirements on the coefficients B,C, etc so that the scattering occurs only into states that satisfy the Pauli principle. This requires that B be an even function of (cos θ), G odd, H even, N odd, and Cbe an even function of cos θ. In the case of C it is more proper to say it is an even function of (cos θ) times(sin θ). In applying the Pauli-principle conditions note that on exchange of the two outgoing protons

$$\vec{K} \to -\vec{P} \text{ and } \quad \vec{P} \to -\vec{K} \text{ and } \quad \vec{n} \to -\vec{n} .$$

Note also that the singlet spin state is antisymmetric on interchange.

Given the density-matrix method discussed by Michel(notes, Ecole d'Eté de Physique Théorique, I957) it is not too difficult, but somewhat tedious, to obtain the expression for I_0, $I_0 P$, etc

in terms of the coefficients B,C, etc. . Here I_o is the differential scattering cross-section (for pp scattering) for unpolarized beam. Since the density matrix is in this case 4 x 4 the first such relation is

$$I_o = (1/4)\, Tr\,(M M^*) .$$

The resulting expressions are

$$I_o = (1/4)|B|^2 + 2|C|^2 + (1/4)|G-N|^2 + (1/2)|N|^2 + (1/2)|H|^2,$$

$$I_o P = 2\, Re\,(C^* N)$$

$$I_o (1-D) = (1/4)|G-N-B|^2 + |H|^2$$

$$I_o R = (1/2)\, Re\,[(G-N)^*(N+H) + B^*(N-H)]\cos(\theta/2) + Im\,[C^*(B+G-N)]\sin(\theta/2)$$

$$I_o A = -(1/2)\, Re\,[(G-N)^*(N+H) + B^*(N-H)]\sin(\theta/2) + Im\,[C^*(B+G-N)]\cos(\theta/2)$$

The last two relations need some comment in that the components of polarization analysis would be different for R and A depending on whether center-of-mass or laboratory coordinate system were used. The expressions given here refer to the laboratory system, if the transformations are made quite non-relativistically.

Phase-Shift Analysis.

Unfortunately we have at one given angle only 5 experimental measurements namely I_o, P, D, R, and A, while we would need

9 measurements to fix the 5 complex numbers B,C, G, H, and N except for one overall phase factor. This means that to get definite results we must correlate knowledge at one angle with that at other angles through a phase-shift analysis and the assumption that waves higher than, say, f-waves, are not prominent in the scattering. For this purpose B,C, and so forth must be expressed in terms of the phase shifts. With some effort, and with several people calculating independently, this has been done. The result is given here only for B, which involves only singlet states and therefore (Pauli principle) only even orbital angular momentum states.

$$B = (1/8ik)\left[(8\alpha_0 - 20\alpha_2 + 27\alpha_4) + (60\alpha_2 - 270\alpha_4)\cos^2\theta + 315\alpha_4\cos^4\theta\right].$$

For simplicity coulomb effects have not been included here even though they were included in the actual computation. In the above expression ,

$$\alpha_\ell = e^{2i\delta_\ell} - 1$$

in which δ are the phase shifts k, is the center-of-mass wave number.

The computation of the phase shifts was carried out by Stapp,Ypsilantis, and Metropolis (Phys.Rev.105, 302 (1957)). They used a high speed computer and were able to include up to h waves in the analysis, showing that g- and h-wave phase shifts are quite small in all acceptable solutions.

Their calculation procedure was essentially that used by Fermi for the meson-nucleon scattering problem. The machine was asked to make successive small charges in a given set of phase shifts until the best possible fit was obtained to the experimental results for I_o, P, D, R, and A at the various angles at which the quantities were measured. The goodness of fit was measured by the smallness of the quantity.

$$\eta = \sum_i \frac{\left[\text{Observable}_i\ (\text{calc.}) - \text{Observable}_i\ (\text{expt.})\right]^2}{(\text{Exptl. error}_i)^2}$$

The machine had 36 experimental measurements with which I4 phase shifts and mixing parameters were to be determined.

Unfortunately 8 sets of phase shifts (8 solutions) were found that gave reasonable agreement with the experiments. By comparing with meson production information in the reaction.

$$p + p \rightarrow \pi^+ + d$$

three solutions could be rejected, leaving 5 solutions acceptable. The argument is given in Gell-Mann and Watson, Annual Reviews of Nuclear Science (I954), and concerns the phase angle T_o of meson production from the ^1S_o state of two protons and the phase angle T_1 of meson production from the $^3P_1$ state. In each case the phase angles are given as relative to the production from the $^1D_2$ state of the two protons.

The five remaining solutions (numbered 1, 2,3, 4 and 6 in Stapp,Ypsilantis, and Metropolis) have been discussed by Bethe and by Gammel and Thaler at the Rochester Conference, I957. Bethe finds, using an argument of Heckrotte, that solution 6 is unacceptable because the scattering amplitudes given cannot explain the observed large polarization in proton-carbon(elastic)scattering. The argument hinges on the fact that the term in the scattering matrix with the coefficient H is a depolarizing term.

Using as the preferred direction the direction of the normal (to the scattering plane) $\vec{n}$, the term H leads from an initial state $\uparrow\uparrow$ to a final state $\downarrow\downarrow$ and so reverses the spin direction of both the incident and the struck proton.

In proton-carbon scattering this term is suppressed by the fact that we require elastic scattering. Thus a large term in H is required to explain the larger polarization in proton-caebon scattering than in the p-p scattering. The argument is slightly complicated by the fact that the carbon scattering depends also on n-p scattering. However, the argument may be used to eliminate solution 6 regardless of assumptions about n-p scattering.

Solutions 1, 2, 3 and 4 have been discussed by Gammel and Thaler. They find that solutions 2 and 4 give unreasonable singlet phase shifts when compared with information at lower energies. They argue that at low energy (for example I8 Mev) the s-wave and a small d-wave scattering are well known and can be used to fix the parameters of a singlet potential. Since there is no tensor force or spin-orbit force in singlet states they argue that a singlet potential concept should be reliable. Since solutions 2 and 4 do not fit such a potential they discard these.

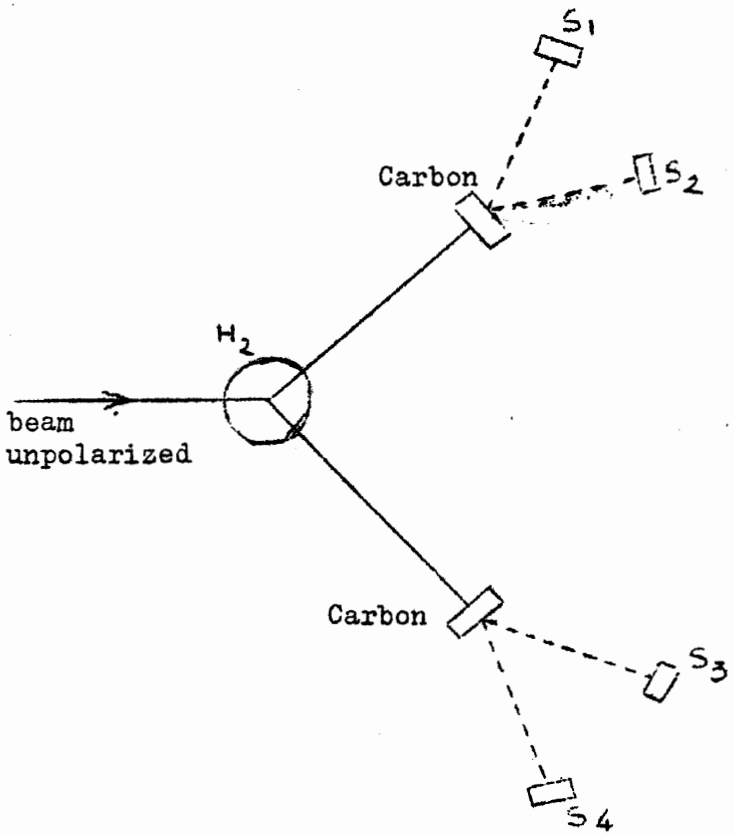
Of the remaining two solutions they find only solution 1 showing the effect of a consistent sign of tensor force and fitting reasonably to lower energy data. They believe only solution 1 of Stapp, Ypsilantis and Metropolis should be acceptable.

Polarization Correlation Experiments.

The arguments for rejecting all but solution 1 are persuasive but not absolutely convincing. We hope that within the coming year additional experimental information may be

available which will make the choice more firm, and perhaps add to the reliability of the whole analysis (which is based in part on some experiments which have been performed only once).

One of the most promising types of experiments is the polarization correlation experiment. In its simplest form it consists of a pp scattering made with an unpolarized proton beam, with the polarisation state of both outgoing protons analyzed by a further scattering of each.



One would compare the coincidences S1S3 (or S2S4) with coincidences S1S4 (or S2S3) to see to what extent the spins of the outgoing protons are parallel or antiparallel. At 90° c.m.angle, this experiment has the interesting property that it would tell directly the fractions of singlet and of triplet scattering-something not done up to the present time. Several varieties of polarization correlation experiment could be performed,some

requiring an incident proton beam that is polarized. It is expected that experiments of this type will be attempted at Liverpool,Berkeley and perhaps Cern, in the next few years.If they are successful there seems no question but that they will succeed in resulving any remaining doubt about the $p p$ scattering amplitudes and phase shifts.

The analysis of $n p$ scattering has not yet been carried as fas as the $p p$ scattering analysis. However, a great deal of experimental information exists and it should be possible to carry the $n p$ analysis at least as far as the $p p$ analysis providing one is willing to use chargo independence and take the ^1S, ^3P, ^1D, ^3F,etc scattering directly from the analysis.

The hope throughout all this work is that we may reach the point of being able to say positively whether all nuclear structure can be explained in terms of 2-body interactions, and, can fully specify the nucleon-nucleon interaction that should be explained in detail through meson theory.

Antinucleon-Nucleon Interaction.

In the course of the coming year it is hoped it may be possible to stop antiprotons in a liquid-hydrogen bubble chamber. Besides giving direct information about the elementary annihilation process(multiplicity of mesons, etc.) such an experiment may give information on the size of the annihilation matrix elements.

To estimate the matrix element size (or the lifetime for antinucleons in nuclear matter) it is necessary to determine how annihilation competes with some reaction. Bethe and Hamilton (Nuovo Cimento 4, 1(1956)) have shown how the ab-

solute rate of annihilation could be established by determining whether antiprotons in hydrogen and helium annihilate from s, p, or d states. There are sufficient selection rules on the anti - proton-proton system so that the characteristics of the annihilation (especially the annihilation into 2-meson or 3-meson final states) can be expected to determine the orbital state from which annihilation occurs.

The selection rules for annihilation have been given by Michel in 1953 (Nuovo Cimento 10, 319 (1953). Besides the usual requirements of parity and angular momentum there is an interesting selection rule based on the operator I shall call C R. C is charge conjugation and involves a reflection with respect to the xz plane in the isotopic spin (I-spin) space whose axes correspond to the components I_x, I_y, I_z of the isotopic spin. R is a rotation of 180° about the I_y axis in the same space, which I shall also call the charge-symmetry operator even though I believe it may differ from the charge-symmetry operator used by some people. The combined operation CR makes a complete inversion in the I-spin space.

Since the meson wave-function is an ordinary vector in this space all three components of the meson wave function change sign under this transformation. Thus any state having n mesons behaves as

$$(-1)^n$$

under this transformation.

The behaviour of the nucleon (or antinucleon) states under R can be obtained from the usual Pauli matrices, representing the rotation by π around direction y by

$$e^{i\pi\sigma_y/2} = \cos(\pi/2) + i\sigma_y \sin(\pi/2) = i\sigma_y$$

which means

$$R p = -n, \quad R n = p, \quad R\bar{n} = -\bar{p}, \quad R\bar{p} = \bar{n}$$

The behaviour on the $\bar{p}p$ system under charge conjugation C has nothing to do with isotopic spin (it is like the behaviour of $e^- e^+$). For example the $^1S_0$ state of $\bar{p}p$ may be seen to be a state for which C = 1 by any of the following arguments:

1. Michel showed it for any state of J = 0 (Ecole d'Eté de Physique Théorique, 1957) .

2. Fermi constructed a neutral pion from this state, for which we know C = 1 because a reflection in the xz plane does not affect φ_y which is the neutral part.

3. This state of positronium decays into two gamma rays and C = 1 for even number of gamma rays.

4. This state is odd on spin interchange, even on space interchange, hence must be even for charge conjugation to obey the Pauli principle.

5. Representing proton creation by a^+ and antiproton creation by b^+ and using arrows to indicate spin direction the operator to create one state starting from the vacuum state would be

$$a^+_\uparrow \, b^+_\downarrow - a^+_\downarrow \, b^+_\uparrow$$

which on charge conjugation becomes

$$b^+_\uparrow \, a^+_\downarrow - b^+_\downarrow \, a^+_\uparrow = - a^+_\downarrow \, b^+_\uparrow + a^+_\uparrow \, b^+_\downarrow$$

because the operators anticommute. Hence C leads to the same operator and C = 1.

Using the property we may construct a table of various decay possibilities in which the table is left blank if the decay is permitted, and when the decay is forbidden one reason for being forbidden is entered in the table. P indicates parity, J angular momentum, C charge conjugation, R charge symmetry, I fn means the I-spin function does not contain that particular final state, I indicates total I-spin

TABLE OF FORBIDDEN DECAYS FOR $\bar{p}p$ SYSTEM:

		P	R	C	CR	$\pi^+\pi^-$	$\pi^0\pi^0$	$\pi^+\pi^-\pi^0$	$\pi^0\pi^0\pi^0$
$^1S_0$	I = 0	−	+	+	+	P J	P J	C R	I fn
	I = 1	−	−	+	−	P J	P J		
$^3S_1$	I = 0	−	+	−	−	C R	C R		I fn
	I = 1	−	−	−	+		I fn	C R	C R
$^1P_1$	I = 0	+	+	−	−	P J	P J		I fn
	I = 1	+	−	−	+	P J	P J	C R	C R
$^3P_0$	I = 0	+	+	+	+			P J	P J
	I = 1	+	−	+	−	C R	I fn	P J	P J
$^3P_1$	I = 0	+	+	+	+	P J	P J	C R	I fn
	I = 1	+	−	+	−	P J	P J		
$^3P_2$	I = 0	+	+	+	+			C R	I fn
	I = 1	+	−	+	−	C R	I fn		

Clearly most decay possibilities are forbidden, so the decay events could be used to determine from which spin-angular momentum states the annihilation were occurring. Depending on the angular momentum states, we could tell whether the annihilations were occurring from ℓ states (hence with a very large matrix element) or from μ states (smaller matrix element). We would be comparing rate of annihilation with the known rate at

which the system emits X-rays as it falls through the successive orbital states, F, D, P, S of the system. Such experiments would decide whether the lifetime of an antinucleon in nuclear matter were as short as 10^{-24} seconds or as long as 10^{-22} seconds.

Following Wigner (Phys.Rev.1932): Be prepared.

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PHYSIQUE DES ELECTRONS DE HAUTE ENERGIE

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PHYSIQUE DES ELECTRONS DE HAUTE ENERGIE

1 - INTRODUCTION

La plus grande partie des expériences analysées dans le cours qui va suivre, ont été réalisées à l'Université Stanford avec l'accélérateur linéaire de 220 pieds (Mark III). Cet accélérateur produit des électrons dont l'énergie pouvait, jusqu'à l'année dernière, atteindre 600 Mev. Il a eté depuis reconstruit entièrement, et il est probable qu'il pourra produire prochainement des électrons de 1 Bev environ.

Les principaux travaux sur la diffusion des électrons par les noyaux ont été faits par Hofstadter et ses collaborateurs. Nous analyserons cependant aussi une expérience sur la mesure absolue de la section efficace de diffusion électron-proton, qui a été effectuée par le groupe de Panofsky. A ce dernier sont dues également les expériences sur l'électroproduction des mésons π et la photoproduction des paires de mésons μ, dont nous parlerons à la fin.

En plus des articles originaux, dont les références seront indiquées dans le texte, nous signalerons immédiatement deux articles de revue qui contiennent un grand nombre de renseignements sur la plupart des problèmes discutés dans ce cours.

- R. HOFSTADTER : Electron Scattering and Nuclear Structure, Rev. Mod. Phys. 28, 214, (1956).
- D. YENNIE, M. LEVY, D. RAVENHALL. Electromagnetic Structure of Nucleons. Rev. Mod. Phys. 29, 1'', (1957).

1 - INTERET DES MESURES DE DIFFUSION DES ELECTRONS

L'importance des expériences sur la diffusion des électrons de grande énergie est qu'elles apportent un grand nombre d'informations sur

les noyaux et la structure nucléaire. La diffusion élastique, analysée jusqu'à des énergies élevées, permet en principe de connaître de façon précise, l'étendue de la distribution de charge des noyaux, dans leur état fondamental, ainsi que la forme de celle-ci. Il en est de même pour la distribution du courant magnétique pour les noyaux doués d'un moment magnétique dans l'état fondamental.

Par ailleurs, la diffusion élastique permet d'obtenir de nombreux renseignements sur les niveaux excités des noyaux. Elle permet aussi de mettre en évidence les effets particuliers dûs aux déformations des noyaux qui ont un moment quadrupolaire important (apparition de niveaux de rotation en accord avec le modèle de Bohr et Mottelson). Enfin, l'utilisation d'électrons d'énergie très élevée (expérience à 400 Mev.), permet d'étudier l'influence des correlations entre protons à l'intérieur du noyau , qui donne à la distribution de charge un aspect granuleux.

En principe, les expériences faites au moyen d'électrons permettent une interprétation plus sure que celles effectuées avec des mésons ou des nucléons, car l'interaction électromagnétique est mieux connue théoriquement que les interactions du type mésoniques. De plus, le fait que la constante de structure fine ($\alpha = e^2/\hbar c = 1/137$) est relativement petite, permet d'avoir davantage confiance dans les calculs basés sur des méthodes de perturbation. Cependant, nous verrons que des difficultés existent, surtout pour des noyaux relativement lourds.

a) parce que Z , le nombre de protons, est élevé, ce qui a pour effet de contrebalancer la petitesse de α , puisque c'est $Z\, e^2/\hbar c = Z/137$ qui est alors le paramètre de développement.

b) parce que l'énergie des électrons est élevée, ce qui rend particulièrement incommodes les méthodes de calcul basées sur un développement en ondes partielles (avec integration numérique exacte des équations différentielles correspondantes).

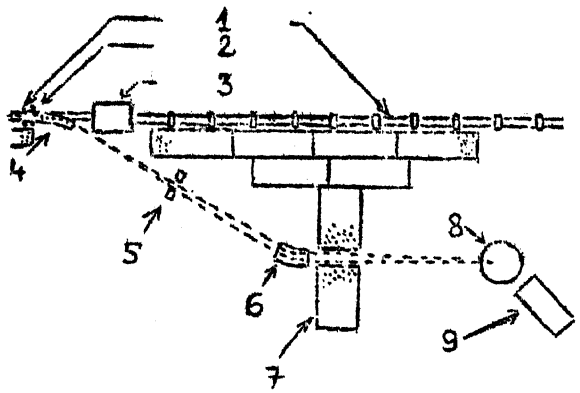
2 - DESCRIPTION RAPIDE DES DISPOSITIFS EXPERIMENTAUX.

Nous ne donnerons ici qu'un aperçu extrêmement bref des méthodes expérimentales. L'accélérateur linéaire de Stanford est longuement décrit dans l'article de M. Chodorow et al. Rev. Scient. Inst. 26, 134,(1953) Les dispositifs de scattering sont discutés en détail dans

l'article de Hofstadter cité plus haut, ainsi que dans plusieurs autres articles (1).

Deux séries distinctes d'expériences ont été faites :

a) A 190 Mev environ, en extrayant le faisceau d'électrons à peu près à la moitié de la longueur totale de l'accélérateur, au moyen d'une double déviation magnétique qui a pour effet de refocaliser le faisceau dans la chambre de diffusion (Fig 1).



1- Accelerator
2- Collimator
3- Beam Stopper
4- Deflecting Magnet
5- Slit
6- Deflecting Magnet
7- Concrete Shielding
8- Scattering Chamber
9- Spectrometer.

(1) : Voir par exemple : Hofstadter, Fechter et McIntyre, Phys. Rev. 92, 978 (1953) ; Chambers et Hofstadter Phys. Rev. 103, 1454 (1956).

L'accélérateur fournit 60 impulsions par seconde, et environ 2 à 3 X 10^9 électrons sont utilisables par impulsion dans la chambre de diffusion. L'énergie couvre une bande de 2 Mev environ. Les principales difficultés de détection des électrons diffusés proviennent de ce que le faisceau ne dure que 0,6 microseconde par impulsion et que toutes les mesures doivent être faites pendant ce temps. Par ailleurs, les compteurs à scintillation sont inutilisables en raison de l'important "background" de rayons γ et de neutrons. Les compteurs employés sont du type de Čerenkov. Le spectromètre magnétique à double focalisation permet de sélectionner les électrons que l'on désire étudier en particulier (pic élastique, spectre inélastique etc...) La chambre à diffusion peut soit être remplie de gaz (diffusion par l'hydrogène par exemple), soit contenir une monture sur laquelle sont placées les feuilles métalliques des substances étudiées.

En général, par ce dispositif, on n'effectue pas de mesures absolues des sections efficaces. Les comptages observés sont comparés à ceux obtenus sur des protons.

b) A 550 Mev, en extrayant le faisceau à la fin du tube accélérateur. Le dispositif est représenté sur la fig 2.

Le principe de la détection est le même que plus haut, sauf que, cette fois, le spectromètre analyseur est d'un type beaucoup plus perfectionné.

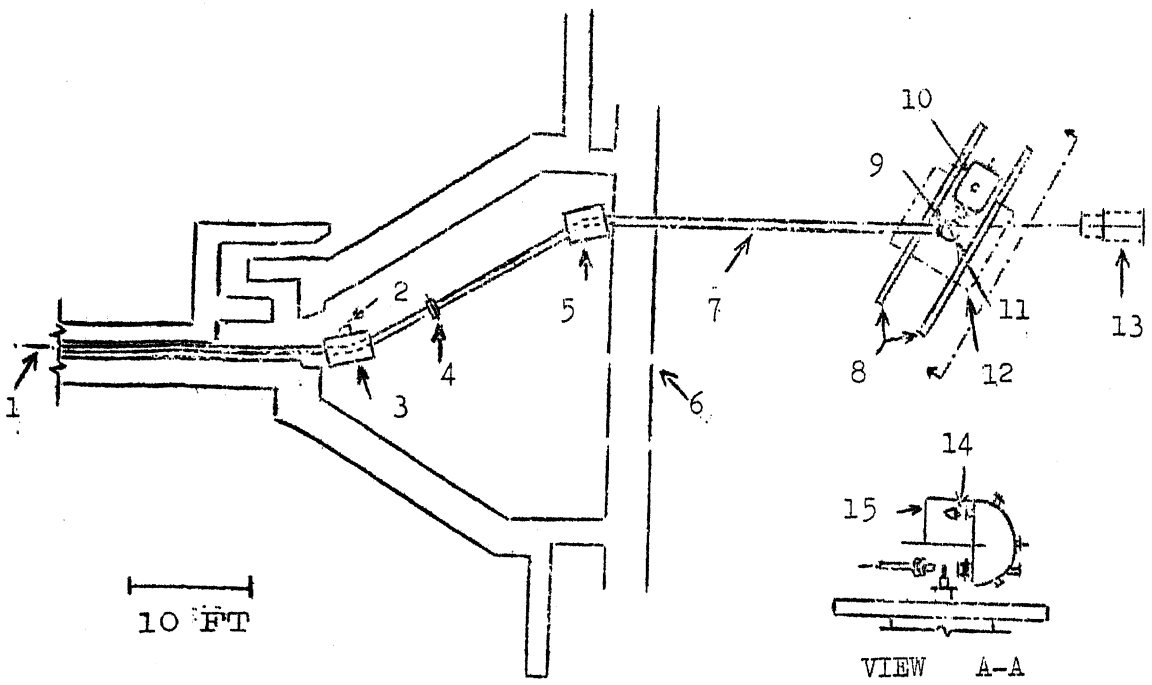
3- FORMULES ELEMENTAIRES DE DIFFUSION ELASTIQUE PAR UN CHAMP COULOMBIEN

La formule classique qui donne la section efficace différentielle de diffusion d'une particule de charge ze par le potentiel coulombien produit par une charge Ze est la formule de Rutherford :

$$\frac{d\sigma_R}{d\Omega} = \frac{z Z e^4}{16 E^2 \sin^4 \frac{\vartheta}{2}} \qquad (1)$$

Cette formule est pratiquement inutilisable pour le problème qui nous préocupe, car les électrons utilisés sont extrèmement relativistes ; de

(fig. 2).



1— Beam
2— Magnetic probe
3— Deflecting Magnet
4— Energy Definning Slit
5— Refocusing Magnet
6— Shield
7— Vacuum Pipe
8— Gunmount ways
9— Secondary Electron Monitor
10— Magnet
11— Target
12— Shield Platform
13— Faraday Cup
14— Detector
15— Shield.

plus il faut tenir compte de leur spin. La section efficace différentielle est, dans ce cas, donnée par la formule de Mott (à l'ordre le plus bas en e^2) :

$$\frac{d\sigma_M}{d\Omega} = \left(\frac{zZe^2}{2m}\right)^2 \frac{1-\beta^2}{\beta^4} \frac{1-\beta^2 \sin^2\frac{\vartheta}{2}}{\sin^4\frac{\vartheta}{2}} \qquad (2)$$

équation qui peut être simplifiée en remarquant que $\beta = v/c \simeq 1$ et que l'énergie des électrons $E = mc^2(1-\beta^2)^{-1/2}$ d'où $1-\beta^2 = (mc^2/E)^2$. On obtient ainsi :

$$\frac{d\sigma_M}{d\Omega} \simeq \left(\frac{zZe^2}{2E}\right)^2 \frac{\cos^2\frac{\vartheta}{2}}{\sin^4\frac{\vartheta}{2}}. \qquad (3)$$

Les formules (2) et (3) sont exprimées dans le système du centre de masse. Dans le système de laboratoire , on obtient :

$$\frac{d\sigma_M}{d\Omega} \simeq \left(\frac{zZe}{2E}\right)^2 \frac{\cos^2\frac{\vartheta}{2}}{\sin^4\frac{\vartheta}{2}} \cdot \frac{1}{1+\frac{2E}{M}\sin^2\frac{\vartheta}{2}} \qquad (4)$$

où M est la masse du centre diffuseur (cette dernière formule est surtout utile dans le cas de la diffusion par les protons ou les neutrons).

II - DIFFUSION DES ELECTRONS PAR LES NUCLEONS.

Dans ce chapitre, nous examinerons successivement les expériences effectuées sur les protons, les neutrons, les deuterons, et les conséquences théoriques que l'on peut en tirer sur la structure des nucléons.

1- PREMIERES EXPERIENCES SUR LA DIFFUSION DES ELECTRONS DE 188 MEV PAR LES PROTONS. (McAllister et Hofstadter, Phys. Rev. 102, 831 1956).

Les résultats expérimentaux de la diffusion des électrons de 188 Mev sur les protons sont représentés sur la fig. 3 où est basée la variation de $d\sigma/d\Omega$ en fonction de l'angle de diffusion ϑ dans le système du laboratoire. En même temps est représentée en pointillés la courbe de diffusion de Mott (Eq. 4). On peut constater qu'aux grands

angles , c'est à dire pour les grandes valeurs du transfert d'impulsion :

$$q = \frac{2E \sin\frac{\vartheta}{2}}{\left[1 + \frac{2E}{M}\sin^2\frac{\vartheta}{2}\right]^{1/2}} \qquad (5)$$

le désaccord entre les deux courbes atteint déjà un rapport 5 environ. On peut envisager plusieurs raisons à ce désaccord :

a) l'existence du moment magnétique "normal" du proton (qui résulte simplement de ce que celui-ci est une particule de spin $\frac{1}{2}$, qui obéit à l'équation de Dirac) ; il n'est pas tenu compte de ce moment magnétique dans la formule de Mott, qui traite bien l'électron comme une particule relativiste de spin $\frac{1}{2}$, mais considère le <u>proton</u> comme une source infiniment lourde dénuée de spin.

b) l'existence d'un moment magnétique "anormal" du proton celui-ci sur lequel nous aurons à revenir, provient du nuage de mésons virtuels qui entourent le "proton nu" (le proton entouré de son nuage sera souvent dans la suite désigné sous le nom de "proton physique"), nuage qui porte une charge et intéragit indépendamment avec le champ électromagnétique ; le moment magnétique anormal du proton est égal en magnétons de Bohr à :

$$\varkappa_p = 1{,}79\ \mu_B \qquad (6)$$

Le moment magnétique total étant par conséquent égal à $\mu_p = 1 + 1{,}79 = 2{,}79$ magnétons de Bohr. Remarquons tout de suite que, pour les mêmes raisons le neutrons aura aussi un moment magnétique anormal.

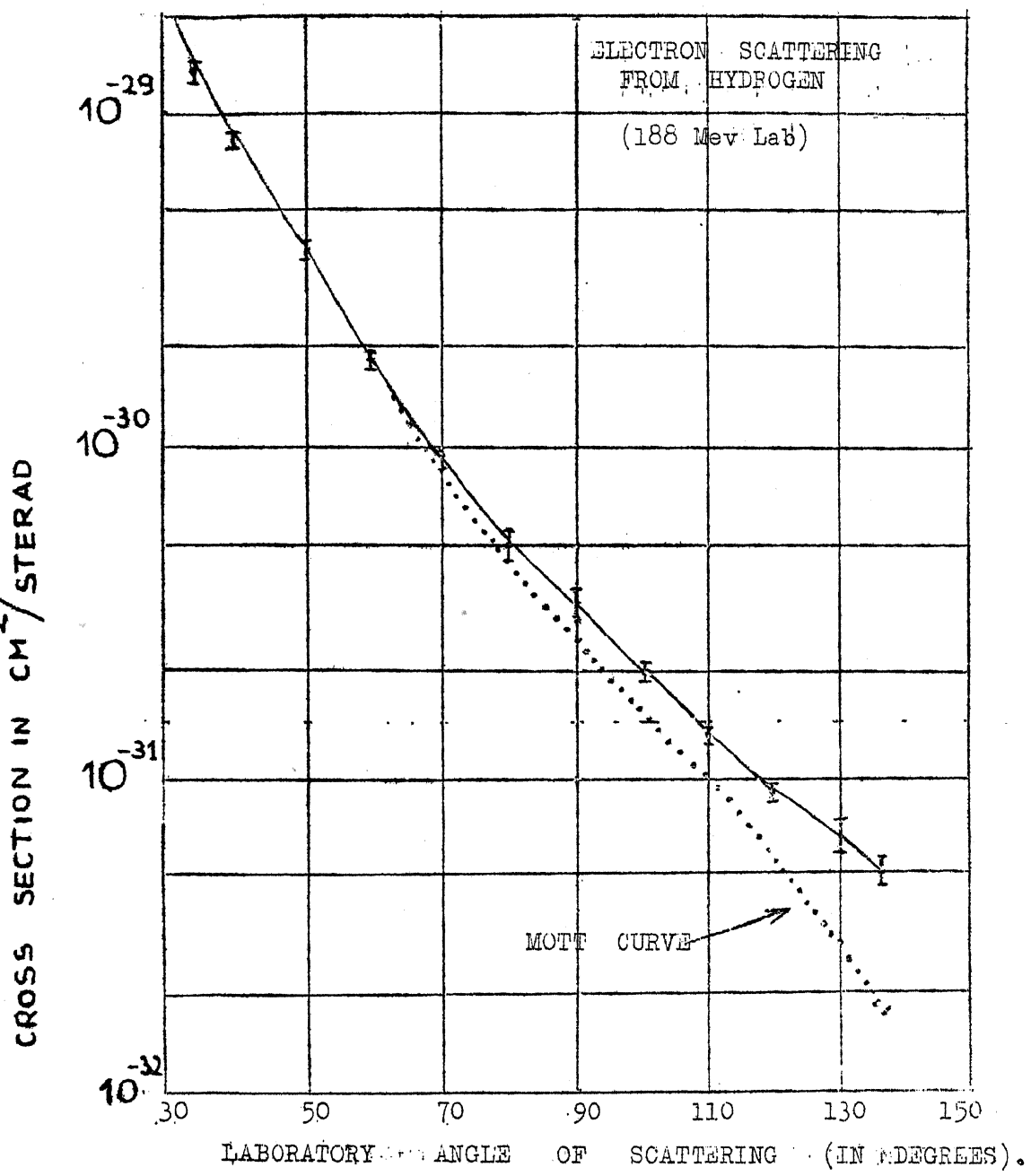
$$\varkappa_n = -1{,}91\ \mu_B \qquad (7)$$

le signe négatif correspondant simplement au fait que le mcmont magnétique du neutron est dirigé dans la direction opposée à celle de son spin.

c) L'influence de l'extension finie du proton.

d) D'autres effets telles que les corrections radiatives, les corrections d'ordre supérieur à la formule de Mott (2ème approximation de Born) etc...

Nous discuterons successivement ces differents effets.


ELECTRON SCATTERING
FROM HYDROGEN
(188 Mev Lab)
MOTT CURVE
CROSS SECTION IN CM²/STERAD
10⁻²⁹
10⁻³⁰
10⁻³¹
10⁻³²
30
50
70
90
110
130
150
LABORATORY ANGLE OF SCATTERING (IN DEGREES).

2- INFLUENCE DU MOMENT MAGNETIQUE DU PROTON (Théorie de ROSENBLUTH, Phys. Rev. 79, 615, 1950).

Il serait très difficile de calculer l'influence du nuage mésique en considérant en détail les differentes intéractions de champ des mésons du nuage avec le champ électromagnétique. Rosenbluth a calculé cet effet en se donnant phénoménologiquement le moment magnétique anormal $\mathcal{H}_p$ et en ajoutant au moment de Dirac du proton un terme de Pauli, destiné à tenir compte de la contribution de $\mathcal{H}_p$. L'interaction électromagnétique du proton s'écrit donc :

$$H_{el}^{(p)} = \left[\gamma_\mu^{(p)} A_\mu + \frac{\mathcal{H}_p}{2M} \sigma_{\mu\nu}^{(p)} F_{\mu\nu} \right] \qquad (8)$$

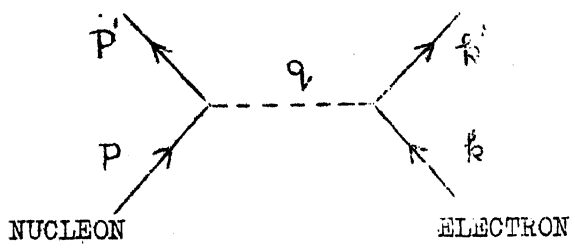
où A_μ est le potentiel vecteur qui décrit le champ magnétique et $F_{\mu\nu}$ les forces de champ correspondantes :

$$F_{\mu\nu} = \frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu} . \qquad (9)$$

Les γ_μ sont les 4 matrices habituelles de la théorie de Dirac et les $\sigma_{\mu\nu}$ constituent le tenseur antisymétrique de Spin :

$$\sigma_{\mu\nu} = \frac{1}{2} [\gamma_\mu, \gamma_\nu] \qquad (10)$$

Utilisant la technique de Feynmann, Rosenbluth calcule la section efficace différentielle à la première approximation de Born , c'est à dire qu'il exclut l'élément de matrice correspondant au diagramme de la fig 4 :

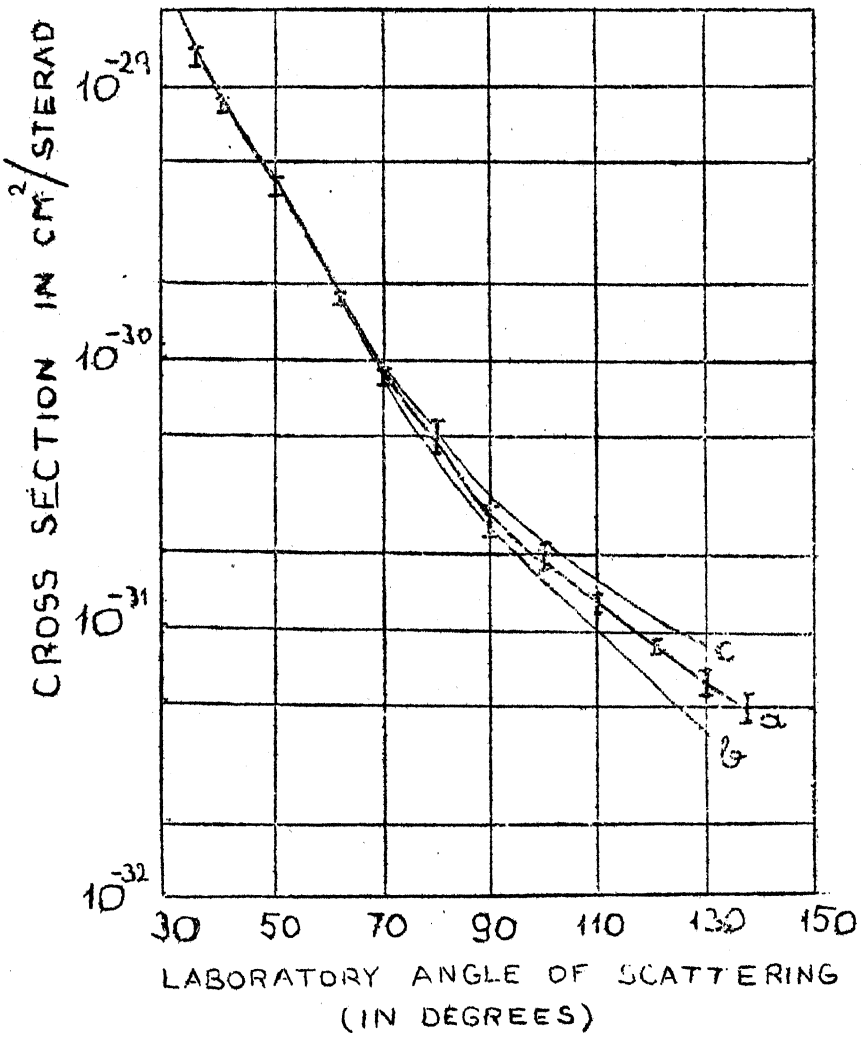


Le résultat est le suivant :

$$\frac{d\sigma(\vartheta)}{d\Omega} = \sigma_M(\vartheta)\left\{1 + \frac{q^2}{4M^2}\left[2(1+\varkappa_p)^2 \operatorname{tg}^2\frac{\vartheta}{2} + \varkappa_p^2\right]\right\} \quad (11)$$

où nous avons posé pour simplifier $\sigma_M(\vartheta)$ égal à la section différentielle de l'équation (4) (avec évidemment $z = Z = 1$).

La fig. 5 représente cette fois en plus des résultats expérimentaux, (courbe a), la section efficace que l'on obtiendrait en faisant $\varkappa_p = 0$ dans la formule (11) (c'est à dire en ne retenant que le terme "normal" de Dirac dans l'expression du courant j_μ), (courbe b), et enfin la section efficace complète donnée par l'équation (11). On voit que les courbes (b) et (c) encadrent la courbe (a) mais qu'aucune ne rend compte de façon satisfaisante des résultats expérimentaux. Il est donc nécessaire de tenir compte de l'extension finie du proton.



3- INFLUENCE DE L'EXTENSION FINIE DU PROTON (Yennie, Levy et Ravenhall Rev. Mod. Phys., 29, 144, 1957).

Indépendemment d'une théorie détaillée de la structure des nucléons (sur laquelle nous reviendrons dans la suite de ce cours), il est possible de décrire phénoménologiquement l'influence de l'extension finie du proton en se basant uniquement sur des arguments très généraux d'invariance :

1) L'invariance relativiste qui veut que le courant $j_\mu^{(p)}$ se transforme comme un quadrivecteur.

2) La loi de conservation du courant

3) Le fait que le nucléon est une particule de Dirac (cette dernière condition n'est pas absolument indispensable ; il suffit de supposer qu'il s'agit d'une particule de spin $\frac{1}{2}$).

Dans ces conditions, la transformée de Fourier du courant peut s'écrire de façon tout à fait générale :

$$j_\mu^{(p)}(P',P) = ie\,\bar{u}(P')\left[\gamma_\mu F_1(q^2) + \frac{\mathcal{H}_p}{2M}\sigma_{\mu\nu}q_\nu F_2(q^2)\right]u(P) \qquad (12)$$

où P et P' sont les quadrivecteurs énergie-impulsion initial et final du proton, u (P) le spineur de Dirac correspondant $q = P' - P$ le quadrivecteur qui décrit le transfert d'énergie-impulsion. F_1 et F_2 enfin sont deux fonctions quelconques de q^2. La loi de conservation du courant s'écrit dans l'espace des impulsions :

$$(P'_\mu - P_\mu)\,j_\mu^{(p)}(P',P) = 0. \qquad (13)$$

Les spineurs u et $\bar{u}$ obeissent aux équations :

$$(\gamma_\nu P_\nu - M)\,u(P) = 0 \qquad (14)$$

et

$$\bar{u}(\gamma_\nu P'_\nu - M) = 0. \qquad (15)$$

Il est facile de montrer que tout autre terme quisatisfait aux conditions (1), (2), (3) ci-dessus, peut se ramener aux deux termes qui sont déjà inclus dans la partie entre crochets de (12) au moyen des équations (14) et (15). Par exemple, un terme de la forme $\bar{u}(p')(p'_\mu + p_\mu)u(p)$ peut s'exprimer en fonction des deux autres par la méthode suivante : Multiplions (14) à gauche par $\bar{u}(p')\gamma_\mu$, (15) à droite par $\gamma_\mu u(p)$ et additionnons. Il vient l'équation :

$$\bar{u}(p')\left[\gamma_\mu\gamma_\nu p_\nu + \gamma_\nu\gamma_\mu p'_\nu\right]u(p) = 2M\,\bar{u}(p')\gamma_\mu u(p) \quad (16)$$

La partie entre crochets du 1er membre peut s'écrire en tenant compte des relations d'anticommutation des matrices γ_μ :

$$(2\delta_{\mu\nu} - \gamma_\nu\gamma_\mu)p_\nu + \gamma_\nu\gamma_\mu p'_\nu = \gamma_\nu\gamma_\mu(p'_\nu - p_\nu) + 2p_\mu$$

ou bien :

$$\gamma_\mu\gamma_\nu p_\nu + (2\delta_{\mu\nu} - \gamma_\mu\gamma_\nu)p'_\nu = -\gamma_\mu\gamma_\nu(p'_\nu - p_\nu) + 2p'_\mu$$

et en prenant la demi-somme de ces deux expressions :

$$p'_\mu + p_\mu \,, \qquad \sigma_{\mu\nu}q_\nu \,.$$

Par conséquent, l'équation (16) peut s'écrire :

$$\bar{u}(p')(p'_\mu + p_\mu)u(p) = 2M\,\bar{u}(p')\left[\gamma_\mu + \frac{1}{2M}\sigma_{\mu\nu}q_\nu\right]u(p) \quad (17)$$

Le second membre a bien exactement la forme du second membre de l'équation(12) .

Les positions F_1 et F_2 peuvent être considérées comme des généralisations des"facteurs de forme" qui apparaissent dans la théorie de la diffusion par des noyaux lourds. Mais il est important de remarquer qu'elles ne pouvaient être égales aux transformées de Fourier des distributions de charge et de moment magnétique du proton. Ceci est dû à plusieurs effets :

a) F_1 est le coefficient de l'opérateur γ_μ qui contribue à la fois à la charge du proton et à son moment magnétique normal.

b) Les effets cinétiques de recul des nucléons viennent compliquer la relation entre les distributions de charge et de moment magnétique, d'une part, les transformées de Fourier des positions F_1 et F_2, d'autre part. Nous prenons dans la suite :

$$F_i(q^2) = \int f_i(r)\, e^{-i\vec{q}\vec{r}}\, d\vec{r} \tag{18}$$

et par conséquent :

$$f_i(r) = (2\pi)^{-3} \int F_i(q^2)\, e^{i\vec{q}\vec{r}}\, d\vec{q}. \tag{19}$$

Si on développe les fonctions F_i en puissances de q^2, on peut introduire des "rayons carrés moyens" des distributions f_i correspondantes par les relations :

$$F_i(q^2) \simeq 1 - \frac{q^2}{6} \langle r_i^2 \rangle + \dots \tag{20}$$

et

$$\langle r_i^2 \rangle = \int r^2 f_i(r)\, d\vec{r}. \tag{21}$$

Pour illustrer ce que nous venons de dire sur la relation entre $f_i(r)$ et les distributions de charge $\rho_{ch}(\vec{r})$ et de moment magnétique $\rho_{mag}(\vec{r})$ nous calculerons, à l'ordre le plus bas en M^{-2} la relation entre $\langle r_1^2 \rangle$ défini par (21) et le rayon carré de la distribution de charge:

$$\langle r_{ch}^2 \rangle = \int r^2 \rho_{ch}(\vec{r})\, d\vec{r} \tag{22}$$

Pour cela il suffit de prendre la forme non relativiste de la quatrième composante j_0 du courant (12), en se limitant aux termes du premier ordre en q^2/M^2:

$$j_0 = e\, \phi_2^* \phi_1 \left[F_1 - \frac{q^2}{8M^2} \left(F_1 + 2\varkappa_p F_2 \right) \right], \tag{23}$$

où ϕ_1 et ϕ_2 sont les grandes composantes des spiners de Dirac. En tenant compte de (20), et en conservant seulement dans (23) les termes en q^2/M^2 on obtient :

$$\langle r_{ch}^2 \rangle \simeq \langle r_1^2 \rangle + \frac{3}{4M^2}(1+2\varkappa_p); \qquad (24)$$

le 2ème terme de (24) est essentiellement dû au recul du proton et son importance a été reconnue en premier lieu par Foldy (Phys. Rev. 87, 688, 1952).

Finalement le résultat du calcul de la section efficace différentielle de diffusion électron-proton , lorsqu'on prend comme courant du proton l'expression (12) et qu'on utilise la première approximation de Born, est :

$$\frac{d\sigma(\vartheta)}{d\Omega} = \sigma_M(\vartheta)\left\{F_1^2 + \frac{q^2}{4M^2}\left[2(F_1+F_2\cdot\varkappa_p)^2 tg^2\frac{\vartheta}{2} + \varkappa_p^2 F_2^2\right]\right\} \qquad (25)$$

équation qui se réduit évidemment à celle de Rosenbluth lorsqu'on fait $F_1 = F_2 = 1$. L'influence de l'extension finie du proton sera analysée par comparaison à cette dernière au moyen du "facteur de forme" $\mathcal{F}(\vartheta, q^2)$ défini par :

$$\frac{d\sigma}{d\Omega} = \sigma_{point}\left[\mathcal{F}(\vartheta, q^2)\right]^2 \qquad (26)$$

où σ_{point} est donnée par l'équation (11). $\mathcal{F}$ dépend en général de l'angle ϑ , sauf si $F_1 = F_2 = F$. Dans ce cas il sera possible de représenter le rapport $(\sigma_{point})^{-1}\left(\frac{d\sigma}{d\Omega}\right)$ en fonction de q^2 sur une seule courbe indépendante de l'énergie des électrons diffusés.

4- ANALYSE DES RESULTATS EXPERIMENTAUX AUX GRANDES ENERGIES.

La structure du proton a été étudiée de façon systématique par Chambers et Hofstadter (Phys. Rev. 103, 1454, 1950)qui ont poussé les mesures de diffusion jusqu'à 500 Mev environ. La première observation à tirer de leurs résultats est que l'effet de l'extension finie du proton devient très important, puisque les facteurs de forme $\mathcal{F}^2$ peuvent décroitre

jusqu'à 0,1. Par ailleurs, les expériences sont suffisamment précises pour permettre non seulement de mesurer les carrés moyens $\langle r_i^2 \rangle$ mais aussi de déterminer quelle <u>forme</u> de distribution s'accorde ou ne s'accorde pas avec les résultats expérimentaux. Nous ne ferons que résumer très rapidement ici leurs conclusions. De tous les modèles qui ont été essayés celui qui s'accorde le mieux avec l'expérience est fourni par la distribution "exponentielle creuse"

$$\varrho = \varrho_0 \, r \, e^{-\alpha r} \tag{27}$$

avec

$$\langle r_1^2 \rangle = \langle r_2^2 \rangle = \left[(0{,}77 \pm 0{,}10) \cdot 10^{-13} \text{ cm} \right]^2 \tag{28}$$

D'autres modèles peuvent également être considérés comme satisfaisants. La distribution Gaussienne, par exemple

$$\varrho = \varrho_0 \, e^{-\beta r^2} \tag{29}$$

avec :

$$\langle r_1^2 \rangle = \langle r_2^2 \rangle = \left(0{,}70 \times 10^{-13} \text{ cm} \right)^2 \tag{30}$$

ou bien la distribution exponentielle $\varrho = \varrho_0 \, e^{-\gamma r}$ avec $\langle r_1^2 \rangle = \langle r_2^2 \rangle = \left(0{,}80 \times 10^{-13} \text{ cm} \right)^2$. Par contre, une distribution du type de Yukawa ($\varrho = \varrho_0 \, e^{-\mu r} / r$) ou bien du type "uniforme" ($\varrho = \text{cte}$ pour $r \leqslant a$) ne s'accordent absolument pas avec l'expérience. On remarquera également que le meilleur accord est toujours obtenu au moyen de représentations identiques pour f_1 et f_2. Il est très important théoriquement de savoir dans quelle mesure ces deux fonctions peuvent s'écarter l'une de l'autre sans contredire les résultats expérimentaux. La conclusion de Chambers et de Hofstadter est qu'aucun des deux rayons ne peut être inférieur à $0{,}6 \times 10^{-13}$ cm ou supérieur à $1{,}5 \times 10^{-13}$ cm.

Dans ces équations, le densité de courant de l'électron s'exprime en fonction du champ $\psi(\vec{x},t)$ de l'électron par la formule habituelle :

$$j_\mu^{(e)}(\vec{x},t) = ie\bar{\psi}(\vec{x},t)\gamma_\mu\psi(\vec{x},t). \qquad (35)$$

Nous considérons deux états du système : un état initial Φ_A qui correspond à la présence d'un électron dont l'énergie-impulsion est représentée par le quadrivecteur p_μ, l'état des photons étant celui du vide ;

: un état final Φ_B, qui ne contient qu'un électron d'énergie-impulsion p'_μ.

Nous cherchons la probabilité de transition de l'état A à l'état B, après interaction avec le champ extérieur et le champ propre (fig. 6) Celle-ci s'obtient à partir de l'élément :

$$M_{AB} = (\Phi_B, S\Phi_A) \qquad (36)$$

de la matrice de collision S qui est donnée par la formule de Dyson :

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \mathcal{P}\{H_{int}(t_1)\dots H_{int}(t_n)\}dt_1\dots dt_n, \qquad (37)$$

le symbole $\mathcal{P}$ signifiant que le produit entre accolades doit être ordonné de façon à ce que les temps $t_1,\dots,t_n$ aillent en croissant de la droite vers la gauche. L'élément d'ordre zéro de la matrice S est évidemment égal à 1. Il correspond à la progagation libre de l'électron considéré dans l'état initial A. A l'ordre 1, seul le terme en H^{ext} va contribuer car l'élément de matrice de $A_\mu(\vec{x},t)$ entre deux états du vide des photons est nul. L'élément correspondant est celui de la diffusion de l'électron par le champ du proton à l'approximation de Born. C'est celui que nous avons calculé par la formule de Mott.

$$M_1 = e\bar{u}(p')\gamma_\mu A_\mu^{ext}(q)u(p). \qquad (38)$$

Au second ordre en e, nous rencontrons deux sortes de contributions non nulles. Celles qui, dans l'équation (37), contiennent un produit de deux termes en H_{ext} :

il s'agit de la deuxième approximation de Born à la diffusion de l'électron par le proton, dont nous savons qu'elles est négligeable. Celles, par ailleurs, qui contiennent un produit de deux termes H_I : il s'agit alors d'un terme d'énergie propre de l'électron dont nous avons à tenir compte. Les termes croisés $H_{ext} H_I$ ne donnent pas de contribution pour la même raison que plus haut (les éléments de matrices de A_μ dans le vide sont nuls). On voit facilement que les termes d'énergie propre peuvent se mettre sous la forme :

$$M_2 = ie^2 \bar{u}(p') u(p) \delta(p'-p) Q, \qquad (39)$$

où Q est un nombre qui est donné par une certaine intégrale divergente dont l'expression ne nous est pas nécessaire pour comprendre le principe de la méthode de renormalisation. Nous allons montrer que ce terme résulte uniquement d'une addition δm à la masse "mécanique" m_o de l'électron. Comme seule la masse totale

$$m = m_o + \delta m \qquad (40)$$

est observable, nous ne devons pas nous attendre au moindre effet mesurable dans la diffusion de l'électron à cet ordre. Ecrivons en effet le Lagrangien de l'électron,

$$\mathcal{L} = \bar{\psi}\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + m_o\right)\psi = \bar{\psi}\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + m\right)\psi - \delta m \bar{\psi}\psi. \qquad (41)$$

Si nous convenons d'inclure dans la partie libre de l'Hamiltonien la masse m, il en résulte une addition à l'Hamiltonien d'interaction :

$$H'_{int} = H_I + H_{ext} + H_{s.e.} \qquad (42)$$

où $H_{s.e}$ (Hamiltonien de self-énergie) s'écrit :

$$H = \delta m \int \bar{\psi}(\vec{x},t)\psi(\vec{x},t)\, d\vec{x}. \qquad (43)$$

Au premier ordre en δm $H_{s.e.}$ produit, par l'intermédiaire des formules (36) et (37) un élément de matrice de diffusion :

$$M_{s.e} = -i\delta m\,(2\pi)^4\,\delta(p-p')\,\bar{u}(p')\,u(p). \qquad (44)$$

Par suite, si nous choisissons pour δm la valeur :

$$\delta m = \frac{1}{(2\pi)^4}\,Q\,e^2, \qquad (45)$$

nous garantissons, par la relation :

$$M_2 + M_{s.e.} = 0. \qquad (46)$$

l'absence de tout effet observable au 2ème ordre en e dans la diffusion de l'électron.

Nous calculons maintenant en 3ème ordre en e les éléments de matrice de scattering en introduisant dans la formule (37) qui définit S l'expression (42) de H'_{int}, en n'oubliant pas toutefois que si H_I et H_{ext} sont du 1er ordre en e, $H_{s.e.}$ est un terme en e^2. Au troisième ordre en e, les effets non nuls proviendront donc de produits de la forme $H_I\,H_I\,H_{ext}$ ou $H_{ext}\,H_{s.e.}$. Les diagrammes correspondant au 1er groupe sont representés sur la fig (7), où l'interaction avec le champ extérieur est indiqué par un gros point noir.

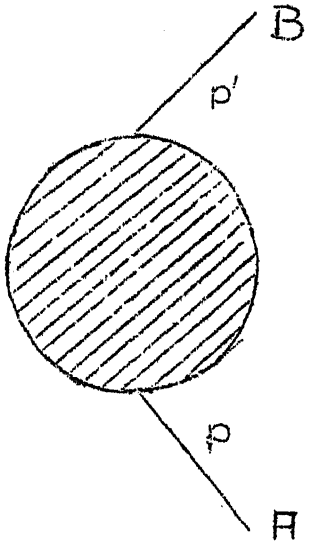
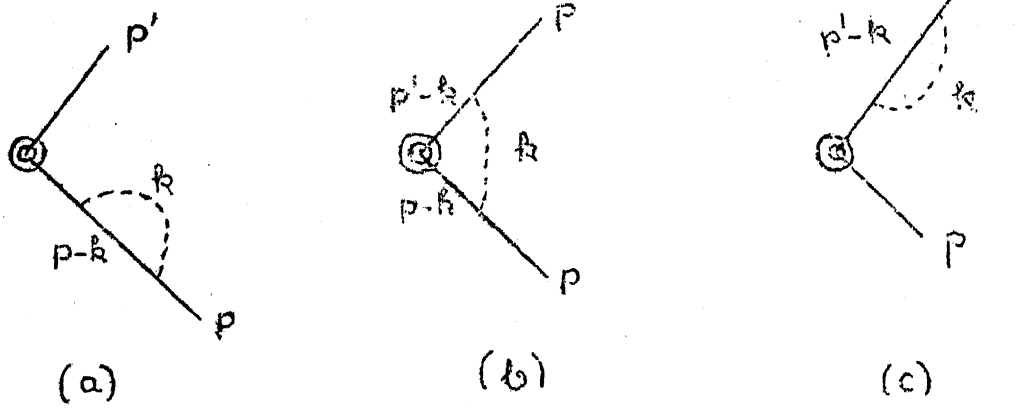


Fig. 6



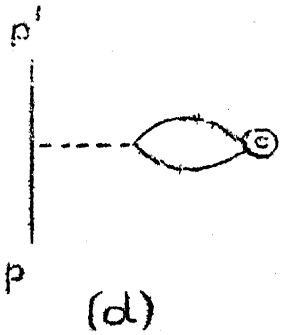


Fig. 7

Chacun des processus représentés sur cette figure diverge quadratriquement mais cette divergence est annulée par l'addition des termes du second groupe, δm étant toujours donné par l'équation (45). Il reste que chacun des termes représentés sur la figure (7) diverge encore logarithmiquement, mais que leur somme est finie, au moins en ce qui concerne la partie des intégrales qui correspond aux grandes fréquences du photon virtuel (région ultra-violette). Il existe encore une divergence pour les petites valeurs de la fréquence du photon (catastrophe infra-rouge) qui provient de ce que dans un terme comme celui-ci :

$$\bar{u}(p')\frac{ie}{(2\pi)^4}\gamma_\nu\int\frac{1}{[i\gamma(p'-k)+m]}\gamma_\mu\frac{1}{[i\gamma(p-k)+m]}\gamma_\nu\frac{d^4k}{k^2}A^{ext}(p'-p)u(p) \quad (47)$$

qui correspond à un facteur près, au terme (b) de la fig (7), les fonctions d'onde $u(p)$ et $\bar{u}(p')$ de l'électron correspondant à des particules libres qui satisfont à l'équation de Dirac habituelle :

$$(i\gamma p+m)u(p) = \bar{u}(p')(i\gamma p'+m) = 0. \quad (48)$$

Par suite, l'élément de matrice (47) peut s'écrire(on pose : $q=p'-p$) :

$$\frac{ie}{(2\pi)^4}\bar{u}(p')\gamma_\nu\int\frac{1}{i\gamma k}\gamma_\mu\frac{1}{i\gamma k}\gamma_\nu\frac{d^4k}{k^2}A_\mu(q)u(p) \quad (49)$$

et l'on voit que l'intégrale en R diverge logarithmiquement pour $k\to 0$.

Pour éliminer la divergence infra-rouge, il est nécessaire de tenir compte des diffusions inélastiques avec émisson de photons dont la fréquence est trop faible pour être détectée, et qui apparaîtront par conséquent comme des diffusions élastiques expérimentalement. Si l'on appelle ΔE la fréquence minimum qui peut être détectée par les appareils de mesure, on obtient finalement pour l'élément de matrice correct jusqu'au 3ème ordre :

$$M^{tot} = M_1 + M_3 = M_1[1-\delta(\vartheta,E)] \quad (50)$$

où $\delta(\vartheta,E)$ est donné, à la limite relativiste de l'électron par l' expression :

$$\delta = \frac{4\alpha}{\pi}\left\{\left[\mathrm{Log}\frac{2E\sin\frac{\vartheta}{2}}{m} - \frac{1}{2}\right]\left[\mathrm{Log}\frac{E}{\Delta E} - \frac{13}{12}\right] + \frac{17}{72} + \phi(\vartheta_1)\right\} \qquad (51)$$

$\phi(\vartheta_1)$ étant défini par l'intégrale :

$$\phi(\vartheta_1) = \frac{1}{2}\sin\frac{\vartheta}{2}\int_{\cos\frac{\vartheta}{2}}^{1}\left[\frac{\mathrm{Log}\frac{1+x}{2}}{1-x} - \frac{\mathrm{Log}\frac{1-x}{2}}{1+x}\right]\frac{dx}{\sqrt{x^2-\cos^2\frac{\vartheta_1}{2}}} \qquad (52)$$

$\phi(\vartheta_1)$ n'est pas calculable analytiquement sauf pour $\phi(0)=0$ et $\phi(\pi)=\frac{\pi^2}{24}$. Il existe une approximation par excès qui vaut toujours aux angles pas trop grands :

$$\phi(\vartheta) \simeq \frac{1-\cos\frac{\vartheta}{2}}{\left[2\cos\frac{\vartheta_1}{2}\left(1+\cos\frac{\vartheta}{2}\right)\right]^{1/2}}\left[\mathrm{Log}\frac{1}{2\left(1-\cos\frac{\vartheta}{2}\right)} + \frac{1-\cos\frac{\vartheta}{2}}{2} + 1\right] \qquad (53)$$

Dans le calcul précédent, on a négligé le recul du proton qui n'apporte que des corrections de l'ordre de E/M, soit aux énergies qui seront considérées au paragraphe suivant, de l'ordre de 10%. Ces corrections doivent être incluses si l'on veut comparer δ aux mesures qui pourraient être faites à des énergies de l'ordre de 500 Mev ou 1 Bev.

Le calcul de δ que nous avons esquissé ne tient compte que des effets à l'ordre le plus bas dans le nombre de photons virtuels. On pourrait se demander si, en raison de la présence du terme logarithmique en $E/\Delta E$ dans l'équation (51), l'approximation n'est pas sujette à critiques malgré la petitesse de la constante $\alpha = 1/137$. Yennie et Suura (Phys. Rev. 105 , 1378, 1957) ont montré que l'inclusion d'un nombre arbitraire de photons virtuels revient simplement à remplacer l'équation (50) par :

$$M^{tot} = M_1 e^{-\delta(\vartheta,E)} \qquad (54)$$

confirmant en celà une hypothèse qui avait été faite initialement par Schwinger.

6- MESURE DE LA SECTION EFFICACE ABSOLUE DE DIFFUSION PAR LES PROTONS A 188 MEV (Tautfest et Panofsky Phys. Rev. 105, 1358, 1957).

La seule mesure absolue de section efficace a été faite par Tautfest et Panofsky, utilisant le même dispositif de diffusion que Hofstadter, sauf qu'ils ont observé non pas les électrons diffusés, mais le recul des protons dans des émulsions photographiques disposée autour du faisceau à l'intérieur de la chambre de diffusion. Dans ces conditions les formules (50) et (51) doivent être modifiées pour deux raisons :

a) elles doivent être exprimées en fonction de l'angle de recul γ des protons ; dans le cas simplifié où l'on fait $E/m \gg 1$ et $E/M \ll 1$, on a la relation :

$$\gamma = \frac{\pi}{2} - \frac{\vartheta}{2}. \qquad (55)$$

b) il est nécessaire de tenir compte de ce qu'une grande partie de la diffusion inélastique est incluse ; la quantité qui limite expérimentallement la mesure n'est pas la fréquence minimum des photons détectables mais le recul minimum q_{min} des protons qui peut être observé, étant données les conditions géométriques de l'expérience. Le recul des protons est donné en fonction de l'énergie par la relation :

$$q = 2E\cos\gamma. \qquad (56)$$

Le calcul du terme correctif a été fait par Schiff (Phys. Rev. 87, 750, 1952.) qui trouve pour l'elément de matrice total :

$$M^{tot} = M_i\left[1 + \Delta(\gamma, E)\right] \qquad (57)$$

dont $\Delta(\gamma, E)$ est donné par la formule :

$$\Delta = \frac{\alpha}{\pi}\left\{\left[\mathrm{Log}\frac{2E\cos\gamma}{m} - \frac{1}{2}\right]\left[\mathrm{Log}\left(x + \frac{1}{x} - 2\right) + x + \frac{11}{6}\right] - \frac{17}{18} - \frac{1}{4}\phi(\pi - 2\vartheta)\right\} \qquad (58)$$

où l'on a posé : $x = q / q_{min} = 2E\cos\gamma / q_{min}$
et où $\phi(\vartheta)$ est toujours donné par l'équation (52).

La quantité mesurée par Tautfest et Panofsky est le nombre total N_t de traces de protons correspondant à des reculs compris entre 54° et 78°, c'est à dire la "section efficace intégree"

$$N_t = G \int_{54°}^{78°} \left(\frac{d\sigma}{d\Omega}\right)_{point} F^2(q^2)(1+\Delta) \sin\gamma \, d\gamma \qquad (59)$$

$F^2(q^2)$ étant le facteur de forme commun aux distributions de charge et de moment magnétique déterminé par Hofstadter et ses collaborateurs et G un coefficient ne dépendant que des propriétés géométriques du système de détection. Le dispositif dans la chambre de scattering est représenté en détail sur la figure (8). Plusieurs difficultés particulières ont dû être surmontées dans cette expérience. Les plus notables ont été :

a) le fait que les sels d'argent des émulsions se trouvaient réduits par l'hydrogène de la chambre, d'où apparition de grains d'argent métallique qui rendaient tout scanning impossible ; cette difficulté a été surmontée en refroidissant l'ensemble de la chambre de diffusion à -30° C ; il était en outre très important de maintenir la température uniforme, avec des variations extrêmes inférieures à 0,5°

b) l'obscurcissement des plaques par la radiation caractéristique de l'hydrogène bombardé par les électrons ; le seul moyen de limiter cette difficulté s'est trouvé être la réduction du temps d'exposition, de façon à n'avoir pas plus de 10^{14} électrons dans les plaques.

Il est très important dans cette expérience, de disposer les plaques symetriquement par rapport au faisceau de façon à additionner les évènements observés dans des bandes d'émulsion symétriques, ce qui élimine à peu près complètement les erreurs dûes au centrage du faisceau. La fig. (9) indique les différents facteurs géométriques qui interviennent. En fonction de ces quantités, le nombre dN_t de traces se trouvant dans l'intervalle compris entre γ et $\gamma + d\gamma$ est donné par la relation :

$$dN_t = N_p N_e \frac{d\sigma}{d\Omega} L z \left[\int_{y_1}^{y_2} \frac{dy}{d^2} \sin\gamma\right] d\gamma \qquad (60)$$

où $y_2 - y_1 = W$ et N_p et N_e sont respectivement le nombre de protons par unité de volume et le nombre d'électrons total arrivant sur la cible.

Pour apprécier la difficulté de cette expérience et l'ordre de grandeur de la précision à laquelle on peut raisonnablement mesurer une section efficace absolue , il est intéressant de faire la liste des erreurs qui interviennent dans le calcul de $dN_t/d\gamma$ défini par l'équation (60) :

1) Erreur sur N_p : provient essentiellement des variations de pression et de température, estimées chacune à 0,2 % .

2) Erreur sur N_e : due principalement aux variations du système intégrateur du faisceau, estimé à 1% au maximum.

3) Erreur géométrique : (définition mécanique des dimentions du porteur de plaques , séparation de celles-ci, surface totale scannée, etc...) estimées à 1,2% .(erreur sur le facteur d'angle solide).

4) Centrage du faisceau : erreur négligeable.

5) Erreur d'observation ("scanning efficiency") : certainement inférieure à 1%.

6) Mesure de "range" et détermination de l'énergie primaire : 1,46% en tout.

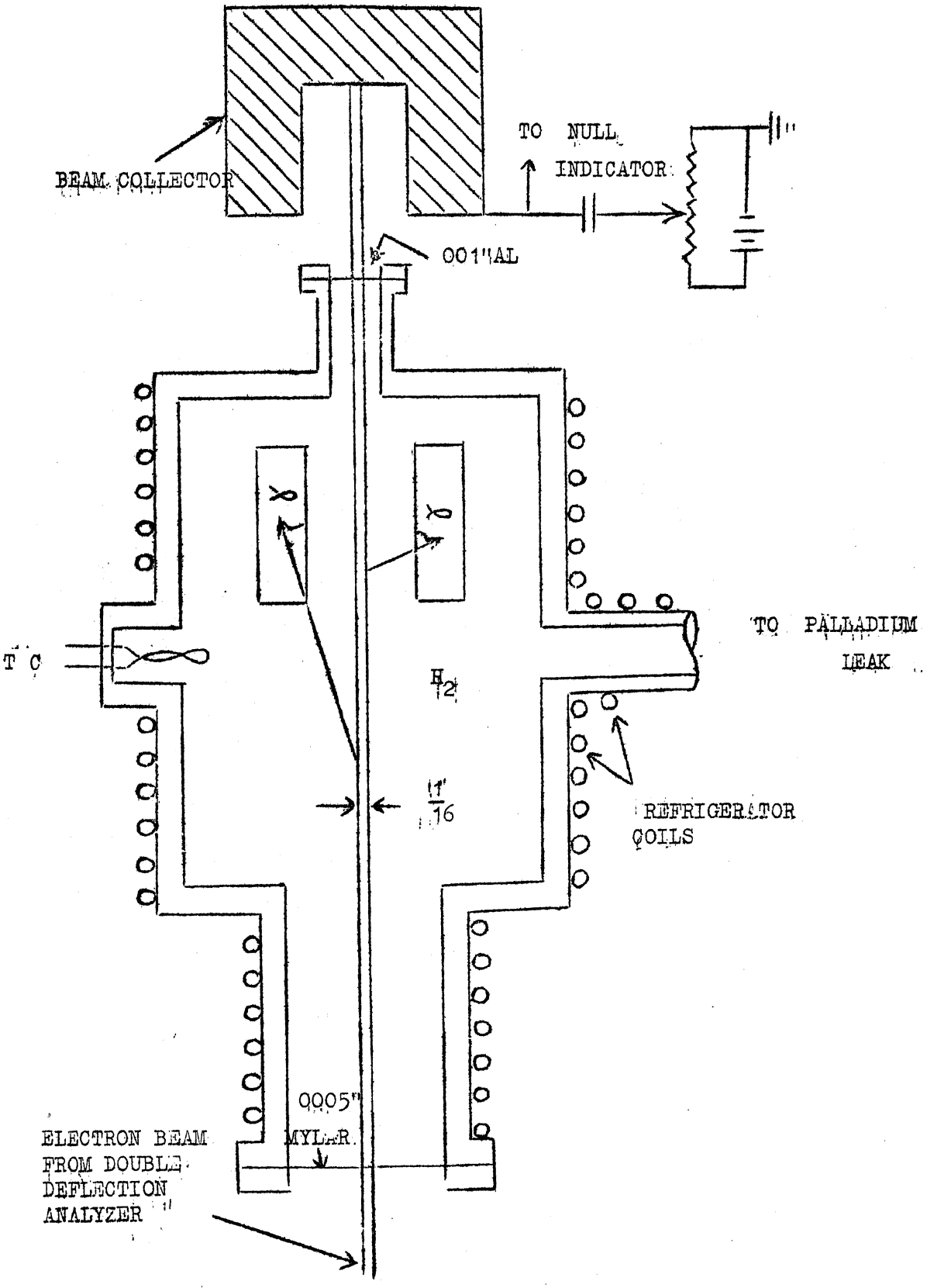
7) Erreur statistique, provenant de la division des 2350 traces dans l'intervalle 54°-78° en intervalle de 2° : incertitude estimée à 2, 06%.

8) Background : incertitude corrigée sur les résultats expérimentaux avec une erreur possible de 0,2%.

9) Autres erreurs (négligeables) : longueur scannée, variation de la densité de l'hydrogène en raison du chauffage par le faisceau ; contamination possible du gaz de la chambre par du deuterium.

En définitive, la combinaison de ces erreurs a été estimée statistiquement à $\pm$ 2,1% (erreur probable). La section efficace théorique, définie par (59), a évidemment été corrigée pour tenir compte du recul des protons. Finalement le résultat de Tautfest et Panofsky s'écrit :

$$\frac{N_t(exp)}{N_t(th.)} = 0,988 \pm 0,021. \qquad (61)$$


BEAM COLLECTOR
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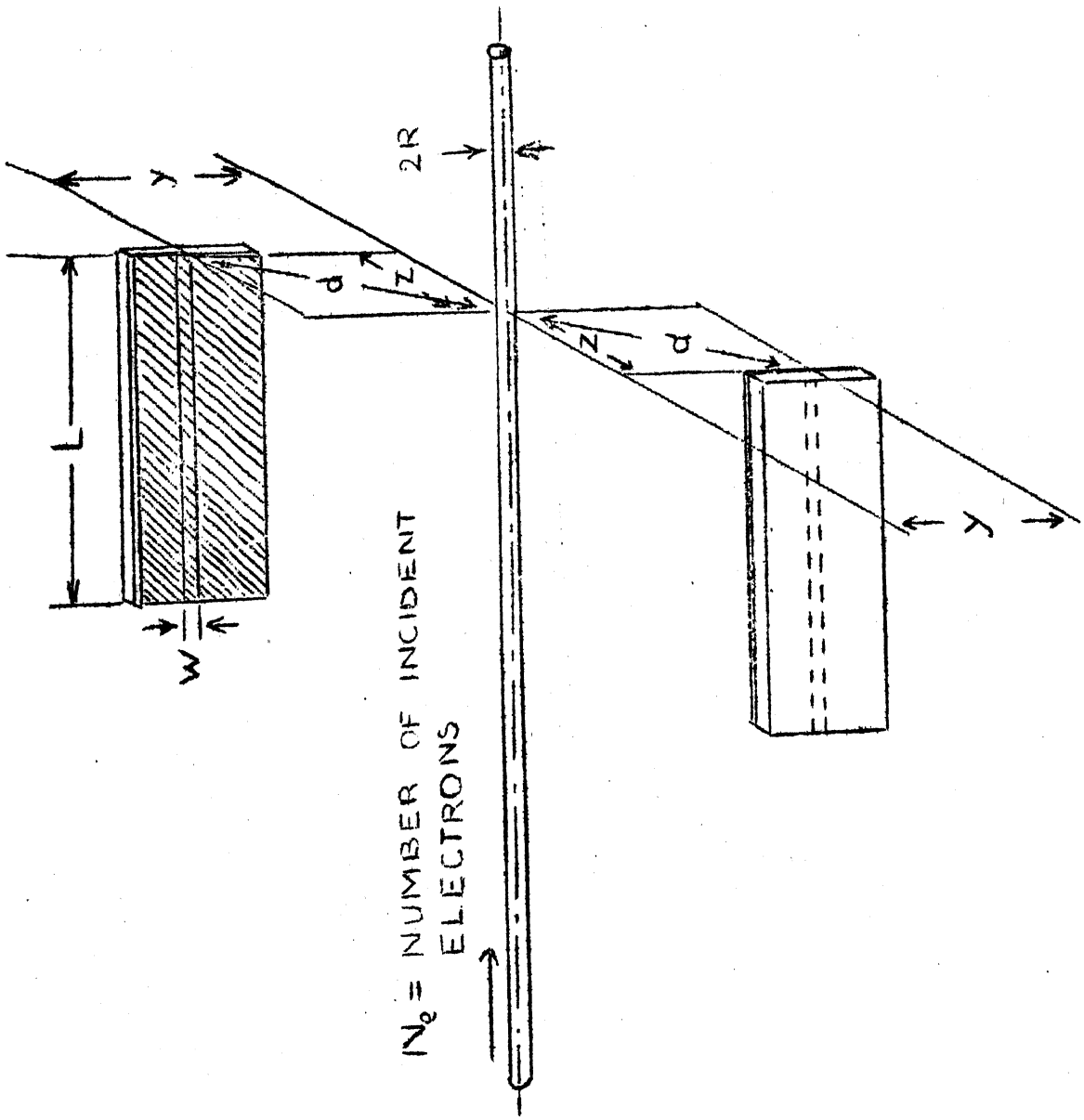
2R
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N_e = NUMBER OF INCIDENT ELECTRONS

Il faut remarquer que les corrections radiatives n'interviennent dans ce résultat que pour

$$\Delta = -2,74\% . \qquad (62)$$

A une énergie de 500 Mev, cette correction serait beaucoup plus importante (30% environ). Le facteur de forme ne diffère de son coté de l'unité que de -2,93%.

7- INTERACTION ELECTRON-NEUTRON.

Il est très important de comparer maintenant les résultats obtenus par diffusion des électrons sur les protons avec les mesures qui ont été faites indépendemment sur l'interaction électron-neutron. Historiquement, ces mesures furent les premières qui donnèrent, par diffusion de neutrons très lents sur les atomes, des informations sur la structure des nucléons. Les premières mesures sont dues à Fermi et Marshall(Phys. Rev. 72, 1139, 1947)et à Hamermesh, Ringo et Wattenberg (Phys. Rev. 85, 583, 1952). Nous parlerons un peu en détail des mesures plus récentes de Hughes Harvey, Goldberg et Stafne (Phys.Rev. 90, 497, 1953) qui furent les premières à fournir une valeur relativement précise de l'interaction électron neutron. L'expérience consiste à mesurer l'angle de réflexion critique de neutrons de grande longueur d'onde (λ) sur une surface de séparation de bismuth et d'oxigène liquide. Cet angle critique Ω_c est donné en fonction des longueurs de diffusion des neutrons par le bismuth , l'oxygène et les électron a_{Bi} , a_o et a_e respectivement, au moyen de la relation :

$$\frac{\pi}{\lambda^2}\,\Omega_c^2 = N_o a_o - N_{Bi} a_{Bi} + a_e \left\{ N_{Bi} Z_{Bi} - N_o Z_o \right\} \qquad (63)$$

où N_{Bi} , N_o sont respectivement les nombres de noyaux de bismuth et d'oxygène par cm^3.

On trouve : $\Omega_c = 3,64 \pm 0,04$ minutes, et par conséquent (en utilisant des mesures directes de a_o et a_{Bi}).

$$a_e = 1,40 \times 10^{-16} \text{ cm.}$$

L'interaction électron-neutron s'exprime traditionnellement et conventionellement par la profondeur V_o du puits carré d'un potentiel de portée égale au rayon classique de l'electron $r_o = e^2/m$, qui fournirait la même longueur de diffusion. L'équation (63) conduit par conséquent à :

$$V_o = -3860 \pm 370 \text{ eV}. \tag{64}$$

D'autres mesures encore plus récentes, employant des méthodes différentes, sont dues à Crouch, Krohn et Ringo (Phys. Rev. 102, 1321, 1956) qui trouvent $V_o = -3900 \pm 800$eV, et à Melkonian, Rustad et Havens (Bull. Am. Phys. Soc. Série II, 1, 62,1956) qui obtiennent le résultat relativement précis suivant :

$$V_o = -4165 \pm 265 \text{ eV} \tag{65}$$

Si l'on prend la moyenne des deux résultats (64) et (65) on peut admettre pour V_o la valeur :

$$V_o = -4000 \pm 300 \text{eV}. \tag{66}$$

Les experiences précédentes ayant été faites à très basse énergie, il est permis de développer les facteurs de forme en puissances de q^2/M^2, où q^2 est le carré du transfert d'impulsion, et M la masse des nucléons. La formule (24), s'écrit dans le cas du neutron (qui n'a évidemment pas de moment magnétique de Dirac) :

$$\langle r^2 \rangle_{ch.} = \frac{3}{4M^2}(2\partial\mu_n) + \langle r_{1n}^2 \rangle \tag{67}$$

où $\partial\mu_n$ est le moment magnétique anormal $-1{,}91\ \mu_B$ et $\langle r_{1n}^2 \rangle$ le rayon carré moyen du facteur de forme F_1 correspondant au neutron. Celui-ci peut avec nos définitions, être négatif car, pour le neutron nous avons :

$$F_{1n} \simeq -\frac{q^2}{6}\langle r_{1n}^2 \rangle + \text{etc.} \tag{68}$$

La quantité $\langle r^2 \rangle_{ch}$ est liée à l'écart δV par rapport au potentiel résultant d'une distribution ponctuelle par la relation :

$$\frac{2\pi e^2}{3} \langle r^2 \rangle_{ch} = \int \delta V \, d\vec{r}. \qquad (69)$$

L'intégrale sur δV est à son tour, exprimable en fonction de V_o en supposant que celui-ci s'étend à l'intérieur d'une sphère de rayon $r_o = e^2/m$

$$\int \delta V \, d\vec{r} = \frac{4\pi}{3} r_o^3 V_o \qquad (70)$$

d'où l'on déduit :

$$V_o = \frac{e^2}{2 r_o^3} \langle r^2 \rangle_{ch} = \frac{e^2}{2 r_o^3} \left[\frac{3 \mathcal{E}_n}{2 M^2} + \langle r_{in}^2 \rangle \right]. \qquad (71)$$

Le premier terme qui figure à la droite de (71) est le terme bien connu de Foldy :

$$V_o^{(1)} = \frac{3 m^3}{4 e^4} \frac{\mathcal{E}_n}{M^2} = -4070 \text{ eV}. \qquad (72)$$

Il en résulte que le second terme de (71) est sensiblement égal à :

$$V_o^{(2)} = 0 \pm 300 \text{ eV},$$

d'où l'on tire :

$$\langle r_{in}^2 \rangle = (0{,}000 \pm 0{,}006) \cdot 10^{-26} \text{ cm}^2. \qquad (73)$$

Il existe donc un contraste frappant entre l'extension de la distribution de charge du proton et celle du neutron. Si l'on admet l'image, que nous examinerons de façon plus critique dans la suite, selon laquelle l'extension des nucléons est due en majeure partie à leur nuage mésique (les effets de recul du "coeur" étant considérés comme petits), le résultat que nous venons d'obtenir semble être en contradiction avec l'indépendance de charge des forces nucléaires, qui suppose essentiellement que le couplage du proton, avec son champ mésique, est identique à celui du neutron.

Avant de discuter de façon plus détaillée de l'extension des nucléons prédite par la théorie du méson, nous continuerons notre analyse des résultats expérimentaux, en considérant la diffusion des électrons par le deutérium.

8- DIFFUSION ELASTIQUE DES ELECTRONS PAR LE DEUTERIUM.

La diffusion élastique et inélastique (électro-désintégration) a été étudiée par McIntyre, Hofstadter et leurs collaborateurs. Pour l'énergie de 190 Mev environ, les résultats ont été publiés dans : Phys. Rev. 98, 158, 1955. Les résultats préliminaires pour 500 Mev ont paru dans Phys. Rev. 103, 1464, 1956. Une étude systématique de la diffusion à haute énergie a, enfin été faite par McIntyre et Dhar :(Electron scattering from the deuteron and the neutron-proton potential), (sous presse à la Phys. Rev.). Les mesures ont été effectuées à 400 Mev pour des angles de diffusion compris entre 30° et 90° et à 500 Mev pour des angles compris entre 30 et 80°. Les sections efficaces n'ont pas été déterminées en valeur absolue, mais seulement par comparaison avec l'hydrogène.

Si $f_{1p}(r)$ et $f_{1n}(r)$ représentent respectivement, dans le deutéron, l'extension de la charge du proton et du neutron, et si $\psi_D(\vec{r})$ est la fonction d'onde de l'état fondamental, la distribution de charge (exprimée en unités égales à la charge de l'électron) est égale à :

$$\rho(\vec{r}) = \int \left[f_{1p}(\vec{r} - \vec{r}') + f_{2p}(\vec{r} - \vec{r}') \right] |\psi_D(\vec{r}')|^2 d\vec{r}' \quad (74)$$

et, par suite, le facteur de forme correspondant peut s'écrire :

$$F(q^2) = \left[F_{1p}(q^2) + F_{1n}(q^2) \right] F_D(q^2) \quad (75)$$

où $F_D(q^2)$ est la transformée de Fourier de $|\psi_D(\vec{r})|^2$

Si nous développons l'expression entre crochets de (75) en puissances de q^2, nous avons d'après (20) et (68) :

$$F(q^2) \simeq \left[1 - \frac{q^2}{6} \left(\langle r_{1p}^2 \rangle + \langle r_{1n}^2 \rangle \right) + \text{etc.} \right] F_D(q^2). \qquad (76)$$

Un calcul des distributions de charges statiques du nuage mésique de chaque nucléon conduirait rigoureusement au résultat $\langle r_{1p}^2 \rangle = \langle r_{1n}^2 \rangle$ et par suite à $F(q^2) = F_D(q^2)$. Nous avons vu au paragraphe précédent que ceci est en contradiction avec les résultats expérimentaux sur la diffusion des électrons par les nucléons libres. Nous supposons, pour commencer, que dans le deutéron, conformément à la théorie du méson, les contributions des nuages mésiques s'annulent identiquement et que le facteur de forme se réduit effectivement à F_D. Cette dernière fonction a été calculée par Jenkins (Phys. Rev. 102, 1586, 1956) qui ont la section efficace de diffusion élastique sous la forme :

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{2E} \right)^2 \frac{\cos^2 \frac{\vartheta}{2}}{\sin \frac{\vartheta}{2}} \frac{1}{1 + \frac{E}{M} \sin^2 \frac{\vartheta}{2}} F_D^2(q^2) \qquad (77)$$

et la fonction d'onde du deuteron dans l'état fondamental :

$$\Phi_{1m}(\vec{r}) = \frac{1}{\sqrt{4\pi}} \cdot \frac{1}{r} \left[u(r) + \frac{1}{\sqrt{8}} S_{12}\, w(r) \right] \chi_m \qquad (78)$$

où χ_m est la fonction de spin correspondant aux 3 valeurs du nombre quantique magnétique +1, 0, ou -1 ; $u(r)$ et $w(r)$ sont respectivement les fonctions radiales des états S et D et S_{12}, l'opérateur tensoriel de spin habituel :

$$S_{12} = \frac{3(\vec{\sigma}_1 \vec{r})(\vec{\sigma}_2 \vec{r})}{r^2} - (\vec{\sigma}_1 \vec{\sigma}_2). \qquad (79)$$

Dans ces conditions F_D^2 se divise en trois parties

$$F_D^2(q^2) = F_S^2(q^2) + F_Q(q^2) + F_M(q^2) \times \frac{q^2}{6} \left[\frac{2}{\cos^2 \frac{\vartheta}{2}} - 1 \right] \qquad (80)$$

où F_S est la contribution de la partie de la distribution de charge qui a la symétrie sphérique :

$$F_S(q^2) = \int_0^\infty (u^2+w^2)\, j_0(\tfrac{1}{2}qr)\, dr \qquad (81)$$

F_Q est un terme quadrupolaire :

$$F_Q(q^2) = \int_0^\infty 2w\left(u-\frac{w}{\sqrt{8}}\right) j_2\left(\frac{qr}{2}\right) dr \qquad (82)$$

et F_M un terme d'origine magnétique :

$$F_M(q^2) = \int dr \left\{ \left[(\mu_n+\mu_p)(u^2+w^2) - \frac{3}{2}(\mu_p+\mu_n-\tfrac{1}{2})w^2 \right] j_0\left(\frac{qr}{2}\right) \right. \qquad (83)$$

$$\left. + \frac{3w}{\sqrt{2}}\left[(\mu_n+\mu_p)\left(u+\frac{2w}{\sqrt{2}}\right) + \frac{3w}{\sqrt{8}} \right] j_2\left(\frac{qr}{2}\right) \right\}$$

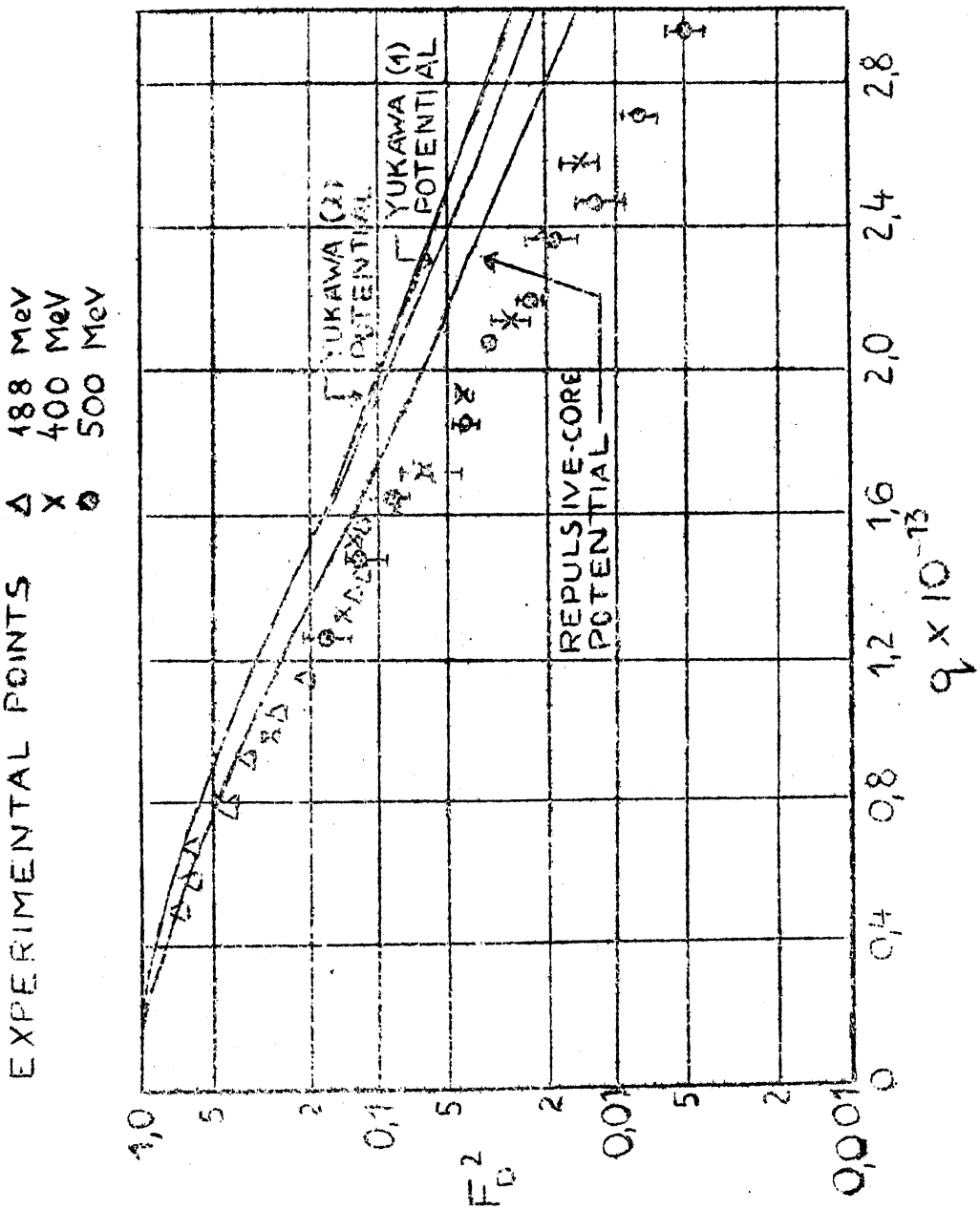
où μ_p et μ_n sont les moments magnétiques totaux du proton et du neutron, et $j_i(x)$ les fonctions de Bessel sphériques habituelles.

En pratique, les termes qui font intervenir $w(r)$ en facteur sont très petits (ils représentent une correction de quelques millièmes seulement). Si on les néglige, on peut écrire F_D^2 sous la forme :

$$F_D^2(q^2) = \left[1 + \frac{q^2}{6}\left(\frac{2}{\cos^2\frac{\theta}{2}} - 1\right)(\mu_p+\mu_n)^2 \right] F_S^2(q^2). \qquad (84)$$

La comparaison détaillée de ces formules avec les résultats expérimentaux se heurte à la difficulté que l'on ne connait pas avec certitude, l'interaction neutron-proton, et par suite, la fonction d'onde du deutéron dans l'état fondamental ($^3S_1 + ^3D_1$). McIntyre et Dhar ont étudié systématiquement les différents patentiels qui permettent de rendre compte correctement de l'énergie de liaisonet du moment électrique quadrupolaire de la diffusion neutron-proton aux basses énergies dans l'état triplet (" effective range"). Ce sont, tout d'abord, deux types phénoménologiques de potentiels de Yukawa déterminés par Feshbach et Schwinger (Phys. Rev. 84, 194, 1951), qui ne different entre eux que par une prédiction légèrement différente du pourcentage P_D d'état D dans

l'état fondamental. Ensuite ils ont considéré le potentiel de Gartenhaus, qui contient un coeur répulsif aux environ de $r_0 = 0,5 \times 10^{-13}$ cm et qui se déduit de la théorie mésique avec source fixe de Chew et Low (Phys. Rev. 100, 900, 1955). Les valeurs correspondantes de F_D^2 sont représentés sur la figure(10) en même temps que les points expérimentaux obtenus entre 188 Mev et 500 Mev (on remarquera que F_D^2 ne doit dépendre que de q^2, et non pas de l'énergie particulière des électrons incidents à laquelle les mesures sont faites):



On constate que pour les valeurs de q qui approchent de $3 \times 10^{13}\ cm^{-1}$, le désaccord entre l'expérience et le modèle le plus favorable (potentiel à coeur répulsif) correspond à un facteur 3 environ. On remarquera également que pour des transferts d'impulsion aussi grands, les prédictions théoriques peuvent varier d'un facteur 2 d'un modèle à l'autre. En tout cas, les résultats expérimentaux indiquent que la distribution de charge du deuteron est nettement plus étendue que celle prédite par les trois modèles considérés plus haut. Pour s'affranchir de l'incertitude sur l'interaction neutron-proton, McIntyre et Dhar se sont demandé quelle était l'interaction qui, pour une portée effective $^3r_0$ fixée à sa valeur expérimentale $^3r_0 = (1,70 \pm 0,03) \times 10^{-13}\ cm$, donnera la fonction d'onde du deuteron la plus étendue. On voit assez facilement qu'il suffit de prendre une interaction de la forme :

$$V = -V_0\,\delta(\vec{r} - \vec{a}) \qquad r \geq a$$
$$V = +\infty \qquad r < a \qquad (85)$$

c'est à dire une combinaison d'un coeur répulsif et d'une force attractive de portée nulle. Il apparait, cependant, que même cette interaction extrême conduit quand même à un désaccord de l'ordre de 100% ($\sigma_{th}/\sigma_{exp} \simeq 2$) avec les résultats expérimentaux. Il est donc nécessaire d'admettre que le proton et le neutron n'ont pas, dans le deuteron, des nuages mésiques de charge opposée et d'extension comparable. S'inspirant des résultats obtenus sur les nucléons libres, McIntyre et Dhar prennent dans la formule (75), $F_{1n} = 0$ et F_{1p} égal à celui déterminé par Hofstadter sur les protons libres. Ils s'efforcent de décider alors quel est le potentiel neutron-proton qui s'accorde le mieux avec les résultats expérimentaux. Il semble que le meilleur accord soit obtenu avec l'un des potentiels du type Yukawa de Feshbach et Schwinger. Cependant, la grande latitude qui existe sur la forme de distribution de charge du proton (§5) permet en réalité d'obtenir un accord satisfaisant avec tous les trois potentiels considérés. Ceci est illustré par la table suivante :

DIFFUSION ELECTRON-DEUTERON		
Espèce de deutéron supposée	Rayon et forme de proton nécessaire	
	Exponentielle creuse	Gaussienne
Gartenhaus	0,68 ± 0,02	0,66 ± 0,02
Yukawa I	0,80 ± 0,02	0,77 ± 0,02
Yukawa II	0,84 ± 0,02	0,80 ± 0,02
DIFFUSION ELECTRON-PROTON		
	0,78 ± 0,05	0,72 ± 0,05

Il convient de remarquer enfin qu'il est peu vraisemblable que la distribution de charge du proton ne soit pas déformée par la présence d'un neutron même ponctuel, puisque celui-ci possède un moment magnétique anormal.

9- DIFFUSION INELASTIQUE DES ELECTRONS PAR LE DEUTERIUM

La diffusion inélastique des électrons sur le deutérium, qui a été étudié par Blankenbecler, Hofstadter, et Yearian (à paraître dans Phys. Rev.), permet en principe de connaître la distribution des impulsions des nucléons dans le noyau, puisqu'on peut la considerer comme une diffusion élastique sur les nucléons en mouvement à l'intérieur du deutéron. La formule de Rosenbluth montre qu'aux grands angles et pour les grandes énergies (valeurs élevées de q^2), c'est la diffusion par le moment magnétique anormal qui prédomine. Par ailleurs, la diffusion par le neutron s'écrit de façon analogue :

$$\sigma_n = \sigma_M(\vartheta)\frac{q^2}{4M^2}\left[2\varkappa_n \operatorname{tg}^2\frac{\vartheta}{2} + \varkappa_n^2\right]F_2^{(n)}(q^2) \qquad (86)$$

et l'on peut s'attendre à ce que, pour les grandes valeurs de q^2, le rapport σ_p/σ_n soit de l'ordre de $(\varkappa_p/\varkappa_n)^2 \simeq 2$, à condition que les facteurs de forme magnétiques soient comparables. Si l'extension de la distribution magnétique du proton est très supérieure à celle du neutron on pourra s'attendre à ce que la section efficace de diffusion par les neutrons soit, pour les grands transferts d'impulsions, comparable ou même supérieure à celle des protons.

Jenkins a calculé également la section efficace de diffusion inélastique(électro désintégration du deutéron) qui est donnée par l'expression suivante :

$$\sigma_D^{in} = \sigma_M\left\{1 - F_D^2 + \frac{q^2}{4M^2}\left[2(\mu_p^2 + \mu_n^2 - 3F_D^2)\operatorname{tg}^2\frac{\vartheta}{2} + \mu_p^2 + \mu_n^2 - 3F_D^2\right]\right\} \qquad (87)$$

qui se réduit, si q^2 est grand et $F_D^2 \ll 1$ à :

$$\sigma_D^{in} = \sigma_M\left\{1 + \frac{q^2}{4M^2}\left[2(\mu_p^2 + \mu_n^2)\operatorname{tg}^2\frac{\vartheta}{2} + \mu_p^2 + \mu_n^2\right]\right\} \qquad (88)$$

et par conséquent à :

$$\sigma_D^{in}(\Omega) \simeq \sigma_p + \sigma_n . \qquad (89)$$

On peut, en principe, déterminer σ_n (et, par suite $F_2^{(n)}$) en soustrayant, aux grands angles, la section efficacepour des protons libres de la valeur obtenue pour la section efficace inélastique. La fig. (11) représente pour E=500 Mev et θ=135°, le spectre inélastique des électrons diffusés sur du polyéthylène deutéré (courbe en trait plein) et, après soustraction de la diffusion par le carbone, le spectre inélastique correspondant au deutérium.

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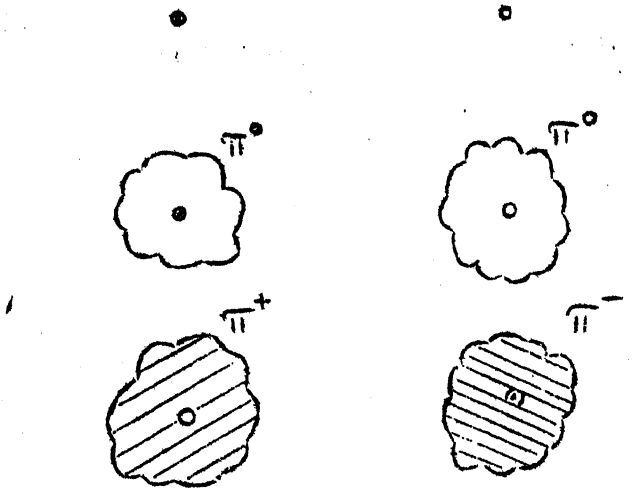
Il est évidemment beaucoup plus large que le pic élastique (50 mev, au lieu de 5mev environ), en raison du mouvement des nucléons dans le noyau. La soustraction du pic de diffusion libre des protons permet de calculer σ_n à 135°. Les expériences préliminaires de Blankenbecler et al, dont la précision est encore insuffisante, semblent indiquer cependant que le carré moyen du rayon de la distribution magnétique du neutron est à peu près du même ordre que celle du neutron ($\langle r_{2n}^2 \rangle^{1/2} \sim 0{,}6 \cdot 10^{-13}$ cm). Il semblera donc que la mystérieuse annulation de l'extension finie du neutron, mise en évidence aux § 7 et 8, ne concerne en fait que la distribution de charge, et non pas son moment magnétique anormal.

10- COMPARAISON QUALITATIVE DES DISTRIBUTIONS DE CHARGE DU PROTON ET DU NEUTRON. INFLUENCE DES MESONS K.

Nous supposons que les nucléons physiques sont composés d'un nucléon nu (ou coeur) et d'un nuage mésique, principalement dû aux mésons π virtuels (l'effet des mésons K sera discuté ensuite). L'hypothèse de la symétrie de charge de l'interaction nucléon-méson π implique que, chaque fois que l'état du proton physique contient un méson d'une certaine charge, il y a un état physique correspondant pour le neutron, qui contient un méson de charge opposée. (Fig. 12).



Ceci est vrai quel que soit le nombre de mésons π qui composent le nuage Utilisant une méthode due à Lack (Phys. Rev. 87, 1100, 1952), on peut utiliser ce fait pour éliminer la contribution du nuage mésique à la distribution de charge des nucléons. Si $\rho_c(\vec{r})$ est la distribution du coeur, on a exactement (dans la mesure où l'ypothèse de la symétrie de charge est valable) :

$$\rho_c(\vec{r}) = \rho_p(\vec{r}) + \rho_n(\vec{r}) \qquad (90)$$

et, par suite, le rayon carré moyen de la distribution ρ_c s'écrit :

$$\langle r^2 \rangle_c = \langle r^2 \rangle_{1,p} + \langle r^2 \rangle_{1,n} + \frac{3}{4M^2}\left[1 + 2(\mu^n + \mu^p)\right] \qquad (91)$$

Il en résulte que :

$$\langle r^2 \rangle_c^{1/2} = 0,77.10^{-13}\ \text{cm}. \qquad (92)$$

Ceci doit être comparé à la longueur de Compton des nucléons qui est de $0,21 \times 10^{-13}$ cm et au rayon de la distribution de charge du proton, qui est de $0,84 \times 0,^{13}$ cm. On voit que le rayon du coeur est égal à 3 fois et demie la longueur de Compton des nucléons et aux 3/10 du rayon de la distribution de charge du proton observée par Hofstadter et Chambers. Nous verrons qu'il est très difficile d'interpréter ce résultat sur la base de la théorie du méson, même en incluant les effets de recul. Contentons nous d'indiquer dès maintenant que la valeur (92) du rayon du coeur semble être en contradiction avec les valeurs pratiquement égales et opposées des moments magnétiques anormaux des nucléons, qui tendent à montrer que la contribution du coeur est extrèmement faible.

L'effet du nuage des mésons π peut, dans une certaine mesure, être contrebalancé par celui des mésons K virtuels, dont une théorie complète doit nécessairement tenir compte. Cette remarque ingénieuse due à Sandri (Phys. Rev. 101, 1616, 1956) est basée sur le comportement différent des mésons K^+ et K^- dont l'étrangeté (voir par exemple Gell Mann, Nuov. Cim. Suppl. 848, 1956) est différente. Pour comprendre qualitativement cette influence, comparons, dans l'hypothèse du couplage faible (dissociation des nucléons avec émission d'un seul méson virtuel), le comportement des deux

types de mésons. Pour les mésons π (si l'on élimine les émissions de mésons neutres) les deux reactions sont :

$$p \rightarrow n + \pi^+ \quad | \quad n \rightarrow p + \pi^-$$

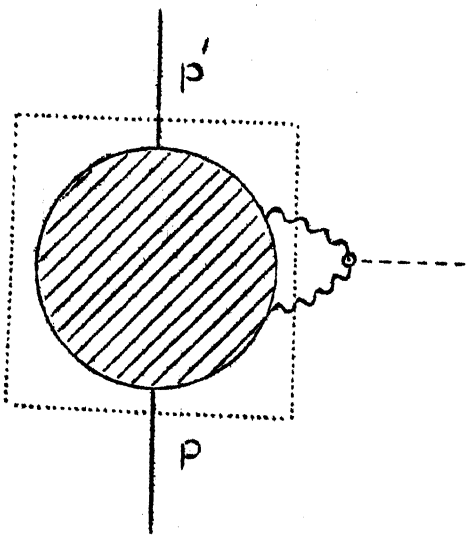
Pour les mésons K, au contraire, on a :

$$p \rightarrow \Sigma^0 + K^+ \quad | \quad n \rightarrow \Sigma^- + K^+.$$

l'émission d'un seul K^- étant interdite par la conservation de l'étrangeté. On voit donc que les nuages de π et de K ont tendance à ajouter leurs charges dans le cas des protons et à les retrancher dans le cas des neutrons. Il convient de remarquer cependant que la longueur d'onde de Compton des mésons K étant très inférieure à celle des π cet effet risque de ne pas être très important. Une estimation basée sur l'interaction K-nucléons à basse énergie observée dans les émulsions photographiques conduit à une contribution à l'intéraction électron-neutron due aux mésons K de l'ordre de +1 Kev. Cet effet va donc dans la bonne direction, mais semble très insuffisant pour supprimer le désaccord existant actuellement. On peut remarquer également que si les effets de recul des "coeurs" devaient s'averer importants, les contributions dues au recul des protons et des Σ^- dans le cas de la dissociation du neutron, se compenseraient en grande partie.

11.- STRUCTURE DU NUCLEON DANS LA THEORIE DU MESON AVEC SOURCE FIXE.

Dans la théorie du méson avec source fixe de Chew et Low (Phys. Rev. 101, 1597, 1956), il est possible de calculer exactement en fonction des amplitudes de diffusion méson-nucléon, toutes les valeurs moyennes (dans l'état du nucléon physique) des opérateurs qui s'expriment sous forme bilinéaire en fonction des opérateurs de création et d'annihilation des mésons. Ceci a été réalisé pour la première fois par Miyazawa qui a appliqué cette technique très élégante au calcul des moments magnétiques anormaux des nucléons (Phys. Rev. 101, 1564, 1956). On peut comprendre qualitativement la raison de ce phénomène si l'on se reporte à la fig. 13, qui représente schématiquement à l'ordre le plus bas de la constante de structure fine électromagnétique (mais à tous les ordres en ce qui concerne le couplage avec le champ mésique), l'intéraction du champ électromagnétique avec le nuage mésique (dans tout ce paragraphe, les intéractions avec le "coeur" des nucléons sont négligés).



On voit que la partie de la figure qui est entourée d'un trait pointillé n'est pas autre chose qu'une diffusion d'un méson de quantité de mouvement, initiale $\vec{K}$ et finale $\vec{K}'$. Il convient de remarquer que cependant expérimentalement les amplitudes de diffusion ne sont connues que sur la couche d'énergie($\omega_k = \omega_{k'}$) tandis que dans la fig. 13, les mésons n'intervenant que de façon virtuelle, il n'est pas nécessaire de conserver l'énergie. C'est pourquoi il n'est possible de calculer les intéractions électromagnétiques des mésons π à partir de la diffusion méson-nucléon, que dans la théorie de la source fixe, où les éléments de matrice peuvent s'extrapoler très facilement, hors de la couche d'énergie. Miyazawa a montré également (Phys. Rev. 104, 1741, 1956) que si l'on se borne aux échanges ne faisant pas intervenir plus de deux mésons virtuels, on peut calculer exactement les forces nucléon-nucléon à partir des amplitudes de diffusion méson-nucléon (fig. 14). Le diagramme (a) se ramène au potentiel déduit par la théorie des perturbations au deuxième ordre, si l'on remplace la constante de couplage non renormalisée f_0 par la constante renormalisée f. Le diagramme (b) se décompose en une partie qui est proportionnelle à f^4 et qui correspond au terme de perturbation du 4ème ordre, et une partie indépendante de f qui s'exprime en fonction des amplitudes de diffusion.

Nous écrirons l'Hamiltonien de Chew et Low sous la forme :

$$H = H_0 + H_I \qquad (93)$$

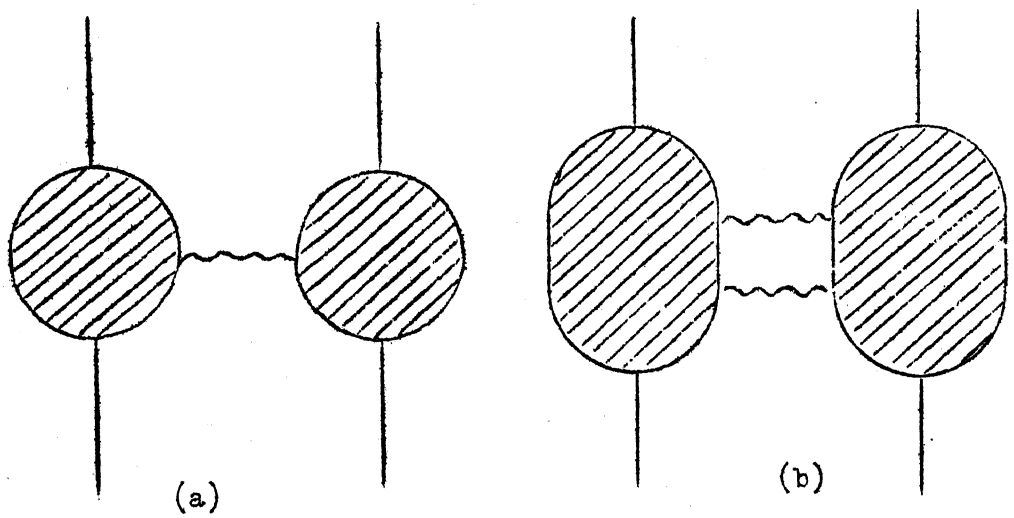


Fig. 14

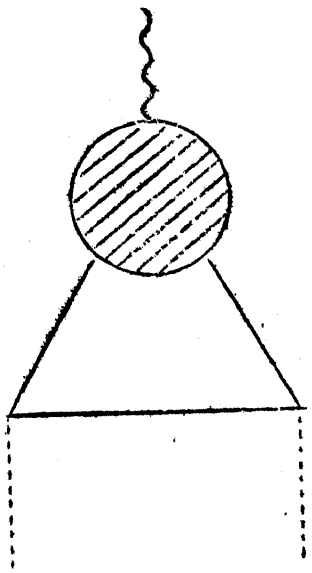


Fig. 15

où

$$H_o = \sum_{k\alpha} a^+_{k\alpha} a_{k\alpha} \omega_k \qquad (94)$$

est l'Hamiltonien des mésons libres ; $\omega_k = (k^2+\mu^2)^{1/2}$ μ =masse des mésons π , les α (1, 2, 3,) correspondent au spin isotopique.
L'Hamiltonien d'intéraction s'écrit :

$$H_I = \sum_{k,\alpha} \left(V_{k\alpha} a_{k\alpha} + V^+_{k\alpha} a^+_{k\alpha} \right) \qquad (95)$$

où l'on pose :

$$V_{k\alpha} = i f_o \frac{(\vec{\sigma}\vec{k})}{\sqrt{2\omega_k}} \tau_\alpha v(k). \qquad (96)$$

Les $a_{k\alpha}$ et $a^+_{k\alpha}$ sont respectivement les opérateurs d'annihilation et de création de mésons d'impulsion k dans l'état α ; $\vec{\sigma}$ et $\vec{\tau}$ sont les spin et spin isotopique du nucléon, représenté par une fonction de source dont la transformée de Fourier est $v(k)$.

Supposons que nous voulions calculer la distribution de charge des mésons. Celle-ci est donnée par la valeur moyenne, dans l'état du nucléon physique, de l'opérateur (Wentzel, Quantum théory of field) :

$$\varrho(\vec{r}) = \frac{ie}{2} \sum_{k,k'} \frac{e^{i(\vec{k}+\vec{k}')\vec{r}}}{\sqrt{\omega_k \omega_{k'}}} \left[(\omega_k - \omega_{k'}) \left(a_{k1} a_{k2} - a^+_{-k1} a^+_{-k2} \right) \right. \qquad (97)$$

$$\left. - (\omega_k + \omega_{k'}) \left(a^+_{-k1} a_{k2} - a_{k1} a^+_{-k2} \right) \right]$$

où seuls, évidemment, interviennent les opérateurs des mésons chargés. On voit que $\varrho(\vec{r})$ dépend bilinéairement des $a_{k\alpha}$ et $a^+_{k\alpha}$. Le calcul, et la détermination de la distribution de charge des protons, ainsi que de l'intéraction électron-neutron, a été fait indépendemment par Fubini (Nuov. Cim. 3, 1425, 1956), Treiman et Sachs (Phys. Rev. 103, 435, 1956) et Salzman (Phys. Rev. 105, 1076, 1957) . Nous en indiquerons le principe en calculant la valeur moyenne dans l'état du nucléon physique d'un produit de deux opérateurs $a_k a_{k'}$, (nous laissons tomber, pour simplifier l'écriture les indices α).

Le nucléon physique est représenté par la fonction propre d'énergie la plus basse de l'Hamiltonien total :

$$H\Psi_0 = 0 \tag{98}$$

et nous proposons de calculer :

$$P_0(k,k') = (\Psi_0, a_k a_{k'} \Psi_0). \tag{99}$$

A partir des équations (94) et (95), on vérifie facilement que :

$$[H, a_k] = -\omega_k a_k - V_k^+. \tag{100}$$

En appliquant les deux membres de (100) à Ψ_0, et en tenant compte de (98), on obtient la relation :

$$(H+\omega_k) a_k \Psi_0 = V_k^+ \Psi_0 \tag{101}$$

que l'on peut écrire aussi formellement :

$$a_k \Psi_0 = -\frac{1}{H+\omega_k} V_k^+ \Psi_0 \tag{102}$$

Si nous ajoutons $a_k \omega_{k'}$, aux deux membres de (100) nous obtenons l'équation :

$$(H+\omega_k+\omega_{k'}) a_k = -V_k^+ + a_k (H+\omega_{k'}). \tag{103}$$

En appliquant les deux membres de (103) à $a_{k'}$, et en utilisant encore l'équation (100) (avec $k \to k'$), on obtient ;

$$\begin{aligned}(H+\omega_k+\omega_{k'}) a_k a_{k'} &= -V_k^+ a_{k'} + a_k (H+\omega_{k'}) a_{k'} = \\ &= -V_k a_{k'} + a_k [-V_{k'}^+ + a_{k'} H].\end{aligned} \tag{104}$$

Nous appliquons les 2 membres de (104) à Ψ_0 , et nous tenons compte de (98) et (102) :

$$(H+\omega_k+\omega_{k'})\,a_k a_{k'}\Psi_0 = V_k^+ \frac{1}{H+\omega_{k'}} V_{k'}^+ \Psi_0 + V_{k'}^+ \frac{1}{H+\omega_k} V_k^+ \Psi_0 \,, \qquad (105)$$

(ne pas oublier que a_k et $V_{k'}^+$ commutent). Ceci peut s'écrire, en opérant à gauche avec $(H+\omega_k+\omega_{k'})^{-1}$,en multipliant par Ψ_0^* , et en tenant compte de (98) :

$$P_0(k,k') = \frac{1}{\omega_k+\omega_{k'}}\left[\left(\Psi_0, V_k^+ \frac{1}{H+\omega_{k'}} V_{k'}^+ \Psi_0\right) + \text{terme obtenu en échangeant k et k' (que nous désigneront simplement par ex. dans la suite).}\right. \qquad (106)$$

Pour nous débarasser du terme en $(H+\omega_k)^{-1}$ à droite de (106), nous utiliserons la règle de multiplication des éléments de matrice en prenant pour base complète et orthonormale l'ensemble des vecteurs propres "entrants" ("incoming" state vectors) $\Psi_n^{(-)}$ de l'Hamiltonien total H :

$$H\Psi_n^{(-)} = E_n \Psi_n^{(-)} \,. \qquad (107)$$

Nous avons donc :

$$P_0(k,k') = \frac{1}{\omega_k+\omega_{k'}}\left[\sum_n \frac{1}{E_n+\omega_{k'}} \left(\Psi_0, V_k^+ \Psi_n^{(-)}\right) \cdot \left(\Psi_n^{(-)}, V_{k'}^+ \Psi_0\right) + \text{ex.}\right] \qquad (108)$$

Nous considérons un élément de la matrice S entre un état à un méson q et un état quelconque, et nous l'exprimons en fonction des éléments de la matrice de réaction par la relation :

$$(n|S|q) = \delta_{nq} - 2\pi i\, \delta(E_q - E_n)\, T_q(n). \qquad (109)$$

On peut alors montrer (voir l'article de Chew et Low cité plus haut) que :

$$T_q(n) = (\Psi_n^{(-)} V_q \Psi_0), \tag{110}$$

et que : $$T_q^+(n) = (\Psi_0, V_q^+ \Psi_n^{(-)}) = -(\Psi_0, V_q \Psi_n^{(-)}) \tag{111}$$

d'où l'équation :

$$P_0(k,k') = -\frac{1}{\omega_k + \omega_{k'}} \sum_n \left[\frac{1}{E_n + \omega_{k'}} T_k^+(n) T_{k'}(n) + ex. \right] \tag{112}$$

Dans la somme située à droite de (112) le premier terme qui correspond aux états du nucléon physique conduit, en ce qui concerne la distribution de charge, au même résultat que celui de la théorie des perturbations, avec remplacement de f_0 par la constante de couplage renormalisée f. Les autres termes sont exprimables en fonction des amplitudes de diffusion méson-nucléon (en utilisant la relation de fermeture).
Il est commode, de plus, de décomposer les éléments de matrice de réaction en élément qui se réfèrent à des états de spin isotopique i et de spin total j bien définis. Si l'on pose, comme d'habitude :

$$\begin{aligned} \delta_1(k) &\equiv \delta\left(k;\ j=\tfrac{1}{2}, i=\tfrac{1}{2}\right) \\ \delta_2(k) &\equiv \delta\left(k;\ j=\tfrac{1}{2}, i=\tfrac{3}{2}\right) = \delta\left(k; j=\tfrac{3}{2}, i=\tfrac{1}{2}\right) \\ \delta_3(k) &\equiv \delta\left(k;\ j=\tfrac{3}{2}, i=\tfrac{3}{2}\right) \end{aligned} \tag{113}$$

où les δ sont les déphasages dela diffusion dans les états p et si l'on écrit :

$$h_j(k) = \frac{e^{i\delta_j} \sin\delta_j}{k^2 v^2(k)} \tag{114}$$

les amplitudes de diffusion correspondantes, on obtient pour la distribution de charge $\rho(\vec{x})$ (où un certain nombre de valeurs moyennes du type (112) entervienneent) l'expression suivante :

$$\rho(\vec{r}) = \rho_0(\vec{r}) + \rho_1(\vec{r}) \tag{115}$$

où l'on pose :

$$\rho_0(\vec{r}) = -\frac{4e}{(2\pi)^5}\left(\frac{f}{\mu}\right)^2 \tau_3 \int d\vec{k}\, d\vec{k}' \frac{v(k)\, v(k') e^{i(\vec{k}+\vec{k}')\vec{r}}}{\omega_k\, \omega_{k'}(\omega_k + \omega_{k'})} (\vec{k}.\vec{k}') \tag{116}$$

et :

$$\rho_1(\vec{r}) = -\frac{8e}{3(2\pi)^6}\tau_3 \int d\vec{k}\, d\vec{k}' \frac{v(k) v(k') e^{i(\vec{k}+\vec{k}')\vec{r}}}{(\omega_k + \omega_{k'})} \cdot (\vec{k}\vec{k}') \cdot (117)$$

$$\cdot \int d\vec{k}'' \frac{v^2(k'')\left\{|h_1(\omega'')|^2 + |h_2(\omega'')|^2 - 2|h_3(\omega'')|^2\right\}}{(\omega_k + \omega_{k''})(\omega_{k'} + \omega_{k''})\omega_{k''}}.$$

On remarquera que comme $2|h_3|^2$ est en moyenne beaucoup plus grand que $|h_1|^2 + |h_2|^2$ (en raison de la résonnance dans l'état $\left(\frac{3}{2}, \frac{3}{1}\right)$), le terme en $\rho_1(\vec{r})$ est de signe opposé à $\rho_0(\vec{r})$, ce qui montre que les corrections d'ordre supérieur (ou corrections de diffusion) à la distribution de charge, tendront à diminuer celle-ci.

Salzman (référence plus haut) a appliqué le calcul précédent à la diffusion électron proton à 180 mev, en négligeant toutefois les corrections de diffusion (117). Il montre qu'un bon accord peut être obtenu avec les mesures d'Hofstadter et al, en prenant pour impulsion de cut-off k_{max} [$v(k) = 1$ pour $k < k_{max}$, $v(k) = 0$ pour $k > k_{max}$ et pour constante f les mêmes valeurs que Chew et Low. Il convient de remarquer cependant qu'à l'énergie considérée, les résultats expérimentaux sont exprimables uniquement en fonction du carré moyen de la distribution de charge $\langle r^2 \rangle_{ch}$ qui, comme nous le verrons plus loin, est modifié d'environ 20% par les corrections de scattering. Il serait intéréssant de déterminer l'influence de ces corrections à des énergies plus élevées, et de comparer la forme de la distribution prévue avec

celle qu'ont obtenu Chambers et Hofstadter à 500 Mev.

Treiman et Sachs ont utilisé la même technique pour le calcul de l'intéraction électron-neutron, ou plus exactement du terme :

$$V_0^{(2)} = + \frac{e}{2 r_0^3} \int r^2 \rho(\vec{r})\, d\vec{r} \qquad (118)$$

défini par l'équation (71). Cette intéraction peut s'exprimer en fonction des sections efficaces totales de diffusion de mésons positifs et négatifs $\sigma^+(K)$ et $\sigma^-(K)$ grace aux relations :

$$\sigma^+(K) = \frac{8\pi^2\omega}{K} \left\{ 2 |h_3(K)|^2 + |h_2(K)|^2 \right\} \qquad (119)$$

$$\sigma^-(K) = \frac{8\pi^2\omega}{3K} \left\{ 2 |h_3(K)|^2 + 5 |h_2(K)|^2 + 2 |h_1(K)|^2 \right\}$$

d'où l'on tire :

$$\sigma^-(K) - \sigma^+(K) = \frac{16\pi^2\omega}{K} \left\{ |h_2(K)|^2 + |h_1(K)|^2 - 2 |h_3(K)|^2 \right\} \qquad (120)$$

On reconnait au second membre de (120) la même combinaison des $|h_j|^2$ que celle qui apparait à droite de (117). Le calcul de la valeur moyenne de r^2 fait intervenir deux différenciations par rapport aux variables k et k' ce qui conduit à l'expression suivante : $V_0^{(2)} = V_{01}^{(2)} + V_{02}^{(2)}$

$V_{01}^{(2)}$ résulte une fois de plus du terme de perturbation :

$$V_{01}^{(2)} = \frac{e}{2\pi r_0^3} \left(\frac{f}{\mu}\right)^2 \int dK \frac{k^4}{\omega^7} \left[2\omega_K^4 \left(\frac{dv}{dk}\right)^2 + 5 \left(3\omega_K^2 - 2K^2\right) v^2(K) \right] \qquad (122)$$

et $V_{02}^{(2)}$ des corrections de diffusion :

$$V_{02}^{(2)} = - \frac{e}{16\pi^3 r_0^3} \int dK \frac{k^4 v^2(K)}{\omega_K} \int dK' \frac{\sigma^+(K) - \sigma^-(K')}{\omega'_K v^2(K')} U(\omega'_K, \omega_K) \qquad (123)$$

où l'on a posé :

$$U(\omega'_K, \omega_K) = \frac{1}{(\omega+\omega')^2} \left[\frac{2\omega^2 + 3\mu^2}{\omega^4} - \frac{2}{v} \frac{d^2v}{dK^2} - 2 \left(\frac{3\omega^2 + \mu^2}{K\omega^2 v} \right) \frac{dv}{dK} \right] \qquad (124)$$

$$+ \frac{1}{(\omega+\omega')^3} \left[2 \left(\frac{3\omega^2 + 2\mu^2}{\omega^3} \right) + \frac{4K}{\omega v} \frac{dv}{dK} \right] - \frac{4}{\omega+\omega'} \frac{k^2}{\omega^2}$$

Deux difficultés interviennent dans l'évaluation de (123) :

1) En principe, il devrait être inutile d'inclure un cut-off dans l'intégration sur k', puisque la différence $\sigma^+(k) - \sigma^-(k)$ doit forcément tendre vers zéro aux très grandes énergies. Cependant, comme les sections efficaces de diffusion sont mal connues au delà de quelques centaines de Mev, et que l'on ne doit pas s'attendre à ce que la théorie statique du méson soit valable au delà, Treiman et Sachs utilisent un cut-off égal à 5 ou 6 fois la masse du méson π (μ dans nos notations).

2) L'existence de dérivées de la fonction de cut-off $v(k)$ rend impossible l'emploi d'une fonction du type carré, car la discontinuité à k_{max} introduirait des singularités sans signification physique. Une autre méthode consisterait donc à prendre une fonction de cut-off arrondie sur les bords, en modifiant, en conséquence, le paramètre qui détermine sa largeur de façon à obtenir des expressions correctes pour les sections efficaces de diffusion méson-nucléon. Treiman et Sachs se contentèrent de faire v = 1 dans (123) et(124) , $dv/dk = d^2v/dk^2 = 0$ et d'introduire un cut-off k_{max} dans les deux intégrations. Leur résultat final s'exprime par le tableau suivant :

$V_0^{(2)}$ correspondant à la distribution statique du nuage mésique :

cutt-off	$V_{01}^{(2)}$	$V_{02}^{(2)}$	$V_0^{(2)}$
5 μ	- 10,9 kev	+ 2,0 kev	- 8,9 kev
6 μ	- 12,4 kev	+ 2,4 kev	- 10,0 kev

On voit qu'il subsiste un très grand désaccord avec la valeur expérimentale que nous avons donnée pour $V_0^{(2)}$ (0 ± 300eV).

12- INFLUENCE DU RECUL DU COEUR DES NUCLEONS SUR LA DISTRIBUTION DE CHARGE

Dans toute la discussion qui précède, nous avons constamment négligé les effets du recul du coeur du nucléon qui, si nous en jugeons par la formule (92), doivent être très importants. Dans le terme de "recul", on inclut en réalité deux type d'effets très différents :

1) L'influence des créations de paires virtuelles de nucléons. Ce phénomène est en général considéré comme négligeable (ce qui est à la base même du succès de la théorie de la source fixe) car la probabilité de produire une paire de nucléons (même virtuels) est très faible. C'est ce que, dans le langage de la théorie du méson on appelle l'effet de "pair damping". Il est confirmé par trois types d'évidences expérimentalles :

a) la faible section efficace de production des anti-nucléons à Berkeley (par comparaison à ce que l'on obtiendrait par de simples considérations basées sur les volumes dans l'espace des phases);

b) la durée de vie relativement longue du méson π^0, qui se désintègre en deux rayons γ par un diagramme du type de la fig.(15) (mais cette évidence est moins certaine, à la suite des nouvelles mesures de la durée de vie du π^0); (voir p. 47 bis)

c) la petite contribution des mésons dans l'état s à la diffusion méson - nucléon et aux forces nucléaires (voir, par exemple, l'article de Miyazawa, (Phys. Rev. 104, 1741 1956). Notons toutefois, sans insister, qu'aucune de ces évidences n'est à l'abri de discussion, mais c'est plutôt leur concordance qui est impressionnante, ainsi que l'ampleur du succès de la théorie de Chew et Low, qui néglige explicitement tout phénomène de création de paires.

Malgré ce que nous venons de dire, Tamm (Communication au Congrès International de Physique Théorique de Seattle, Septembre 1956) a proposé une explication de l'étendue du coeur des nucléons basée sur la création de paires virtuelles. Supposons par exemple, que dans le nuage mésique, et assez loin du coeur, supposé pour l'instant quasi ponctuel, une paire de nucléons-antinucléons soit créée. Nous savons depuis les expériences de Berkeley, qu'il existe une force attractive à longue portée entre les nucléons et les anti-nucléons. Il y aura donc une

certaine probabilité pour que l'anti-nucléon ainsi créé s'annihile avec le coeur initialement présent, laissant comme nouveau coeur l'autre nucléon de la paire. Ce phénomène pouvant se reproduire un grand nombre de fois, le coeur du nucléon voyagera en pratique dans la totalité du nuage mésique, donnant ainsi au carré moyen $\langle r^2 \rangle_c$ une valeur pratiquement égale à celle correspondant au nucléon physique. Cette explication est ingénieuse, mais repose sur l'existence d'une forte probabilité de création de paires virtuelles (l'annihilation étant, de toute façon, très probable) qui n'est pas, pour le moment, confirmée expérimentallement.

2) Un autre effet qui peut agrandir le rayon du coeur des nucléons provient du phénomène purement dynamique de recul après émission et absorption d'un grand nombre de mésons. On trouvera une discussion qualitative de ce phénomène dans l'article de Yennie et al. (Rev. Mod. Phys. 29, 144, 1957). Nous parlerons plutôt ici d'un travail plus détaillé de Suura (Contribution of recoil to charge distribution of nucléons), à paraître à la Phys. Rev.,qui est basé sur une modification de la théorie de lasource fixe de Chew et Low qui inclut les effets dynamiques du nucléon. On suppose que le champ mésique ϕ_α (α = 1, 2, 3, correspondant au spin isotopique) est couplé aux nucléons par l'Hamiltonion :

$$H = H_\pi - i\vec{\alpha}\cdot\vec{\nabla} + \beta M + g\,\gamma_5\,\tau_\alpha\,\phi_\alpha \qquad (126)$$

où H_π est l'Hamiltonien du champ mésique libre, M la masse des nucléons, $\vec{\alpha}$ et β les matrices de Dirac habituelles. Nous appelons $\vec{x}$ les coordonées du nucléon source et $\vec{z}$ celles du champ mésique.
La densité de charge est donnée par la formule :

$$\rho(\vec{x},\vec{z}) = \frac{1}{2}(1+\tau_3)\,\delta(\vec{z}-\vec{x}) + \rho_\pi(\vec{z}) \qquad (127)$$

où ρ_π est la densité habituelle du nuage mésique.
Nous considérons l'état Φ_p du système qui représente un nucléon physique d'impulsion $\vec{p}$ et qui satisfait aux relations :

$$H\Phi_p = E_p\Phi_p \qquad (128)$$
$$\vec{P}\Phi_p = \vec{p}\,\Phi_p$$

où $E_p=(p^2+M^2)^{1/2}$ et où $\vec{P}$ est l'opérateur impulsion totale:

$$\vec{P} = -i\vec{\nabla} + \vec{p}_\pi . \tag{129}$$

L'élément de matrice dans l'état du nucléon physique de l'intéraction avec le champ électromagnétique scalaire A_o s'écrit :

$$M_e = \int (\Phi_{p'}(\vec{x}), \int \varrho(\vec{z},\vec{x}) H_o(\vec{z}) d\vec{z}\, \Phi_p(\vec{x}))\, d\vec{x} \tag{130}$$

On élimine maintenant les coordonnées du centre de gravité $\vec{X}$, puis celles du nucléon au moyen de la transformation canonique :

$$\Phi_o(\vec{x}) = e^{-i\vec{p}_\pi \cdot \vec{x}}\, \Psi_o \tag{131}$$

où $\Phi_o(\vec{x})$ est l'état du nucléon en repos défini par :

$$\Phi_p = e^{i\vec{p}\vec{X}}\, \Phi_o . \tag{132}$$

Le nouvel Hamiltonien s'écrit :

$$H = -i\vec{\alpha}\vec{\nabla} + \beta M - \vec{\alpha}\cdot\vec{p}_\pi + H_\pi + g\gamma_5 \tau_\alpha \Phi_\alpha . \tag{133}$$

Les coordonnées du centre de gravité étant définies par :

$$\vec{X} = \vec{x} + \vec{y} \tag{134}$$

avec :

$$\vec{y} = H^{-1}\vec{R} - \frac{i}{4}[\vec{\alpha}, H^{-1}] \tag{135}$$

et :

$$\vec{R} = \int H_\pi \vec{z}\, d\vec{z} . \tag{136}$$

La nouvelle densité de charge s'écrit :

$$\varrho(\vec{z},\vec{x}) = \frac{1}{2}(1+\tau_3)\,\delta(\vec{z}-\vec{x}) + \varrho_\pi(\vec{z}-\vec{x}) \tag{137}$$

Si l'on définit la transformée de Fourier $\rho(q)$:

$$\rho(q) = \frac{1}{2}(1+\tau_3) + \int \rho_\pi(z) e^{i\vec{q}\vec{z}}\, d\vec{z} \qquad (138)$$

sa valeur moyenne dans le nouvel état Ψ_0 est égale :

$$\langle \rho(q) \rangle_0 = \left(\Psi_0, e^{i\vec{q}\vec{y}}\, \rho(q) \Psi_0\right) \qquad (139)$$

où nous avons posé $\vec{q} = \vec{p} - \vec{p}'$.

En fonction de cette valeur moyenne de $\rho(q)$ on peut définir (par un développement en série de q^2) les rayons carrés moyens des distributions de charge du proton ou du neutron par les relations :

$$\begin{aligned} \langle r^2 \rangle_p &= \langle r^2 \rangle_c + \langle r^2 \rangle_{\pi,S} - \langle r^2 \rangle_{\pi,R} \\ \langle r^2 \rangle_n &= -\langle r^2 \rangle_{\pi,S} + \langle r^2 \rangle_{\pi,R} \end{aligned} \qquad (140)$$

(on vérifie bien la relation $\langle r^2 \rangle_p + \langle r^2 \rangle_n = \langle r^2 \rangle_c$) où le carré moyen du rayon du coeur est donné par :

$$\langle r^2 \rangle_c = \left(\Psi_0, \vec{y}^2 \Psi_0\right) \qquad (141)$$

$\vec{y}$ étant défini en fonction de H par (135) et (136), et où $\langle r^2 \rangle_{\pi,S}$ et $\langle r^2 \rangle_{\pi,R}$ sont respectivement le carré moyen du rayon de la distribution statique du nuage des mésons π :

$$\langle r^2 \rangle_{\pi,S} = \left(\Psi_0, S \Psi_0\right) \qquad (142)$$

avec :

$$S = \int \rho_\pi(z)\, z^2\, d\vec{z} \qquad (143)$$

et le carré moyen du <u>recul du nuage mésique</u> :

$$\langle r^2 \rangle_{\pi,R} = 2\left(\Psi_0, \vec{y}\vec{T}\, \Psi_0\right) \qquad (144)$$

où nous avons posé :

$$\vec{T} = \int \rho(z)\, \vec{z}\, d\vec{z}. \qquad (145)$$

Il reste à évaluer les valeurs moyennes S, y^2 et $\vec{y}.\vec{T}$. Le premier opérateur est une expression bilinéaire des opérateurs de création et d'annihilation du champ ϕ (d'après ce que nous avons vu plus haut) tandis que y^2 et $\vec{y}.\vec{T}$ sont des fonctions quadrilinéaires, qui, lorsque seuls des mésons dans l'état p sont présents dans les nuages se réduisent à des fonctions bilinéaires des $a_{R\alpha}$ et $a^+_{R\alpha}$. En négligeant, comme au paragraphe précedent, les corrections de scattering et les dérivées de la fonction de source, Suura obtient pour $\langle r^2\rangle_c$, $\langle r^2\rangle_{\pi,S}$ et $\langle r^2\rangle_{\pi,R}$ les valeurs suivantes :

$$\langle r^2\rangle_{\pi,S} = \langle r^2\rangle_{\pi,R} \qquad (146)$$

d'où l'on tire $\langle r^2\rangle_n \sim 0$ (en accord avec l'expérience) et :

$$\langle r^2\rangle_p \sim \langle r^2\rangle_c \simeq 0{,}17 \cdot 10^{-26}\ \text{cm}^2 \qquad (147)$$

ce qui est environ quatre fois trop faible. La conclusion semblerait être que les effets de recul réduisent simultanément les rayons du proton et du neutron.

13- VALIDITE DE L'ELECTRODYNAMIQUE QUANTIQUE A HAUTE ENERGIE.

Nous venons de voir que l'ensemble de l'interprétation des intéractions des neutrons et protons avec les électrons parait se heurter, même si l'on tient compte des effets de recul, à une contradiction. Yennie et al. (référence citée plus haut) se sont donc demandés s'il ne fallait pas voir là l'indication que l'électrodynamique quantique cesse d'être valable à très haute énergie ou, en d'autres termes, que la loi d'intéraction entre deux charges ponctuelles cesse d'être, aux distances très petites, prouvée par la loi de Coulomb (effet qui doit être distinct de l'influence des corrections radiatives dont il a déjà été tenu compte)

Si par exemple, le potentiel Coulombien est donné par une expression de la forme :

$$V_c(r) = e^2 \int f(\vec{r}-\vec{r}') \cdot \frac{d\vec{r}'}{r'}, \qquad (148)$$

où $f(\vec{r}-\vec{r}')$ est une fonction universelle (à déterminer) de portée assez petite, la transformée de Fourier de V_c s'écrit :

$$V_c(q) = -4\pi e^2 \frac{C(q^2)}{q^2} \qquad (149)$$

avec :

$$C(q^2) = \int f(r) e^{i\vec{q}\vec{r}} d\vec{r} \qquad (150)$$

et la formule de Rosenbluth s'écrira :

$$\sigma(\vartheta) = \sigma_M(\vartheta) C^2(q^2) \left\{ F_1^2 + \frac{q^2}{4M^2} \left[2(F_1 + \varkappa_p F_2)^2 \operatorname{tg}^2 \frac{\vartheta}{2} + \varkappa_p^2 F_2^2 \right] \right\}. \qquad (151)$$

On voit par suite, que les expériences de diffusion des électrons ne permettront pas de séparer C, F_1 et F_2 mais détermineront simplement les produits CF_1 et CF_2. Si nous appelons $\langle r_0^2 \rangle$ le carré moyen observé de la distribution de charge du proton sera donné par :

$$\langle r^2 \rangle_{ch}^{(p)} = \langle r_0^2 \rangle + \langle r_{1p}^2 \rangle + \frac{3}{4M^2}(1 + 2\varkappa_p). \qquad (152)$$

Par contre, le neutron étant ponctuellement (ou globalement) neutre, le terme $\langle r_0^2 \rangle$ ne figurera pas dans le carré moyen de sa distribution de charge :

$$\langle r^2 \rangle_{ch}^{(n)} = \langle r_{1n}^2 \rangle + \frac{3}{4M^2} \cdot 2\varkappa_n. \qquad (153)$$

On voit que si l'on prend $\langle r_0^2 \rangle$ de l'ordre 0,4 et 0,5 X 10^{-26} cm^2, il n'y aura plus de contradiction. Cette modification de la loi d'intéraction de deux charges ponctuelles n'affecteraient en rien les observations faites, aux basses énergies, sur le déplacement des raies de . . .

l'hydrogène ou bien le moment magnétique anormal de l'électron. En effet, la totalité de l'extension observée du proton ne produit, dans le spectre de l'hydrogène qu'un déplacement égal à :

$$\Delta E_\alpha \simeq \frac{3}{4\pi} |\psi_\alpha(0)|^2 \int \delta V d\vec{r} = \frac{e^2}{2} \langle r^2 \rangle_{ch} |\psi_\alpha(0)|^2 \quad (154)$$

où $\psi_\alpha(0)$ est la fonction d'onde de l'atome d'hydrogène dans l'état considéré. Pour l'état $^2S_{1/2}$, en prenant pour $\langle r^2 \rangle_{ch}$ la valeur de Chambers et Hofstadter, on obtient :

$$\Delta E(^2S_{1/2}) \simeq 0{,}1 \text{ Mc} \quad (155)$$

ce qui est très inférieur à l'incertitude actuelle sur la valeur du déplacement de Lamb.

Il faut remarquer, cependant, qu'un facteur du type $G(q^2)$ dans l'équation (149) ne pourrait pas résulter d'une simple modification de la fonction de propagation des photons due à l'inclusion des champs d'autres particules qui sont actuellement négligés en électrodynamique. En effet, on peut montrer, de façon générale que la forme du propagateur renormalisé du photon (Källen, Helv. Phys. Acta 25, 417, 1952); Lehman (Nuov. Cim. 11, 342, 1957); J. Schwinger (Cours de Stanford 1956) conduit nécessairement à l'expression de :

$$G(q^2) = \left\{ 1 - q^2 \int_0^\infty \frac{\sigma(k^2)\, dk^2}{k^4 (q^2 + k^2)} \right\}^{-1} \quad (156)$$

où $\sigma(k^2)$ est une fonction positive ou nulle, dont on peut voir facilement qu'elle est une fonction croissante de q^2, ce qui va en sens contraire de l'effet recherché. Il faudrait donc admettre une modification fondamentale des concepts qui sont à la base de la théorie des champs actuelle.

Les considérations qui précèdent ne doivent, pour le moment, être considérées que comme des spéculations. Il serait très important d'examiner d'abord, de plus près, l'influence des créations de paires virtuelles ou bien, de réaliser des expériences qui mettent indépendam-

ment à l'épreuve la validité de l'électrodynamique quantique aux grandes énergies.

III. DIFFUSION DES ELECTRONS DE HAUTE ENERGIE PAR LES NOYAUX.

L'analyse de la diffusion des électrons de haute énergie par les protons, neutrons et deutérons, nous avait été facilitée par la validité de l'approximation de Born (dans le cas où $Z = 1$) pour les énergies considérées. Dans le cas des noyaux lourds, des difficultés apparaissent, qui proviennent de ce que l'approximation de Born suppose réalisée la condition :

$$\frac{|V|R}{\hbar v} \ll 1 \qquad (1)$$

où V est le potentiel, R le rayon moyen de la région diffusante et v la vitesse des particules incidentes. Dans le cas des électrons on a dès que $E \gtrsim 10$ mev $v \simeq c$ et l'inégalité (1) se réduit à une condition indépendante de l'énergie :

$$\frac{|V|R}{\hbar c} \sim \frac{Z}{137} \ll 1. \qquad (2)$$

Nous commencerons par discuter le cas des noyaux légers pour lesquels dans une certaine mesure, la situation est similaire à celle que nous avons rencontrée au Chap. II. La seule différence provient de l'existence de niveaux d'excitation des noyaux, avec possibilité de diffusion inélastique, ou d'effets de dispersion dans la diffusion élastique (excitation à des niveaux différents du niveau fondamental dans les états intermédiaires).

1.- DIFFUSION ELASTIQUE ET INELASTIQUE A L'APPROXIMATION DE BORN.

Nous donnerons dans ce paragraphe, un traitement simplifié de l'approximation de Born pour des électrons de grande énergie. Un traitement plus détaillé sera donné ultérieurement en relation avec une méthode de calcul plus complète de l'amplitude de diffusion. Nous considérons le noyau initialement dans l'état fondamental ψ_0 et, après diffusion, dans l'état ψ_f (qui pourra, dans le cas de diffusion élastique, coincider avec ψ_0). Au 1er ordre en $e^2/\hbar c$, l'élément de matrice de l'intéraction Coulombienne avec l'électron s'écrit :

$$V_{f0}^{(1)} = \sum_{i=1}^{Z} \iint \psi_f^* \, e^{-i\vec{k}_f \vec{r}} \, a_{f0} \, \frac{e^2}{|\vec{r} - \vec{R}_i|} \, \psi_0 \, e^{i\vec{k}_0 \vec{r}} \, d\vec{r} \, d\vec{R}_1 \ldots d\vec{R}_A^{(3)}$$

où les $\vec{R}_1, \ldots, \vec{R}_A$ sont les coordonnées de tous les nucléons ; $\vec{k}_0$ et $\vec{k}_f$ sont les impulsions initiales et finales de l'électron et a_{f0} est l'élément de matrice de l'opérateur unité de Dirac :

$$a_{f0} = u^*(\vec{k}_f) u(\vec{k}_0) \qquad (4)$$

provenant de la 4ème composante du courant (puisque le potentiel vecteur se réduit à sa partie scalaire A_0). Si nous prenons comme nouvelle variable $\vec{r}' = \vec{r} - \vec{R}_i$ et posons :

$$\vec{q} = \vec{k}_0 - \vec{k}_f \qquad (5)$$

(transfert d'impulsion), nous voyons apparaître la transformée de Fourier du potentiel Coulombien :

$$\int \frac{e^2}{r'} e^{i\vec{q}\vec{r}'} \, d\vec{r}' = \frac{4\pi e^2}{q^2} \qquad (6)$$

et $V_{f0}^{(1)}$ s'écrit simplement :

$$V_{f0}^{(1)} = \frac{4\pi e^2 a_{f0}}{q^2} \sum_{i=1}^{Z} \int \psi_f^* (\vec{R}_1 \ldots \vec{R}_A) e^{i\vec{q}\vec{R}_i} \psi_0 (\vec{R}_1 \ldots \vec{R}_A) \, d\vec{R}_1 \ldots d\vec{R}_A^{(7)}$$

La section efficace de diffusion inélastique, le noyau restant dans l'état (f), s'obtient par sommation S_f sur les spins de l'électron dans l'état final (états d'énergie positive seulement) et moyenne, sur les spins de l'état initial $\frac{1}{2} S_0$. On obtient ainsi :

$$\sigma_{f0}^{(1)} = \left(\frac{2e^2 E_f}{q^2}\right)^2 \cos^2 \frac{\vartheta}{2} \left| \sum_{i=1}^{Z} \int \psi_f^* e^{i\vec{q}\vec{R}_i} \psi_0 \, d\tau_N \right|^2 . \qquad (8)$$

Pour la diffusion élastique (f = 0), on peut exprimer facilement (8) en fonction de la densité de charge des protons dans l'état fondamental :

$$\rho(\vec{r}) = \int \psi_0^* (\vec{R}_1 \ldots \vec{R}_A) \sum_{i=1}^{Z} \delta(\vec{r} - \vec{R}_i) \psi_0 (\vec{R}_1 \ldots \vec{R}_A) \, d\tau_N \qquad (9)$$

qui est évidemment normalisée de la façon suivante :

$$\int \rho(\vec{r}) \, d\vec{r} = Z \qquad (10)$$

Si nous posons, en effet :

$$F(q) = \int \rho(\vec{r}) e^{i\vec{q}\cdot\vec{r}} \, d\vec{r}, \qquad (11)$$

nous obtenons, à partir de (9) :

$$\begin{aligned} F(q) &= \sum_{i=1}^{Z} \int e^{i\vec{q}\vec{r}} |\psi_0^*|^2 \, \delta(\vec{r} - \vec{R}_i) \, d\vec{r} \, d\tau_N \\ &= \sum_{i=1}^{Z} \int \psi_0^* \, e^{i\vec{q}\vec{R}_i} \psi_0 \, d\tau_N \end{aligned} \qquad (12)$$

qui n'est pas autre chose que le terme entre $|\ldots|$ du second membre de (8); dans le cas où f = 0. F (q), qui est la transformée de Fourier de la distribution de charge, constitue ce que nous appelons le <u>facteur de forme du noyau</u>. La section efficace de diffusion élastique s'écrit :

$$\sigma_{el}(\vartheta) = \left(\frac{e^2}{2E}\right)^2 \frac{\cos^2 \frac{\vartheta}{2}}{\sin^2 \frac{\vartheta}{2}} F^2(q). \qquad (13)$$

Comme $F(0) = Z$ (d'après (10) et (11)), on voit que l'expression (13) se réduit à la formule de Mott quand $q \to 0$ (noyau ponctuel).

La diffusion inélastique pourra également s'exprimer par une formule analogue à (13), en définissant une <u>probabilité de transition à un proton de la densité de charge</u> :

$$\rho_{f0}^{(1)} = \sum_{i=1}^{Z} \int \psi_f^* (R_1 \dots R_A)\, \delta(\vec{r} - \vec{R}_i)\, \psi_0 (R_1 \dots R_A)\, d\tau_N \qquad (14)$$

et un facteur de forme inélastique :

$$F_{f0}(q) = \int \rho_{f0}^{(1)}(\vec{r})\, e^{i\vec{q}\vec{r}}\, d\vec{r}, \qquad (15)$$

$$\sigma_{f0}^{(in)} = \left(\frac{2e^2 E_f}{q^2}\right)^2 \cos^2 \frac{\vartheta}{2} \left| F_{f0}(q) \right|^2. \qquad (16)$$

Remarquons que :

$$F_{f0}(0) = \int \rho_{f0}^{(1)}(\vec{r})\, d\vec{r} = Z \delta_{f0} \qquad (17)$$

(pas de diffusion inélastique pour le noyau ponctuel).

A l'approximation de Born; nous pourrons obtenir des informations intéressantes sur les corrélations entre protons à l'intérieur du noyau (granulation de la distribution de charge) si nous sommons sur l'ensemble de la diffusion élastique et des différents termes de diffusion inélastique avec excitation de tous les niveaux nucléaires. La formule (8) permet d'écrire en effet :

$$\sum_{f=0}^{\infty} \sigma_{f0}^{(1)} = \sum_{f=0}^{\infty} \left(\frac{2e^2 E_f}{q^2}\right)^2 \cos^2 \frac{\vartheta}{2} \sum_{i=1}^{Z} \sum_{j=1}^{Z} \int \psi_0^* (\vec{R}_1' \dots \vec{R}_A') \cdot e^{-i\vec{q}\vec{R}_j'} \cdot \psi_f (\vec{R}_1' \dots \vec{R}_A') \psi_f^* (\vec{R}_1 \dots \vec{R}_A) e^{i\vec{q}\vec{R}_i} \cdot \psi_0 (\vec{R}_1 \dots \vec{R}_A)\, d\tau_N\, d\tau_N' . \qquad (18)$$

Si nous négligeons la variation de E_f et de q en fonction de l'indice f (ce qui signifie que nous traitons les énergies d'excitation du noyau comme petites par rapport à l'énergie des électrons incidents, nous pouvons appliquer à (18) la règle de somme :

$$\sum_{f=0}^{\infty} \Psi_f(\vec{R}'_1 \ldots \vec{R}'_A)\Psi_f(\vec{R}_1 \ldots \vec{R}_A) = \prod_{i=1}^{A} \delta(\vec{R}'_i - \vec{R}_i) \qquad (19)$$

d'où la relation :

$$\sum_{f=0}^{\infty} \sigma_{f0}^{(1)} = \left(\frac{e^2}{2E}\right)^2 \frac{\cos^2 \frac{\vartheta}{2}}{\sin \frac{\vartheta}{2}} \sum_{i=1}^{Z}\sum_{j=1}^{Z} \int \Psi_0^*(\vec{R}_1 \ldots \vec{R}_A) e^{i\vec{q}(\vec{R}_i - \vec{R}_j)} \cdot \Psi_0(\vec{R}_1 \ldots \vec{R}_A)\, d\tau_N \qquad (20)$$

Nous distinguerons, dans le double somme à droite de (20) les termes où $i \neq j$ (en nombre de $Z(Z-1)$. Les premiers sont simplement égaux chacun à l'unité ; puisque Ψ est normalisée. Les seconds peuvent s'exprimer simplement en fonction de la densité de charge pour deux protons (ou fonction de corrélation de deux protons) à l'intérieur du noyau dans l'état Ψ_0 :

$$\rho_{00}^{(2)}(\vec{r}, \vec{r}') = \sum_{i \neq j} \int \Psi_0(\vec{R}_1 \ldots \vec{R}_A)\delta(\vec{r} - \vec{R}_i)\delta(\vec{r}' - \vec{R}_j)\Psi_0(R_1 \ldots R_N)\, d\tau_N \qquad (21)$$

qui satisfait aux relations :

$$\rho_{00}^{(2)}(\vec{r}, \vec{r}') = \rho_{00}^{(2)}(\vec{r}', \vec{r}), \qquad (22)$$

$$\int \rho_{00}^{(2)}(\vec{r}, \vec{r}')\, d\vec{r}' = (Z-1)\rho^{(1)}(\vec{r}) \qquad (23)$$

où $\rho_{oo}^{(1)}(\vec{r})$ est la densité de charge pour un proton défini par (9). L'équation (20) peut s'écrire à l'aide de (21) :

$$\sum_f \sigma_{fo}^{(1)} = \left(\frac{e^2}{2E}\right)^2 \frac{\cos^2\frac{\vartheta}{2}}{\sin^4\frac{\vartheta}{2}} \left[Z + \int \rho_{oo}^{(2)}(\vec{r},\vec{r}')\, e^{i\vec{q}(\vec{r}-\vec{r}')}\, d\vec{r}\, d\vec{r}'\right]. \quad (24)$$

En utilisant les équations (17) et (23) on voit facilement que l'expression (24) tend vers la section efficace de diffusion élastique ponctuelle quand $q \to 0$. L'équation (24) permet, à partir de la somme sur toutes les sections efficaces élastiques et inélastiques, de déterminer à haute énergie la fonction de corrélation $\rho_{oo}^{(2)}$. Ceci est mis plus clairement en évidence si l'on admet pour $\rho_{oo}^{(2)}$ une forme séparable (Downs, Phys. Rev. 101, 820, 1956) :

$$\rho_{oo}^{(2)}(\vec{r},\vec{r}') = \rho\left(\frac{\vec{r}+\vec{r}'}{2}\right) g(\vec{r}-\vec{r}'). \quad (25)$$

Ce qui permet d'écrire :

$$\sum_f \sigma_{fo}^{(1)} = Z\left(\frac{e^2}{2E}\right)^2 \frac{\cos^2\frac{\vartheta}{2}}{\sin^4\frac{\vartheta}{2}} \left[1 + G(q)\right] \quad (26)$$

où $G(q)$ est la transformée de Fourier de $g(\vec{r})$. Il est raisonnable de supposer, comme nous l'avons fait dans (26), que :

$$\int f(\vec{r})\, d\vec{r} = Z$$

et que $G(0) = Z - 1$, ce qui conduit à :

$$\sum_f \sigma_{fo}^{(1)} = \sigma_M(\vartheta)\left[1 + \frac{G(q)-G(0)}{Z}\right]. \quad (27)$$

PHYSIQUE DES ELECTRONS DE HAUTE ENERGIE

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2.- INFLUENCE DES CORRECTIONS D'ORDRE SUPERIEUR (EFFETS DE DISPERSION).

(L.I. Schiff, Phys. Rev. 98, 765 (1955) ; B.W. Downs, Phys. Rev. 101, 820 (1956).

Lorsque la condition de validité de l'approximation de Born est remplie ($Ze^2/\hbar v \ll 1$), on peut considérer que la deuxième approximation de Born n'apporte qu'une correction négligeable à la diffusion par un centre fixe. Il n'en est pas à priori de même pour un système diffuseur qui, comme le noyau, est susceptible d'être excité à des niveaux d'énergie supérieurs au niveau fondamental ; même dans le cas de la diffusion élastique, cette excitation intervient à la deuxième approximation de Born dans les états intermédiaires. C'est ce qu'on appelle l'effet de la dispersion.

A la deuxième approximation de Born, la section efficace de diffusion est donnée par la relation :

$$\sigma^{(1)} + \sigma^{(2)} = \left(\frac{k_f}{2\pi}\right)^2 \frac{1}{2} \sum_f \sum_0 \left| V_{f0}^{(1)} + V_{f0}^{(2)} \right|^2 \qquad (28)$$

$V_{f0}^{(1)}$ (élément de matrice du 1er ordre du potentiel d'interaction a été défini au paragraphe précédent ; $\sum_f$ et $\sum_0$ représentent respectivement (pour un faisceau incident non polarisé) une sommation sur les spins de l'état final et une moyenne sur les spins de l'état initial, ces sommations ne s'étendent qu'aux états d'énergie positive. A l'ordre le plus bas, c'est le terme d'interférence de (28) qui nous intéresse, ce qui fait que $\sigma^{(2)}$ est donné par l'expression

$$\sigma^{(2)} \simeq \left(\frac{k_f}{2\pi}\right)^2 \frac{1}{2} \sum_f \sum_0 V_{f0}^{(1)*} V_{f0}^{(2)} + \text{c.c.} \qquad (29)$$

(c.c. signifiant : expression complexe conjuguée). Nous appellerons ε_n les niveaux d'énergie du noyau (ε_0 étant l'état fondamental) et E_ν les états intermédiaires de l'électron. Dans ces conditions, l'élément de matrice peut s'écrire :

$$V_{f0}^{(2)} = -\sum_n \sum_\nu{}' \int_{(C)} \frac{d\vec{k}_\nu}{(8\pi)^3} \frac{1}{\varepsilon_n - \varepsilon_0 + E_\nu - E_0} . \qquad (30)$$

$$\cdot \sum_{i=1}^{Z} \iint \psi_f^* e^{-i\vec{k}_f\cdot\vec{r}} a_{fv} \frac{e^2}{|\vec{r}-\vec{R}_i|} \psi_n e^{i\vec{k}_v\cdot\vec{r}} d\tau_N d\tau$$

$$\cdot \sum_{j=1}^{Z} \iint \psi_n^* e^{-i\vec{k}_v\cdot\vec{r}'} a_{vo} \frac{e^2}{|\vec{r}'-\vec{R}_j|} \psi_o e^{i\vec{k}_o\cdot\vec{r}'} d\tau_N' d\tau' \qquad (30)$$

où $\sum_v'$ représente une sommation sur les quatre états de spin (correspondent à des énergies positives ou négatives) de l'électron intermédiaire et le contour C est choisi de façon à sélectionner le pôle ($k_v > 0$) qui correspond à un choix correct suivant la théorie des trous

Dans la région extrêmement relativiste, nous pouvons écrire (en posant $E_n = E_0 + \varepsilon_0 - \varepsilon_n$)

$$\frac{1}{2}\int_f\int_0\sum_v' \bar{a}_{f0} \frac{1}{\varepsilon_n - \varepsilon_0 + E_v - E_0} a_{fv} a_{vo} =$$

$$= \frac{1}{2}\frac{1}{(E_v^2 - E_n^2)} \int_f\int_0\sum_v' a_{0f} a_{fv} (E_n + E_v) a_{vo} = \frac{E_n(1+\cos\vartheta) + |E_v|(\cos\vartheta_{ov} + \cos\vartheta_{fv})}{2(E_v^2 - E_n^2)} \qquad (31)$$

où ϑ_{ov} est l'angle entre $\vec{k}_o$ et $\vec{k}_v$ et ϑ_{fv} l'angle entre $\vec{k}_f$ et $\vec{k}_v$. Après substitution dans (30), nous obtenons donc pour $\sigma^{(2)}$:

$$\sigma^{(2)} = -\left(\frac{k_f}{2\pi}\right)^2 \frac{4\pi e^2}{q^2} F_{fo}^*(\vec{q}) \int_C \frac{d\vec{k}_v}{8\pi^3} \frac{1}{2(E_v^2 - E_n^2)} \cdot$$

$$\cdot \left[E_n(1+\cos\vartheta_1) + |E_v|(\cos\vartheta_{ov} + \cos\vartheta_{fv})\right] \cdot$$

$$\cdot \sum_{i=1}^{Z} \iint \psi_f^* \frac{e^2}{|\vec{r}-\vec{R}_i|} \psi_n e^{i(\vec{k}_v - \vec{k}_f)\cdot\vec{r}} d\tau_N d\tau$$

$$\cdot \sum_{j=1}^{Z} \iint \psi_n^{*\prime} \frac{e^2}{|\vec{r}'-\vec{R}'|} \psi_o' e^{i(\vec{k}_o - \vec{k}_v)\cdot\vec{r}'} d\tau_N' d\tau + c.c. \qquad (32)$$

La méthode la plus avantageuse pour calculer (32) consiste à effectuer l'intégration sur les variables intermédiaires de l'électron ($\vec{k}_v$) et à utiliser ensuite de façon approchée (comme nous l'avons fait

au paragraphe précédent) la relation de fermeture pour effectuer la sommation sur les niveaux nucléaires.

L'intégration sur k_ν donne :

$$\sigma^{(2)} = -\frac{e^6 k_f^2}{8\pi^2 q^2} F_{f_0}^{*}(q) \sum_n \sum_{i=1}^{Z} \sum_{j=1}^{Z} \int \dots \int [A_n] e^{i\left[E_n \rho + \vec{k}_0 \cdot \vec{r}' - \vec{k}_f \cdot \vec{r}\right]} \cdot \frac{1}{\rho |\vec{r} - \vec{R}_i| |\vec{r}' - \vec{R}_j'|} \psi_f^* \psi_n \psi_n'^* \psi_0 \, d\tau_N \, d\tau_N' \, d\tau \, d\tau' + c.c. \tag{33}$$

où $\vec{\rho} = \vec{r} - \vec{r}'$ et $[A_n]$ un facteur égal à :

$$A_n = E_n (1 + \cos\vartheta_i + \cos\vartheta_{0\rho} + \cos\vartheta_{f\rho}) + \frac{i}{\rho} (\cos\vartheta_{0\rho} + \cos\vartheta_{f\rho})$$

$\vartheta_{0\rho}$ et $\vartheta_{f\rho}$ étant respectivement les angles de $\vec{k}_0$ et $\vec{k}_f$ avec $\vec{\rho}$.

Pour appliquer la règle de somme à l'expression (33), nous supposerons que les énergies d'excitation du noyau $\mathcal{E}_n - \mathcal{E}_0$ sont faibles devant E_0 (énergie des électrons incidents) et que par conséquent E_n varie peu avec n. Nous remplacerons E_n par une énergie moyenne $\bar{E}$, qui, en pratique, sera peu différente de E_0. Nous avons donc une relation du type :

$$\sum_n \int f(E_n) \psi_n(\vec{R}_i) \psi_n^*(\vec{R}_j') g(\vec{R}_j') \, d\tau_N' = f(\bar{E}) \, g(\vec{R}_j) \tag{34}$$

et par suite :

$$\sigma^{(2)} = -\frac{e^6 k_f^2}{8\pi^2 q^2} F^*(\vec{q}) \sum_{i=1}^{Z} \sum_{j=1}^{Z} \int \dots \int [\bar{A}] e^{i\left(\bar{E}\rho + \vec{k}_0 \cdot \vec{r}' - \vec{k}_f \cdot \vec{r}\right)} \cdot \frac{1}{\rho |\vec{r} - \vec{R}_i| |\vec{r}' - \vec{R}_j'|} \psi_f^* \psi_0 \, d\tau_N \, d\tau \, d\tau' + c.c. \tag{35}$$

Nous distinguerons dans $\sigma^{(2)}$ deux parties qui correspondent respectivement, dans la somme à droite de (35), à $i = j$ et à $i \neq j$

$$\sigma^{(2)} = \sigma_1^{(2)}(i=j) + \sigma_2^{(2)}(i \neq j). \tag{36}$$

Le premier terme s'exprime facilement en fonction de la densité de charge correspondant aux transitions à un proton définies précédemment

$$\sigma_1^{(2)} = -\frac{e^6 k_f^2}{8\pi^2 q^2} F_{f0}^*(q) \int \dots \int [A]\, e^{i(\vec{E}\varepsilon + \vec{k}_0\vec{r}' - \vec{k}_f\vec{r})} \quad (37)$$

$$\frac{1}{\varepsilon |\vec{r} - \vec{R}||\vec{r}' - \vec{R}|} \rho_{f_0}^{(1)}(\vec{R})\, d\vec{R}\, d\vec{r}\, d\vec{r}' + c.c.$$

ce qui donne, si l'on pose : $\vec{S} = \vec{r} - \vec{R}$ et $\vec{S}' = \vec{r}' - \vec{R}$, $\vec{\varepsilon} = \vec{S} - \vec{S}'$ (l'intégration sur $\vec{R}$ se faisant alors facilement et donnant $F_{f0}(\vec{q})$)

$$\sigma_1^{(2)} = -\frac{e^6 k_f^2}{8\pi^2 q^2} |F(q)|^2 \iint [A]\, e^{i(\vec{E}\varepsilon + \vec{k}_0\vec{S}' - \vec{k}_f\vec{S})} \quad (38)$$

$$\cdot \frac{d\tau\, d\tau'}{\varepsilon S S'} + c.c.$$

L'intégrale qui apparait à droite de (38) (si l'on pose $E = E_0$) n'est pas autre chose que la deuxième approximation de Born qui correspond à une diffusion ponctuelle. Dans le cas de la diffusion <u>élastique</u> ($f = 0$), nous avons donc facilement

$$\sigma^{(1)} + \sigma_1^{(2)} = \left(\frac{e^2}{2E}\right)^2 \frac{|F(q)|^2}{\sin^4 \frac{\vartheta}{2}} \left[\cos^2\frac{\vartheta}{2} + \frac{\pi e^2}{\hbar c}\left(\sin\frac{\vartheta}{2} - \sin^2\frac{\vartheta}{2}\right)\right] \quad (39)$$

La correction qui se trouve à l'intérieur du crochet n'est jamais supérieure à $\frac{\pi}{4}\frac{e^2}{\hbar c}$, c'est-à-dire à plus de 1 %. En réalité le terme intéressant, qui contient justement les effets de dispersion est le second terme de (35), celui qui correspond à $i \neq j$, et que l'on peut exprimer en fonction de la densité de charge correspondant à des transitions à deux protons :

$$\rho_{f0}^{(2)}(\vec{r}, \vec{r}') = \int \dots \int \psi_f^*(\vec{R}_1 \dots \vec{R}_A) \sum_{i \neq j} \delta(\vec{r} - R_i)\, \delta(\vec{r} - R_j) \quad (40)$$

$$\psi_0(\vec{R}_1, \dots, \vec{R}_A)\, d\tau_1 \dots d\tau_A$$

qui est telle que

$$\int \rho_{f0}^{(2)}(\vec{r}, \vec{r}')\, d\vec{r}' = (Z-1)\, \rho_{f0}^{(1)}(\vec{r}), \quad (41)$$

ce qui donne :

$$\sigma_2^{(2)} = -\frac{e^6 k_f^2}{8\pi^2 q^2} F^*(q) \int\cdots\int [\bar{A}]\, e^{i(\vec{E}\xi + \vec{k}_0 \vec{r}\,' - \vec{k}_f \vec{r})} \cdot \frac{1}{\rho |\vec{r} - \vec{R}||\vec{r}\,' - \vec{R}'|} \rho_{fo}^{(2)}(\vec{R}, \vec{R}')\, d\vec{R}\, d\vec{R}'\, d\vec{r}\, d\vec{r}\,' + c.c. \tag{42}$$

expression que nous pouvons évaluer en changeant les 4 variables d'intégration : $\vec{S} = \vec{r} - \vec{R}$, $\vec{S}' = \vec{r}\,' - \vec{R}'$, $\vec{S} = \frac{1}{2}(\vec{R} + \vec{R}')$, $\vec{\rho} = \vec{r} - \vec{r}\,'$, et par conséquent

$$\sigma_2^{(2)} = -\frac{e^6 k_f^2}{8\pi^2 q^2} F^*(q) \int\cdots\int [\bar{A}]\, e^{i(\vec{q}\vec{s} + \vec{E}\xi - \vec{K}_0 \vec{\rho} + \frac{1}{2}(\vec{S} + \vec{S}'))} \cdot \frac{1}{\rho s s'} \rho_{fo}^{(2)}(\vec{R}, \vec{R}')\, d\vec{S}\, d\vec{\xi}\, d\vec{s}\, d\vec{s}\,' + c.c. \tag{43}$$

où nous avons posé $\vec{K}_0 = \frac{1}{2}(\vec{k}_0 + \vec{k}_f)$. Cette expression a été analysée par Schiff et Downs dans différents cas particuliers. Il est difficile de l'estimer de façon générale quand on ne connait pas la forme explicite de la fonction de corrélation $\rho_{fo}^{(2)}$. Pour des angles suffisamment grands, satisfaisant à l'inégalité :

$$k_0 R \left(1 - \cos\frac{\vartheta}{2}\right) \gg 1 \tag{44}$$

et dans le cas de la diffusion élastique ($f = 0$), Schiff donne pour $\sigma_2^{(2)}$ l'expression approchée :

$$\sigma_2^{(2)} \simeq \frac{768\, e^6 k_0 (Z-1)}{R^3 q^6} |F(q)|^2 \operatorname{ctg}^2 \frac{\vartheta}{2} \tag{45}$$

où R est en gros le rayon de la distribution de charge du noyau (de façon plus précise la distance au bout de laquelle $\rho_{oo}^{(2)}$ varie de façon appréciable). Pour des électrons de 200 Mev, l'inégalité (44) correspond à $\vartheta \gg 148°$ pour le carbone et $\vartheta \gg 88°$ pour l'or (où l'approximation de Born cesse, de toutes façons, d'être valable) C'est dire que l'estimation (45) n'est pas très utile et ne permet de donner qu'un ordre de grandeur. Le rapport $\sigma_2^{(2)} / \sigma^{(1)}$ est égal à :

$$\frac{\sigma_2^{(2)}}{\sigma^{(1)}} \simeq \frac{48(Z-1)e^2}{(k_0R)^3} \frac{1}{\sin^4\frac{\vartheta}{2}} \qquad (45)$$

(puisque $q \simeq 2k_0 \sin\frac{\vartheta}{2}$). L'inégalité (44) permet d'écrire :

$$\frac{1}{\sin^2\frac{\vartheta}{2}} \ll \frac{k_0R}{1+\cos\frac{\vartheta}{2}} < k_0R \qquad (\text{pour } \vartheta \leq \pi)$$

et par suite

$$\frac{\sigma_2^{(2)}}{\sigma^{(1)}} \ll \frac{48(Z-1)e^2}{(k_0R)} \qquad (46)$$

L'expression à droite de (46) est égale à 0,64 pour le carbone et à 3,88 pour l'or. C'est dire que les corrections de dispersion peuvent ne pas être négligeables. Nous verrons dans la suite une méthode beaucoup plus puissante (parce que libérée des incertitudes de l'approximation de Born) pour les évaluer.

3.- ANALYSE DES RESULTATS EXPERIMENTAUX SUR LA DIFFUSION ELASTIQUE PAR LES NOYAUX LEGERS.

A l'approximation de Born, nous avons vu que la section efficace de diffusion élastique est proportionnelle au facteur de forme du noyau, qui n'est autre que la transformée de Fourier de la distribution de charge dans l'état fondamental. Les formules qui définissent $F(q)$ et $\rho(r)$ se simplifient dans le cas où le noyau à la symétrie sphérique :

$$F(q) = \frac{4\pi}{q} \int_0^\infty \rho(r) \sin qr \cdot r\, dr \qquad (47)$$

et

$$\rho(r) = \frac{1}{2\pi^2 r} \int_0^\infty F(q) \sin qr \; q\, dq. \qquad (48)$$

Plusieurs méthodes ont été employées pour analyser les résultats expérimentaux. Hofstadter et ses collaborateurs emploient une table de douze modèles (Table I de l'article de la Rev. Mod. Phys. 28, 214 (1956) exprimés en fonction des variables réduites $X = qa$ et $y = r/a$ où a est choisi de

façon à être exactement égal à la racine carrée du rayon carré moyen :

$$a^2 = 4\pi \int_0^\infty \rho(t)\, t^4\, dt. \qquad (49)$$

Par exemple, le modèle gaussien sera tel que le facteur de forme

$$F(qa) \equiv F(x) = e^{-\frac{x^2}{6}} \qquad (50)$$

corresponde à la densité de charge

$$4\pi a^2 \rho\left(\frac{t}{a}\right) = 3\sqrt{\frac{6}{\pi}}\, e^{-\frac{3}{2}y^2}, \qquad (51)$$

et ainsi de suite pour tous les autres modèles.

Une méthode pour déterminer lequel des modèles convient pour représenter les mesures expérimentales (qui, rappelons-le ne sont jamais déterminées en valeur absolue) consiste à réprésenter graphiquement en fonction de q le rapport :

$$G(q) = \left(\frac{\sigma_{exp}}{\sigma_M}\right)^{1/2} \qquad (52)$$

sur une échelle doublement logarithmique et à superposer les représentations graphiques des différents modèles que l'on désire essayer. Un déplacement le long de l'axe des abscisses correspond à une modification du rayon carré moyen tandis qu'un déplacement le long de l'axe des coordonnées correspond à une normalisation arbitraire de la section efficace.

Une autre méthode, due à Ravenhall, consiste à normaliser $G(q)$ de façon à ce que son intégrale (déterminée graphiquement) soit égale à l'unité, puis à déterminer $\rho(t)$ par transformée de Fournier graphique de $G(q)$. Les résultats expérimentaux obtenus jusqu'ici n'ont cependant pas une précision suffisante pour rendre vraiment utile cette détermination directe de la distribution de charge.

Nous résumerons brièvement ci-dessous les résultats obtenus pour la diffusion <u>élastique</u> sur quelques noyaux légers :

a) He^4 (particule α) : A 188 Mev, les mesures ont été faites par Mc Allister et Hofstadter, Phys. Rev. 102, 851 (1956). A 400 Mev, les mesures de Blankenbecler et Hofstadter n'ont pas encore été publiées (voir Hofstadter loc. cit. pp. 237-238). Le noyau ayant une symétrie sphérique (spin zéro), seule la distribution de charge peut être déterminée par la diffusion élastique. L'influence des moments magnétiques individuels des nucléons ne peut être détectée que dans la diffusion inélastique à grand angle (voir notre discussion de la diffusion inélastique sur le deutérium). A 400 Mev et pour $\vartheta = 90°$, le rapport entre la section expérimentale et la formule de Mott est de l'ordre de 1000 ($F^2 \sim 0{,}001$). Le modèle qui s'accorde le mieux avec les résultats est le modèle gausien avec $a \simeq 1{,}61 \times 10^{-13}$ cm. Un calcul très approximatif du à Dalitz et Ravenhall, qui utilisent les fonctions d'onde déterminées par Clark (Pro. Phys. Soc. A 67, 323 (1954) au moyen d'une méthode variationnelle fournit pour la particule α un rayon égal aux 2/3 du rayon observé. Il serait probablement nécessaire de recommencer ce calcul en utilisant de meilleures fonctions d'onde.

b) Li^6, Li^7, Be^9 : Mesures à 188 mev dues à Streib (Phys. Rev. 100, 1797 (1955) et analysées par le modèle "exponentiel modifié" qui correspond à $F(x) = \left(1 + \frac{x^2}{18}\right)^{-3}$ où encore à $\rho(y) = \frac{27}{\sqrt{2}}\left(1 + \frac{y}{\sqrt{18}}\right)e^{-\frac{y}{\sqrt{18}}}$.

Il trouve, pour Li^6 $a = (2.78 \pm 0{,}02)\ 10^{-13}$ cm
et pour Li^7 $a = (2.71 \pm 0{,}02)\ 10^{-13}$ cm

On remarque que le rayon du Li^6 est supérieur à celui du Li^7. Ceci n'est pas dû, comme on pourrait le croire à première vue, à l'effet du moment magnétique du Li^7, qui a été déjà éliminé au moyen d'une formule du genre de Rosenbluth.

Il est probable que cela correspond plutôt à une structure assez "lâche" du nucléon situé à l'extérieur de la couche fermée du Li^6, analogue en cela au deutéron. Streib a mesuré de façon très précise le rapport des rayons carrés moyens des deux r :

$$\frac{a(Li^6)}{a(Li^7)} = 1{,}026 \pm 0{,}008.$$

Un calcul de Ferrell et Visscher (Bull. Ann. Phys. Soc. 1, 17, (1956) basé sur les fonctions d'onde des nucléons sur la couche p, donne pour Li^6 $a = 2.8 \times 10^{-13}$ cm (très bon accord) mais pour Li^7 $a = (2.3 \pm 0,2)\ 10^{-13}$ cm (un peu faible).

Les résultats expérimentaux sur la diffusion élastique par le Be^9 donnent un bon accord avecle même modèle (exponentiel modifié) et $a = (3.04 \pm 0,07) \times 10^{-13}$ cm.

Il est intéressant de déterminer la variation de r_0, rayon équivalent d'un modèle uniforme :

$$R = r_0 A^{1/3} \cdot 10^{-13} \text{ cm}$$

(a et r_0 sont liés par la relation

$$a = \sqrt{\tfrac{3}{5}}\, r_0 A^{1/3}.)$$

Ceci correspond aux valeurs suivantes :

Li^6 : $r_0 = 1.98$

Li^7 : $r_0 = 1.83$

Be^9 : $r_0 = 1.89$

On remarquera la valeur anormalement faible de r_0 pour le Li^7.

c) Carbone 12 : Le C^{12} a été étudié en détail, tant du point de vue de la diffusion élastique que de celui de la diffusion inélastique correspondant à l'excitation des trois premiers niveaux. (Fregeau et Hofstadter, Phys. Rev. 99, 1503, (1955) ; Fregeau, Phys. Rev. 104, 225 (1956). La fig. 16 représente le spectre des électrons diffusés à 187 Mev et pour des angles respectifs de 80º (a) et 90º (b). Outre le pic élastique que l'on reconnait à l'extrême droite, on peut constater l'importance de la diffusion inélastique correspondant à l'excitation du premier niveau, à 4,43 Mev (transition $2^+ \longrightarrow 0^+$). A 90º par exemple, la section efficace inélastique d'excitation à 4.43 Mev (0.179 ± 0,021 microbarn par steradian) est pratiquement égale à la section efficace de diffusion élastique (0,199 ± 0,014). On distingue aussi sur la fig. 16 les deux pics moins importants correspondant respectivement à l'excitation du niveau de 7,68 Mev (probablement

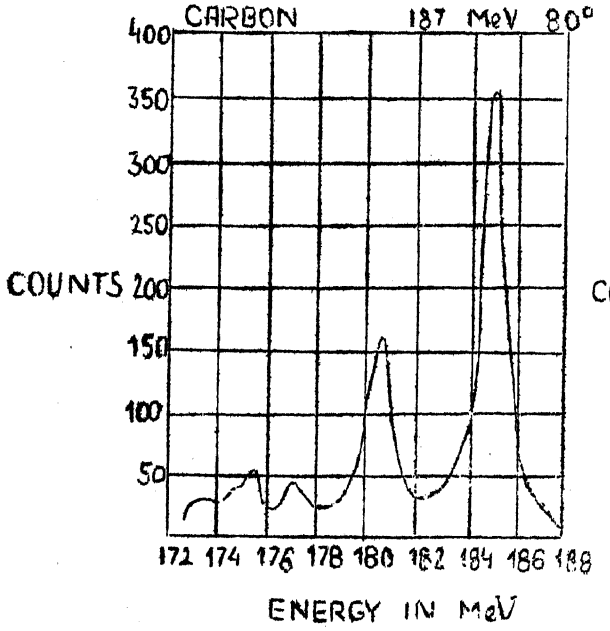


Fig. 16 a

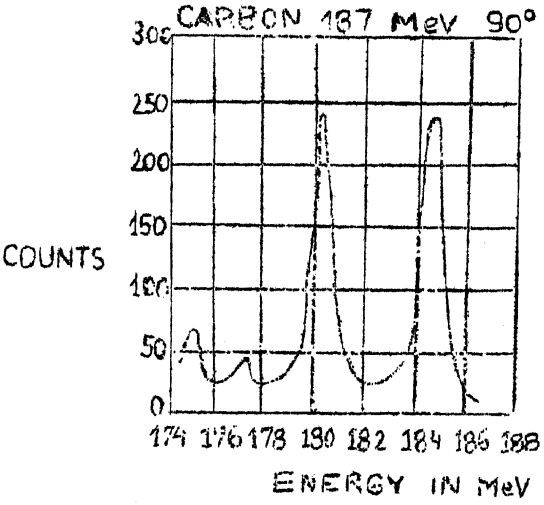


Fig. 16 b

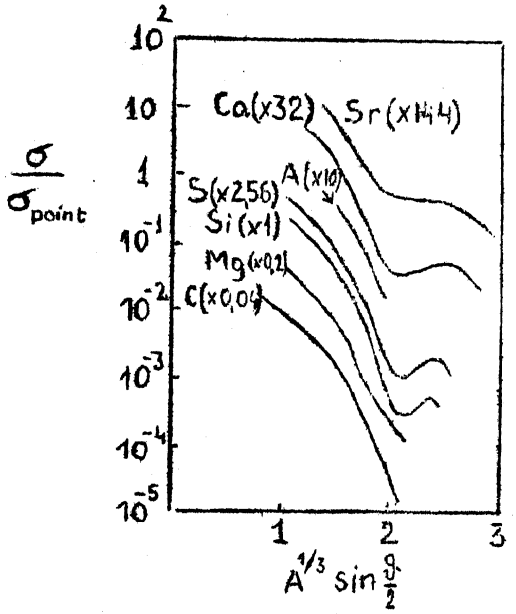


Fig. 17

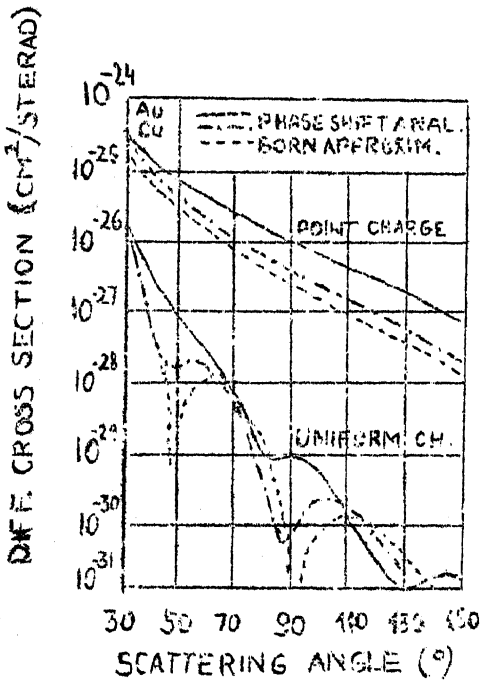


Fig. 18

$0^+ \rightarrow 0^+$) et de 9,6 Mev (probablement $2^+ \rightarrow 0^+$). Nous reviendrons ultérieurement sur l'interprétation de la transition $0^+ \longrightarrow 0^+$ de 7,68 Mev. Donnons, pour le moment, quelques indications sur les calculs qui ont été faits pour la diffusion élastique et l'excitation du niveau de 4,43 Mev (Ravenhall, non publié ; Morpurgo, Nuovo Cimento 3, 430 (1956)).

Dans le modèle à particules indépendantes, où les fonctions d'onde correspondant à chaque nucléon i sont désignées par $f_{n_i}(\vec{r})$, la densité de charge dans l'état fondamental peut s'écrire :

$$\rho_0(\vec{r}) = \sum_1^Z \left| f_{n_i}(\vec{r}) \right|^2 \qquad (53)$$

tandis que la densité de charge correspondant à une transition de un proton $0 \longrightarrow f$ est égale à :

$$\rho_{f0}(\vec{r}) = f^*_{n_z}(\vec{r}) f_{n'_z}(\vec{r}) \qquad (54)$$

où z est l'indice du proton qui est responsable de la transition considérée. La comparaison de (53) et de (54) montre que le rapport $\sigma_{inel.}/\sigma_{el}$ est en principe de l'ordre de $1/Z^2$, ce qui - en ce qui concerne la transition à 4.43 Mev, par exemple - est certainement contraire à l'expérience. Nous devons donc rechercher l'explication de cette contradiction dans des effets d'interférence.

Morpurgo prend pour les $f_n(\vec{r})$ les fonctions de l'oscillateur harmonique correspondant à un puits infini. Le seul paramètre arbitraire est donc la "largeur" des fonctions radiales :

$$R_{1\ell}(\vec{r}) = N_\ell \, e^{-\frac{\nu r^2}{2}} \, r^{\ell+1} \qquad (55)$$

avec

$$N_\ell^2 = \frac{\sqrt{\nu} \; \nu^{\ell+1} \; 2^{\ell+2}}{\sqrt{\pi} \; 1 \cdot 3 \ldots (2\ell+1)} \qquad (56)$$

Dans le cas du couplage j- j', la fonction d'onde de l'état initial ψ_0 correspond à la configuration $(1s_{1/2})^4 (1p_{3/2})^8$.
Le niveau de 4,43 Mev (2^+) correspond à une fonction d'onde $J = 2$, $T = 0$

formée à partir des configurations $(1s_{1/2})^4 (1p_{3/2})^7 ((1p_{1/2})$. Le sous-niveau $M_j = 2$ a, par exemple, pour fonction propre :

$$\frac{1}{\sqrt{2}} \left(1\bar{p}^{(+)}_{3/2,-3/2} \cdot 1p^{(+)}_{1/2,1/2} + 1\bar{p}^{(-)}_{3/2,-3/2} \cdot 1p^{(+)}_{1/2,1/2} \right)$$

où les indices + et - désignent les protons et neutrons.

Dans le cas du couplage L = S, la construction des fonctions d'onde est plus compliquée. Elle peut se faire à partir du Be^8 en échangeant les trous et les particules.

Le résultat est pratiquement le même dans les deux théories. La seule différence réside dans l'apparition d'un facteur numérique différent (1 ou $\sqrt{3}$ suivant le modèle), dans la densité de charge de transition à 4.43 Mev. On trouve en effet :

$$\rho_0(\vec{r}) = \frac{\nu\sqrt{\nu}}{3\pi\sqrt{\pi}} \left(1 + \frac{4}{3} r^2 \nu \right) e^{-\nu r^2} \qquad (56 \text{ bis})$$

et, pour la transition en sous état Mj = 2 :

$$\rho_{fo}^{(M_j=2)} = -\frac{\Lambda}{2} \frac{\nu^2}{\pi} \frac{\nu\sqrt{\nu}}{\sqrt{\pi}} \sqrt{\frac{1}{3}} e^{-\nu r^2} (x+iy)^2, \qquad (57)$$

où $\Lambda = 1$ pour le couplage j - j et $\Lambda \sqrt{\frac{14}{15}} \times 3 \simeq 3$ pour le couplage L - S
Après sommation de tous les Mj (en ce qui concerne la diffusion inélastique) on trouve pour les carrés des facteurs de forme :

$$|F_{el}(q^2)|^2 = \left(1 - \frac{q^2}{9\nu} \right) e^{-\frac{q^2}{2\nu}}, \qquad (58)$$

$$|F_{in}(q^2)|^2 = \frac{\Lambda^2}{18 Z^2} \left(\frac{q^2}{\nu} \right)^2 e^{-\frac{q^2}{2\nu}}. \qquad (59)$$

Le rapport des sections efficaces est donc égal à

$$\frac{d\sigma_{in}}{d\sigma_{el}} = \frac{9}{2} \frac{A^2}{Z^2} \frac{\left(\frac{q^2}{9\nu} \right)^2}{\left(1 - \frac{q^2}{9\nu} \right)^2} \qquad (60)$$

On voit que le coefficient en $\frac{1}{Z^2}$ est compensé par l'apparition d'un

terme de "résonance" au dénominateur. Morpugo donne, pour la comparaison avec l'expérience, les chiffres suivants :

Rapport $d\sigma_{in} / d\sigma_{el}$

E en Mev	80	150	187
Exp.	$(2.5 \pm 2)\ 10^{-3}$	$1,4 \times 10^{-1}$	9×10^{-1}
Couplage L.S	3.6×10^{-3}	$0,9 \times 10^{-1}$	3.6×10^{-1}
Couplage j - j	1.2×10^{-3}	$0,3 \times 10^{-1}$	1.2×10^{-1}

On constate que l'ordre de grandeur est correct, l'accord étant un peumeilleur dans le couplage L - S, les résultats théoriques étant de toutes façons systématiquement inférieurs aux résultats expérimentaux.

En ce qui concerne la diffusion élastique une analyse plus détaillée de la distribution angulaire a été faite par Fregeau qui a essayé systématiquement des distributions du type :

$$\rho_0(\vec{r}) \simeq \rho_0 (1 + \gamma v + ^2)\, e^{-v+^2} \qquad (61)$$

ce qui, en coordonnées réduites, correspond à

$$4\pi a^2 \rho(y) = \frac{8 k^3}{\sqrt{\pi}(2+3\alpha)} (1 + \alpha k^2 y^2)\, e^{-k^2 y^2} \qquad (62)$$

ou encore à :

$$F(x) = \left[1 - \frac{\alpha x^2}{2k^2(2+3\alpha)} \right] e^{-\frac{x^2}{4k^2}}, \qquad (63)$$

où nous avons posé :

$$k = \left[\frac{3(2+5\alpha)}{2(2+3\alpha)} \right]^{1/2} \qquad (64)$$

On voit que l'apparition d'un paramètre γ dans (61) ou α dans (62) et (63) correspond à un degré de liberté supplémentaire (en plus du rayon moyen a déjà inclus dans les coordonnées réduites) qui peut être, dans

une certaine mesure, fixé par la grande précision des résultats expérimentaux. Fregeau trouve que le meilleur accord est justement obtenu avec $\gamma = 4/3$, qui correspond exactement au modèle à particules indépendantes. Cependant des valeurs de γ comprises entre 1 et 2 sont tout de même acceptables, en modifiant légèrement les valeurs de a correspondantes. Celle qui s'accorde à $\gamma = 4/3$ est

$$a = 1.635 \times 10^{-13} \text{ cm} \qquad (65)$$

d) Mg^{24}, Si^{28}, S^{32}, A^{40} et Sr^{88} : un certain nombre de noyaux pair-pair a été étudié par Helm (Thèse Stanford, 1956) surtout dans le but de mesurer la diffusion inélastique correspondant aux transitions 0 - 2. En ce qui concerne la diffusion élastique, le fait le plus caractéristique est l'apparition de minima de diffraction dans les distributions angulaires. Ceci peut se comprendre si l'on songe, par exemple, au facteur de forme correspondant à une distribution uniforme :

$$4\pi a^3 \rho(y) = \frac{9}{5}\sqrt{\frac{3}{5}} \quad \text{pour} \quad y \leq \sqrt{\frac{5}{3}} \qquad (66)$$
$$= 0 \quad \text{pour} \quad y > \sqrt{\frac{5}{3}}$$

ou, en coordonnées x :

$$F(x) = 5\sqrt{\frac{5}{3}}\,\frac{1}{x^3}\left[\sin x\sqrt{\frac{5}{3}} - x\sqrt{\frac{5}{3}}\cos x\sqrt{\frac{5}{3}}\right] \qquad (67)$$

Les oscillations du facteur de forme correspondent évidemment à une diffraction sur la paroi abrupte de la distribution de charge. A l'approximation de Born, on voit apparaitre des zeros dans la section efficace qui, même pour les noyaux lourds, ne sont pas observés (voir notre discussion au paragraphe suivant). Malgré tout, on peut constater qualitativement que le paramètre qui détermine l'apparition de minima et maxima est proportionnel à

$$x \simeq qa \simeq 2E r_0 A^{1/3} \sin\frac{\vartheta}{2}. \qquad (68)$$

Ceci est mis en évidence par la fig. 17 qui représente les rapports σ/σ_m pour un certain nombre de noyaux en fonction de $A^{1/3}\sin\frac{\theta}{2}$. On voit

que le premier minimum de diffraction apparait exactement au même endroit. (On comprend également pourquoi il n'y a pas de figures de diffraction pour les noyaux légers). De façon à reproduire analytiquement les oscillations du facteur de forme sans pour cela annuler la section efficace, Helm utilise un modèle composite où la distribution de charge est donnée sous la forme d'une convolution :

$$\rho(r) = \int \rho_0(r')\,\rho_1(r-r')\,d\vec{r}' \qquad (69)$$

ρ_0 étant la distribution de charge uniforme mais correspondant à une largeur b inférieure à celle de la distribution totale :

$$\rho_0(r) = \frac{3}{4\pi b^3} \quad \text{pour} \quad r \leq b, \qquad (70)$$
$$= 0 \quad \text{pour} \quad r > b.$$

ρ_1 peut être l'une quelconque des distributions de Hoftadster. En pratique, Helm a considéré la distribution gaussienne (ce qui donne pour le ρ total une distribution dite gaussienne-uniforme ou gU) ou bien la distribution uniforme (ρ = uniforme - uniforme ou uU). Le facteur de forme correspondant à (69) s'exprime évidemment sous la forme d'un produit :

$$F(q) = F_0(q)\,F_1(q) \qquad (71)$$

ou F_0 et F_1, sont les transformées de Fourier de ρ_0 et de ρ_1.

A partir du moment où les valeurs de q qui interviennent dans les expériences sont suffisamment grandes (grandes énergies ou grands angles) ou lorsque les noyaux sont suffisamment lourds pour avoir un assez grand rayon moyen, un seul paramètre ne peut plus suffire à caractériser la distribution de charge. Ceci correspond au fait que le développement de $F(q)$ en puissance de $(qa)^2$ ne peut plus se limiter aux deux premiers termes :

$$F(q) \simeq 1 - \frac{q^2a^2}{6} + \cdots \qquad (72)$$

Hahn, Ravenhall et Hofstadter (Phys. Rev. 101, 1131 (1956) introduisent donc, pour caractériser toutes les distributions, deux paramètres :

- une "demi largeur" c, qui est égale à la distance du centre au point où la distribution $\rho(r)$ a décru de moitié par rapport à sa valeur pour $r = 0$;
- une "épaisseur" ou "paramètre de peau" t égale à la distance entre les deux points où la distribution est égale respectivement à 90 % et à 10 % de sa valeur au centre.

Dans le cas de la distribution gaussienne-uniforme (ρU) le tableau suivant donne les différents paramètres correspondant à chaque noyau :

Paramètres de la distribution ρU (en unités de 10^{-13}cm)

Noyau	r_0	$r_1 = c\,A^{-1/3}$	t
C^{12}	1.35	0.95	2.2
Mg^{24}	1.33	0.99	2.6
Si^{28}	1.29	0.97	2.8
S^{32}	1.30	1.03	2.6
Ca^{40}	1.28	1.08	2.4
Sr^{88}	1.20	1.08	2.3

4.- DETERMINATION DES DISTRIBUTIONS DE CHARGE DES NOYAUX LOURDS - GENERALITES

Comme nous l'avons dit précédemment, l'approximation de Born cesse d'être valable pour les noyaux lourds car le nombre Z ne peut plus alors être considéré comme très petit par rapport à $137 = \frac{\hbar c}{e^2}$. Plusieurs méthodes ont été employées qui permettent une détermination plus exacte de la distribution de charge, et qui sont basées sur une analyse en ondes partielles de l'onde diffusée. Yennie, Ravenhall et Wilson (Phys. Rev. 92, 1325 (1953) ; 95, 500 (1954) partent d'un modèle de distribution de charge déterminée , calculent le potentiel correspondant par intégration de l'équation de Poisson, puis résolvent numériquement l'équation de Dirac analysée

en onde partielle. E. Baranger (Phys. Rev. 93, 1127 (1954)) adopte une méthode semi-analytique, basée sur l'approximation W K B. Enfin, J. Reignier (Bull. Acad. Roy. de Belgique, Classe des Sciences, XLI, 151, (1955) ; XLII, 173 (1956)) utilise une méthode entièrement analytique. Ces différentes méthodes sont résumées et comparées dans l'appendice de ce cours, dont la rédaction est due à J. Reignier.

La fig. 18, due à Yennie, Ravenhall et Wilson illustre la comparaison entre la résolution numérique et l'approximation de Born dans le cas du cuivre et de l'or pour une distribution ponctuelle et une distribution uniforme.(pour le cuivre, il s'agit d'une énergie de 225 Mev environ, et pour l'or de 150 Mev ; l'approximation de Born, qui ne dépend que de $qA^{1/3}$ environ est la même dans ces deux cas). On voit que c'est surtout au voisinage des minima de diffraction que l'approximation de Born devient très mauvaise.

Les méthodes d'analyse en ondes partielles (qu'elles soient numériques ou analytiques) ne sont praticables que lorsque l'énergie n'est pas trop élevée.

Au delà de 180 Mev, en pratique, le nombre de valeurs du moment angulaire dont il est nécessaire de tenir compte devient si élevé que la méthode devient beaucoup trop longue et impraticable. C'est pourquoi il est important de trouver des méthodes d'approximation pour le calcul des sections efficaces de collision à très haute énergie qui n'aient pas les défauts de l'approximation de Born. Nous étudierons dans les paragraphes suivants une nouvelle méthode très importante qui est due à Schiff.

5.- CALCUL DE LA SECTION EFFICACE DE DIFFUSION AUX GRANDES ENERGIES. CAS DE L'EQUATION DE SCHRODINGER.

L'approximation de Schiff est en réalité valable de façon plus générale, pour les courtes longueurs d'onde. Un premier article étudie la diffusion de particules douées de masse et obéissant soit à l'équation de Schrödinger, soit à l'équation de Dirac (Phys. Rev. 103, 443 (1956). Un deuxième article étend la même méthode à la diffusion de radiation obéissant aux équations de Maxwell.(Phys. Rev. 104, 1681 (1956). Nous nous

bornerons, dans ce cours, au premier problème, et, dans ce paragraphe, au cas de l'équation de Schrödinger, que nous écrivons :

$$\left\{ \Delta + k^2 - U(\vec{r}) \right\} \psi(\vec{r}) = 0, \qquad (73)$$

où $k^2 = 2mE$ et $U = 2mV(\vec{r})$, $V(\vec{r})$ étant le potentiel diffusant. L'approximation de Schiff, ou <u>approximation de la phase stationnaire</u> (Eckart, Rev. Mod. Phys. <u>20</u>, 399 (1948)), repose sur une évaluation approchée d'une intégrale du type :

$$I(\vec{\varsigma}) = \int \frac{g(\vec{\varsigma})}{\varsigma} e^{i(k\varsigma - \vec{k}\vec{\varsigma})} \, d\vec{\varsigma} \qquad (74)$$

dans le cas où les dimensions du système diffuseur R sont telles que $k\varsigma \simeq kR \gg 1$. On voit facilement en effet, si l'on prend l'axe polaire dirigé suivant k que l'exponentielle à droite de (74) oscillera rapidement dans ce cas, et donnera une contribution nulle en moyenne, sauf si le rayon vecteur est contenu à l'intérieur d'un paraboloïde de révolution axé sur $\vec{k}$ tel que $\vartheta \simeq (k\varsigma)^{-1/2}$. (Fig 19)

Prenons pour variables ς , φ et $\mu = \cos\vartheta$ et écrivons :

$$I(\vec{k}) = \int_0^{\infty} \int_0^{2\pi} \int_{-1}^{+1} g(\varsigma, \mu, \varphi) e^{ik\varsigma(1-\mu)} \varsigma \, d\varsigma \, d\varphi \, d\mu \qquad (75)$$

Une intégration par parties par rapport à μ permet d'écrire :

$$I = \int_0^{\infty} \int_0^{2\pi} \left\{ \left[\frac{i}{k\varsigma} g(\varsigma, \mu, \varphi) e^{ik\varsigma(1-\mu)} \right]_{-1}^{+1} - \int_{-1}^{+1} \frac{i}{k\varsigma} \frac{\partial g}{\partial \mu} e^{ik\varsigma(1-\mu)} \, d\mu \right\} \varsigma \, d\varsigma \, d\varphi . \qquad (76)$$

Le 2ème terme entre accolades est de l'ordre de $\frac{1}{k^2}$, car il contient la dérivée $\frac{\partial g}{\partial \mu}$ qui ne s'étend, en pratique, qu'au paraboloïde de la fig. 19. Dans le premier terme, seule la limite $\mu = +1$ donne une contribution non oscillante. Nous avons donc approximativement (après intégration sur φ , puisque lorsque $\vartheta = 0$, g ne peut plus dépendre de φ) :

$$I \simeq \frac{2\pi i}{k} \int_0^{\infty} g\left(\varsigma \cdot \frac{\vec{k}}{k}\right) d\varsigma + G\left(\frac{1}{k^2}\right). \qquad (77)$$

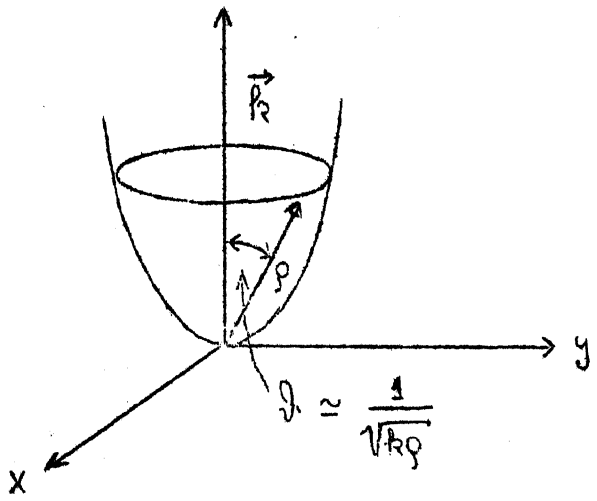

$\vec{k}$
ρ
y
x
$\vartheta \simeq \frac{1}{\sqrt{k\rho}}$

Fig. 19

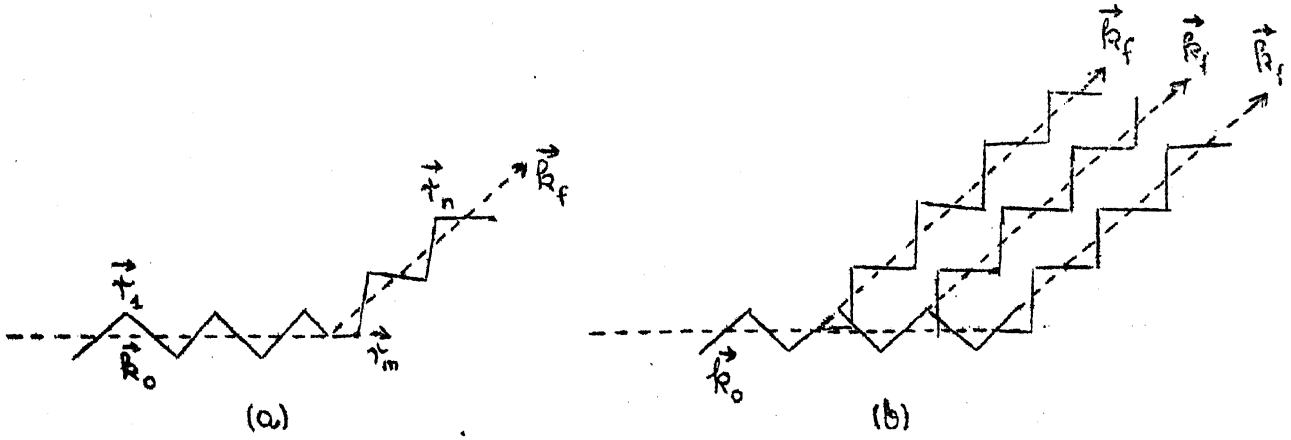

$\vec{r}_1$
$\vec{k}_0$
$\vec{r}_n$
$\vec{k}_f$
$\vec{r}_m$
(a)
$\vec{k}_f$
$\vec{k}_f'$
$\vec{r}_n'$
$\vec{k}_0$
(b)

Fig. 20

Revenons à l'équation (73) et appelons $\vec{k}_0$ et $\vec{k}_f$ les directions initiales et finale de l'électron (dans le cas de la diffusion élastique $|\vec{k}_0| = |\vec{k}_f| = k$). L'équation (73) peut être remplacée par l'équation intégrale (cf. Mott and Massey, Théory of atomic collisions) :

$$\psi(\vec{r}) = e^{i\vec{k}_0\cdot\vec{r}} + \int G(|\vec{r}-\vec{r}'|)U(\vec{r}')\psi(\vec{r}')d\vec{r}' \quad (78)$$

où $G(r)$ est la fonction de Green solution de l'équation :

$$(\Delta + k^2)G = \delta(\vec{r}) \quad (79)$$

et qui est égale à :

$$G(r) = -\frac{1}{4\pi\rho}e^{ik\rho}. \quad (80)$$

Nous nous intéresserons à la valeur asymptotique de la fonction d'onde :

$$\psi \simeq e^{i\vec{k}_0\cdot\vec{r}} + \frac{e^{ikr}}{r} f(\hat{k}_f, \hat{k}_0) \quad (81)$$

où $\hat{k}_f$ et $\hat{k}_0$ sont des vecteurs unitaires portés par k_f et k_0. L'amplitude de diffusion s'obtient par résolution de l'équation (78) par approximations successives :

$$f(\hat{k}_f, \hat{k}_0) = \sum_{n=1}^{\infty} \int\cdots\int e^{-i\vec{k}_f\cdot\vec{r}_n} U(\vec{r}_n) G(|\vec{r}_n - \vec{r}_{n-1}|) U(\vec{r}_{n-1}) \cdot \quad (82)$$

$$\cdot G(|\vec{r}_{n-1} - \vec{r}_{n-2}|) U(\vec{r}_{n-2}) \cdots U(\vec{r}_2) G(|\vec{r}_2 - \vec{r}_1|) U(\vec{r}_1) e^{i\vec{k}_0\cdot\vec{r}_1} d\vec{r}_1 \cdots d\vec{r}_n$$

1º Premier type d'approximation (grands angles) : Nous poserons dans (82) :

$$\vec{\rho}_1 = \vec{r}_2 - \vec{r}_1, \quad \vec{\rho}_2 = \vec{r}_3 - \vec{r}_2, \quad \cdots, \quad \vec{\rho}_{n-1} = \vec{r}_n - \vec{r}_{n-1}, \quad (83)$$

et nous écrirons l'argument des exponentielles qui apparaissent sous le signe somme de la façon suivante :

$$-\vec{k}_f\cdot\vec{r}_n + \vec{k}_0\cdot\vec{r}_1 = -\vec{k}_f\cdot\vec{\rho}_{n-1} - \vec{k}_f\cdot\vec{\rho}_{n-2} \cdots - \vec{k}_f\cdot\vec{\rho}_m + \vec{q}\cdot\vec{r}_m \quad (84)$$
$$- \vec{k}_0\cdot\vec{\rho}_{m-1} - \vec{k}_0\cdot\vec{\rho}_{m-2} \cdots - \vec{k}_0\cdot\vec{\rho}_1$$

(aucune approximation jusqu'ici), $\vec{r}_m$ étant l'un quelconque des points $\vec{r}_1 \ldots \vec{r}_n$ et $\vec{q} = \vec{k}_0 - \vec{k}_f$. Nous fe ons maintenant sur les n - 1 intégrales sur $\vec{\rho}_i$ ($i = 1, 2, \ldots n-1$) l'approximation de la phase stationnaire (77), l'intégration sur $\vec{r}_m$ étant laissée sans changement. Pour un choix particulier de $\vec{r}_m$ ceci revient à décomposer la diffusion en m - 1 diffusions de petit angle ($\vartheta \ll (k\rho)^{-\frac{1}{2}}$) autour de $\vec{k}_0$, une diffusion de grand angle en $\vec{r}_m$, et n - m diffusions de petit angle autour de $\vec{k}_f$ (fig 20 (a)). Nous sommerons ensuite toutes les contributions correspondant à chaque $\vec{r}_m$ (fig. 20 (b)) et l'on voit que ceci ne peut se faire sans recouvrement des domaines d'intégration que pour des angles totaux de diffusion grands par rapport à $(kR)^{-\frac{1}{2}}$ (R étant le rayon du centre diffuseur) :

$$\vartheta = (\hat{k}_0 \cdot \hat{k}_f) \gg (kR)^{-1/2} \qquad (85)$$

Nous obtenons ainsi pour f l'expression suivante :

$$f(\hat{k}_f, \hat{k}_0) = -\frac{1}{4\pi} \sum_{n=1}^{\infty} \sum_{m=1}^{n} \left(-\frac{i}{2k}\right)^{n-1} \cdot \qquad (86)$$

$$\cdot \int d\vec{r}_m \int_0^\infty d\rho_1 \ldots \int_0^\infty d\rho_{n-1}\, e^{i\vec{q}\cdot\vec{r}_m}\, U\left[\vec{r}_m + \hat{k}_f(\rho_{n-1} + \rho_{n-2} + \ldots + \rho_m)\right] \cdot$$

$$\ldots U\left[\vec{r}_m + \hat{k}_f \rho_m\right] U(\vec{r}_m)\, U(\vec{r}_m - \hat{k}_0 \rho_{m-1}) \ldots .$$

$$\ldots U\left[\vec{r}_m - \hat{k}_0(\rho_{m-1} + \rho_{m-2} + \ldots + \rho_1)\right].$$

Nous prenons d'abord m - 1 variables

$$\left.\begin{aligned} S_1 &= \rho_1 + \rho_2 + \ldots + \rho_{m-1} \\ S_2 &= \rho_2 + \rho_3 + \ldots + \rho_{m-1} \\ &\ldots\ldots\ldots\ldots \\ S_{m-1} &= \rho_{m-1} \end{aligned}\right\} \quad S_i = \rho_i + S_{i+1} \qquad (87)$$

L'intégration sur les m - 1 premières variables $\rho_1, \ldots, \rho_{m-1}$ peut donc s'écrire :

$$\int_0^\infty d\rho_1 \cdots \int_0^\infty d\rho_{m-1}\, U(\vec{r}_m - \hat{k}_0 \rho_{m-1}) \cdots U(\vec{r}_m - \hat{k}_0 (\rho_{m-1} + \cdots + \rho_1))$$

$$= \int_0^\infty ds_{m-1} \int_{s_{m-1}}^\infty ds_{m-2} \cdots \int_{s_2}^\infty ds_1\, U(\vec{r}_m - \hat{k}_0 s_1) \cdots U(\vec{r}_m - \hat{k}_0 s_{m-1})$$

$$= \frac{1}{(m-1)!} W_0^{m-1}(\vec{r}_m, \hat{k}_0) \qquad (88)$$

où nous avons posé :

$$W_0(\vec{r}, \hat{k}_0) = \int_0^\infty U(\vec{r} - \hat{k}_0 s)\, ds \qquad (89)$$

Nous écrirons de même :

$$\left.\begin{aligned} s_m &= \rho_m \\ s_{m+1} &= \rho_m + \rho_{m+1} \\ &\cdots \\ s_{n-1} &= \rho_m + \rho_{m+1} + \cdots + \rho_{n-1} \end{aligned}\right\} \; s_{i+1} = s_i + \rho_{i+1} \qquad (90)$$

L'intégration sur les n - m variables $\rho_m \cdots \rho_{n-1}$ est donc égale à

$$\int_0^\infty d\rho_m \int_0^\infty d\rho_{m+1} \cdots \int_0^\infty d\rho_{n-1}\, U(\vec{r}_m + \hat{k}_f \rho_m) \cdots U(\vec{r}_m + \hat{k}_f (\rho_m + \cdots + \rho_{n-1}))$$

$$= \int_0^\infty ds_m \int_{s_m}^\infty ds_{m+1} \cdots \int_{s_{n-2}}^\infty ds_{n-1}\, U(\vec{r}_m + \hat{k}_f s_m) \cdots U(\vec{r}_m + \hat{k}_f s_{n-1})$$

$$= \frac{W_f^{n-m}(\vec{r}_m, \hat{k}_f)}{(n-m)!} \qquad (91)$$

où W_f est défini par :

$$W_f(\vec{r}, \hat{k}_f) = \int_0^\infty U(\vec{r} + \hat{k}_f s)\, ds. \qquad (92)$$

Nous avons donc finalement

$$f(\hat{k}_f, \hat{k}_0) = -\frac{1}{4\pi} \sum_{n=1}^{\infty} \sum_{m=1}^{n} \left(\frac{-i}{2k}\right)^{n-1} \int d\vec{r}\, e^{i\vec{q}\cdot\vec{r}}\, U(\vec{r}) \cdot$$

$$\cdot \frac{\left[W_0(\vec{r}, \hat{k}_0)\right]^{m-1}}{(m-1)!} \frac{\left[W_f(\vec{r}, \hat{k})\right]^{n-m}}{(n-m)!} \qquad (93)$$

La sommation sur m se fait facilement par la formule du binôme

$$\sum_{m=1}^{n} \frac{a^{m-1} b^{n-m}}{(m-1)!\,(n-m)!} (n-1)! = (a+b)^{n-1}$$

et par conséquent :

$$f(\hat{k}_f, \hat{k}_0) = -\frac{1}{4\pi} \int d\vec{r}\, e^{i\vec{q}\cdot\vec{r}}\, U(\vec{r}) \sum_{n=1}^{\infty} \frac{1}{(n-1)} \left[-\frac{i}{2k}(W_0 + W_f)\right]^{n-1}$$

$$= -\frac{1}{4\pi} \int d\vec{r}\, e^{i\vec{q}\cdot\vec{r}}\, U(\vec{r})\, e^{-\frac{i}{2k}(W_0 + W_f)} \qquad (94)$$

On vérifie facilement que cette amplitude de diffusion satisfait à la condition de micro-réversibilité :

$$f(\hat{k}_f, \hat{k}_0) = f(-\hat{k}_0, -\hat{k}_f) \qquad (95)$$

qui revient à changer W_0 et W_f à droite de l'équation (94) et, dans le cas où V (r) a la symétrie sphérique,

$$f(\hat{k}_f, \hat{k}_0) = f(\hat{k}_0, \hat{k}_f) \qquad (96)$$

(que l'on obtient en changeant aussi $\vec{r}$ en $-\vec{r}$ dans l'intégration).

2º Deuxième type d'approximation (petits angles) : Dans le cas où

$$\vartheta = (\hat{k}_0 \cdot \hat{k}_f) \ll (kR)^{-1/2} \qquad (97)$$

les n - 1 domaines d'intégration sur les variables ϱ introduites précédemment se recouvrent, et il ne faut prendre que l'une seulement des contributions. On trouve alors :

$$f(\hat{k}_f,\hat{k}_0)=\frac{ik}{2\pi}\int_{-\infty}^{+\infty}dx\int_{-\infty}^{+\infty}dy\,e^{i(q_x x+q_y y)}\left[1-e^{-\frac{i}{2k}\int_{-\infty}^{+\infty}U(x,y,z)dz}\right] \quad (98)$$

et, dans le cas où $V(\vec{r})$ a au moins une symétrie cylindrique autour de Oz (direction de propagation) de sorte que U ne dépend que de z et de $b=\sqrt{x^2+y^2}$ (la symétrie sphérique est un cas particulier de celui-ci) on peut simplifier l'équation (98) sous la forme :

$$f(\hat{k}_f,\hat{k}_0)=ik\int_0^{+\infty}db\;b\;J_0(qb).\left[1-e^{-\frac{i}{2k}\int_{-\infty}^{+\infty}\bar{U}(b,z)dz}\right] \quad (99)$$

ou $q=|\vec{k}_0-\vec{k}_f|$ et J_0 est la fonction de Bessel d'ordre zero.
L'équation (98) permet d'obtenir une bonne approximation de la section efficace totale qui, comme on le sait, est reliée à la partie imaginaire de l'amplitude de diffusion en avant par la relation :

$$\sigma_{tot}=\frac{4\pi}{k}\,Im\,(\hat{k}_0,\hat{k}_0) \quad (100)$$

Ceci donne immédiatement :

$$\sigma_{tot}=2\int_{-\infty}^{+\infty}dx\int_{-\infty}^{+\infty}dy\left\{1-\cos\left[\frac{1}{2k}\int_{-\infty}^{+\infty}\bar{U}(x,y,z)dz\right]\right\} \quad (101)$$

3º Approximation pour les angles intermédiaires (Méthode de Saxon et Schiff)

La limitation de la validité des approximations précédentes aux angles grands ou petits par rapport à $(kR)^{-\frac{1}{2}}$ peut être très ennuyeuse. Par exemple, pour A = 100 et $E\simeq 400$ Mev, on a $(kR)^{-\frac{1}{2}}\simeq 80º$. Une méthode plus puissante, mais d'une grande complexité, a été développée dans le cas de une dimension (ou de 3 dimensions avec analyse en ondes partielles) par D. Saxon : "Formulation of High Energy Potential Scattering Problems" à paraître à la Phys. Rev.) puis étendue au cas de 3 dimensions pour l'équation de Schrödinger par Saxon et Schiff: "Theory of High Energy Potential Scattering" (à paraître au Nuovo Cimento). Nous donnerons ci-dessous les résultats de leur première approximation (valable pour tous les angles). Posons dans la suite

$$\delta_0(\vec{r})=-\frac{1}{2k}\,W_0(\vec{r},\hat{k}_0)=-\frac{1}{2k}\int_0^{\infty}U(\vec{r}-\hat{k}_0 s)ds \quad (102)$$

et de même :

$$\delta_f(\vec{r}) = -\frac{1}{2k} W_f(\vec{r}, \hat{k}_f) = -\frac{1}{2k}\int_0^\infty U(\vec{r}+\hat{k}_f s)\,ds. \quad (103)$$

Considérons ensuite un vecteur $\vec{r}'$ qui a pour coordonnées x y z' (x, y, z étant les coordonnées de $\vec{r}$) (fig. 21). L'amplitude de diffusion est alors donnée par :

$$f^{(1)}(\hat{k}_f, \hat{k}_0) = -\frac{1}{4\pi}\int U(\vec{r})\, e^{i(\vec{q}\cdot\vec{r}+\delta_0(\vec{r}))}\, d\vec{r} \quad (104)$$

$$-\frac{i}{8\pi k}\int U(\vec{r})\, e^{i\delta_0(\vec{r})}\left\{\vec{\nabla}^2\int_z^\infty e^{i\vec{q}\cdot\vec{r}'}\left[e^{i\delta_f(\vec{r}')}-1\right]dz'\right\}d$$

On vérifie que pour les grands et petits angles cette expression se réduit aux équations (94) et (97). Une méthode d' itération permet d'améliorer l'approximation précédente autant de fois que l'on veut.

6.- CAS DE L'EQUATION DE DIRAC.

Nous considérons le cas de l'équation de Dirac :

$$\left(-i\vec{\alpha}\cdot\vec{\nabla}+\beta m+E\right)\psi = V\psi \quad (105)$$

où E est, cette fois, l'énergie totale de la particule diffusée, et $\vec{\alpha}$ et β les matrices de Dirac habituelles. L'équation (105) peut se transformer en équation intégrale en opérant d'abord à gauche avec l'opérateur $(E+i\vec{\alpha}\cdot\vec{\nabla}-\beta m)$ ce qui donne :

$$(\Delta+k^2)\psi = (E+i\vec{\alpha}\cdot\vec{\nabla}-\beta m)V\psi \quad (106)$$

et, par suite, en utilisant la fonction de Green (55) :

$$\psi(\vec{r}) = a_0 e^{i\vec{k}_0\cdot\vec{r}} + \int G(|\vec{r}-\vec{r}'|)(E+i\vec{\alpha}\cdot\vec{\nabla}'-\beta m)V(\vec{r}')\psi(\quad (107)$$

où a_0 est l'amplitude du spineur de Dirac qui caractérise l'onde incidente. On cherche une amplitude de diffusion $f_D(\hat{k}_f, \hat{k}_0)$ en écrivant asymptotiquement :

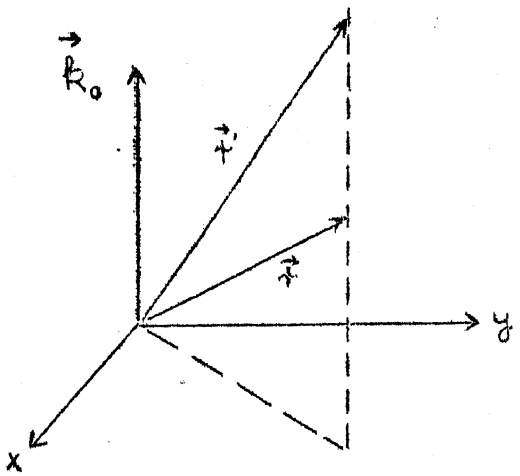


Fig. 21

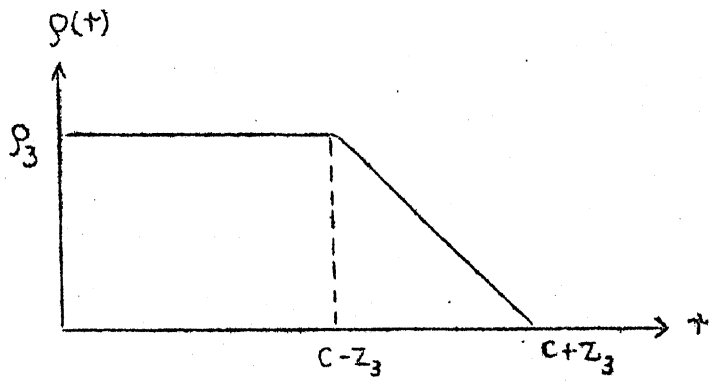


Fig. 22

$$\psi(\vec{r}) = a_0 e^{i\vec{k}_0\cdot\vec{r}} + \frac{1}{r} e^{ikr} f_D(\hat{k}_f, \hat{k}_0) \qquad (108)$$

f_D est un spineur. Ce qui nous intéresse, c'est en réalité, sa projection $\bar{a}_f f_D$ sur l'amplitude du spineur a_f qui représente l'état final après diffusion (particule libre de quantité de mouvement $\vec{k}_f$). La section efficace différentielle s'obtient ensuite en sommant sur les spins de l'état final et en faisant une moyenne sur les spins de l'état initial (énergies positives seulement) :

$$\sigma_D(\vartheta) = \frac{1}{2} \int_f \int_0 |\bar{a}_f f_D|^2 \qquad (109)$$

Les méthodes d'approximations du paragraphe précédent s'appliquent exactement de la même façon à l'équation (107). Nous n'entrerons pas pour le moment dans le détail de ces calculs que nous retrouverons sous une forme un peu plus compliquée, au paragraphe suivant (voir si l'on veut Schiff, Phys. Rev., 103, 443 (1956)).

7.- INFLUENCE DES CORRELATIONS ENTRE PROTONS SUR LA DIFFUSION (INELASTIQUE OU ELASTIQUE) A GRANDE ENERGIE (Schiff, Nuovo Cimento 5, 1223 (1957)).

Nous sommes maintenant mieux armés pour reprendre le problème de l'influence des corrélations entre protons que nous avons abordé au § 2. Nous considérons le système électron + noyau, dont la fonction d'onde Ψ obéit à l'équation :

$$H\Psi = (E_0 + E)\Psi, \qquad (110)$$

où E_0 est l'énergie du noyau dans l'état fondamental et E celle des électrons incidents. L'hamiltonien du système peut s'écrire :

$$H = H_N + i\vec{\alpha}\cdot\vec{\nabla} - m\beta + V(\vec{r}; \vec{R}_1, \ldots, \vec{R}_Z) \qquad (111)$$

H_N est l'hamiltonien du noyau dont les fonctions propres ψ_a (correspondent aux niveaux E_a) forment un ensemble complet et orthonormé

$$H_N \psi_a = E_a \psi_a \tag{112}$$

Le potentiel d'interaction dépend de la position $\vec{r}$ de l'électron et de celles $\vec{R}_1 \ldots \vec{R}_z$ des z protons :

$$V(\vec{r}; \vec{R}_1, \vec{R}_2, \ldots, \vec{R}_z) = -\sum_{i=1}^{z} \frac{e^2}{|\vec{r} - \vec{R}_i|} \tag{113}$$

Nous développons Ψ en série des ψ_a :

$$\Psi = \sum_a u_a(\vec{r})\, \psi_a(\vec{R}_1, \ldots, \vec{R}_A) \tag{114}$$

Les amplitudes $u_b(\vec{r})$ sont solutions d'un ensemble d'équations différentielles linéaires du premier ordre :

$$(-i\vec{\alpha}\cdot\vec{\nabla} + \beta m + \varepsilon_b)\, u_b(\vec{r}) = \sum_a V_{ba}(\vec{r})\, u_a(\vec{r}) \tag{115}$$

où nous avons posé :

$$\varepsilon_b = E - E_b + E_0 \tag{116}$$

et où les éléments de matrice de l'intéraction sont définis par :

$$V_{ba}(\vec{r}) = \int \ldots \int \psi_b^*(\vec{R}_1 \ldots \vec{R}_A) V(\vec{r}; \vec{R}_1 \ldots \vec{R}_z) \psi_a(\vec{R}_1 \ldots \vec{R}_a) \cdot d\tau_1 \ldots d\tau_A \tag{117}$$

l'intégration portant sur tous les nucléons du noyau (et non pas seulement sur les protons).

Asymptotiquement, l'amplitude $u_b(\vec{r})$ a la forme :

$$u_b(\vec{r}) = a_0 \delta_{b0} e^{i\vec{k}_0 \cdot \vec{r}} + \frac{1}{r} e^{ik_b r} f_b(k_b, k_0), \tag{118}$$

où

$$k_b = (\varepsilon_b^2 - m^2)^{1/2} \tag{119}$$

est la quantité de mouvement d'un électron diffusé inélastiquement après excitation du noyau au niveau E_b. Nous appellerons :

$$G_a(\rho) = -\frac{1}{4\pi\rho} e^{ik_a\rho} \qquad (120)$$

la fonction de Green correspondant à k_a. Après multiplication des deux membres de (115) par $\varepsilon_b - m\beta + i\vec{\alpha}\vec{\nabla}$, on peut remplacer cette dernière équation par l'équation intégrale :

$$u_b(\vec{r}) = a_0 \delta_{b0} e^{i\vec{k}_0\cdot\vec{r}} + \sum_a \int G_b(|\vec{r}-\vec{r}'|)(\varepsilon_b - \beta m + i\vec{\alpha}\cdot\vec{\nabla}'). \cdot V_{ba}(\vec{r}')u_a(\vec{r}')d\vec{r}' \qquad (121)$$

ce qui donne, pour l'amplitude f_b obtenue par approximation successives, la série suivante :

$$f_b(\vec{k}_b,\vec{k}_0) = -\frac{1}{4\pi}(\varepsilon_b - \vec{\alpha}\cdot\vec{k}_b - \beta m)\sum_{n=1}^{\infty}\int d\tau_1 \ldots \int d\tau_n\, e^{-i\vec{k}_b\cdot\vec{r}_n}$$

$$\cdot \sum_{a_{n-1}} V_{ba_{n-1}}(\vec{r}_n)(\varepsilon_{a_{n-1}} + i\vec{\alpha}\vec{\nabla}_n - m\beta) G_{a_{n-1}}(|\vec{r}_n - \vec{r}_{n-1}|) \qquad (122)$$

$$\cdot \sum_{a_{n-2}} V_{a_{n-1}a_{n-2}}(\vec{r}_{n-1})(\varepsilon_{a_{n-2}} + i\vec{\alpha}\vec{\nabla}_{n-1} - m\beta) G_{a_{n-2}}(|\vec{r}_{n-1} - \vec{r}_{n-2}|)\ldots$$

$$\cdot \sum_{a_1} V_{a_2a_1}(\vec{r}_2)(\varepsilon_{a_1} + i\vec{\alpha}\vec{\nabla}_2 - m\beta) G_{a_1}(|\vec{r}_2 - \vec{r}_1|) V_{a_1 0}(\vec{r}_1) e^{i\vec{k}_0\vec{r}_1}.$$

La différence essentielle avec l'équation analogue (82) (compte tenu des modifications dues à l'équation de Dirac - équation (107) est qu'il y a cette fois, après chaque intégration sur les variables intermédiaires de l'électron $\vec{r}_1 \ldots \vec{r}_n$, une sommation sur les niveaux intermédiaires $a_1 \ldots a_{n-1}$ du noyau. Pour pouvoir nous débarrasser de ces sommations, nous ferons l'approximation suivant laquelle les énergies d'excitation $E_{a_i} - E_{a_{i-1}}$ sont faibles devant l'énergie E des électrons, c'est-à-dire que les ε_{a_i} dépendent peu des a_i. Par ailleurs, nous ferons l'approximation de haute énergie correspondant aux grands angles, car c'est surtout pour les grands transferts d'impulsion que nous pourrons déceler les effets de correlation qui nous intéressent. Nous avons donc, de façon analogue à (86) :

$$f_b(\vec{k}_b,\vec{k}_o) = -\frac{1}{4\pi}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\frac{-i}{2k_b}\right)^{n-m}\left(\frac{-i}{2k_o}\right)^{m-1}\cdot$$

$$\int d\vec{r}_m \int_0^\infty d\rho_1 \dots \int_0^\infty d\rho_{n-1}\, e^{i\vec{q}_b\vec{r}_m}\left\{V(\vec{r}_m - \hat{k}_b(\rho_{n-1}+\dots+\rho_m))\right. \quad (123)$$

$$\dots V(\vec{r}_m+\hat{k}_b\rho_m)\,V(\vec{r}_m)\,V(\vec{r}_m-\hat{k}_o\rho_{m-1})\dots$$

$$\left.\cdot\, V(\vec{r}_m-\hat{k}_o(\rho_{m-1}+\dots+\rho_1))\right\}_{bo}(\varepsilon_b-\vec{\alpha}\vec{k}_b-\beta m)^{n-m+1}(\varepsilon_o-\vec{\alpha}\vec{k}_o-\beta m)^{m-1}a_o$$

où $\vec{q}_{kb}=\vec{k}_o-\vec{k}_b$ et où le symbole $\{\ \}_{bo}$ signifie qu'il faut prendre l'élément de matrice du produit des V entre accolades entre les fonctions d'onde nucléaires ψ_b et ψ_o. Comme au § 6, c'est en réalité le produit $\bar{a}_b f_b$ qui nous intéresse. a_o et $\bar{a}_b$ sont solutions des équations de Dirac dans l'espace des impulsions :

$$(\varepsilon_o+\vec{\alpha}\cdot\vec{k}_o+\beta m)\,a_o(\vec{k}_o)=0 \qquad (124)$$

et

$$\bar{a}_b(\vec{k}_b)(\varepsilon_b+\vec{\alpha}\vec{k}_b+\beta m)=0. \qquad (125)$$

Par suite le produit

$$\bar{a}_b(\varepsilon_b-\vec{\alpha}\cdot\vec{k}_b-\beta m)^{n-m+1}(\varepsilon_o-\vec{\alpha}\cdot\vec{k}_o-\beta m)^{m-1}a_o$$

$$*\,(2\varepsilon_b)^{n-m+1}(2\varepsilon_o)^{m-1}\bar{a}_b a_o \qquad (126)$$

Nous avons donc finalement :

$$\bar{a}_b f_b(\vec{k}_b,\vec{k}_o) = -\frac{\varepsilon_b}{2\pi}\int d\vec{r}\left\{V(\vec{r})e^{i\vec{q}_b\cdot\vec{r}-\frac{i}{v_o}\int_0^\infty V(\vec{r}-\hat{k}_o s)ds-\frac{i}{v_b}\int_0^\infty V(\vec{r}+\hat{k}_b s)ds}\right\}\bar{a}_b a_o \qquad (127)$$

où $v_o=\varepsilon_o/k_o$ et $v_b=\varepsilon_b/k_b$ sont les vitesses initiales et finales de l'électron. En pratique nous prendrons $v_o=v_b=c=1$ ce qui donne finalement pour la section efficace inélastique $0\to b$:

$$\sigma_{bo}(\vartheta)=\left(\frac{\varepsilon_b}{2\pi}\cos\frac{\vartheta}{2}\right)^2\left|\int d\vec{r}\left\{V(\vec{r})e^{i\vec{q}\cdot\vec{r}-\int_0^\infty V(\vec{r}-\hat{k}_o s)ds-\int_0^\infty V(\vec{r}+\hat{k}_b s)ds}\right\}_{bo}\right|^2 \qquad (128)$$

Afin de pouvoir reconnaitre les effets de corrélation entre deux, trois, etc.... protons, nous voudrions développer l'exponentielle à droite de (128) sans pour cela perdre la supériorité de notre approximation sur celle de Born (§§ 1 et 2). Nous considérerons donc la densité de charge statique dans l'état fondamental $\rho_{00}^{(1)}(\vec{r})$ et le potentiel coulombien modifié $V_c(\vec{r})$ qui se déduit de la résolution de l'équation de Poisson correspondante. Nous écrirons :

$$V(\vec{r}) = V_c + V - V_c \qquad (129)$$

et développons l'exponentielle en puissances de $(V - V_c)$. Nous obtenons ainsi, en deuxième ordre, des termes de la forme :

$$\int d\vec{r} \left\{ V(\vec{r}; R_1, \ldots, R_z) \int_0^\infty V(\vec{r} - \hat{k}_0 s; R_1, \ldots, R_z)\, ds \right\}_{b_0}$$
$$\cdot \exp\left[i\vec{q}\cdot\vec{r} - i\int_0^\infty V_c(\vec{r} - \hat{k}_0 s)\, ds - i\int_0^\infty V_c(\vec{r} + \hat{k}_\beta s)\, ds \right]$$

sur lesquels il est très difficle d'identifier des corrélations entre deux protons simplement à partir de la dépendance de σ_{b_0} en fonction de Q_k^2. Il n'en est pas de même si nous cherchons, comme au § 1, la somme des sections efficaces sur tous les états b (y compris S = 0, qui correspond à la diffusion élastique). La règle de somme permet de simplifier l'expression finale en se débarrassant de la somme sur b. Ceci implique que l'on néglige la variation des ε_β et des $\vec{q}_\beta$ en fonction de b, ce qui est une beaucoup moins bonne approximation que celle que nous avons faite pour les états intermédiaires a_i (car nous considérons maintenant des diffusions de grand angle, avec de grandes valeurs de q_b, alors qu'auparavant, l'approximation de grande énergie impliquait essentiellement de petits angles de diffusion intermédiaires). Quoiqu'il en soit, nous avons finalement (en appelant $\vec{k}_f$ la direction finale de l'électron, $|k_f| = k$, $\vec{q} = \vec{k}_0 - \vec{k}_f$)

$$\sum_\beta \sigma_{b_0}(\vartheta) = \left(\frac{E\cos\frac{\vartheta}{2}}{2\pi}\right)^2 \left\{ \iint d\vec{r}\, d\vec{r}'\, \bar{V}(\vec{r})\bar{V}(\vec{r}')\, e^{i\vec{q}\cdot(\vec{r}-\vec{r}')} \right.$$
$$\left. \cdot e^{-i\int_0^\infty \left[\bar{V}(\vec{r} - \hat{k}_0 s) + \bar{V}(\vec{r} + \hat{k}_f s) - \bar{V}(\vec{r}' - \hat{k}_0 s) - \bar{V}(\vec{r}' + \hat{k}_f s) \right] ds} \right\}_{00} \qquad (130)$$

Si nous posons :

$$u_c(\vec{r},\vec{r}') = e^{-i\int_0^\infty [V_c(\vec{r}-\hat{k}_0 s)+V_c(\vec{r}+\hat{k}_f s)-V_c(\vec{r}'-\hat{k}_0 s)-V_c(\vec{r}'+\hat{k}_f s)]\,ds} \tag{131}$$

fonction qui est telle que

$$u_c(\vec{r},\vec{r}) = \mathrm{I} \tag{132}$$

nous obtenons au Ier ordre du développement en $(V - V_0)$:

$$\sum_b \sigma_{b0}(\vartheta) = \left(\frac{E}{2\pi}\cos\frac{\vartheta}{2}\right)^2 \iint d\vec{r}\,d\vec{r}'\,u_c(\vec{r},\vec{r}')\,e^{i\vec{q}\cdot(\vec{r}-\vec{r}')} \cdot \left\{V(\vec{r};R_1\ldots R_Z)\,\bar{V}(\vec{r}';R_1\ldots R_Z)\right\}_{00} \tag{133}$$

qui peut s'écrire, en vertu de (113)

$$\sum_b \sigma_{b0}(\vartheta) = \left(\frac{e^2 E\cos\frac{\vartheta}{2}}{2\pi}\right)^2 \sum_{i=1}^{Z}\sum_{j=1}^{Z}\iint d\vec{r}\,d\vec{r}'\,u_c(\vec{r},\vec{r}')\left\{\frac{1}{|\vec{r}-\vec{R}_i||\vec{r}'-\vec{R}_j|}\right\}_{00} e^{i\vec{q}\cdot(\vec{r}-\vec{r}')} \tag{134}$$

et, en prenant comme variables d'intégration $\vec{\rho} = \vec{r} - \vec{R}_i$ et $\vec{\rho}' = \vec{r}' - \vec{R}_j$

$$\sum_b \sigma_{b0}(\vartheta) = \left(\frac{e^2 E\cos\frac{\vartheta}{2}}{2\pi}\right)^2 \sum_{i=1}^{Z}\sum_{j=1}^{Z}\iint d\vec{\rho}\,d\vec{\rho}'\,\frac{1}{\rho\rho'}\,e^{i\vec{q}\cdot(\vec{\rho}-\vec{\rho}')} \cdot \left\{u_c(\vec{\rho}+\vec{R}_i,\vec{\rho}'+\vec{R}_j)\,e^{i\vec{q}\cdot(\vec{R}_i-\vec{R}_j)}\right\}_{00} \tag{135}$$

Si l'on néglige la variation de u_c au tour de $\vec{R}_i$ et $\vec{R}_j$, on obtient de façon analogue au § 1,

$$\sum_b \sigma_{b0}(\vartheta) = \left(\frac{e^2}{2E}\right)^2 \frac{\cos^2\frac{\vartheta}{2}}{\sin^4\frac{\vartheta}{2}} \left\{\sum_i\sum_j e^{i\vec{q}\cdot(\vec{R}_i-\vec{R}_j)}\,u_c(\vec{R}_i,\vec{R}_j)\right\}_{00} \tag{136}$$

et, en fonction des densités $\rho_{00}^{(1)}$ et $\rho_{00}^{(2)}$, on obtient en tenant compte de (132)

$$\sum_b \sigma_{b0}(\vartheta) = \left(\frac{e^2}{2E}\right)^2 \frac{\cos^2\frac{\vartheta}{2}}{\sin^4\frac{\vartheta}{2}} \left\{Z + \iint \rho_{00}^{(2)}(\vec{r},\vec{r}')\,u_c(\vec{r},\vec{r}')\cdot e^{i\vec{q}\cdot(\vec{r}-\vec{r}')}\,d\vec{r}\,.\,d\vec{r}'\right. \tag{137}$$

Cette expression est très semblable à celle que nous avions obtenu à la fin du § 1, sauf que le facteur $u_c(\vec{r}, \vec{r}')$ tient compte de la non-validité de l'approximation de Born.

8.- RESULTATS EXPERIMENTAUX CONCERNANT LA DIFFUSION ELASTIQUE PAR LES NOYAUX LOURDS.

1º Noyaux à symétrie sphérique : de toutes les distributions de charge de noyaux lourds analysées jusqu'à 180 Mev par la méthode des ondes partielles, seule celle de l'or (Au^{197}) a été étudiée en détail. Trois modèles ont été utilisés :

- une distribution du type de Fermi

$$\rho(r) = \frac{\rho_1}{e^{\frac{(r-c)}{z_1}} + 1}$$

dont le "paramètre de peau" t correspond à $t = 4{,}40\, z_1$)

- une distribution dite "gaussienne modifiée"

$$\rho(r) = \frac{\rho_2}{e^{(r^2-c^2)/z_2^2} + 1}$$

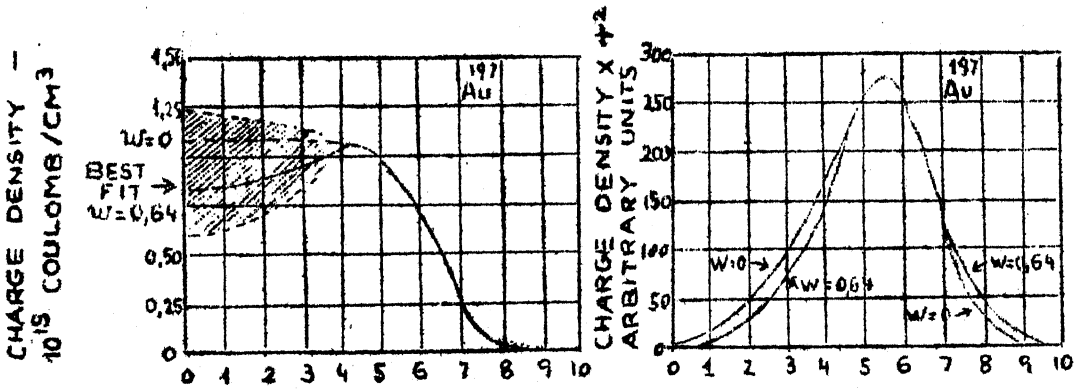
qui est telle que $t = 2.20\, \frac{z_2}{c}$,

- une distribution trapézoidale (Fig. 22) telle que $t = 1.60\, z_3$

Pour examiner de plus près la forme de la densité au centre, une distribution de Fermi modifiée du type

$$\rho(r) = \rho_1 \frac{\left(1 + w \frac{r^2}{c^2}\right)}{e^{\frac{r-c}{z_1}} + 1}$$

a été employée en variant le paramètre W (fig. 23). En pratique, W peut être varié dans un assez grand domaine, car seul le produit $4\pi \rho^2$ (densité d'une couche sphérique) est déterminé expérimentalement. Cependant $W = 0.64$ semble représenter le meilleur accord (Hahn, Ravenhall et Hofstadter, Phys. Rev. 101, 1131 (1956)).



(a) (b)

Fig. 23

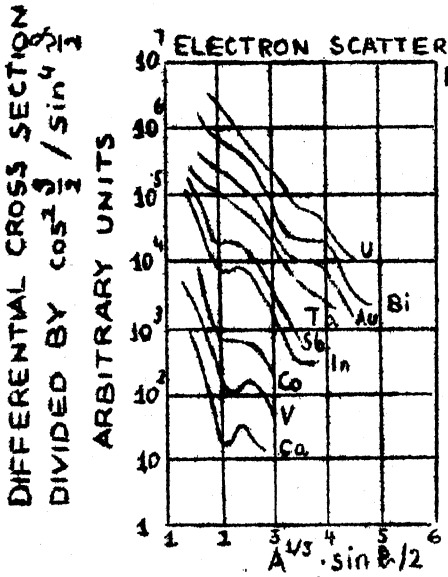


Fig. 24

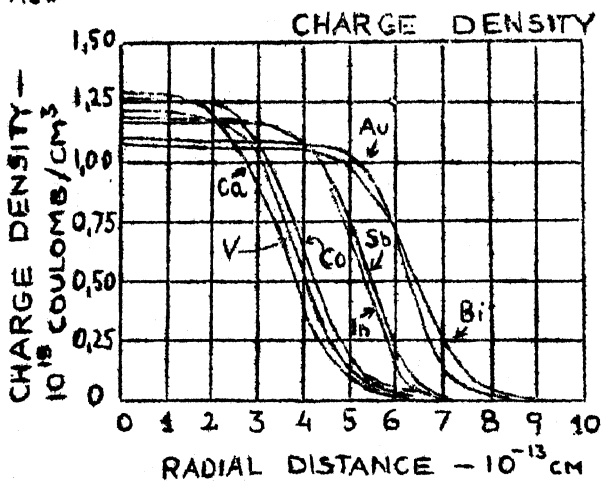


Fig. 25

En ce qui concerne les autres noyaux lourds étudiés à 183 Mev, le fait le plus saillant qui caractérise la diffusion élastique est la régularité des minima et maxima de diffraction lorsque les carrés des facteurs de forme sont représentés en fonction de $A^{1/3} \sin \frac{\vartheta}{2}$ (fig 24). La fig. 25 représente les distributions de charge du type de Fermi pour différents noyaux. On remarquera l'augmentation de la densité centrale moyenne quand Z décroit.

2º Noyaux déformés : ce qui caractérise les sections efficacesde diffusion sur les noyaux déformés (formes ellipsoidales qui, conformément au modèle de Bohr et Mottelson, sont associées aux noyaux ayant un moment quadrupolaire Q_o important) c'est la disparition presque complète des minima et maxima de diffraction(fig. 26). Ceci s'explique qualitativement si l'on remarque que les effets de diffractionsont des effets de paroi, et que ceux-ci sont d'autant plus marqués que le paramètre de peau t est plus petit. Dans le cas de noyaux déformés, la moyenne sur toutes les orientations de l'axe du spin a pour effet d'augmenter l'épaisseur effective de la distribution de charge d'une quantité de l'ordre de la différence des axes principaux.

D'autres effets apparaissent qui ont été étudiés théoriquement par Downs (Thèse Stanford 1955) :

a) diffusion avec renversement du spin nucléaire (spin-flip) qui s'ajoute de façon incohérente à la diffusion ordinaire (il n'y a de possibilité d'interférence que dans le cas des noyaux orientés).

b) diffusion inélastique avec excitation des niveaux très bas qui sont présents pour ces noyaux, et qui apparait expérimentalement comme quasi élastique.

Downs indique que l'observation de la section efficace de diffusion élastique en voisinage d'un (faible) minimum de diffractionpeut permettre de déterminer le moment quadrupolaire (cf. fig. 27 pour le tantale

3º Rapport de sections efficaces pour des noyaux voisins : une méthode très ingénieuse a été développée par Hahn, Hofstadter et Ravenhall (Phys. Rev. 105, 1353 (1957) pour déterminer le rapport des sections efficaces de dispersion élastique correspondant à des noyaux voisins en observant celles-ci

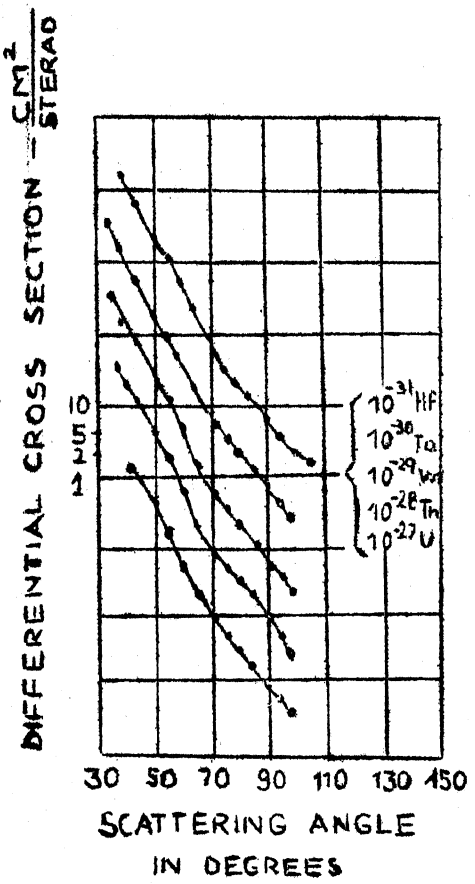


Fig. 26

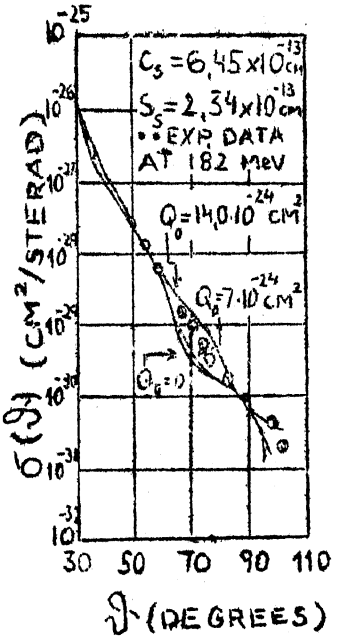


Fig. 27

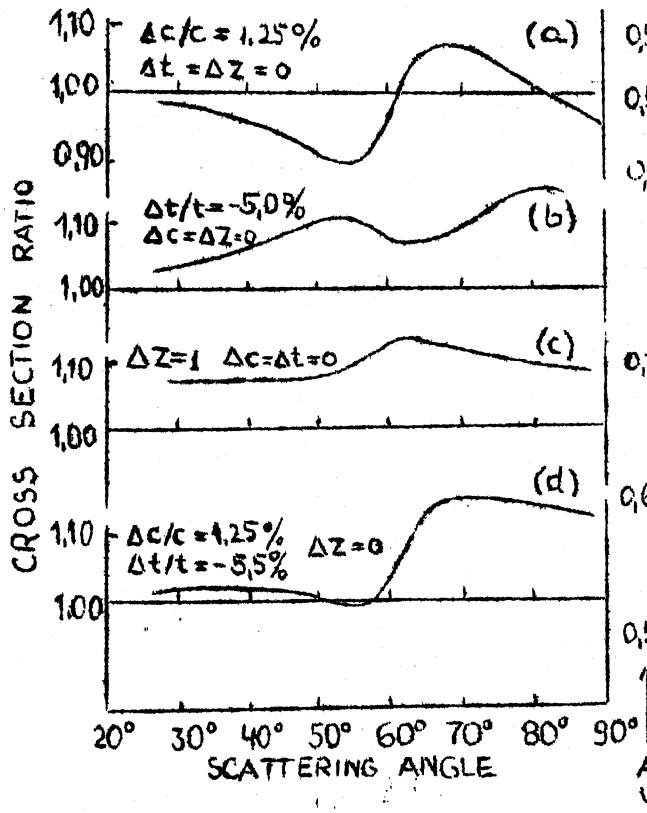


Fig. 28

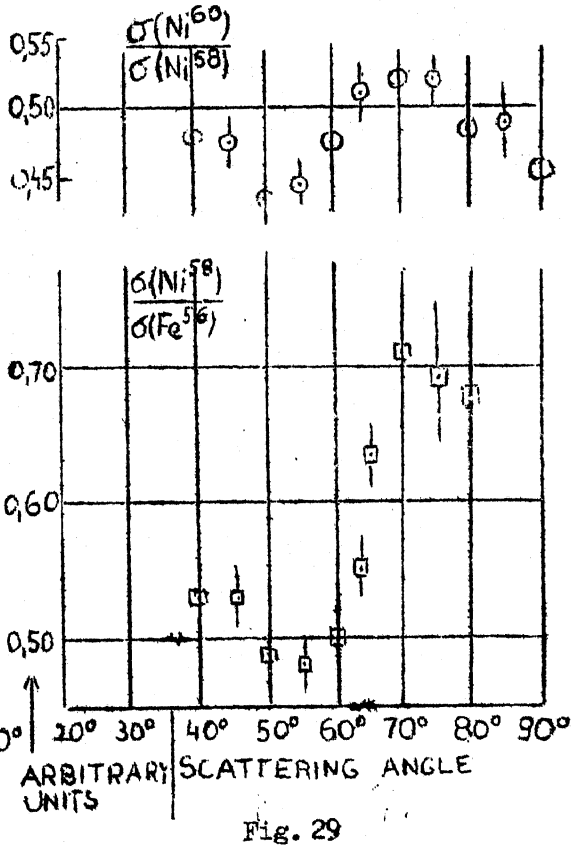


Fig. 29

au voisinage d'un minimum de diffraction. La fig. 28 montre par exemple l'influence, dans le cas du Ni (nombre de protons $Z = 28$ magique) d'une variation des différents paramètres c , t ou Z. On voit que la variation peut, au voisinage de 60º (à 183 Mev), atteindre 15 % pour des variations relatives des paramètres de l'ordre de 1 à 2 %. La fig. 29 montre le rapport des sections efficaces pour deux paires de noyaux voisins Ni^{60} et Ni^{58} d'une part, Ni^{58} et Fe^{56} d'autre part.

9.- RESUME DES EFFETS DE DIFFUSION INELASTIQUE.

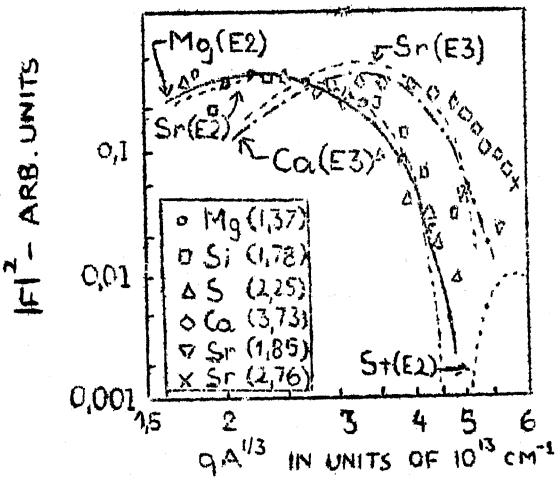
Nous ne pouvons que résumer brièvement les effets de diffusion inélastique que nous avons déjà évoqués au cours de ce chapitre et du précédent.

a) Excitation des niveaux nucléaires : (exemple du C^{12} déjà discuté). Le rapport de ces excitations avec les transitions multipolaire dans les noyaux a été étudié par Schiff, Phys. Rev. 96, 765 (1954). Sherman et Ravenhall (Phys. Rev. 103, 949 (1956) ont étudié en détail la transition $0^+ - 0^+$ (de 7,68 Mev) du C^{12}. Helm (these déjà citée) obtient un résultat intéressant en représentant le rapport $\sigma / \sigma_M \sim |F|^2$ inélastique en fonction de $qA^{1/3}$ pour divers éléments (fig. 30). On voit que quel que soit l'élément, les points expérimentaux se placent approximativement sur 2 courbes correspondant à des transitions (E 2) et (E 3), indiquant ainsi l'existence de courbes universelles correspondant à chaque valeur de la transition électrique multipolaire correspondante.

b) Electro-désintégration (avec émission d'un nucléon). Exemple de la particule α (fig. 31, qui représente la sphère d'un électron diffusé à 400 Mev et 60º) où l'on peut déterminer la distribution des impulsions des nucléons dans le noyau en comparant le pic élastique des protons au pic inélastique élargi par les collisions de l'électron incident avec l'un des nucléons en mouvement du noyau.

Cas particulier : Dislocation complète du noyau ou break-up

Fig. 30


Mg(E2)
Sr(E3)
Sr(E2)
Ca(E3)
Sr(E2)
$|F|^2$ - ARB. UNITS
0,1
0,01
0,001
1,5
2
3
4
5
6
$qA^{1/3}$ IN UNITS OF 10^{13} cm^{-1}
Mg (1,37)
Si (1,78)
S (2,25)
Ca (3,73)
Sr (1,85)
Sr (2,76)

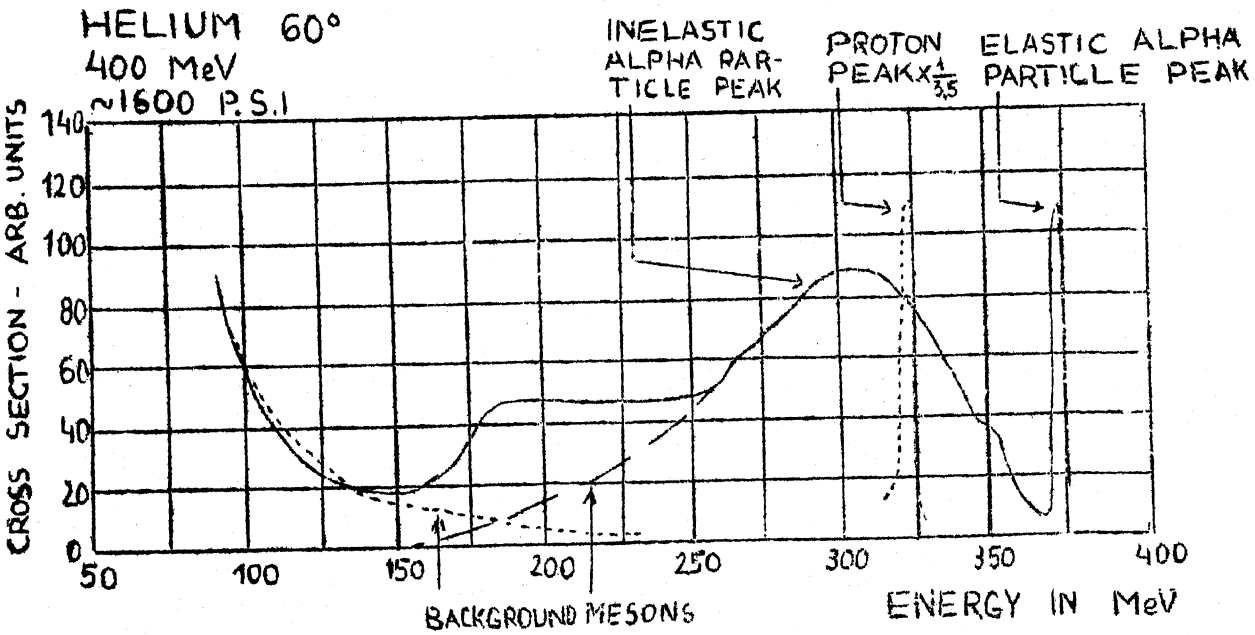

HELIUM 60°
400 MeV
~1600 P.S.I
INELASTIC ALPHA PARTICLE PEAK
PROTON PEAK x 1/3,5
ELASTIC ALPHA PARTICLE PEAK
CROSS SECTION - ARB. UNITS
140
120
100
80
60
40
20
0
50
100
150
200
250
300
350
400
BACKGROUND MESONS
ENERGY IN MeV

Fig. 31

c) Effets radiatifs : les corrections radiatives correspondant aux effets virtuels (élastiques) ont été discutés.

Pour l'émission de radiation réelle (Bremsstantung) le problème est classique, il n'est pas utile de le discuter ici.

d) Production de mésons : à partir de 150 Mev environ, il y a possibilité de produire des mésons π . Ceci est un phénomène qui par sa nature est beaucoup plus complexe qu'un effet purement électro-magnétique. Nous le discuterons en détail au chapitre suivant.

IV - PRODUCTION DE MESONS PAR DIFFUSION INELASTIQUE DES ELECTRONS DANS L'HYDROGENE

Parmi les phénomènes inélastiques qui accompagnent la diffusion des électrons, la production des mésons π occupe une place particulièrement importante, à cause des conséquences possibles sur la théorie des mésons. L'expérience la plus importante a été faite par Panofsky et ses collaborateurs (Panofsky, Newton et Yodh, Physical Review 98, 75I (I955); Panofsky, Woodword et Yodh, Physical Review, I02,392(I956); Yodh et Panofsky,Physical Review I05, 73I (I957)),qui ont étudié la réaction:

$$e + p \rightarrow e' + n + \pi^+ \qquad (I)$$

au moyen d'électrons d'énergie initiale de 600 Mev.

1.- Discussion des expériences et résultats expérimentaux.

La méthode a consisté à comparer directement l'électro - production et la photoproduction des mésons π . Le dispositif expérimental est représenté dans la Fig.32. La cible d'hydrogène liquide est bombardée soit par des électrons de 600 Mev., soit par des photons produits par Brehmsstrahlung sur une cible de carbone ou de cuivre. Si l'on fixe la quantité de mouvements des mésons π produits (dispositif expérimental) , on peut définir la quantité de mouvement. k des photons qui auraient pu produire ces mésons π . L'expérience mesure: a) le nombre de mésons Y_k produits par absorption dans l'hydrogène d'un photon d'impulsion k ;

b) le nombre Y_e de mésons produits par diffusion inélastique d'électrons d'impulsion initiale fixe p_1

Soit $\sigma(k)$ la section efficace différentielle de photoproduction, $N_k\, dk$ le nombre de photons incidents dont l'impulsion est comprise entre k et $k+dk$, et N_0 le nombre de photons par unité de surface de la cible. On a alors:

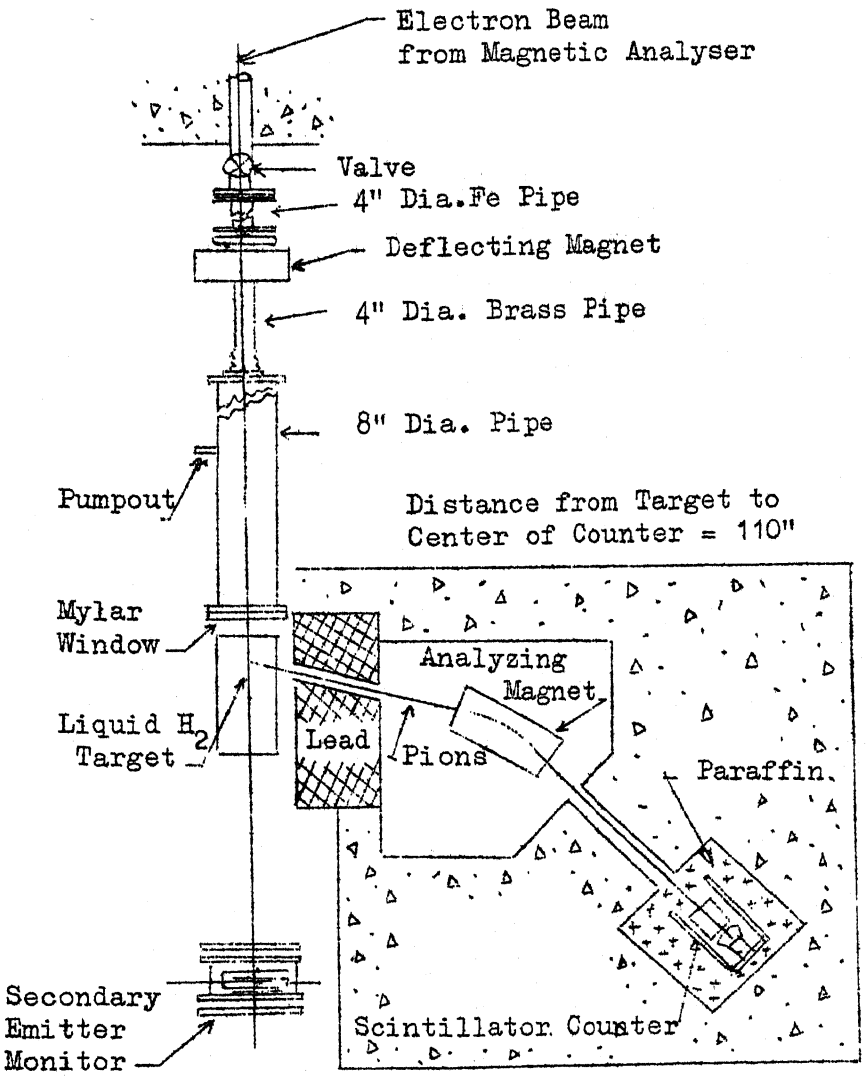


Fig. 32. Meson Detection Geometry

$$Y_k = N_0 \int k\, N_k\, \sigma(k) \frac{dk}{k} \qquad (2)$$

l'intégrale étant étendue à la largeur Δk de la bande d'énergie utile correspondant au dispositif expérimental. D'autre part, soit $\sigma_e (E_o, E)$ la section efficace de production d'un méson par un électron d'énergie initiale E_o et d'énergie finale E. On a :

$$Y_e = N_o \int \sigma_e (E_o, E)\, dE \qquad (3)$$

On peut définir également une "longueur de radiation équivalente" X_e qui est l'épaisseur d'un radiateur de photons telle que le nombre de mésons produits par ces photons soit égal au nombre de mésons produits directement par les électrons. Ou encore, on peut introduire le nombre N_e tel que $N_e \left(\frac{dk}{k}\right)$ est "le nombre de photons virtuels" compris entre k et $k + dk$:

$$Y_e = N_o \int N_e\, \sigma(k) \frac{dk}{k} \qquad (4)$$

N_e et X_e sont reliés par la relation :

$$X_e = \frac{N_e}{k\, N_k} \qquad (5)$$

Les résultats expérimentaux sont donnés dans le tableau ci-dessous où θ est l'angle des mésons π avec les électrons incidents dans le système du laboratoire, et T_π l'énergie cinétique des mésons π (E_o = 600 Mev).

θ	T_π	N_e
75°	60 Mev	0,0202 ± 0,0007
75°	147 Mev	0,0177 ± 0,0007
75°	170 Mev	0,0145 ± 0,0020
135°	70 Mev	0,0178 ± 0,0007

2.- Détermination théorique du rapport des sections efficaces de photo - et d'électro-production des mésons π .

Le nombre N_e de photons virtuels équivalents (défini par les équations (4) et (5) peut être déterminé avec une bonne approximation par une extension de la méthode de Weizsäcker- Williams due à Dalitz et Yennie (Phys.Rev. .. Cette détermination est indépendante de l'interaction entre les mésons et les nucléons.. Cette dernière n'intervient que dans de petites corrections que nous discuterons au paragraphe suivant.

L'interaction entre le champ électromagnétique et le système méson-nucléon peut, pour un transfert d'impulsion-énergie $(\vec{k}, k_o)$ s'écrira:

$$H'(\vec{k}) = J_\mu(k_o, \vec{k})\, A_\mu(k_o, \vec{k}) \qquad (6)$$

où J_μ est l'élément de matrice de l'opérateur courant entre un état initial correspondant au nucléon seul et un état final décrivant le système méson plus nucléon. $A_\mu(k_o, \vec{k})$ est la composante de Fourier du potentiel de Møller:

$$A_\mu(k_o, \vec{k}) = e\, \frac{\bar{u}(\vec{p} - \vec{k})\, \gamma_\mu\, u(\vec{p})}{k_o^2 - k^2} \qquad (7)$$

où $k_o = \varepsilon(\vec{p}) - \varepsilon(\vec{p} - \vec{k})$,avec $\varepsilon(\vec{p}) = (p^2 + M^2)^{1/2}$,

et $u(\vec{p})$ est le spineur de Dirac habituel.

Le courant J_μ peut se décomposer en une composante transversale et une composante longitudinale en tenant compte de la relation de conservation.

$$k_\mu J_\mu = k_o J_o - \vec{k} \cdot \vec{J} = 0 \qquad (8)$$

et de la condition de Lorentz satisfaite par A_μ

$$k_\mu A_\mu = \frac{e\,\bar{u}(\vec{p}-\vec{k})\,\gamma_\mu\, k_\mu\, u(\vec{p})}{k_0^2 - k^2} \qquad (9)$$

$$= \frac{+ie\,\bar{u}(\vec{p}-\vec{k})\left[i\gamma(p-k)+m-i\gamma p-m\right]u(\vec{p})}{k_0^2-k^2} = E$$

compte tenu de ce que $u(p)$ et $\bar{u}(\vec{p}-\vec{K})$ sont solutions de l'équation de Dirac. Cette dernière relation s'écrit aussi:

$$k_0 A_0 - \vec{k}.\vec{A} = 0 \qquad (10)$$

Par suite, H' défini par (6) peut s'écrire:

$$H' = \vec{j}.\vec{A} - \frac{1}{k_0^2}(\vec{k}.\vec{j})(\vec{k}.\vec{A}) \qquad (11)$$

Soit J_ℓ la composante longitudinale

$$\vec{J}_\ell = \vec{k}\left(\frac{\vec{k}.\vec{J}}{k^2}\right) \qquad (12)$$

et J_t la composante transversale

$$\vec{J}_t = \vec{J} - \vec{J}_\ell \qquad (13)$$

on peut écrire finalement:

$$H' = (\vec{m}_t + \vec{m}_\ell)\,\vec{A} \qquad (14)$$

avec:

$$\vec{m}_t = \vec{j}_t \qquad \text{et} \qquad \vec{m}_\ell = \vec{j}_\ell\left(1 - \frac{k^2}{k_0^2}\right) \qquad (15)$$

Dans le cas où le nucléon est considéré comme infiniment lourd le nombre $N_\ell(\vec{p}, \vec{k}_f)$ (avec $\vec{k}_f = \vec{p} - \vec{p}'$, $\vec{p}$ et $\vec{p}'$ étant les impulsions initiale et finale de l'électron) peut s'écrire sous la forme:

$$N_e = N_e^{(t)} + N_e^{(l)} \tag{16}$$

où la composante transversale est égale à:

$$N_e^{(t)}(p, k_f) = \frac{\alpha}{\pi} \int_{(p-p')^2}^{(p+p')^2} \frac{k_f^2 \, d(k^2)}{(k_0^2 - k^2)^2} \left\{ \frac{\left[(p+p')^2 - k^2\right]\left[k^2 - (p-p')^2\right]}{4p^2 k^2} + \frac{k^2 - k_0^2}{2p^2} \right\} \frac{\langle J_t^2(k^2)\rangle}{\langle J_t^2(k_f^2)\rangle} \tag{17}$$

tandis que la composante longitudinale est donnée par la formule

$$N_e^{(l)}(p, k_f) = \frac{\alpha}{\pi} \int_{(p-p')^2}^{(p+p')^2} \frac{k_f^2 \, dk^2}{4k_0^2 p^2} \left\{ \frac{(p+p')^2}{k^2} - 1 \right\} \frac{\langle J_l^2(k^2)\rangle}{\langle J_t^2(k_f^2)\rangle} \tag{18}$$

Il faut remarquer que dans la photoproduction, seul $J_t(k_f^2)$ (composante transversale sur la courbe d'énergie) intervient. La valeur la plus simple de N_e (dite: valeur standard) s'obtient en faisant $\langle J_t^2(k^2)\rangle = \langle J_t^2(k_f^2)\rangle$ dans l'équation(I7) et en négligeant $N_e^{(l)}$. Ceci donne:

$$N_e^{(st.)}(p, p') = \frac{\alpha}{\pi} \left\{ \frac{\varepsilon^2 + \varepsilon'^2}{p^2} \log\left[\frac{\varepsilon\varepsilon' + pp' + m^2}{m(\varepsilon - \varepsilon')}\right] - \frac{(\varepsilon+\varepsilon')^2}{2p^2} \log\frac{p+p'}{p-p'} - \frac{p'}{p} \right\} \tag{19}$$

Les contributions à l'équation (19) proviennent surtout des petits angles. Par exemple, pour $p = 600$ Mev/c et $k_f = 200$ Mev., 95 % de l'intégrale provient d'angles de scattering inférieurs à 6°. Il convient de remarquer enfin que la valeur standard contribue encore 90% de l'effet observé, les corrections mésiques n'étant jamais supérieur à I0%.

3.- Corrections dues à l'interaction méson-nucléon.

Dalitz et Yennie ont calculé les corrections mésiques dans le cas de la théorie phénoménologique de Brueckner et Watson (Phys.Rev.86, 923 (I952)) et de Gell-Mann et Watson (Ann.Rev.Nucl.Sci.,4, 2I9 (I952)),

ainsi que dans le cas de la théorie de Chew et Low (Phys. Rev. 101, 1597 (1956)). La méthode dans les deux cas consiste à partir d'une forme particulière du courant $\vec{J}$ défini au paragraphe précédent. Dans le cas de la théorie de Chew et Low par exemple, le courant $\vec{J}$ est décomposé en une partie qui provient principalement de la diffusion S :

$$\vec{J}_0 = \frac{e\sqrt{2}g}{2M}\left[\vec{\sigma} - (2\vec{q}-\vec{k})\frac{\vec{\sigma}.(\vec{q}-\vec{k})}{\mu^2+(q-k)^2}\right] \qquad (20)$$

(avec $q = p-p'$), et une partie qui provient de la modification du courant due à la résonance dans l'état $P_{3/2}$:

$$\vec{J}_1 = \frac{e g \sqrt{2}}{2M} D_0 \left(2i\vec{q}\times\vec{k} + (\vec{\sigma}.\vec{k})\vec{q} - \vec{\sigma}(\vec{q}.\vec{k})\right)$$

$$\times \mu \frac{e^{i\alpha_{33}} \sin\alpha_{33}}{q^3} F(k^2) \qquad (21)$$

où $D_0 = \frac{1}{3}\left(\frac{\mu}{4m}\right)\left(\frac{4\pi}{f^2}\right)(g_p - g_n) = 0,8 \pm 0,1$ (22)

g_p et g_n étant les rapports giromagnétiques du proton et du neutron et $\frac{f^2}{4\pi}$ la constante de couplage de Chew et Low. $F(k^2)$ est le facteur de forme du moment magnétique nucléaire (qui est égal à 1 dans le présent calcul.

La Figure (33) représente les résultats du calcul à 600 Mev dans le cas des deux théories en même temps que les points expérimentaux pour les deux angles de production 75° et 135°. Il convient de remarquer qu'un test beaucoup plus sensible de la théorie consisterait à mesurer le spectre de diffusion inélastique des électrons, qui est, comme l'ont montré Dalitz et Yennie, très sensible au mode particulier d'intéraction méson-nucléon que l'on suppose. Il serait souhaitable qu'une semblable expérience soit faite dans un proche avenir.

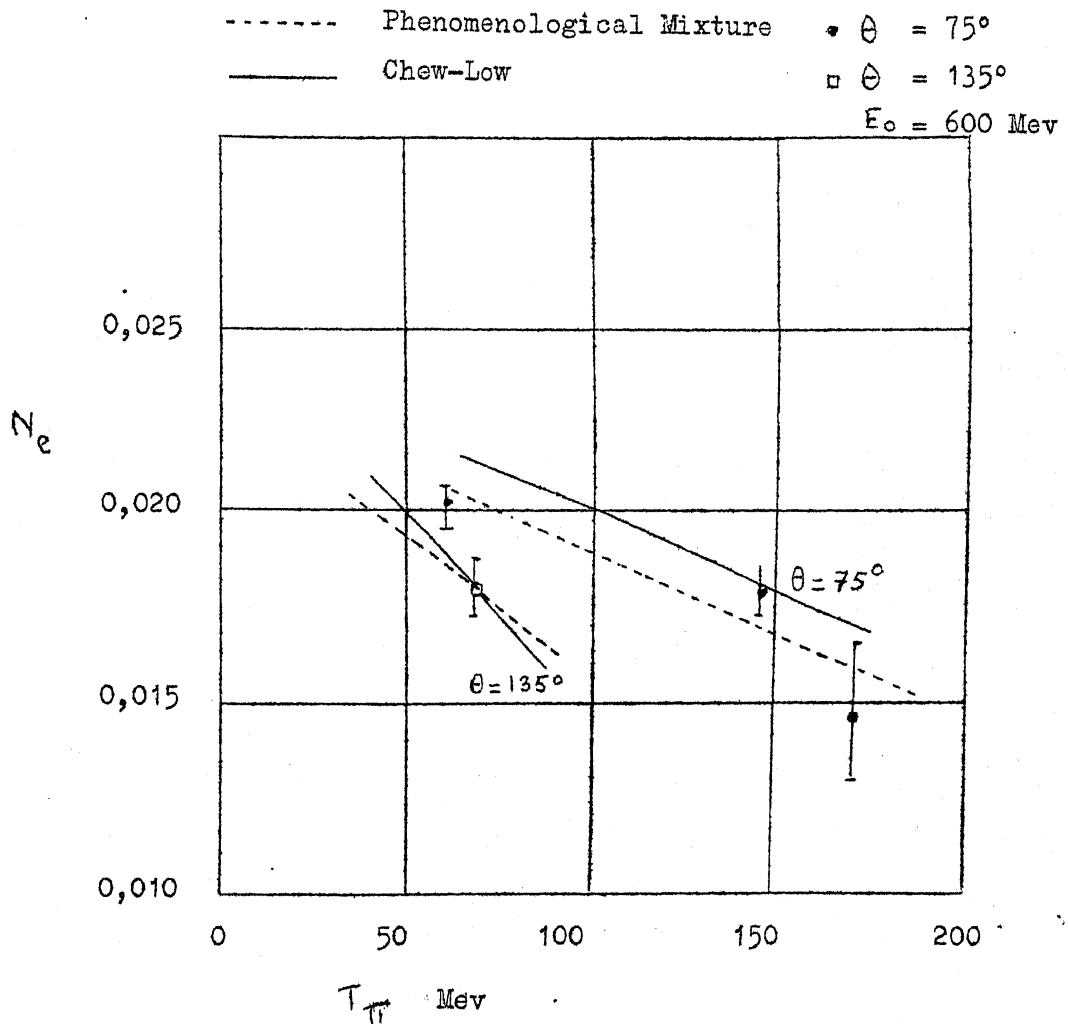

Phenomenological Mixture
Chew-Low
θ = 75°
θ = 135°
E₀ = 600 Mev
0,025
0,020
0,015
0,010
N_e
θ=75°
θ=135°
0
50
100
150
200
T_π Mev

Fig. 33

APPENDICE

Diffusion élastique - Méthode des ondes partielles

par

J. REIGNIER

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1 - INFLUENCE DU VOLUME FINI DU NOYAU SUR L'AMPLITUDE DE DIFFUSION

Nous considérons la diffusion d'électrons de haute énergie ($E \gg 1$) (*) par une répartition statique d'électricité à symétrie sphérique représentant un noyau. Ce modèle convient pour les noyaux moyens et lourds pas trop déformés, et aux énergies inférieures à environ 200 MeV. L'analyse en ondes partielles montre que la section efficace différentielle de diffusion peut s'écrire (1) (2)

$$\frac{d\sigma}{d\Omega} = \frac{1}{\cos^2 \frac{\vartheta}{2}} |f(\vartheta)|^2, \qquad (1)$$

où l'amplitude de diffusion $f(\vartheta)$ est donnée par

$$f(\vartheta) = \frac{1}{2iE} \sum_{j+\frac{1}{2}=1}^{\infty} (j+\tfrac{1}{2}) e^{2i\eta_j} \left\{ P_{j-\frac{1}{2}}(\cos\vartheta) + P_{j+\frac{1}{2}}(\cos\vartheta) \right\}; \qquad (2)$$

E est l'énergie de l'électron, $P_n(x)$ est le polynôme de Legendre d'ordre n en la variable x et η_j est le déphasage de l'onde partielle de moment angulaire total j.

(*) Les unités sont $\hbar = 1$, $C = 1$, $m_e = 1$.

(1) D.R. Yennie, D.G. Ravenhall, R.N. Wilson P.R. 95 (1954) 500

(2) N.F. Mott and H.S.W. Massey "The Theory of Atomic Collisions" Oxford (1952) IV § 4. Dans les notations de Mott et Massey, l'approximation de haute énergie $E \gg 1$ entraine $\eta_n = \eta_{-n-1}$.

Le potentiel est coulombien à l'infini et les solutions $\phi_{1,j}(r)$ et $\phi_{2,j}(r)$ de l'équation radiale de Dirac

$$\left(\frac{d}{dr} - \frac{j+1/2}{r}\right)\phi_{1,j} = -(E-V)\phi_{2,j} \qquad (3)$$

$$\left(\frac{d}{dr} - \frac{j+1/2}{r}\right)\phi_{2,j} = (E-V)\phi_{1,j}$$

($V(r)$ est l'énergie potentielle) ont pour forme asymptotique

$$\phi_{1,j}(r) \sim \sin\left\{Er + a\log 2Er - (j-\tfrac{1}{2})\tfrac{\pi}{2} + \eta_j\right\} \qquad (4)$$

$$\phi_{2,j}(r) \sim \cos\left\{Er + a\log 2Er - (j-\tfrac{1}{2})\tfrac{\pi}{2} + \eta_j\right\}$$

($a = Z/137$, où Z est le nombre atomique du noyau diffuseur). Les déphasages η_j dépendent de la répartition considérée. Une difficulté propre au potentiel coulombien est que les déphasages ne tendent pas vers une limite finie quand j augmente ; ils se comportent sensiblement comme - a log j. Par contre, si nous désignons par η_j^R le déphasage de l'onde partielle j dans le cas d'un noyau ponctuel (*), les déphasages supplémentaires

$$\delta_j = \eta_j - \eta_j^R \qquad (5)$$

dus au volume fini du noyau tendent rapidement vers zéro quand j augmente. On écrit généralement (2) sous la forme

$$f(\vartheta) = f_p(\vartheta) + \frac{1}{2iE}\sum_{(j+\frac{1}{2})=1}(j+\tfrac{1}{2})e^{2i\eta_j^R}\left(e^{2i\delta_j}-1\right)\left\{P_{j-\frac{1}{2}}(\cos\vartheta) + P_{j+\frac{1}{2}}(\cos\vartheta)\right\} \qquad (6)$$

$f_p(\vartheta)$ est l'amplitude de diffusion par un noyau ponctuel ; elle nécessite des précautions spéciales pour être nommée [1] mais elle

---

(*) L'indice R rappelle qu'il s'agit du déphasage de la solution régulière à l'origine ($\varphi_j^R(r)$)

est la même pour tous les modèles des noyaux de charge Ze et ne doit être nommée qu'une fois. Au contraire, la seconde partie de $f(\vartheta)$ dépend du modèle considéré mais converge assez rapidement ; le nombre de termes nécessaires dans la série croit à peu près linéairement avec l'énergie ; avec la précision actuelle, il faut par exemple 10 termes dans le cas de la diffusion d'électrons de 183 MeV par l'or[3].

2 - CALCUL DES DEPHASAGES

En tout point r_1 extérieur au noyau $\left(V(r_1) = -\alpha/r_1 \right)$ les solutions $\phi_{1,j}(r)$ et $\phi_{2,j}(r)$ du problème considéré sont une combinaison linéaire des solutions régulières $\left(\varphi^R_{1,j}(r) \text{ et } \varphi^R_{2,j}(r)\right)$ et irrégulières $\left(\varphi^I_{1,j}(r) \text{ et } \varphi^I_{2,j}(r)\right)$ à l'origine du problème à noyau ponctuel $V(r) = -\alpha/r$ de 0 à ∞) :

$$\phi_{1,j}(r_1) = C_j\, \varphi^R_{1,j}(r_1) + D_j\, \varphi^I_{1,j}(r_1) \qquad (7)$$

$$\phi_{2,j}(r_1) = C_j\, \varphi^R_{2,j}(r_1) + D_j\, \varphi^I_{2,j}(r_1)$$

Remplaçons dans (7) les fonctions par leur forme asymptotique (4) avec les déphasages correspondants (η_j , η^R_j , η^I_j) ; nous obtenons la relation

$$\operatorname{tg} \delta_j = \frac{\sin(\eta^I_j - \eta^R_j)}{C_j/D_j + \cos(\eta^I_j - \eta^R_j)} \qquad (8)$$

Introduisons maintenant dans (7) la fonction

$$\gamma_j(r) = -\frac{\phi_{2,j}(r)}{\phi_{1,j}(r)} \quad ; \qquad (9)$$

nous obtenons C_j/D_j sous la forme d'une fonction homographique de

(3) B. Hahn, D.G. Ravenhall, R. Hofstadter P.R. 101 (1956) 1131 / 1.

$$\frac{C_j}{D_j} = - \frac{\varphi^I_{1,j}(r_1)\,\gamma_j(r_1) + \varphi^I_{2,j}(r_1)}{\varphi^R_{1,j}(r_1)\,\gamma_j(r_1) + \varphi^R_{2,j}(r_1)} \qquad (10)$$

Les formules (8) et (10) montrent que le déphasage supplémentaire de l'onde partielle j dû au volume fini du noyau ne dépend de la répartition de la charge électrique dans le noyau que par la valeur de la fonction $\gamma_j(r)$ à l'extérieur du noyau ; les autres grandeurs intervenant dans le calcul ne dépendent du noyau que par sa charge totale Ze. Notre problème se ramène donc au calcul de la valeur en un point extérieur au noyau des fonctions $\gamma^*_j(r)$ des ondes partielles sensiblement affectées par l'extension finie du noyau.

3 - CALCUL DE $\gamma_j(r)$.- DISCUSSION

La plupart des auteurs arrêtent l'analyse à ce stade ; ils résolvent les équations radiales de Dirac et calculent $\gamma^*_j(r)$ d'après la définition (9)[4]. Sauf dans les cas les plus simples comme par exemple la répartition en surface ou la répartition homogène, les équations radiales de Dirac sont intégrées numériquement. On obtient ainsi des résultats très précis mais dont la dépendance en le potentiel est totalement masquée. On peut faire apparaitre cette dépendance en déterminant $\gamma^*_j(r)$ analytiquement. Diverses méthodes ont été proposées [5][6][7]. En particulier, nous avons montré [7b] qu'on

---

(4) On trouvera les références à ces travaux dans les articles suivants :
K.W. Ford and D.L. Hill Ann. Rev. Nuc. Sc. 5 (1955) 25
R. Hofstadter R.M.P. 28 (1956) 214

(5) A.R. Bodmer P.P.S. 66A (1953) 1041

(6) G.N. Fowler P.P.S. 68A (1955) 559

(7) J. Reignier a) Bull. Acad. Roy. Belg. Cl. Sc. 41 (1955) 151
b) " " " " " " 42 (1956) 173

Des résultats plus complets seront présentés prochainement comme thèse de doctorat à la Faculté des Sciences de l'Université Libre de Bruxelles.

peut obtenir une bonne approximation de $\gamma_j(+)$ tout en ne calculant qu'une seule intégrale dépendant de la répartition de la charge électrique dans le noyau. Cette solution approchée s'écrit :

$$\gamma_j(+) = -\frac{J_{j+1}[U(K,+)+]}{J_j[U(K,+)+]} \quad (11)$$

($J_\nu(x)$ est la fonction de Bessel d'indice ν en la variable x.

avec

$$U(K,+) = E - \frac{\int_0^+ V(+')+'\left[J_j^2(K+') + J_{j+1}^2(K+')\right]d+'}{\int_0^+ +'\left[J_j^2(K+') + J_{j+1}^2(K+')\right]d+'} \quad (12)$$

$$E-V(+) \leq K \leq E-V(0).$$

L'approximation est d'autant meilleure que la variation d'énergie potentielle du centre du noyau au point r est plus faible. Dans le cas de l'or et d'une répartition homogène de la charge par exemple, cette solution approchée permet de calculer les déphasages δ_j avec une précision de l'ordre de 10^{-3} radian.

Il en résulte que le déphasage de l'onde partielle j ne dépend du potentiel dans le noyau que par

$$\int_0^{+_1} V(+)+\left[J_j^2(K+) + J_{j+1}^2(K+)\right]d+ \quad (13)$$

où r_1 est un point extérieur au noyau. En intégrant deux fois par parties, en utilisant l'équation au potentiel et le fait que le potentiel est coulombien en r_1, on obtient que ce déphasage ne dépend de la densité $\rho(+)$ de la charge électrique dans le noyau que par [8]

$$\int_0^\infty \rho(+)\, J_j(K+)+^2\, d+ \quad (14)$$

(8) J. Reignier Nuc. Ph. 3 (1957) 340

avec

$$\mathcal{J}_j(x) \equiv \int_0^x \left\{ J_j^2(x') + J_{j+1}^2(x') - \frac{2j+1}{x'} J_j(x') J_{j+1}(x') \right\} dx' \quad (15)$$

ou encore, par $\langle \mathcal{J}_j(K\tau) \rangle$ en désignant ainsi la valeur moyenne de $\mathcal{J}_j(K\tau)$ prise sur le noyau avec la fonction de poids $\rho(\tau)$.

A basse énergie, où une ou deux ondes partielles seulement sont affectées par l'extension finie du noyau, des mesures précises de section efficace de diffusion doivent permettre de déterminer $\langle \mathcal{J}_{1/2}(K\tau) \rangle$ et $\langle \mathcal{J}_{3/2}(K\tau) \rangle$ qui se réduisent d'ailleurs approximativement dans ce cas à $\langle \tau^2 \rangle$ et $\langle \tau^4 \rangle$. Aux énergies plus élevées, l'analyse se complique par le fait que des ondes partielles de plus en plus nombreuses sont affectées par l'extension finie du noyau. Toutefois, de par la forme des fonctions $\mathcal{J}_j(K\tau)$, il est apparent que les valeurs superficielles de la densité sont toujours plus importantes que les valeurs centrales. De ce fait, la diffusion aux énergies élevées peut donner une image en principe de plus en plus détaillée de la densité aux valeurs pas trop petites de r et en particulier à la surface du noyau mais ne peut pas préciser le centre de la distribution d'électricité. Il convient d'ailleurs de rappeler que le modèle statique utilisé cesse probablement d'être valable au delà d'environ 200 Mev.

TABLE DES MATIERES

PION - NUCLEON INTERACTIONS

J.C. POLKINGHORNE
University of Edinburgh

Université de Grenoble

Cours professé à
l'ECOLE D'ETE DE PHYSIQUE THEORIQUE
Les Houches (Haute-Savoie), France

Juillet 1957

J.C. POLKINGHORNE

PION-NUCLEON INTERACTION

C O R R I G E N D A

Page		
3	line 7	Replace Karplus by Kroll.
	eq. 2.4	" ω_k by $\sqrt{\omega}_k$
9	eq. 2.25	" $F_{qp}(z)$ by $F_{qp}(x)$
		" $G_{qp}(z)$ by $G_{qp}(x)$
11	eq. 2.31	Add factors k k' to right hand side.
	eq. 2.33	Replace k' by k.
12	Delete the last line.	
14	line 14	Replace $\lambda\alpha(\omega)$ by $\lambda\alpha/\omega$
15	eq. 2.48	Add - sign to right hand side.
19	eqs. 2.58, 2.59	Add factor 1/2.
28	eq. 3.20	Should read : $A(-\omega) = A(\omega)$.
30	section 3-6 line 3	Δ is missing.
31	eq. 3.30	Replace by $\omega_a = \left(M_\mu - \frac{\Delta^2}{4}\right) \Big/ \sqrt{M^2 + \frac{\Delta^2}{4}}$
31	line 14	Replace by : A partial wave decomposition in the centre of mass system, and continuing $P_n(\cos\theta_c)$ to $\cos\theta_c < -1$

CHAPTER 1

INTRODUCTION

1.1 Properties of Pions

The properties that are important for our purpose are :

(i) three charge states π^+, π^0, π^-

(ii) $m(\pi^+) = m(\pi^-) = 273.3 \pm .2$ electron masses

$m(\pi^-) - m(\pi^0) = 8.8 \pm .6$ electron masses

(iii) Spin of the charged π's is determined from detailed balancing arguments applied to

$$p + p \longleftrightarrow D + \pi^+$$

This gives S = 0. The π^0 also has spin 0 since it decays into 2γ, which is impossible for spin 1.

(iv) Parity is determined from theobservation that the process

$$\pi^- + D \rightarrow 2N$$

is observed. This gives π^- negative parity.

1.2 Charge Symmetrical Theory and Isobaric Spin.

The three pions are thought of as belonging to an isobaric triplet, $\underset{\sim}{\pi}$, of isospin 1 and the proton and neutron as belonging to an isobaric doublet, ψ, of isopin 1/2. Kemmer proposed an interaction

$$g \bar{\psi} \underset{\sim}{\tau} \psi \cdot \underset{\sim}{\pi} \qquad (1.1)$$

which gives the observed symmetry of the nuclear forces.

If φ creates π^+ or destroys π^-

$\varphi^+ \ldots \quad \pi^- \ldots \quad \pi^+$,

then

$$\varphi_1 = \frac{\varphi + \varphi^+}{\sqrt{2}}, \quad \varphi_2 = \frac{\varphi - \varphi^+}{\sqrt{2}\, i}$$

and the interaction (1.1) may be written as

$$\sqrt{2}\, g\, \tau_- \varphi + \sqrt{2}\, g\, \tau_+ \varphi^+ + g\, \tau_3 \varphi_3 \qquad (1.2)$$

This Hamiltonian makes the total isobaric spin, I, a good quantum numbe in pion-nucleon reactions.

CHAPTER II

CHEW - LOW THEORY - (Chew & Low, P.R. 101, 1956, 1570, 1579)

2. 1. Introduction

The nucleon is assumed to be infinitely heavy and so is described completely by Q and σ. Spin-flip and charge exchange are the only possible processes for the nucleon. Its interaction with the π is assumed to be a Yukawa-type interaction, i.e. mesons are created or annihilited singly.

It is perhaps surprising that so crude a theory proves useful. The explanation lies probably in the Low Energy Limit theorems(Thirring, Phil. Mag. 41, 1193; Karplus and Ruderman, P.R.93, 233; Klein, P.R. 99, 998). Those state (roughly speaking) that ' all theories coincide at sufficiently low energies.' Essentially this is a manifestation of the invariance properties of the theory. Of course we only have access to energies $> \mu$ but since $\mu/M \sim 1/7$ there is hope of a region of validity.

The Hamiltonian of the theory is

$$H = H_0 + H_1 \,, \tag{2.1}$$

where

$$H_0 = \sum_k \omega_k \, a_k^+ \, a_k \,,$$

$$\left[a_k \,, a_{k'}^+ \right] = \delta_{k k'} \,, \tag{2.2}$$

$$\omega_k = \sqrt{\mu^2 + k^2} \,,$$

is the free Hamiltonian and

$$H_1 = \sum_k V(k) \, a_k + V^+(k) \, a_k^+ \,, \tag{2.3}$$

$$V(k) = \frac{i f_0}{\sqrt{2} \mu \omega_k} \, \vec{\sigma} \cdot \vec{k} \, \epsilon_k \, v(k) \,, \tag{2.4}$$

is the interaction Hamiltonian. $V(k)$ describes the "spread" of the nucleon and provides a cut-off for the theory. Eq. (2.4) gives the only Yukawa-type invariant for this theory involving pseudoscalar pions. It gives an interaction only for ℓ-wave pions. This follows from the conservation of angular momentum and parity:

$$\underset{\sim}{J} = \underset{\sim}{\ell} + \underset{\sim}{1/2} = \underset{\sim}{1/2}$$

final a.m. after π is emitted — initial a.m. of nucleon alone,

giving $\ell \neq 0$, . But the parity of the initial state is $+$ and that of the final is $-(-)^{\ell}$ and so $\ell = 1$.

2.2. SCATTERING THEORY

Lippman & Schwinger, P.R. 79. 469;

Gell-Mann & Goldberger, P.R. 91, 398;

Wick, R.M.P., 27, 339.

Consider the Schrodinger equation

$$(H - E)\,|\Psi\rangle = 0 \,, \tag{2.5.}$$

and assume the existence of the real nuclear solutions $|\Psi_0\rangle$ satisfying

$$H\,|\Psi\rangle = 0 \,. \tag{2.6.}$$

We wish to find the solutions $|\Psi^{(+)}_{k}\rangle$ corresponding to nucleon $+$ outgoing meson. They satisfy

$$(H - E_k)\,|\Psi^{(+)}_{k}\rangle = 0 \,, \tag{2.7.}$$

with

$$E_k = \sqrt{\mu^2 + k^2} = \omega_k \tag{2.8.}$$

Write

$$|\Psi_k^{(+)}\rangle = a_k^+ |\Psi_0\rangle + |\chi_k^{(+)}\rangle \qquad (2.9)$$

Since

$$[H_0, a_k^+] = \omega_k a_k^+ ,$$

$$[H_1, a_k^+] = V_k ;$$

$$(H - E_k)|\chi_k^{(+)}\rangle = -(H - E_k) a_k^+ |\Psi_0\rangle$$

$$= - V_k |\Psi_0\rangle \qquad (2.10)$$

This gives

$$|\chi_k^{(+)}\rangle = \frac{1}{H - E_k - i\varepsilon} \cdot V_k |\Psi_0\rangle , \qquad (2.11)$$

outgoing waves

or, introducing a sum over a complete set of states,

$$|\chi_k^{(+)}\rangle = \sum_n \frac{-1}{E_n - E_k - i\varepsilon} |\Psi_n^{(-)}\rangle \langle \Psi_n^{(-)} | V_k | \Psi_0 \rangle$$

$$= \sum_n \frac{-1}{E_n - E_k - i\varepsilon} |\Psi_n^{(-)}\rangle T_k(n) , \qquad (2.12)$$

where

$$T_k(n) = \langle \Psi_n^{(-)} | V_{(k)} | \Psi_0 \rangle \qquad (2.13)$$

$T_k(n)$ must now be related to the S matrix elements for π-N scattering. These are given by

$$\langle k' | S | k \rangle = \langle \Psi_{k'}^{(-)} | \Psi_k^{(+)} \rangle . \qquad (2.14)$$

Now $$|\Psi_k^{(+)}\rangle - |\Psi_k^{(-)}\rangle = \sum_n \left[\frac{1}{E_n - E_k + i\varepsilon} - \frac{1}{E_n - E_k - i\varepsilon} \right] |\Psi_n^{(-)}\rangle T_k(n)$$

$$= -2\pi i \sum_n \delta(E_n - E_k) |\Psi_n^{(-)}\rangle T_k(n) \qquad (2.15)$$

where the symbolic identity

$$\frac{1}{a \pm i\varepsilon} = P\frac{1}{a} \mp i\pi\delta(a) \qquad (2.16)$$

has been used. Substitution of (2.15) into (2.14) gives

$$\langle k'|S|k\rangle = \delta_{k'k} - 2\pi i\,\delta(E_{k'}-E_k)\,T_k(k') \qquad (2.17)$$

$T_k(n)$ resembles Møller's wave matrix for the static theory considered here it has one important property, not possessed by the wave matrix, which proves of the greatest use in what follows. This is its trivial dependence on k (see eqns. (2.4) and (2.13)). This enables us when using $T_k(n)$ to move on and off the energy shell at will.

2.3 The Low Equation.

cf. Low, P.R. 97, 1932.

We shall now derive an equation satisfied by $T_k(k')$, which is the celebrated Low equation for the static theory.

$$\begin{aligned} T_k(k') &= \langle \Psi^{(-)}_{k'} | V(k) | \Psi_0 \rangle \\ &= \langle \Psi_0 | a_{k'}\, V(k) | \Psi_0 \rangle \\ &\quad - \langle \Psi_0 | V^{(+)}(k') \frac{1}{H-\omega_{k'}+i\varepsilon} V_k | \Psi_0 \rangle . \end{aligned} \qquad (2.18)$$

Now

$$\left[a_{k'}, H \right] = \omega_{k'} a_{k'} + V^{+}(k') ,$$

and so

$$-H a_{k'}|\Psi_0\rangle = \omega_{k'} a_{k'} |\Psi_0\rangle + V^{+}(k')|\Psi_0\rangle ,$$

Since

$$H|\Psi_0\rangle = 0$$

Therefore

$$(H+\omega_{k'})\, a_{k'}|\Psi_0\rangle = -V^+_{k'}|\Psi_0\rangle ,$$

or

$$a_{k'}|\Psi_0\rangle = \frac{-1}{H+\omega_{k'}} V^+_{k'}|\Psi_0\rangle , \qquad (2.19)$$

the inverse operator being uniquely defined since $H+\omega_{k'}$ is positive definite. Eqns. (2.18) and (2.19) yield

$$T_k(k') = -\langle\Psi_0|V(k)\frac{1}{H+\omega_{k'}}V^+(k')|\Psi_0\rangle$$
$$-\langle\Psi_0|V^+(k')\frac{1}{H-\omega_{k'}-i\varepsilon}V(k)|\Psi_0\rangle ,$$

which on introducing a sum $\sum_n |\Psi^{(-)}_n\rangle\langle\Psi^{(-)}_n|$ becomes

$$T_k(k') = -\sum_n \left\{ \frac{T^+_{k'}(n)\, T_k(n)}{E_n-\omega_{k'}-i\varepsilon} + \frac{T^+_k(n)\, T_{k'}(n)}{E_n+\omega_{k'}} \right\} \qquad (2.20)$$

N.B. This is a matrix equation in nucleon spin and isospin spaces.

The Low equation (2.20) has the following properties:

(i) Unitarity

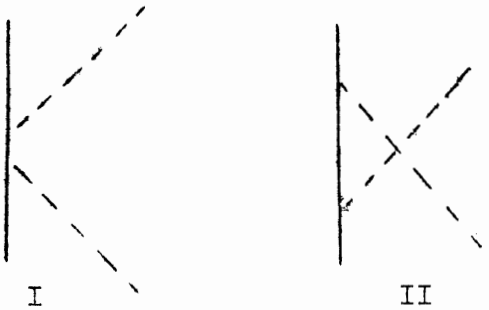
From (2.17) the unitarity of the S matrix implies and is implied by

$$T^+_{k'}(k) - T_k(k') = 2\pi i \sum_n \delta(E_n-\omega)\, T^+_{k'}(n)\, T_k(n) , \qquad (2.21)$$

with $\omega_{k'} - \omega_k = \omega$.

Eqn. (2.21) follows immediately from (2.20).

(iii) Crossing Symmetry



I II

This important symmetry possessed by all known field theories was fully noticed and recently by Gell-Mann and Goldberger. It is an expression of the symmetry inherent in field theories between the emission of a particle and the absorption of an anti-particle. It relates the contribution of graphs I and II.

In the present not completely relativistic theory the crossing may be most readily expressed by first defining a function of a complex variable Z given by

$$t_{k'k}(Z) = -\sum_n \left\{ \frac{T^+_{k'}(n)\, T_k(n)}{E_n - Z} + \frac{T^+_k(n)\, T_{k'}(n)}{E_n + Z} \right\} \tag{2.22}$$

(Then $T_k(k')$ is given by $\lim_{Z \to \omega_{k'} + i\varepsilon} t_{k'k}(Z)$).

The function $t_{k'k}(Z)$ has two important properties:

(a) a reality condition

$$t_{k'k}(Z) = t^+_{k\,k'}(Z^*) , \tag{2.23}$$

(b) the crossing theorem

$$t_{k'k}(Z) = t_{k\,k'}(-Z) . \tag{2.24}$$

(iii) Energy Spectrum

From physical reasons we know that the energy spectrum E_n has a singularity at $E_n = 0$ (the nucleon) and a continuous range $E_n \geq \mu$ (the nucleon + meson(s)).

The singularity at $E_k = 0$ gives a pole at $Z = 0$ with residue

$$T^+_{k'}(0)\, T_k(0) - T^+_k(0)\, T_{k'}(0)$$

Removing the trivial $|k|$ dependence,

$$T_k(0) = \langle \Psi_0 |\, \sigma_k\, \sigma_k \,| \Psi_0 \rangle = Z \langle u |\, \sigma_k\, \sigma_k \,| u \rangle,$$

where Z is a numerical factor and $|u\rangle$ is a free nucleon spinor. This follows from invariance arguments. If we write a renormalized coupling constant

$$f = Z f_0$$

the behaviour of $t_{k'k}(Z)$ in the neighbourhood of $Z = 0$ is given by f and the free nucleon matrix elements of the Born approximation. (This is the low energy limit theorem. Our dealings with the rest of the Low equation are really aimed at finding an extrapolation procedure from $Z \geq \mu$ to $Z = 0$ in order to use this fact).

The one-meson threshold at $Z = \mu$ implies a branch point there, similarly at $Z = 2\mu$ for the two-meson threshold, etc. Thus, we must have a cut from μ to ∞ and, by crossing symmetry, from $-\mu$ to $-\infty$.

2.4 Solution of the Low Equation.

It is rather remarkable that the three properties discussed above completely specify the Low equation in the sense that any function possessing these properties will provide a solution of (2.20). We shall show this initially only for $\omega_k = \omega_{k'}$, but the trivial k-dependance of $T_k(k')$ will enable us to remove that restriction.

The energy spectrum properties (iii) enable us to write $t_{gp}(Z)$ in the form

$$t_{gp}(Z) = \frac{R_{gp}}{Z} + \int_{\mu}^{\infty} \left\{ \frac{F_{gp}(x)}{x - Z} + \frac{G_{gp}(x)}{x + Z} \right\} dx \qquad (2.25)$$

where $R_{gp} = f$. (Born Approximation). Eq. (2.25) implies that

$$2\pi i F_{gp}(x) = \left(\lim_{Z \to x+i\varepsilon} - \lim_{Z \to x-i\varepsilon} \right) t_{gp}(Z) , \quad (2.26)$$

$$x \geq 1 ,$$

$$2\pi i G_{gp}(x) = \left(\lim_{Z \to -x-i\varepsilon} - \lim_{Z \to -x+i\varepsilon} \right) t_{gp}(Z) . \quad (2.27)$$

Let us define

$$T_k(k') = \lim_{Z \to \omega_{k'} + i\varepsilon} t_{k k'}(Z) , \quad (2.28)$$

then the reality condition (2.23) gives

$$F_{kk'}(\omega_{k'}) = \frac{1}{2\pi i} \left[T_k(k') - T^{+}_{k'}(k) \right] , \quad (2.29)$$

$$\omega_k = \omega_{k'} ,$$

while the crossing relation (2.24) gives

$$G_{kk'}(\omega_{k'}) = \frac{1}{2\pi i} \left[T_{k'}(k) - T^{+}_{k}(k') \right] , \quad (2.30)$$

$$\omega_k = \omega_{k'} .$$

The unitary condition (2.2I) then enables us to transform (2.29) and (2.30) to yield expressions that in the appropriate limit (2.28) reduce eq. (2.25) to the Low equation.

2.5. The one Meson Approximation.

The most difficult of the three conditions to satisfy is the unitary condition.

This is considerably simplified however if we make the approximation of neglecting all states in the sum over n except those corresponding to a nucleon, or a nucleon + single meson. We may hope that this approximation is a good one to somewhat above the threshold for meson production, say up to 300 Mev.

It is appropriate to write

$$t_{kk'}(Z) = -V(k)V(k')\frac{2\pi}{\sqrt{\omega_k \omega_{k'}}}\sum_\alpha P_\alpha(k.k')h_\alpha(Z), \quad (2.31)$$

where P_α are the projection operators

$$\begin{aligned}
P_{11} &= \tfrac{1}{3}\tau_k \tau_{k'}.(\sigma.\hat{k}),(\sigma.\hat{k}'),\\
P_{13} &= \tfrac{1}{3}\tau_k \tau_{k'}\left[3\hat{k}.\hat{k}' - (\sigma.\hat{k})(\sigma.\hat{k}')\right], \quad (2.32)\\
P_{31} &= \left(\delta_{kk'} - \tfrac{1}{3}\tau_k \tau_{k'}\right)(\sigma.\hat{k})(\sigma.\hat{k}'),\\
P_{33} &= \left(\delta_{kk'} - \tfrac{1}{3}\tau_k \tau_{k'}\right)\left[3\hat{k}.\hat{k}' - (\sigma.\hat{k})(\sigma.\hat{k}')\right],
\end{aligned}$$

with $\hat{k} = k/|k|$, $\hat{k}' = k'/|k'|$.

The corresponding phase shifts are given by

$$\underset{Z \to \omega_{k'} + i\varepsilon}{Lt} \quad h_\alpha(Z) = \frac{e^{i\delta_\alpha(\omega_{k'})}\sin\delta_\alpha(\omega_{k'})}{k'^3 \, V^2(k')}, \quad (2.33)$$

and in this approximation the requirement of unitarity is simply that the δ_α are real. In the static theory there is complete symmetry between angular momentum and isotopic spin and so

$$\delta_{13} = \delta_{31}, \quad (2.34)$$

and it is convenient to write

$$h_1 \equiv h_{11}, \quad h_2 \equiv h_{13} \equiv h_{31}, \quad h_3 \equiv h_{33}. \quad (2.35)$$

The crossing theorem becomes

$$h_\alpha(Z) = A_{\alpha\beta}\, h_\beta(-Z) \quad , \tag{2.36}$$

with

$$A_{\alpha\beta} = 1/9 \begin{bmatrix} 1 & -8 & 16 \\ -2 & 7 & 4 \\ 4 & 4 & 1 \end{bmatrix}$$

The Born Approximation residues at $Z = 0$ are λ_α where

$$\lambda_\alpha = 2/3\, f^2 \begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix} \tag{2.37}$$

The Low equation may be written

$$h_\alpha(\omega) = \frac{\lambda_\alpha}{\omega} + \frac{1}{\pi}\int d\omega_{k'}\, k'^3 V^2(k') \times \left\{ \frac{|h_\alpha(\omega_{k'})|^2}{\omega_{k'} - \omega - i\varepsilon} + A_{\alpha\beta}\, \frac{|h_\beta(\omega_{k'})|^2}{\omega_{k'} + \omega} \right\} \tag{2.38}$$

In order to solve (2.38) it is convenient to consider

$$H_\alpha(Z) = \frac{\lambda_\alpha}{Z}\left[h_\alpha(Z)\right]^{-1} \tag{2.39}$$

The unitarity condition (2.I3) yields

$$\left(\underset{Z \to \omega_{k'} + i\varepsilon}{Lt} - \underset{Z \to \omega_{k'} - i\varepsilon}{Lt} \right) H_\alpha(Z) = -2i\, \frac{\lambda_\alpha k'^3}{\omega_{k'}} V^2(k') \tag{2.40}$$

$\omega_{k'} \geq 1$

If $h_\alpha(Z)$ has no zeros(and this will be true for sufficiently small f^2) it is then easy to see that

$$H_\alpha(Z) = 1 - \frac{Z}{\pi}\lambda_\alpha \int \frac{d\omega_{k'}}{\omega_{k'}^2} \frac{k'^3 V^2(k')}{\omega_{k'} - Z} - \frac{Z}{\pi}\int \frac{d\omega_{k'}}{\omega_{k'}^2} \frac{k'^3 V^2(k')}{\omega_{k'}+Z} \times F_\alpha(\omega_{k'}), \tag{2.4I}$$

where $F_\alpha(\omega_{k'})$ is a determinate function obtained from the crossing theorem, but not known in closed form. This furnishes a solution of (2.38) which has the property that it is the analytic continuation of the perturbation solution in f^2. Castillejo, Dalitz and Dyson(P.R. IOI, 453) have shown that in fact an infinite number of solutions exist. These are generated by giving $h_\alpha(Z)$ an arbitrary number of zeros on the real axis and $H_\alpha(Z)$ arbitrary positive residues at these points.

It would be nice to be able to reject all these extra solutions on physical grounds. Haag (N.Cim. 5, 203) has given arguments for doing so. The most persuasive relates to the mutual orthogonality of the resulting eigenstates.On the other hand,Dyson(R.R.I06,I57) has constructed a model in which all solutions have a physical meaning. Neither discussion satisfies crossing symmetry.

2.6. The Effective Range Approximation.

Eqns. (2.35), (2.39), (2.4I), yield

$$e^{i\delta_\alpha(k)}\sin\delta_\alpha(k) = \frac{\lambda_\alpha/\omega_k\, k^3 V^2(k)}{+ - \frac{\omega_k}{\pi}\Delta_\alpha(\omega_k) - \frac{i\lambda_\alpha}{\omega} k^3 V^2(k)\ )} \tag{2.42}$$

$$\omega_k \geq 1,$$

where

$$\Delta_\alpha(\omega_k) = \lambda_\alpha P \int_\mu^\infty \frac{d\omega_{k'}}{\omega^2_{k'}} \frac{k'^3 V^2(k')}{\omega_{k'} - \omega_k}$$

$$+ \int_\mu^\infty \frac{d\omega_{k'}}{\omega^2_{k'}} \frac{k'^3 V^2(k')}{\omega_{k'} + \omega_k} F_\alpha(\omega_{k'}) \quad (2.43)$$

The integrals are $\Delta_\alpha(\omega_k)$ would be linearly divergent without the cut-off and so most of their contribution comes from $\omega_{k'} \sim \omega_{max}$. For small ω_k they may therefore be taken as almost constant. In this approximation

$$g_\alpha(\omega_k) \equiv \frac{\lambda_\alpha}{\omega} k^3 V^2(k) \cot \delta_\alpha = 1 - \omega \Gamma_\alpha \quad (2.44)$$

Γ_α is the effective range. Eq. (2.44) gives a means of extrapolation from $\omega = \mu$ to $\omega = 0$. The relation has been verified for the experimental results for the 33 phase shift by Lindenbaum and Yan (P.R.100, 306). They obtain a value

$$f^2 = .08. \quad (2.45)$$

Some other consequences of this approximation are:

(i) For sufficiently low energies the phase shifts have the same signs as those given by Born approximation.

(ii) We can obtain certain sum rules. For $\omega < \mu$ no imaginary term appears in $h_\alpha(\omega)$ and we write

$$h_\alpha(\omega) = \frac{\lambda_\alpha(\omega)}{1 - \omega \Gamma_\alpha}$$

Substitution into (2.38) and letting $\omega \longrightarrow 0$ yields

$$\lambda_\alpha \Gamma_\alpha = \frac{1}{\pi} \int \frac{d\omega_{k'} \; k'^3 \sigma^2(k')}{\omega_{k'}} (\delta_{\alpha\beta} + A_{\alpha\beta}) |h_\beta(\omega_{k'})|^2. \quad (2.46)$$

(not summed over α!)

In particular this yields the important result

$$\Gamma_3 > 0 \qquad (2.47)$$

Reference to (2.44) shows that this implies that γ_3 can vanish for ω sufficiently large i.e. a (33) resonance is possible. We use the known position of the (33) resonance to fix the value of ω_{max} .

Other sum rules involving such quantities as f/f_0 have been obtained by Cini and Fubini (N.Cim. 3, 764). These can only be fitted to the experimental data by a special choice of cut off shape. These results are interesting as illustrating the limitations of this theory. We only trust predictions like (2.47) which are independent of any details of the cut off.

Salzman has found a numerical solution of (2.38) using a gaussian cut off centred at 7μ .

2.7 Phot-Meson Production

Perhaps the greatest achievement of the Chew-Low theory has been the way it has interrelated results in meson-nucleon scattering with those in photo-meson production.

To first order in the matrix element for photo-meson production is

$$M_k(q) = \langle \Psi \bar{q} \left| \int d\tau\, j \cdot A_k \right| \Psi_0 \rangle , \qquad (2.48)$$

where the vector potential A_k for a photon of momentum k and polarization ε is

$$A_k(x) = \frac{1}{\sqrt{2k}}\, \varepsilon\, e^{ik\cdot x} . \qquad (2.49)$$

Usually one divides j into (meson current + nucleon current + interaction current). Here, however, it is more convenient to write

$$j = j^S + j^V + j^\pi \qquad (2.50)$$

j^π contains all terms depeading on the meson field operators:

j^S and j^V commute with a_g , a_g^+ and have scalar and vector isobaric spin dependence respectively (I.E. proportional to 1 and τ_3).

This definition is made unambiguous by requiring that

$$\langle \Psi_0 | j | \Psi_0 \rangle = \langle \Psi_0 | j^S + j^V | \Psi_0 \rangle \qquad (2.51)$$

The contribution of j^V to M may be obtained at once.

Invariance requires that

$$\int j^V e^{ikx} dx = i \tau_3 \left[A k F'(k^2) + B \sigma_\Lambda k F(k^2) \right]$$

and since

$$\operatorname{div} j^V = 0$$

then

$$A = 0$$

and we will define B by the condition

$$F(0) = 1$$

Then $\langle \Psi_0 | j^V | \Psi_0 \rangle = \lim_{k \to 0} B \langle \Psi_0 | i \tau_3 \sigma_\Lambda k | \Psi_0 \rangle$

$$= \lim_{k \to 0} (u_0 | i \tau_3 \sigma_\Lambda k | u_0) \frac{M_p - M_N}{2}$$

and so

$$B = \frac{1}{Z} \quad \frac{\mu_p - \mu_N}{2} \tag{2.52}$$

Then

$$M_k^V(q) = -\frac{1}{\sqrt{2k}} \langle \Psi_q^{(-)} | i\tau_3 \sigma \cdot k \wedge \varepsilon | \Psi_0 \rangle \times \frac{f_0}{f} \quad \frac{\mu_p - \mu_N}{2} F(k^2)$$

$$= -\frac{1}{f} \frac{F(k^2)}{V(k)} \sqrt{\frac{\omega_k}{k}} \quad \frac{\mu_p - \mu_N}{2} T_k'(q). \tag{2.53}$$

where $T_k'(q)$ is the scattering amplitude for a meson of momentum $k \wedge \varepsilon$ and isotopic spin in the 3 direction. This remarkable direct connection between the photoproduction and scattering matrix elements does hot hold, unfortunately for j^S and j^π

Let us know consider

$$M_k^S(q) = \langle \Psi_q^{(-)} | - \int j^S \cdot A_k | \Psi_0 \rangle.$$

Techniques similar to those used in section 2.3. enable us to manipulate this to become

$$M_k^S(q) = -\sum_n \left\{ \frac{T_q^+(n) M_k^S(n)}{E_n - \omega_q - i\varepsilon} + \frac{M_k^{S+}(n) T_q(n)}{E_n + \omega_q} \right\} \tag{2.54}$$

The sum over a consists of an integral from μ to ∞ plus the nucleon contribution at $E_n = 0$. Invariance together with (2.5I) enable us to evaluate this latter as

$$\frac{T_g^+(0)\, M_k^S(0) + M_k^{S+}(0)\, T_g(0)}{\omega_g}$$

$$= \frac{1}{\omega_g}\left[V_g, \frac{i\sigma\cdot k\wedge\varepsilon}{\sqrt{2k}}\right]\frac{\mu_p-\mu_N}{2}\, G(k^2), \quad (2.55)$$

with $G(k^2)$ some invariant form function.

Finally we must consider

$$M_k^{\vec{\pi}}(g) = \langle \Psi_g^{(-)} | -\int j^{\vec{\pi}}\cdot A_k \,|\, \Psi_0 \rangle$$

We apply similar techniques to those used for M^S. There is no single nucleon term because of (2.5I) but an inhomogeneous term arises from the fact that a_g does not commute with $j^{\vec{\pi}}$. Accordingly we obtain

$$M_k^{\vec{\pi}}(g) = \langle \Psi_0 | \left[a_g , -\int j^{\vec{\pi}}\cdot A_k \right] | \Psi_0 \rangle$$

$$-\sum_n \left\{ \frac{T_g^+(n)\, M_k^{\vec{\pi}}(n)}{E_n - \omega_g - i\varepsilon} + \frac{M_k^{\vec{\pi}+}(n)\, T_g(n)}{E_n + \omega_g} \right\} \quad (2.56)$$

The evaluation of the first term in (2.56) is rather difficult.

We can use the other decomposition of the current and write

$$\left[a_g , j^{\vec{\pi}} \right] = \left[a_g , j \right]$$

$$= \left[a_g , j^{int} + j^{meson} \right].$$

In order to evaluate j^{int} we take the interaction (2.4) and replace ∇ by $\nabla - ieA$ for the charged mesons. (This does not give a gauge invariant interaction for an extended source but we shall neglect the extra terms).

Then

$$j^{int} \cdot A_k = e f^\circ (\tau_2 \phi_1 - \tau_1 \phi_2)\, \sigma \cdot A_k \qquad (2.57)$$

This gives for an inhomogeneous term evaluated between free field spinors.

$$\frac{ef}{\sqrt{4k\omega_g}} \left[\tau_g, \tau_3 \right] \sigma \cdot \xi \qquad (2.58)$$

Clearly this term produces S -wave charged pions. Rather more involved manipulations applied to j. meson yield a total inhomogeneous term

$$M_k^{\pi(0)} = \frac{ef}{\sqrt{4k\omega_g}} \left[\tau_g, \tau_3 \right] \left\{ \sigma \cdot \varepsilon - 2 \frac{\sigma \cdot (g-k) g \varepsilon}{(g-k)^2 + \mu^2} \right\} \qquad (2.59)$$

2.8 Comparison with Experiment.

The combination of expressions (2.53), (2.54), (2.55), (2.56) and (2.59) is rather too complicated for direct comparison with experiment. We take only $M^V + M^{\pi(0)}$

We neglect

(a) M^S because $\mu_p + \mu_N < \mu_p - \mu_N$.

(b) Rest of M^π because this merely represents the scattering corrections to $M^{\pi(0)}$ and these are expected to be small.

We then find that:

-a- only charged S -wave pions are produced. The production amplitude is

$$\pm \sqrt{2}\, \frac{ief}{\sqrt{4k\omega_g}}\, \sigma \cdot \varepsilon$$

and comparison with experiment yields

$$f^2 = .07. \qquad (2.60)$$

-b- The neutral p-wave amplitude may be best investigated by looking at the 'neutral' cross-section at a photon energy of 324 Mev. This produces a π at the (33) resonance energy. Comparison with experiment yields

$$f^2 = .08 \qquad (2.6I)$$

We note the excellent agreement of (2.45), (2.60) and(2.6I).

2.9 S-wave pions.

Drill, Friedman and Zachariasen, P.R. I04, 236.

The only way of introducing S-wave pions into the static model is through the adhoc terms:

$$H^S = \lambda_0 \phi^2 + \lambda \tau \cdot \phi \wedge \pi \qquad (2.62)$$

in the interaction Hamiltonian. Both terms are required to give the strong isobaric spin dependence of the S-wave phase shifts. (If the ratio λ_0/λ is calculated from the N.R. reduction of γ_5 theory this isobaric dependence is not correctly obtained). D F Z are able to fit both scattering and photo-production data reasonably well. Bincer (P.R. I05, I399) has calculated double S-wave pion production by a p-wave pion on this theory and finds it not in disagreement with experiment.

2.I0 Other Matters.

Other interesting papers on the Chew-Low theory include Cutkosky and Zachariasen, P.R.I03, II08.

Photo-production of a pair of π's(one S-wave and one p-wave).

Piyazawa, P.R. I04, I74I.

Since one can mowe on and off the energy shell so easily it is possible to relate the matrix elements for nucleon anomalous magnetic moment, etc., with the scattering phase shifts.

Salzman, P.R. I05, I076.

It is possible to investigate the outer nucleon structure (pion cloud) predicted by this theory. The results are in reasonable agreement with the Stanford experiments.

This list is by no means exhaustive.

CHAPTER III

CAUSALITY AND DISPERSION RELATIONS

3.I Scattering in the Heisenberg Representation

Low, P.R. 97, 1392;

CF. Low & Gell-Mann, P.R. 84, 350.

$|\phi\rangle$ and $\underset{\sim}{O}$ refer to the Interaction Representation

$|\psi\rangle$ and O refer to the Heisenberg Representation

Then we observe that

$$\left(\phi_{p'}\left|\sum\frac{(-i)^n}{\lfloor n}\int_{-\infty}^{\infty}\bar{T}\left(\underset{\sim}{H}(x_1)\dots\underset{\sim}{H}(x_n)\,\underset{\sim}{O}(x)\,\underset{\sim}{O}(y)\right)dx_1\dots dx_n\,\phi_p\right.\right)$$

$$=\left(\phi_{p'}\left|\underset{\sim}{S}(\infty,x_0)\,\underset{\sim}{O}(x)\,\underset{\sim}{S}(x_0,y_0)\,\underset{\sim}{O}(y)\,\underset{\sim}{S}(y_0,-\infty)\right|\phi_p\right)$$

supposing

$$x_0 > y_0$$

$$=\langle\psi_{p'}\,|\,\bar{T}\,(O(x)\,O(y))\,|\psi_p\rangle. \tag{3.I}$$

Let us consider the scattering of a single meson by a nucleon. We shall suppose that the interaction Hamiltonian is given by

$$\underset{\sim}{H}(x)=\underset{\sim}{j}(x)\,\underset{\sim}{\phi}(x)\,, \tag{3.2}$$

Where $\underset{\sim}{H}(x)$ does not contain the meson field explicitly.
(It is easy to modify the argument if terms like $\lambda \phi^4$ are present). Then the Matrix element is given by

$$(p', k' | S | p, k) = (p' | a_{k'} \sum \frac{(-i)^n}{\lfloor n} \int T(\underset{\sim}{H}(x_1) \dots \underset{\sim}{H}(x_n)) a_k^+ | p)$$

$$= (-i)^2 \int \frac{dx\, dy\, e^{ikx - k'y}}{\sqrt{4 \omega_n \omega_{k'}}} (p' | \sum \frac{(-i)^n}{\lfloor n} \int T(\underset{\sim}{H}(x_1) \dots \underset{\sim}{H}(x_n) j(x) j(y)) | p),$$

where we have used

$$[a_{k'}, \phi(x)] = \frac{e^{-ik'x}}{\sqrt{2 \omega_{k'}}}$$
$$[\phi(x), a_k^+] = \frac{e^{ikx}}{\sqrt{2 \omega_k}} \qquad (3.3)$$

Therefore, from (3.I),

$$(p', k' | S | p, k) = (-i)^2 \int \frac{e^{ikx - ik'y}}{\sqrt{4 \omega_k \omega_{k'}}} \langle \Psi_{p'} | T(j(x) j(y)) | \Psi_p \rangle dx\, dy. \qquad (3.4)$$

The generalization of (3.4) for processes involving extra mesons is obvious.

3.2 General Notions

Gell-Mann, Goldberger & Thirring, P.R.95, 1612.

Goldberger, P.R.99, 979, 986.

Causality is the requirement that the bell does not ring till after the button has been pressed. For relativistic field theory this condition finds its expression in the condition that field quantities commute (anticommute) on a spacelike surface.

The causality condition we shall use is

$$[j(x), j(y)] = 0, \quad (x-y)^2 > 0. \qquad (3.5)$$

The consequences of this condition find their expression in the form of dispersion relations. We can readily understand this in terms of the following simple argument:

Consider a "response fuction" $f(t)$ satisfying

$$f(t) = 0 \quad , \quad t < 0 .$$

Then

$$\varepsilon(t) f(t) = f(t)$$

and the Fourier transform of this is just

$$\hat{f}(\omega) = \frac{-i}{\pi} P \int_{-\infty}^{\infty} \frac{\hat{f}(\omega')}{\omega' - \omega} d\omega' . \qquad (3.6)$$

(3.6) gives a relation between the real and imaginary parts of $\hat{f}(\omega)$ However, one has to be a little careful about what happens at $t = 0$. Perhaps $f(t)$ behaves like $\delta(t)$ or even $\delta'(t)$. If $f(t) \sim \delta(t)$ at $t = 0$, $\hat{f}(\omega) \sim 1$ at $|\omega| = \infty$ and the integral in (3.6) does not converge. However, we can replace (3.6) by the (then convergent) equation

$$\hat{f}(\omega) - \hat{f}(\omega_0) = \frac{-i(\omega_0 - \omega)}{\pi} P \int_{-\infty}^{\infty} \frac{\hat{f}(\omega') d\omega'}{(\omega' - \omega)(\omega' - \omega_0)} . \qquad (3.6')$$

There are two equivalent conditions on $\hat{f}(\omega)$ that ensure its satisfying (3.6). They are:

(i) Its Fourier transform $f(t)$ satisfies

$$f(t) = 0 \quad , \quad t < 0$$

(ii) It is the limit as tends to the real axis from above of a function $f(z)$ regular in the upper half plane.

These and other conditions have been carefully and rigorously discussed by Toll (P.R.104, 1760).

3.3 The Comparison Theorem

The expression (3.4) is not directly suitable for exploiting the causality condition (3.5). Instead it proves possible to consider matrix elements of retarded commutators. It is a rather remarkable result that for positive energies the matrix elements of the T-product and of the retarded commutator are equal. We shall call this result the Comparison theorem and prove it for the simplest case of the forward scattering of mass-less scalar particles by fermions.

Take a frame of reference with the fermions at rest,

$$p - p' = (M, \underset{\sim}{0}).$$

Let the mesons be specified by

$$k - k' = (\omega, \omega n), \qquad n \text{ some unit vector.}$$

After the -function giving overall momentum and energy conservation has been removed, the scattering amplitude is given by

$$F(\omega) = \int e^{i(\omega x_0 - \omega \underset{\sim}{n}.\underset{\sim}{x})} dx$$

$$\langle \Psi | T(j(x) j(0)) | \Psi \rangle . \qquad (3.7)$$

Since

$$T(j(x) j(0)) = \theta(x) j(x) j(0) \qquad (3.8)$$

$$+ \theta(-x) j(0) j(x),$$

equation (3.7) can be rewritten, after introducing a sum over a complete set of intermediate states, as

$$F(\omega) = \frac{1}{2\pi i} \int e^{-i\omega \underset{\sim}{n}\cdot\underset{\sim}{x}} \sum_n \left\{ \frac{\langle \Psi | j(\underset{\sim}{x}) | \Psi_n \rangle \langle \Psi_n | j(\underset{\sim}{0}) | \Psi \rangle}{E_n + E_0 - \omega - i\varepsilon} - \frac{\langle \Psi | j(\underset{\sim}{0}) | \Psi_n \rangle \langle \Psi_n | j(\underset{\sim}{x}) | \Psi \rangle}{E_0 - E_n - \omega + i\varepsilon} \right\} \tag{3.9}$$

If we consider the matrix elements of the retarded commutator,

$$\theta(x) \left[j(x), j(0) \right] \tag{3.10}$$

these are given by

$$M(\omega) = \frac{1}{2\pi i} \int e^{-i\omega \underset{\sim}{n}\cdot\underset{\sim}{x}} \sum_n \left\{ \frac{\langle \Psi | j(\underset{\sim}{x}) | \Psi_n \rangle \langle \Psi_n | j(0) | \Psi \rangle}{E_n - E_0 - \omega - i\varepsilon} - \frac{\langle \Psi | j(\underset{\sim}{0}) | \Psi_n \rangle \langle \Psi_n | j(\underset{\sim}{x}) | \Psi \rangle}{E_0 - E_n - \omega + i\varepsilon} \right\} \tag{3.11}$$

(3.9) and (3.II) differ only by the sign of $i\varepsilon$ in the second energy denominator. However, for all states having non-vanishing matrix elements with $j(x)|\Psi\rangle$

$$E_n > E_0 . \tag{3.12}$$

Thus this denominator does not vanish for $\omega > 0$ and so the sign of $i\varepsilon$ is irrelevant. Thus

$$F(\omega) = M(\omega) \quad , \quad \omega > 0 , \tag{3.13}$$

proving the comparison theorem.

(A proof of this result for the general case and not using sums over intermediate states can be given (Polkinghorne, Proc. Camb.Phil. Soc., to be published).)

We can in fact write:

$$M(\omega) = D(\omega) + i A(\omega) \qquad (3.I4)$$

where $D(\omega)$ and $A(\omega)$ are the real and imaginary parts respectively, and are the transforms of

D $$\varepsilon(x)\left[j(x), j(0)\right],$$

A $$-i\left[j(x), j(0)\right].$$

$D(\omega)$ represents dispersive scattering and $A(\omega)$ represents absorptive scattering (i.e.through a real intermediate state).

We easily see that

$$F(\omega) = D(\omega) + i\varepsilon(\omega) A(i). \qquad (3.I5)$$

3.4 Causality condition and Dispersion Relations.

Using the comparison theorem we consider

$$M(\omega) = \int dx\, e^{i\omega(x_0 - \underset{\sim}{n}\cdot\underset{\sim}{x})} \qquad (3.I6)$$

$$\langle \Psi | \Theta(x)\left[j(x), j(0)\right] | \Psi \rangle .$$

The causality condition implies that $\theta(x)\left[j(x).j(0)\right]$ vanishes unless x lies in the forward light cone. In this case the factor multiplying ω in the exponent is always positive and so $M(\omega)$ satisfies a dispersion relation:

$$M(\omega) = \frac{i}{\pi} P \int_{-\infty}^{\infty} \frac{M(\omega')}{\omega - \omega'} d\omega' \qquad (3.I7)$$

The real part of (3.I7) yields

$$D(\omega) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{A(\omega')}{\omega' - \omega} d\omega' . \qquad (3.I8)$$

No further information is obtained by taking the imaginary part since if D and A satisfy(3.I8) they automatically satisfy

$$A(\omega) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{D(\omega')}{\omega - \omega'} d\omega' . \qquad (3.I9)$$

For this simple case we have the crossing relation

$$A(-\omega) = A(\omega) \qquad (3.20)$$

and so (3.I8) can be rewritten

$$D(\omega) = \frac{\omega}{\pi} P \int_{0}^{\infty} \frac{A(\omega')}{\omega'^2 - \omega^2} d\omega' . \qquad (3.2I)$$

Finally, using the optical theorem which states that

$$A(\omega) = \frac{\sigma(\omega)}{4\pi\hbar} \qquad (3.22)$$

with $\sigma(\omega)$ the total cross-section (3.2I) becomes

$$D(\omega) = \frac{\omega}{4\pi^2} P \int_0^\infty \frac{\sigma(\omega')}{k'(\omega'^2 - \omega^2)} d\omega'. \qquad (3.23)$$

This is a relation between physically observable quantities.

3.5 Forward Scattering of Bosons with Mass.

So far we have considered the forward elastic scattering of zero-mass bosons. In this and the following sections we shall relax these restrictions. If the boson has mass μ for forward scattering we must consider

$$M(\omega) = \int dx\, e^{i\omega x_0 - i\sqrt{\omega^2-\mu^2}\,|\underline{x}|} \langle \Psi | \theta(x) [j(x), j(0)] | \Psi \rangle. \qquad (3.24)$$

The square root in the exponential causes trouble for . Karplus and Ruderman (P.R.98, 77I) pointed out that vanishes in this region except for 'bound state terms" corresponding to the physical nucleon. There occur at

$$\omega_\nu^2 = \left(\frac{\mu^2}{2M}\right)^2 \qquad (3.25)$$

and give δ-function singularities in $A(\omega)$ at these values. Thus the only contribution to the integral in (3.2I) that arises from the difficult region $\omega^2 < \mu^2$ arises from these bound state contributions. Bugoljubov proposed therefore to eliminate them by considering

$$M'(\omega) = (\omega^2 - \omega_\nu^2) M(\omega) \qquad (3.26)$$

This is a casual function whose $A'(\omega)$ vanishes in

There is therefore no trouble in proving that

$$(\omega^2 - \omega_v^2)\, D(\omega) = (\omega_0^2 - \omega_v^2)\, D(\omega_0)$$

$$= \frac{\omega}{\pi} P \int_{\mu}^{\infty} \frac{A(\omega')(\omega'^2 - \omega_0^2)(\omega^2 - \omega_0^2)}{(\omega'^2 - \omega^2)(\omega'^2 - \omega_0^2)}\, d\omega' \qquad (3.27)$$

The term in ω_0 is introduced to make the integral in (3.27) convergent. If now we let $\omega_0 \to \omega_v$ we obtain

$$D(\omega) = \frac{f^2 \omega_v}{\omega^2 - \omega_0^2} f^2 + \frac{\omega}{\pi} P \int_{\mu}^{\infty} \frac{A(\omega')\, d\omega'}{(\omega'^2 - \omega^2)} \qquad (3.28)$$

where

$$f^2 = \text{residue of } D(\omega) \text{ at } \omega = \omega_v \qquad (3.29)$$

which can be shown to be positive. This is just the coupling constant. "Bound state" terms are therefore of great importance as the means of introducing coupling constants into dispersion theory.

3.6 Scattering with Momentum Transfer.

Takeda and Caps, P.R. 103, 1877.

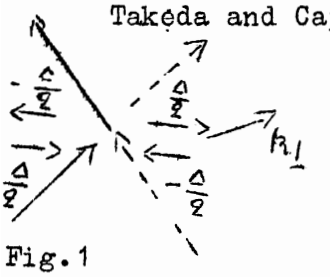


Fig.1

It is convenient to use the Breit frame sketched in Fig.I for scattering with momentum transfer . We obtain a dispersion relation for fixed

Two difficulties arise in this case:

(i) $A(\omega)$ vanishes, apart from the "bound state" contribution, only in the region $\omega^2 < \omega_a^2$, where

$$\omega_a = \mu - \frac{\Delta^2}{4\mu} \tag{3.30}$$

Therefore the difficulty of proving the dispersion relations is acute. This has been solved by Bogoljubov (unpublished lecture notes) by considering analytic continuation from an imaginary mass to a real mass. The argument is extremely involved and depends upon delicate techniques in the theory of functions of many complex variables.

(ii) Not all the quantities appearing in the integrand on the right-hand side of the dispersion relation are directly physically measurable since only energies

$$\omega \geq \sqrt{\mu^2 + \frac{\Delta^2}{4}} \tag{3.3I}$$

are accessible experimentally. For energies below this we must construct a continuation method into this 'unphisical range'. This may be done using a partial wave decomposition and continuing $P_n(\cos\theta)$ to $\cos\theta < -1$.

When the dispersion relations for the charge independent theory are thus evaluated in the one-meson approximation we obtain the Low equation with cut-off function equated to unity.

3.7 Inelastic Scattering.

Polkinghorne, N. Cim. 4, 2I6.

Dispersion relations may also be obtained for processes involving the creation of mesons.

For

$$N + \pi \longrightarrow N + n\pi \tag{3.32}$$

the comparison function needed is just the Fourier transform of

$$\sum_p \theta(x_1 > x_2 > \ldots > x_n) \times \left[j(x_1) \left[j(x_2) \ldots \left[j(x_n), j(0) \right] \ldots \right] \right], \tag{3.33}$$

the sum being taken over all permutations p of $x_1 \ldots x_n$. This satisfies the causality condition that it vanishes unless $x_1, \ldots, x_n$ all lie in the forward light cone.

It is also possible to get dispersion relations for the process

$$N + m\pi \longrightarrow N + n\pi \tag{3.34}$$

though the formalism is very complicated. The point of considering (3.34) is not that these processes are readily observed but that all field theory might be formulated using dispersion relations in terms (almost*) of physically observable quantities only. The dispersion relations provide one connection between the real and imaginary parts of the scattering amplitude and unitarity provides the other. The unitarity condition necessarily introduces the matrix elements for processes such as (3.34).

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* The unphisical ranges prevent this being quite completely true.

3.8 Applications.

Goldberger, Miyazawa & Oehme, P.R. 99, 986.

Anderson, Davidson & Kruse, P.R. 100, 339.

Consider the coherent forward scattering of mesons by nucleons.

The scattering amplitude may be written

$$T_{\alpha\beta}(\omega) = \delta_{\alpha\beta}\, T'(\omega) + \tfrac{1}{2}\left[\tau_\alpha, \tau_\beta\right] T^2(\omega) \,, \qquad (3.35)$$

where the amplitudes for the states $T = 3/2$, $T = 1/2$, are given by

$$T^{3/2} = T' - T^2$$

$$T^{1/2} = T' + 2\, T^2 \qquad (3.36)$$

It is sufficient to consider the non-charge-exchange scattering of π^+ and π^- by P. This yields

$$T' = \tfrac{1}{2}\left(T_- + T_+\right) \qquad (3.37)$$

$$T^2 = \tfrac{1}{2}\left(T_- - T_+\right)$$

Let $D_\pm$, $A_\pm$, be the real and imaginary parts of $T_\pm$. Since

$$D_{\alpha\beta}(-\omega) = -D_{\beta\alpha}(\omega) \,,$$

$$A_{\alpha\beta}(-\omega) = A_{\beta\alpha}(\omega) \,; \qquad (3.38)$$

$$D'(-\omega) = D'(\omega) \,, \qquad A'(-\omega) = -A'(\omega) \,;$$

$$D^2(-\omega) = -D^2(\omega) \qquad A^2(-\omega) = A^2(\omega) \,. \qquad (3.39)$$

Finally we obtain the dispersion relations

$$D_{\pm}(k) - \tfrac{1}{2}\left(1+\frac{\omega}{\mu}\right) D_{\pm}(0) - \tfrac{1}{2}\left(1-\frac{\omega}{\mu}\right) D_{\mp}(0)$$

$$= \frac{k^2}{4\pi^2} P \int_{\mu}^{\infty} \frac{d\omega'}{k'} \left[\frac{\sigma_{\pm}(\omega')}{\omega'-\omega} + \frac{\sigma_{\mp}(\omega')}{\omega'+\omega} \right]$$

$$\pm \frac{2 f^2}{\mu^2} \frac{k^2}{\omega \mp \mu^2/2M} \qquad (3.40)$$

In order to compare this with experiment we use

$$D^{3/2}(k) = D_{+}(k) = \frac{k}{2 k_a^2} \left(\sin 2\alpha_3 + 2 \sin 2\alpha_{33} + \sin 2\alpha_{31} + \ldots \right),$$

$$D^{1/2}(k) = \tfrac{3}{2} D_{-}(k) - \tfrac{1}{2} D_{+}(k) = \frac{k}{2 k_a^2} \left(\sin 2\alpha_1 + 2 \sin 2\alpha_{13} + \sin 2\alpha_{11} \right),$$

where k_a is the pion wave number in the centre of mass system. The effective range approximation enables us to write

$$D_{+}(0) = \lambda\!\!\!^{-}_{c} \left(1 + \mu/M\right) a_3 ,$$

$$D_{-}(0) = \lambda\!\!\!^{-}_{c} \left(1 + \mu/M\right) \left(\tfrac{2}{3} a_1 + \tfrac{1}{3} a_3\right).$$

a_1 and a_3 are the -wave effective ranges, and $\lambda\!\!\!^{-}_{c}$ is the pion compton wavelenght. Totalcrosssections are inserted on the right hand side of (3.40) with the approximation that the crosssection above I.3 Bev is a constant. The only set of phase shifts that satisfy the equations are the Bethe phase shifts.

The remaining ambiguity between Bethe phase shifts (δ) and their Yang counterparts (δ') given by

$$\delta_{33} - \delta_{31} = -\delta'_{33} + \delta'_{31} ,$$

$$2e^{2i\delta_{33}} + e^{2i\delta_{31}} = 2e^{2i\delta'_{33}} + e^{2i\delta'_{31}} ,$$

can be removed by considering a dispersion relation for the spin flip amplitude derived from taking a limiting case of the momentum transfer dispersion relations (Gilbert and Screaton, P.R. 104, 1758).

Recently Puppi and Stanghellini (N.Cim.5, 1305) have reanalysed the low energy π^- scattering data. They find that to fit the dispersion relations they require

$$f^2 = .08 \qquad (3.4I)$$

for this case in contrast to $f^2 \sim .08$ for π^+ scattering. Should this effect persist the theorists will be hard put to it to find an explanation.

I. - SCATTERING PROBLEMS
II. - ELEMENTARY PARTICLES

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PART I - SCATTERING PROBLEMS

CHAPTER I. CLASSICAL THEORY OF OPTICAL DISPERSION

I - INTRODUCTION: The phenomenon of scattering is one of the most direct and easily accessible effects of the interaction between the physical agent (in its aspect of field or particle) being scattered and that serving as "obstacle". Above all, it is essentially a phenomenon at the atomic scale: optical scattering, e.g., depends on the statistical fluctuations in the distribution of the atomic scattering centres in the medium . It has therefore been a powerful tool in the study of the fundamental interactions between the elementary constituents of matter and of the constitution of atomic systems. It will be sufficient to recall that the discovery of the atomic nucleus by Tutherford in 1911 was the direct outcome of the analysis of α particle scattering by atoms.

A universal feature of the scattering process is the occurrence of resonances, i.e. of states of the system scatterer + scattered object in which the exchange of energy between the two partners is so enhanced that the life-time of this coupled state is considerably increased. This results on the one hand in a selective absorption of the impinging beam by the scatterer, on the other in an anomaly in the dispersion of the scattering power (i.e. its dependence on the energy of the system). In the last few years a formal theory of scattering has been developed to a great degree of generality, in order to bring out these features and to trace this origin in very simple and deep-lying properties of physical systems.

It is, however, in the study of optical dispersion, the oldest example of scattering phenonenon, that all the essential points of the theory have been recognized for the first time. The discussion of this problem , which only required elementary consideration, may therefore be regarded as the most convenient introduction to the modern aspects of the subject.

2 - OPTICAL SCATTERING.

- The elementary mechanism of optical scattering is quite simple. The electric field of the impinging light wave excites an electric polarization of the system of electric charges constituting the scatterer, which varies in time with the same frequency as the light wave. This oscillating dipole moment is the source of a secondary light wave, which interferes with the incident wave, producing as a net result a disturbance spread out in all directions with varying intensity.

To put this picture into mathematical language, a simple model of the scatterer will suffice. We consider an electron of charge e, bound to some equilibrium position by an electric force : such a system is characterized by a proper frequency of oscillation[≠] ω_o, depending on the mass m of the particle and the elastic force : the latter is expressed, for a displacement $\vec{x}$, by $-m\omega_o^2\vec{x}$. Let a plane wave of frequency ω and wave number $k\,(=\omega/c)$ travel along the z-direction ; its electric field has the form $\vec{\mathcal{E}} = \vec{E}\, e^{i(kz - \omega t)}$. Under the influence of this electric field, the motion of the electron is determined by the equation :

$$\ddot{\vec{x}} + \omega_o^2 \vec{x} = -\frac{e}{m}\vec{E}\, e^{-i\omega t} \quad , \tag{1}$$

which gives an oscillating dipole moment

$$\vec{p} = e\vec{x} = \frac{e^2}{m}\,\frac{1}{\omega_o^2 - \omega^2}\,\vec{E}\, e^{-i\omega t} \quad . \tag{2}$$

There is thus a linear relation

$$\vec{p} = \gamma_o \vec{\mathcal{E}}_{(z=0)} \quad , \quad \gamma_o = \frac{e^2}{m}\,\frac{1}{\omega_o^2 - \omega^2} \quad , \tag{3}$$

between dipole moment and incident wave-field. The atomic polarizability γ_o exhibits a resonance behaviour when the frequency ω of the light tends to the proper frequency ω_o of the scatterer.

≠ All frequencies are meant as "circular" frequencies, i. e. 2π times the inverse of the period.

The electromagnetic field emitted by the dipole $\vec{p}$ may be calculated from the Hertz vector $\vec{Z}$, which satisfies the Dalembertian equation (generalising the Poisson equation) with a source density $\vec{p}\,\delta(P)$:

$$\Box \vec{Z} = \Delta \vec{Z} + k^2 \vec{Z} = \vec{p}\,\delta(P) . \qquad (4)$$

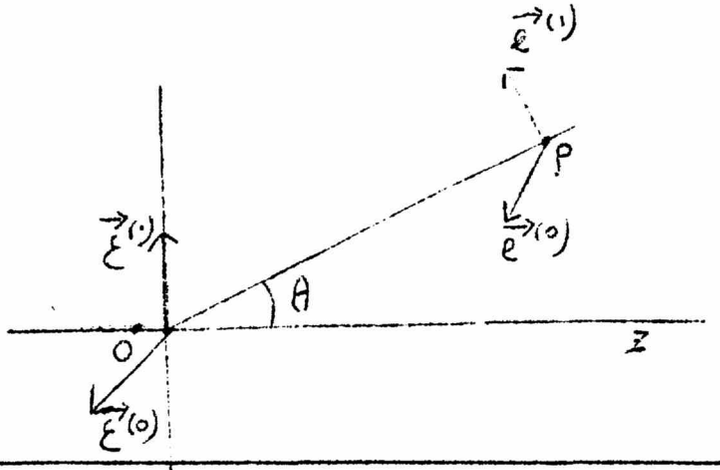
The solution representing an outgoing wave may be expressed in terms of the corresponding Green's function, i. e. the solution U of the equation :

$$\Delta U + k^2 U = -\delta(P) \qquad (5)$$

which has the required asymptotic behaviour. Instead of intreducing a source density represented by a distribution $\delta(P)$, one may speak of a solution of the homogeneous equation $\Delta U + k^2 U = 0$ with a pole at the origin. It is *:

$$U = \frac{1}{4\pi r}\, e^{i k r} , \qquad (6)$$

representing an outgoing spherical wave. Owing to the factor $1/r$ the total intensity transported by the wave remains the same over every spherical surface. With $\vec{Z} = \vec{p}\, U$, the electric field $\vec{\mathcal{E}}_D$ of the scattered wave is given by $\vec{\mathcal{E}}_D = \mathrm{rot}\,\mathrm{rot}\,\vec{Z}$. If we limit ourselves to the wave-zone (i. e. neglect all terms depending on higher powers of $1/r$ than the first), we shall find that $\vec{\mathcal{E}}_D$ is in the plane tangent to the spherical wave-front, and that the magnetic field of the scattered wave is perpendicular to $\vec{\mathcal{E}}_D$ and (in our units) equal in magnitude to it. It is therefore sufficient to consider $\vec{\mathcal{E}}_D$. Now, assume the incident wave to be unpolarized, i. e...



* The factor $1/4\pi$ arises from our use of "rationalized" units in the sense of Heaviside.

the field $\vec{\mathcal{E}}$ to consist of two incoherent components $\vec{\mathcal{E}}^{(0)}, \vec{\mathcal{E}}^{(1)}$, of equal amplitudes and orthogonal to each other. We may then write they electric field of the scattered wave as follows:

$$\vec{\mathcal{E}}_s = |\vec{p}| \left(\vec{e}^{(0)} + \vec{e}^{(1)} \cos\theta \right) k^2 \frac{1}{r}; \qquad (7)$$

in this formula, $\vec{e}^{(0)}, \vec{e}^{(1)}$ are two orthogonal unit vectors in the plane tangent to the scattered wave at the point P at which the field is considered, $\vec{e}^{(1)}$ being in the plane defined by P and the direction Oz of the incident wave; θ is the scattering angle, i.e. the angle between the direction Oz of incidence and the direction OP in which the scattering is observed. With regard to the connection between $\vec{p}$ and the incident field $\vec{\mathcal{E}}$, we may put eq. (7) into the form

$$\vec{\mathcal{E}}_s = a \left(\vec{E}^{(0)} + \vec{E}^{(1)} \cos\theta \right) \frac{e^{ikr - i\omega t}}{r} \qquad (8)$$

with

$$a \equiv \frac{k^2 \gamma_e}{4\pi} = \frac{e^2}{4\pi m c^2} \cdot \frac{\omega^2}{\omega_0^2 - \omega^2} \cdot \qquad (9)$$

Eq. (8) expresses the direct relation between the incident and the scattered wave, by means of the scattering amplitude a, The Green's function $e^{ikr - i\omega t}/r$ and the other factors pertaining to the polarization and angular distribution of the scattered wave. Thus, in this formula, the two stages of process, indicated by our analysis, are clearly represented: the excitation of the scatterer by the scattering amplitudes, the emission of the scattered wave by the Green's function and the other factors.

It is interesting to note that the scattering in general affects the state of polarization (in the present case it produces a partial polarization of an unpolarized wave), and that this is revealed by peculiarities of the angular distribution. Thus, we find here two " partial waves", of indices 0 and 1, with characteristic angular distributions and amplitudes.

The scattering amplitudes have the dimension of a length. Here we find as the characteristic length involved the "classical electron radius" e^2/mc^2 , to which the amplitude reduces (apart from the sign) for $\omega \gg \omega_0$, i.e. when the incident frequency is so large that the electron can be regarded as free during the scattering process. The differential scattering cross-section $\sigma(\theta)d\Omega$ is obtained by calculating the Poynting vector of the scattered wave, multiplied by $r^2 d\Omega$, and dividing it by the energy density of the incident wave, multiplied by its velocity of propagation. This gives

$$\sigma_{sc}(\theta) = \frac{1}{2} a^2 \left(1 + \cos^2\theta\right), \qquad (10)$$

and for the total cross-section

$$\sigma_{sc} = \frac{8\pi}{3} a^2 = \frac{1}{6\pi} k^4 \gamma_0^2 . \qquad (11)$$

The first way of writing σ_{sc} illustrates the meaning of the scattering amplitude as the order of magnitude of the effective radius of a disc-shaped obstacle intercepting the radiation. For radiation of very high frequency, the amplitude, as we have seen, becomes the electron radius, and we get Thomson's formula for the scattering by"free" electrons. For ordinary light, the second expression for σ_{sc} , in which γ_0 (away from the resonance frequency) varies slowly with ω, exhibits the well-known " λ^4 - law" of Rayleigh for the wave-length dependence of the scattering of light by atoms.

3- ENERGY TRANSFER IN OPTICAL SCATTERING

The radiation scattered in all directions by the oscillator is removed by the impinging beam, which accordingly undergoes an absorption. It is instructive to follow in detail the way in which the energy is transferred by the scatterer from the incident beam to the scattered wave.

The emission of radiation by the scatterer is accompanied by a damping of its oscillation, due to the raction of the radiation field upon the oscillating dipole. This radiative reaction may be expressed as an electric field $\vec{\mathcal{E}}_r$ acting upon the moving electron. It has been calculated by Lorentz for the model of a uniformly charged sphere in accelerated motion: the method is quite straightforward, but exceedingly elegant, and leads to a very remarkable result. It consists in calculating the field produced by the motion of the sphere at any point of its interior (due account being taken of the retardation) and summing the forces exerted by this field on all the elements of the moving charge. Apart from terms explicitly depending on the radius of the sphere, but vanishing when this radius tends to zero, on finds that the dissipative part of the electric field produced by the sphere is proportional to the total charge (independently of its extension), and for sufficiently small distances from the original of the motion, also independent of the distance: so that the total force it exerts is again proportional to the charge only. This field has the value:

$$\vec{\mathcal{E}}_r = \frac{1}{6\pi c^3} e \dddot{\vec{x}} \quad , \qquad (12)$$

i.e. for our oscillator

$$\vec{\mathcal{E}}_r = +i \frac{k^3}{6\pi} \vec{p} \qquad (13)$$

Its dissipative character is shown by the fact that it is out of phase with the oscillation of the dipole, as indicated by the factor i.

It is easy to check that the work done on the dipole by the radiative reaction $\vec{\mathcal{E}}_r$ during a period of the oscillation is exactly equal to the energy radiated away during this time. In fact, this amount of work is

$$\mathrm{Re}\int \vec{\mathcal{E}}_r^{*} \cdot e\dot{\vec{x}}\, dt = \frac{1}{6\pi c^3}\,\mathrm{Re}\int \dddot{\vec{p}}^{*}\cdot\dot{\vec{p}}\, dt$$

$$= -\frac{1}{6\pi c^3}\int |\mathrm{Re}\,\ddot{\vec{p}}|^2\, dt,$$

since the integrated term takes the same value after a period; on the other hand, the rate of emission per unit time of dipole radiation has the well-known expression

$$A(\omega) = \frac{1}{6\pi c^3}\,|\mathrm{Re}\,\ddot{\vec{p}}|^2 = \frac{\omega^4}{6\pi c^3}\,|\mathrm{Re}\,\vec{p}|^2 . \qquad (14)$$

This, then, is also the rate at which (in the pure scattering process we are considering) the dipole must absorb energy from the incident beam in order to maintain its state of oscillation stationary. In other words, the absorption cross-section is derived from eq. (I4) in exactly the same way as the scattering cross-section, namely by replacing $\vec{p}$ by * $\gamma_0 \vec{\mathcal{E}}$, and is thus equal to the scattering cross-section:

$$\sigma_{abs} = \sigma_{sc} = \frac{1}{6\pi}\, k^4 \gamma_0^2 \qquad (15)$$

When account is taken of the radiation damping, a more accurate expression is obtained for the polarizability of the oscillator. Instead of $\vec{p} = \gamma_0 \vec{\mathcal{E}}$, one has to write

$$\vec{p} = \gamma_0 (\vec{\mathcal{E}} + \vec{\mathcal{E}}_r),$$

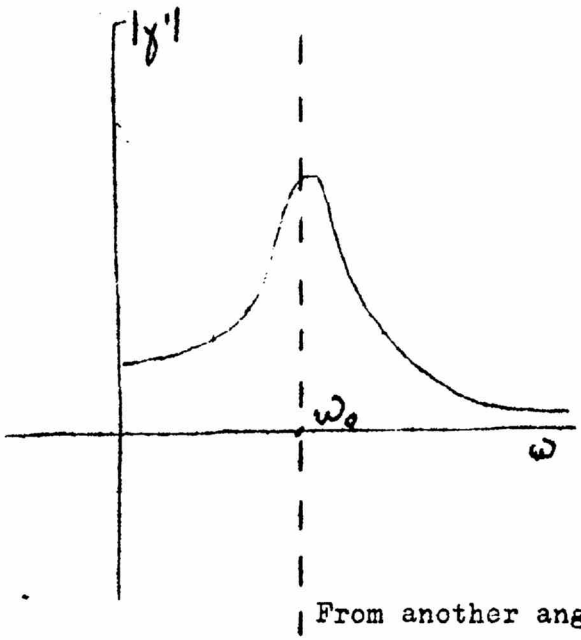
* We shall hear and in the following omit the specification that in such formulae the fields are taken for z = o.

which gives

$$\vec{p} = \gamma' \vec{\mathcal{E}} \quad \text{with} \quad \frac{1}{\gamma_1} = \frac{1}{\gamma_0} - i\frac{k^3}{6\pi}, \qquad (16)$$

or

$$\gamma' = \frac{e^2}{m^2} \frac{1}{\omega_0^2 - \omega^2 - i\omega\Gamma_\omega}, \quad \Gamma_\omega = \frac{e^2\omega^2}{6\pi m c^3}. \qquad (17)$$



The absolute value of γ',

$$|\gamma'| = \frac{e}{m} \cdot \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + \omega^2\Gamma_\omega^2}}$$

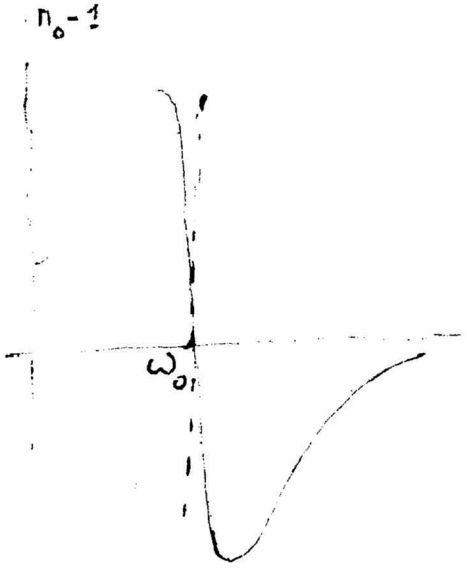
shows the resonance response of the oscillator in a domain of frequencies of width Γ_{ω_0} around the proper frequency ω_0. Moreover, there is a phase-shift between the oscillation of the impinging field and that of the dipole, and therefore also that of the scattered wave: in our case, the phase of γ' passes from 0 to $-\frac{1}{2}\pi$ as the frequency increases from 0 to ω_0.

From another angle, eq (17) illustrates the anomalous dispersion in the resonance region and its connexion with the absorption. Weakly dispersive medium containing N scatterers of our simple oscillator type per unit volume, will have a polarizability $\gamma = N\gamma'$ and a complex refractive index defined by the approximate relation

$$n - 1 = \frac{1}{2}\gamma. \qquad (18)$$

The real part of this relation gives the law of anomalous dispersion

$$n_0 - 1 = N\frac{e^2}{2m} \frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \omega^2\Gamma_\omega^2}. \qquad (19)$$



In the resonance region, we may regard the imaginary part of $1/\gamma'$ in eq(I6) as small compared with $1/\gamma_0$, and write accordingly

$$\gamma' \approx \gamma_0 + i \frac{k^3}{6\pi} \gamma_0^2 . \qquad (20)$$

From (20) and (I8) we derive the approximate expressions, valid in the resonance region;

$$n_0 - 1 \approx \frac{1}{2} N \gamma_0 \quad , \quad n_1 \simeq \frac{1}{2} N \frac{k^3}{6\pi} \gamma_0^2 \qquad (2I)$$

for the real and imaginary parts n_0 , n_1 of the refractive index. The imaginary part n_1 is related to the absorption coefficient $\alpha_{abs} = N \sigma_{abs}$ by

$$\alpha_{abs} = 2 n_1 k . \qquad (22)$$

From (22) and (2I) we find again for the absorption cross-section the expression $\sigma_{abs} = \frac{1}{6\pi} k^4 \gamma_0^2$ identital with that for the scattering cross-section [cf. eq. (I5)] . To make this formula (I5) valid at the resonance $\omega = \omega_0$ as well, we have just to replace in it γ_0 by $| \gamma' |$. We then obtain a representation of the resonance peak of the absorption line, with its natural width given by Γ_{ω_0} .

- 4 SCATTERING AMPLITUDE AND SCATTERING CROSS-SECTION -

When we take account of the radiation damping, the scattering amplitude defined by eq.(I9) also becomes complexe. The imaginary part, as is formally obvious by the definition, is related to the mechanism of absorption and re-emission which, as we have seen, produces the scattering. In fact, from (9), (22) and (I5) one finds for the imaginary part a_1 of a :

$$a_1 = \frac{k}{4\pi}\sigma_{abs} = \frac{k}{4\pi}\sigma_{sc} \qquad (23)$$

This is a very remarkable relation and it is instructive to look for its physical meaning.

Considered alone the scattered wave would give rise to an outward flux of radiation through every spherical surface centred on the scatterer. But under stationary conditions there cannot be any net flux through such a surface. The incoming wave, on the other hand, has of course by itself no net flux through the sphere. There must therefore be some interference between the incoming and the scattered wave, which has the effect of cancelling the total flux of the scattered wave. This will lead to a relation between the scattering cross - section and the scattering amplitude, and it turns out that this is just equation (23).

In order to express the radial flux of the plane incoming wave in a convenient way, let us decompose it into spherical Bessel functions:

$$e^{ikz} = \sum_{\ell} i^{\ell}(2\ell+1)P_{\ell}(\cos\theta)\,j_{\ell}(kr), \qquad (24)$$

where *

$$j_{\ell}(x) = \sqrt{\frac{\pi}{2x}}\,J_{\ell+\frac{1}{2}}(x) \sim \frac{\sin\left(x - \ell\frac{\pi}{2}\right)}{x} \qquad (25)$$

---

* The sign $\sim$ means " has asymptotic form" .

The formula (8) for the scattered wave is of the form

$$\sum_{\ell=0}^{\infty} a_\ell \vec{E}^{(\ell)} P_\ell(\cos\theta) \frac{e^{ikr}}{r} ;$$

obviously, owing to the orthogonality of the zonal harmonics, the condition of no radial flux must hold for each partial wave separately. Thus, in **general**, letting $r \to \infty$, we have to consider the radial flux of a wave of the form

$$F_\ell(r) = i^\ell (2\ell+1) \frac{\sin(kr - \ell\frac{\pi}{2})}{kr} + a_\ell \frac{e^{ikr}}{r} .$$

This flux is equal to

$$\mathcal{S} = Re\left[\frac{c}{ik} F_\ell^* \frac{dF_\ell}{dr}\right] \frac{4\pi r^2}{2\ell+1}$$

(where it suffices to retain the terms in $1/r$ in the derivative). To see this, one has only to replace in the Poynting vector the magnetic field by its value $\frac{1}{ik}$ rot $\vec{E}$ and observe that only the term $\frac{1}{ik}\frac{dF_\ell}{dr}\vec{r}_0 \wedge \vec{E}^\ell$ (where $\vec{r}_0$ is the unit vector in the radial direction) contributes to the radial component of $cR_e\left[\vec{E}^* \wedge \frac{1}{ik} rot\ \vec{E}\right]$. The last factor in $\mathcal{S}$ arises from the integration over angles. Computation gives

$$\mathcal{S} = \frac{4\pi c}{2\ell+1}\left\{|a_\ell|^2 - \frac{2\ell+1}{k} Im\, a_\ell\right\} . \qquad (26)$$

The first term in $\mathcal{S}$ is just c times the scattering cross-section σ_ℓ of the partial wave. The second term is the result of the interference between the incident and the scattered wave; the condition $\mathcal{S} = 0$ shows how it cancels the outward flux of the scattered wave:

$$Im\, a_\ell = k \cdot \frac{|a_\ell|^2}{2\ell+1} = \frac{k}{4\pi}\sigma_\ell . \qquad (27)$$

For the total scattering cross-section one gets from (27)

$$\sigma_{sc} = \frac{4\pi}{k} a_1 \equiv \frac{4\pi}{k} \sum_\ell \mathrm{Im}\ a_\ell \ ; \qquad (28)$$

for a scalar field, the amplitude of which would be represented by $\sum_\ell a_\ell P_\ell(\cos\theta)$, the quantity a_1 is just the imaginary part of the forward amplitude, i.e. of its value for $\theta = 0$. In the optical case, we arrive, as expected, at the relation (23).

Eq. (26) suggests an extension of the fundamental relation (28) to the more general situation when true absorption processes are also possible. In this case, there is a net inward flux $-\mathcal{J}$ which (for the partial wave ℓ) is equal to the number $c\,\sigma_\ell^{\text{true abs}}$ of corresponding true absorption processes taking place inside the sphere per unit time. This gives

$$\frac{4\pi}{k} \mathrm{Im}\ a_\ell = \sigma_\ell + \sigma_\ell^{\text{true abs}} \equiv \sigma_\ell^{\text{total}} \qquad (29)$$

and

$$\frac{4\pi}{k} a_1 = \sigma^{\text{total}} \ , \qquad (30)$$

expressing the most general relation between the imaginary part of the forward scattering amplitude and the total absorption cross-section.

From eq.(27)it is easy to derive the usual expression for the scattering cross-section in terms of the phase-shift characteristic of each partial scattered wave. This phase-shift η_ℓ being introduced by

$$a_\ell = |a_\ell| e^{i\eta_\ell} \ ,$$

one obtains from (27)

$$|a_\ell| = \frac{2\ell+1}{k} \sin\eta_\ell$$

and therefore

$$\sigma_\ell = \frac{4\pi}{k^2} (2\ell+1) \sin^2\eta_\ell \ . \qquad (31)$$

CHAPTER II.- QUANTUM THEORY OF OPTICAL DISPERSION

I - THE CORRESPONDANCE ARGUMENT

The study of optical phenomena has been of decisive importance for the discovery of the fundamental laws of quantum theory. Not only was the existence of the quantum of action recognised by Planck in the course of his investigation of the equilibrium energy distribution of a radiation field, brought about the interaction with a system of oscillators of given temperature , not only were the regularities of the optical spectrum of hydrogen the crucial test of Bohr's quantum postulates , but in the hands of Kramers and Heisenberg, the theory of optical dispersion supplied the clue to the definition of proper quantals variables and the establishments of the commutation laws governing their algebra.

The oscillator model treated in the preceding chapter is not sufficiently refined to serve as a basis for establishing the correspondence between classical and quantal variables. For this purpose, one ought to develop the theory for a system of coupled oscillators, or more generally for a system of charged particles of a multiply periodic character. For such a system, there exists a set of canonically conjugate angle and action variables, in terms of which all physical quantities pertaining to the system can be expressed. The action variables I_k are constants of the motion, and the angle variables are of the form $\varphi_k = \omega_k t$, where the proper frequencies ω_k are obtained from the energy $E(I_1, \dots I_f)$ as $\partial E / \partial I_k$. One may write any quantity , for example the dipole moment of the system, as a multiple Fourier series :

$$\vec{P} = \sum_{n_1, \dots n_f} \vec{P}_{n_1, \dots n_f} \, e^{i(n_1 \omega_1 + \dots + n_f \omega_f)t}$$

The rate of emission of radiation of frequency $\omega_{n_1 \dots n_f} = \sum_k n_k \omega_k$ is then

$$A_{n_1 \dots n_f} = \frac{\omega^4_{n_1 \dots n_f}}{3\pi c^3} \left| \vec{p}_{n_1, \dots n_f} \right|^2 . \qquad (I)$$

According to the quantum postulates, such an emission process is actually a transition between two stationary states, the frequency of the emitted radiation being given in terms of the energy difference of the two states by the quantal relation.

$$\hbar \omega = \Delta E .$$

Now, if we define the stationary states by assigning to the action variables values which are integral multiples of $\hbar$, the transition is characterized by the difference between <u>two</u> sets of integers:

$$\Delta I_k = (n_k - n_{k'}) \hbar \qquad (k = 1, 2, \dots, f).$$

The general relation $\delta E = \sum_k \omega_k \delta I_k$ then leads for the frequency to an expression

$$\omega = \sum_k (n_k - n'_k) \omega_k ,$$

which is readily comparable to the classical one. The correspondence thus established is between the classical set of integers $n \; (\equiv n_1, \dots, n_f)$ and the difference of two sets $n - n'$. Accordingly, the quantal variable corresponding to the Fourier coefficient $\vec{p}_{n_1, \dots n_f}$, must be a function of the two sets $\langle n'_1 \dots n'_f | \vec{p} | n_1 \dots n_f \rangle$ and the quantal formula for the emission rate is

$$\frac{\omega^4_{nn'}}{3\pi c^3} \left| \langle n' | \vec{p} | n \rangle \right|^2 ;$$

the emission probability per unit time of a photon $\hbar \omega_{nn'}$ is therefore

$$A_{n \to n'} = \frac{\omega^3_{nn'}}{3\pi \hbar c^3} \left| \langle n' | \vec{p} | n \rangle \right|^2 . \qquad (2)$$

In the first applications of this correspondence argument, the quantal variables were - faute de mieux -identified with the classical Fourier coefficients. Moreover, the quantum conditions $I_K = m_K \hbar$ were soon found to be generally incorrect and no rational way of improving them suggested itself .

The double problem of finding the right definition of the quantum numbers m and the rules of computation of the quantal variables was only solved when, in dealing with the dispersion theory of multiply periodic systems, Heisenberg realized that one could set up a simple rule for the multiplication of quantal variables, and that, once this quantal algebra was introduced, all formal relations of classical mechanics could be taken over into quantum theory and provided, in particular, a general method for setting up the quantum conditions fixing the stationary states.

We shall not retrace these steps, but rather than rescussitate the once famous theory of multiply periodic systems, directly use the quantal expressions now more familiar.

2 - NATURAL WIDTH OF EMISSION LINES.

The notion of stationary state of definite energy introduced by the quantum postulates is (except for the ground state) an idealization, inasmuch as the lack of definition of the energy arising from the finite life-time of the state is neglected. This neglect is justified by the smallness of the fine structure constant which gives the strength of the interaction of the atomic system with the radiation field.

In the next approximation, the finite life-time of the state, determined by the total probability of emission Γ_K to other, lower-lying states, gives rise to an effective distribution of the energy around the ideal value E_K , of the type

$$w_K(E)\,dE = \frac{\hbar \Gamma_K}{2\pi} \frac{dE}{(E-E_K)^2 + \frac{1}{4}(\hbar\Gamma_K)^2} \qquad (3)$$

as may be seen by studying the time dependence of a general wave -

function of energy E , when the time factors of the eigenfunctions ψ_k are taken of the form

$$e^{-\frac{i}{\hbar}(E_k - \frac{i}{2}\hbar\Gamma_k)t}$$

The parameter Γ_k is then found to have the expected value

$$\Gamma_k = \sum_l A_{k \to l},$$

the sum over the emission probabilities being extended over all the lower lying levels.

The energy distribution (3) of the stationary state is thus of the "dispersion" type, quite similar to the classical line shape; but in quantum theory we must attach a width to each state rather than to each line (when only the ever present effect of radiation is considered, it is called the "natural" width). However, the energy spread of the states between which quantum transition occurs implies a spread $W_{kl}(\omega)\,d\omega$ of the frequency of the emitted photon around the frequency ω_{kl} , which is immediately given by the "convolution" of the energy distributions of the initial and final state:

$$W_{kl}(\omega) = \int_{-\infty}^{\infty} w_k(E)\,dE\; w_l(E - \hbar\omega).$$

Now, the distribution law (3) has the remarkable property of reproducing itself by convolution* : the shape of the emission line is therefore of the same type as in classical theory, the line width being simply the sum of the widths of the two states involved in the transition:

$$W_{kl}(\omega)\,d\omega = \frac{\Gamma_k + \Gamma_l}{2\pi} \frac{d\omega}{(\omega - \omega_{kl})^2 + \frac{1}{4}(\Gamma_k + \Gamma_l)^2} \qquad (5)$$

---

* The formula

$$\int_{-\infty}^{\infty} \frac{(b/\pi)\,dx}{(x-a)^2 + b^2} \cdot \frac{(b'/\pi)}{(x-a'-y)^2 + b'^2} = \frac{(b+b')/\pi}{[y-(a-a')]^2 + (b+b')^2}$$

on which this statement is based, is easily proved by completing the integration path by a semi-circle of infinite radius and applying the theorem of residues.

3 - DISPERSION AND ABSORPTION

The polarizability of an atomic system in the ground state (or any other stationarystate) K is found to be given by the formula

$$\gamma_0 = \frac{e^2}{m} \sum_l{}' \frac{f_{kl}}{\omega_{kl}^2 - \omega^2} , \qquad (6)$$

with

$$f_{kl} = \frac{2m}{3e^2\hbar} \omega_{kl} |\langle l | \vec{p} | k \rangle|^2 ,$$

in which the summation extends over all stationary states l, except the state K itself. This formula holds for any atomic system, in particular also for a single bound electron. If we compare it in this case with the classical formula, we see that the bound electron is equivalent, in its reaction to an external electric field, with a whole set of "virtual" classical oscillators of proprer frequencies ω_{kl}, - Each of these contributes with an oscillator strength f_{kl}, such, however, that the total strength

$$\sum_l{}' f_{kl} = 1 . \qquad (7)$$

Eq.(7) is the expression for the spectnescopic sum rule of Thomas and Kuhn. Historically, it was this empirical relation which enabled Heisenberg to set up the fundamental commutation rule between a position coordinate x and its conjugate momentum $m\dot{x}$. The left hand-side is indeed identical with the diagonal element* in state k of the matrix $\frac{i}{\hbar}[m\dot{\vec{x}}, \vec{x}]$, as is immediately verified if one observes that $\langle k | \dot{\vec{x}} | l \rangle = -i\omega_{kl} \langle k | \vec{x} | l \rangle$.

There is an obvious connexion between the oscillator strength pertaining to a given transition and the corresponding transition probability per unit time due to absorption or stimulated emission of radiation of frequency ω_{kl}. In fact, the latter quantity is given by $B_{kl}\, \varrho(\omega_{kl})$, where

$$B_{kl} = \frac{\pi}{3\hbar^2} |\langle k | \vec{p} | l \rangle|^2 \qquad (8)$$

---

* The sum rule gave only the diagonal element of the commutator. The commutation law in its full operator form was soon found, however, by Dirac and by Born and Jordan.

and $\varrho(\omega)d\omega$ represents the density of radiation energy in the frequency interval $(\omega, \omega+d\omega)$. Therefore,

$$\frac{e^2}{2m} f_{kl} = \frac{1}{\pi} B_{kl} \hbar\omega_{kl}. \tag{9}$$

Now, we may formally write the formula (6) for the polarizability as a Stieltjes integral instead of a sum over the stationary states: this is indeed generally required to take account of the continuous part of the spectrum. Instead of the oscillator strengths f_{kl}, there appear distributions of the form* $f(\omega)\,d\omega$:

$$\gamma_0 = \frac{e^2}{m} \mathcal{P} \int_{-\infty}^{\infty} \frac{f(\omega')\,d\omega'}{\omega'^2 - \omega^2}. \tag{10}$$

The symbol $\mathcal{P}$ indicates that the principal value of the integral is meant: this corresponds to the fact that γ_0 is the real part of the complex polarizability

$$\gamma' = \frac{e^2}{m} \int_{-\infty}^{\infty} \frac{f(\omega')\,d\omega'}{\omega'^2 - \omega^2 - i\omega\Gamma_\omega}$$

when the width Γ_ω is sufficiently small. In fact, if we extend the definition of $f(\omega')$ to negative values of ω' by

$$f(-\omega') = f(\omega'), \tag{11}$$

we may write approximately

$$\gamma' = \frac{e^2}{m} \int_{-\infty}^{\infty} \frac{f(\omega')\,d\omega'/2\omega'}{\omega' - \omega - \frac{1}{2} i\Gamma_\omega},$$

which, in the limit $\Gamma_\omega \to 0$, becomes

$$\gamma' = \frac{e^2}{m} \left\{ \mathcal{P} \int_{-\infty}^{\infty} \frac{f(\omega')\,d\omega'/2\omega'}{\omega' - \omega} + i\pi \frac{f(\omega)}{2\omega} \right\}. \tag{12}$$

* Strictly speaking, we have a different distribution $f_k(\omega)$ for each state. In the following, we shall only consider the ground state and drop the index k.

From eq.(9) we can now derive a relation between the distribution of oscillator strength $f(\omega')$ and the absorption cross-section $\sigma(\omega')$. For any transition frequency $\omega' \equiv \omega_{kl}$, the latter quantity is, by definition, connected with the B_{kl} coefficient by the identity

$$\sigma(\omega_{kl})\, c\, \frac{\varrho(\omega_{kl})\, d\omega_{kl}}{\hbar\omega_{kl}} \equiv B_{kl}\, \varrho(\omega_{kl})$$

whence, by

$$\frac{e^2}{2m} f(\omega') = \frac{c}{\pi} \sigma(\omega'). \qquad (I3)$$

We notice that this relation gives to the imaginary part of γ' in eq. (I2) the expected value $\sigma(\omega)/k$. The deep significance of the relation (I3) appears, however, from the following considerations, originally put forward by Kramers in 1927.

Consider the complex function

$$F(\omega) \equiv n(\omega) - 1 \equiv F_0(\omega) + i F_1(\omega) \qquad (I4)$$

for a weakly dispersive medium. One has

$$F_0(\omega) = \frac{N}{2} \gamma_0'(\omega), \qquad F_1(\omega) = \frac{N\sigma(\omega)\, c}{2\omega}. \qquad (I5)$$

Therefore, if we transform the integral (I0) for γ_0' as we did that for γ', making use of(II), we see from eq.(I3) that the following relation holds between the real and the imaginary part of $F(\omega)$:

$$F_0(\omega) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{F_1(\omega')\, d\omega'}{\omega' - \omega}. \qquad (I6)$$

This equation is strongly suggestive of a familiar property of analytic functions. Assume that $F(\omega)$ is analytic and regular in the upper half-plane of the complex variable ω.

The integral of $F(\omega')/(\omega'-\omega)$ along the real axis, avoiding the point ω by a small half-circle in the upper plane, which is just

$$\mathcal{P}\int_{-\infty}^{\infty} \frac{F(\omega')\,d\omega'}{\omega'-\omega} - i\pi F(\omega), \qquad (I7)$$

is then equal to that taken along any other line going from $-\infty$ to $+\infty$ in the upper half-plane, and the latter may be made to vanish by imposing suitable conditions on $|F(\omega)|$, e.g. that the integral of $|F(\omega)|^2$ along any parallel to the real axis be bounded. From the vanishing of the expression (I7), there follows not only eq. (I6), but also

$$F_1(\omega) = -\frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty} \frac{F_0(\omega')\,d\omega'}{\omega'-\omega}. \qquad (I8)$$

The <u>dispersion relations</u> (I6), (I8) may have important practical applications, inasmuch as they allow conclusions about either the dispersion or the absorption of a medium at any frequency whenever the other property is known over the whole frequency range. In particular the dispersion relations may also be written in terms of the scattering amplitude

$$a(\omega) = a_0(\omega) + i\,a_1(\omega)$$

related to $F(\omega)$, according to eq.(9) of Chapter I, by

$$F(\omega) = \frac{2\pi N}{k^2}\,a(\omega).$$

Combined with the relation(30) of Ch.I between the imaginary part a_1) and the total absorption cross-section, they yield the complex amplitude $a(\omega)$ in terms of the cross-sections for scattering and absorption:

$$a_1(\omega) = \frac{4\pi}{k}\,\sigma^{\text{total}}(\omega),$$

$$a_0(\omega) = \frac{\omega^2}{2\pi^2 c}\,\mathcal{P}\int_0^{\infty} \frac{\sigma_{sc}(\omega')\,d\omega'}{\omega'^2-\omega^2}.$$

and therefore the differential scattering cross-section in the forward direction

$$\frac{d\sigma_{sc}(\omega;\ \vartheta=0)}{d\Omega} \equiv |a_0 + i a_1|^2$$

in terms of these integrated cross-sections.

4 - TIME - DIRECTEDNESS OF PROPAGATION

In view of the importance of the dispersion relations, it is desirable that they should be founded on a less abstract principle than the regularity of the analytic continuation of the dispersion function $F(\omega)$ into the upper half-plane. This mathematical property has indeed a surprisingly simple and deep-lying physical interpretation, which was stressed especially by Kronig. It is based on the remark that a function satisfying the dispersion relations

$$f(\omega)\, e^{i\omega\tau} = \frac{1}{\pi i} P \int_{-\infty}^{\infty} \frac{f(\omega')\, e^{i\omega'\tau}\, d\omega'}{\omega' - \omega} \qquad (I9)$$

is the Fourier transform of a function of t which vanishes for $t < \tau$. This may be seen by noting, that the Fourier transform of the step function

$$E_\tau(t) = \begin{cases} 1 & \text{for } t < \tau, \\ 0 & \text{for } t > \tau \end{cases}$$

is

$$E_\tau(\omega) = -\frac{1}{2\pi i} \frac{e^{-i\omega\tau}}{\omega + i\varepsilon},$$

ε being a non-negative infinitesimal quantity. Hence, if $f_\tau(t)$ is any function with Fourier transform $f(\omega)$, the Fourier transform of the function $F_\tau(t) = f(t)\, E_\tau(t)$ is given by

$$F_\tau(\omega)\, e^{i\omega\tau} = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{f(\omega')\, e^{i\omega'\tau}\, d\omega'}{\omega' - \omega - i\varepsilon}$$

$$= \frac{1}{2}\left[\frac{1}{\pi i} P \int_{-\infty}^{\infty} \frac{f(\omega')\, e^{i\omega' t}\, d\omega'}{\omega' - \omega} + f(\omega)\, e^{i\omega\tau}\right].$$

The condition (I9) is therefore necessary and sufficient for the equality

$$f(\omega) = E_\tau(\omega) \quad \text{or} \quad f(t) = F_\tau(t).$$

Now, any quantity $f(z,t)$ propagated along the z direction in our dispersive medium has a Fourier transform

$$f_z(\omega) = f_0(\omega)\, e^{-i n(\omega) \frac{\omega z}{c}}$$
$$= f_0(\omega)\, e^{-i[n(\omega)-1]\frac{\omega z}{c} + i\omega\frac{z}{c}}.$$

If the quantity $f(z,t)$ is such that $f(0,t)=0$ for $t>0$, its Fourier transform $f_0(\omega)$ is, in virtue of the dispersion relations (I9), regular in the upper half-plane; the same is true of the factor $\exp\left\{-i[n(\omega)-1]\frac{\omega z}{c}\right\}$ because of the dispersion relations (I6), (I8): therefore the form of the function $f_z(\omega)$ shows that the quantity $f(z,t)$ vanishes for $t > \frac{z}{c}$. The argument can be made in the opposite direction, and we thus conclude that the dispersion relations are equivalent to the <u>time-directedness</u> of the phenomenon of propagation of the field: a wave-front $f(0,t)$ does not arrive at $z>0$ before a definite positive time z/c.

The universal character of the last statement leads us to expect that its direct connexion with dispersion relations, which we have recognized by explicit derivation in electro-magnetic phenomena, may also be extended to other fields. The assumed existence of dispersion relations for a field whose structure is still imperfectly known, leads to consequences susceptible of experimental test, which may throw light on the theoretical formulation of the fundamental properties of this field. The investigation of this aspect of scattering theory has already proved its fruitfulness in some important cases and is still actively pursued.

The rigorous formulation of the universal conditions of time-directedness we have been discussing, and of the conditions under which it is equivalent to the existence of dispersion relations, raises delicate mathematical problems which are outside the scope of this elementary

introduction,- There is, however one general point of epistemology in connexion with it to which attention may be called.

The condition which we have denoted as that of "time-directedness"is usually called the "causality" condition. This name was already used, in the early days of relativity theory, in discussions of the way in which a given succession of physical events, implying motion of particles or propagation of fields, is judged by different observers: it was then pointed out that the Lorentz transformation, since it includes the principle of maximum velocity, satisfies the so-called "causality" requirement. Yet, in spite of Einstein's authority, it must be pointed out that the concept of causality is not appropriate to the situation we are envisaging. Causality, as usually understood, refers to the knowledge about a system at a certain time which can be derived from the knowledge we have of it at some other time; but it applies to retrodictions as well as to predictions. At the atomic scale at least, causal relations are essentially reversible in time: the problem is rather how determinate our predictions or retrodictions can be.

The question of the directedness of time is a different one: what is the origin of the irreversible element inherent in our conception of the "course" of any physical process? What is the physical basis of our definition of "earlier" and "later"? An answer which is still widely favoured was given by Boltzmann: according to him the "arrow of time" is a thermodynamical effect. The direction from earlier to later is that of increasing entropy of the part of the universe we observe. Boltzmann, true to his mechanistic view of the world, imagined that large parts of the universe may have been brought by some statistical fluctuation into states far removed from thermal equilibrium; to an observer living in any one of these regions, there will be a trend towards equilibrium which will define the direction of time; the observed lower limits for the time-scale of the evolution and the size of the region taking part in it give an idea of the importance of the "initial" fluctuation which has to be assumed. The difficulty of this conception is that, if at any time a large

fluctuation is present, it is overwhelmingly probable that it has arisen from a still larger one.

In fact, it seems that time-directedness is a much more fundamental feature of our description of the physical world, since it already occurs at the scale of individual atomic processes. If, following Bohr, we realize that physical concepts essentially refer to the interaction between some process in the external world and some "observer" (which may be an appropriate material system under our control), the question of time-directedness is seen in a new light. Indeed, it is inherent in the very process of observation that the perception or registration of any event, owing to the finite velocity of propagation of any interaction, has a definite time-relation to the event, which offers a natural basis for the definition of the concepts of "earlier" and "later". In other words, we say by definition (or convention) that a photon has been emitted by the object seen at an earlier time than that at which it strikes our retina. From this point of view, any statement of some definite time sequence of events of the external world(e.g. the statement that the entropy of the final state in any process is larger than that of the initial state) is a physical law. The definition of the words " initial " and " final " in such statements of physical laws is sought at a deeper level: it is immediately related to the general conception of physics as a description of our interaction with the external world.

CHAPTER III - SCATTERING OF PARTICLES BY SHORT - RANGE POTENTIAL

1.- Introduction.

In nuclear physics, the scattering problem - and its extension to the study of more complicated nuclear reactions- occupies a central position. Bombardment of nuclei by suitable nuclear projectiles has been and remains one of the most powerful and versatile tools for the determination of the properties of the ground states and various states of excitation of the nuclei and the resulting elucidation of the principles governing nuclear structure.

To gain an orientation into the methods by which this problem may be treated, we shall first discuss the simplest case of scattering of neutral scalar particle by a fixed poten - tial of limited range. This finds direct application to those cases in which the interaction between a neutron and a scatterer may be approximated by a short-range potential, as the neutron - proton scattering at not too high energy for each definite spin configuration of the stystem; it also provides a starting point, as we shall see, for the investigation of the elastic scattering of unpolarised neutrons by randomly oriented nuclei.

We are looking for a stationary solution of the wave - equation

$$(E - H)\psi = 0$$

belonging to the energy * $E = k^2$, determined by the wave number k of the incident plane wave. This solution is specified by the condition that its asymptotic form, at large distances from the centre of the scattering potential, must represent the superposition of an incoming plane wave and outgoing spherical wave:

* Here and in the following, we take the quantity $\hbar^2/2M$ (where M is the reduced mass of the neutron) as the unit of energy.

$$\Psi \sim I e^{ikz} + a(\theta)\,\frac{e^{ikr}}{r} \qquad (2)$$

Decomposition in partial waves of definite angular momentum, with eigen functions $P_\ell(\cos\theta)$, reduces the problem to one dimension. Having regard to the expansion of the plane wave

$$e^{ikz} = \sum_\ell i^\ell (2\ell+1)\, P_\ell(\cos\theta)\, j_\ell(kr) \quad , \qquad (3)$$

it is convenient to write for our solution

$$\Psi = \sum_\ell i^\ell (2\ell+1)\, P_\ell(\cos\theta)\, \psi_\ell(r) \qquad (4)$$

The radial components $\psi_\ell(r) = \varphi_\ell(r)/r$ are determined by the differential equation

$$\frac{d^2\varphi_\ell}{dr^2} + \left[k^2 - \frac{\ell(\ell+1)}{r^2} - U(r)\right]\varphi_\ell = 0 \qquad (5)$$

where $U(r)$ represents, in our units, the scattering potential; and they asymptotically reduce to outgoing sperical waves.

The short-range property of the potential allows us to define an "outside" region $r > r_0$ in which the scattered waves are essentially solutions of the field-free radial equation, i.e. linear conbinations of spherical Bessel functions. The combination having the required asymptotic form is the spherical Hankel function of the first kind

$$h_\ell^{(1)}(x) = \sqrt{\frac{\pi}{2x}}\, H_\ell^{(1)}(x) \sim i^{-(\ell+1)}\,\frac{e^{ix}}{x} \quad . \qquad (6)$$

In the outside region, the total radial component is therefore

$$\Psi_{out} = I\, j_\ell(kr) + a_\ell\, h_\ell^{(1)}(kr) \quad . \qquad (7)$$

In the"scattering region" $r < r_0$, the wave $\varphi_\ell(r)/r$ is distorted by the potential; it must be finite at the origin;

$$\varphi_\ell(0) = 0 \quad , \qquad (8)$$

and go smoothly over into the outside solution Ψ_{out} at $r = r_0$. These boundary conditions, with the notation

$$\hat{j}_\ell(x) \equiv x\, j_\ell(x) \quad ; \quad \hat{h}^{(1)}_\ell(x) \equiv x\, h^{(1)}_\ell(x) \tag{9}$$

may be written

$$\begin{aligned} I\,\hat{j}_\ell(kr_0) + a_\ell \hat{h}^{(1)}_\ell(kr_0) &= k\,\varphi_\ell(r_0;k)\,, \\ I\,\hat{j}'_\ell(kr_0) + a_\ell \hat{h}^{(1)\prime}_\ell(kr_0) &= k\,\varphi'_\ell(r_0;k)\,; \end{aligned} \tag{10}$$

the accent denotes the derivative with respect to r. Using the relation

$$\hat{j}_\ell \hat{h}^{(1)\prime}_\ell - \hat{j}'_\ell \hat{h}^{(1)}_\ell = i\,k\,, \tag{11}$$

we may obtain from the conditions (I0)

$$\begin{aligned} a_\ell &= -i\left[\varphi'_\ell(r_0;k)\,\hat{j}_\ell(kr_0) - \varphi_\ell(r_0;k)\,\hat{j}'_\ell(kr_0)\right] \\ I &= i\left[\varphi'_\ell(r_0;k)\,\hat{h}^{(1)}_\ell(kr_0) - \varphi_\ell(r_0;k)\,\hat{h}^{(1)\prime}_\ell(kr_0)\right] \end{aligned} \tag{12}$$

The second of these equations may be regarded as fixing the normalization of the interior solution φ_ℓ in terms of the incident amplitude, or alternatively, giving the value of the latter quantity which corresponds to any normalization of φ_ℓ .

The scattering phenomenom is essentially determined by the ratio of the scattering and incident amplitudes a_ℓ/I, i.e., by the value of the logarithmic derivative. $\varphi'_\ell/\varphi_\ell$ of the interior solution at the boundary of the scattering region. The differen - tial scattering cross-section $\sigma(\theta)d\Omega$ is given by

$$\sigma(\theta) = \frac{1}{k^2}\left|\sum_\ell (2\ell+1) P_\ell(\cos\theta)\ \frac{a_\ell}{I}\right|^2. \qquad (I3)$$

The total cross-section is directly expressed, as we have seen, by the imaginary part of the forward scattering amplitude $a(0)$ cf eq.(2) with our normalization, we get

$$\sigma_t = \frac{4\pi}{k}\ \mathrm{Im}\left[\frac{(a_\ell/I)(2\ell+1)}{ik}\right] \qquad (I4)$$

$$= -\frac{4\pi}{k^2}(2\ell+1)\,\mathrm{Re}\left(\frac{a_\ell}{I}\right)$$

At this stage, the discussion may be pursued in two different directions. Either one endeavours to compute the interior solution φ_ℓ, or at least its logarithmic derivative, in order to obtain numerical values for the cross-sections corresponding to a given form of potential: various methods of approximation have been devised for this purpose. Or one tries to study the general properties of the scattering amplitudes a_ℓ/I, without specifying the form of the potential, with a view to understanding the salient features of the scattering cross-sections, especially

the resonances they exhibit in their variation with the energy of the incident particle. It is this last aspect of the problem which will occupy us in the following.

- 2. - SCATTERING BY AN IMPENETRABLE SPHERE.

Before entering into the subject, however, we shall treat the singular case of scattering by an impenetrable sphere, which would correspond to a potential infinitely repulsive within a sphere of radius R. This may serve as a crude picture of the behaviour of a nucleus for slow neutrons; but the chief reason for considering this problem is that in the general analysis of the scattering amplitude for any short-range potential, a term formally identical to the scattering amplitude for an impenetrable sphere plays an important part.

In this case the interior solution $\varphi_\ell(r)$ reduces to zero, and the boundary conditions (I2) therefore yield (provided only that the derivative $\varphi_\ell'(r,k)$ does not vanish at $r=R$)

$$\frac{a_\ell}{I} = -\frac{\hat{j}_\ell(kR)}{\hat{h}^{(1)}_\ell(kR)} = -\frac{J_{\ell+\frac{1}{2}}(kR)}{1-i H^{(1)}_{\ell+\frac{1}{2}}(kR)} \qquad (I5)$$

This formula contains the full solution of the problem. It is not necessary for our purpose to develop its implications; but for the sake of illustrating the usual type of argument in the discussion of scattering problems, as well as for their own interest, we shall consider in some detail the limiting cases of small and large values of kR.

For small values of its argument, the Besse. funotion $J_p(x)$ varies as x^p . Using the expression

$$H^{(1)}_{\ell+\frac{1}{2}}(x) = J_{\ell+\frac{1}{2}}(x) - i(-1)^{\ell} J_{-(\ell+\frac{1}{2})}(x) \qquad (16)$$

for the Hankel function of index $(\ell + \frac{1}{2})$, we therefore see that the real part (with negative sign) of the scattering amplitudz a_ℓ / I reduces approximately to $\left| J_{\ell+\frac{1}{2}}(x) / J_{-(\ell+\frac{1}{2})}(x) \right|$, which is proportional to $x^{2\ell+1}$. The contributions σ_ℓ to the scattering cross-section from the successive partial waves thus decrease rapidly as ℓ increases, and only the S - wave scattering ($\ell = 0$) is important. For it, the scattering amplitude is

$$\frac{a_0}{I} = -i e^{-ikR} \sin kR \qquad (17)$$

and the scattering cross-section accordingly

$$\sigma_0 = \frac{4\pi}{k^2} \sin^2 kR \qquad (18)$$

In the limit of very small energies of the scattered particles, this becomes $\sigma_0 = 4\pi R^2$, four times the geometrical cross-section of the sphere.

If $kR \gg 1$, the main contribution to the scattering process will come from the partial waves for which $\ell \lesssim kR$: for the other partial waves correspond to classical trajectories remaining outside the scattering sphere -(their distance of nearest approach to the centre is given by $\sqrt{\ell(\ell+1)}/k$). For the partial waves of low angular momentum, we may use the asymptotic form of the Bessel and Hankel functions, valid when the modulus of the argument is much larger than that of the index (itself $\gg 1$):

$$\hat{J}_\ell(x) \sim \sin\left(x - \ell\frac{\pi}{2}\right) \quad , \quad \hat{H}^{(1)}_\ell(x) \sim i^{-(\ell+1)} e^{ix} \qquad (19)$$

We thus get for the total cross-section the estimate

$$\sigma \approx \frac{4\pi}{k^2} \sum_{\ell=0}^{[kR]} (2\ell+1) \sin^2\left(kR - \ell\frac{\pi}{2}\right)$$

This can be written

$$\sigma \approx \frac{4\pi}{k^2}\left[S^{(e)} \sin^2 kR + S^{(o)} \cos^2 kR \right] ,$$

where $S^{(e)}$, $S^{(o)}$ denote the sums $\sum(2\ell+1)$ extended over the even and odd values of ℓ , respectively, from 0 to the nearest integer $[kR]$ to kR .For the large values of kR , these two sums are approximately equal, and their common value is $\frac{1}{2} k^2 R^2$; therefore,

$$\sigma \approx 2\pi R^2 \qquad (kR \gg 1) \qquad (20)$$

It is at first sight surprising that one should get twice the geometrical cross-section in the limit in which one expects to find the classical particle behaviour. We are dealing, however, with one of the exceptional cases of "optical" phenomena in which diffraction effects modify the geometrical shadow even in the limit of vanishingly small wave-length. The diffraction cross-section, which has to be added to the geometrical cross-section of the "opaque" sphere, is immediately found in this case by an application of Babinet's principle. The latter tells us that the intensity of Fraunhofer diffraction by an opaque disk is the same as that by the complementary circular opening in an infinite opaque diaphragm. But since all the waves passing through the opening are diffracted, the corresponding cross-section is just the geometrical one.*

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*: This simple argument is restricted to the case of large values of kR, like Babinet's principle on which it rests. In fact the derivation of this principle requires the assumptions, approximately fulfilled only if $kR \gg 1$, that on the surface alternatively covered by the two complementary scatterers, the amplitudes of the waves vanish at any point occupied by scattering matter and keep their undisturbed values at every point where no such matter is present. Consider, then, the general relation, expressed by Huygens'principle, between the amplitude $a(P)$ at any point P and the values $a(P')$ of the amplitude on any surface Σ dividing space into an " external " region containing P and an " internal " region; by means of the Green's function $G(P',P)$ which vanishes everywhere on Σ, this relation may be written as

$$a(P) = \int_{\Sigma} a(P') \, \mathrm{grad}_{P'} G(P',P) \cdot \vec{n}_{P'} \, d\sigma_{P'}$$

where $\vec{n}_{P'}$ denotes the unit vector of the external normal at P'. This relation is rigorously satisfied, in the absence of any scatterer on Σ, by the undisturbed amplitude a_0. On the above assumptions we have also for the amplitudes a_1, a_2 corresponding to the two complementary scatterers on Σ

$$a_i(P) = \int_{\Sigma_i} a_0(P') \, \mathrm{grad}_{P'} G(P',P) \cdot \vec{n}_{P'} \, d\sigma_{P'} \qquad (i = 1,2)$$

see page 33 -

It is true that the angles of deflexion of the rays diffracted near the surface of the sphere are of the order $(kR)^{-1}$ and thus tend to zero in the limit of vanishing wave-length. But at the same time the diffracted flux contained in a cone with this aperture increases in such a way that it gives a finite contribution to the cross-section, approximately the same as that of the classically reflected part of the incident beam. This may be seen by analyzing the contribution of small-angle deflections to the scattering by means of the expression(I3) for the differential cross-section. To the same approximation as above, this expression becomes

$$\sigma(\theta) = \frac{1}{k^2}\left|\sum_{\ell=0}^{[kR]}(2\ell+1)P_\ell(\cos\theta)\, i^\ell \sin\left(kR - \ell\frac{\pi}{2}\right)\right|^2$$

$$= \frac{1}{k^2}\left| S^{(e)}(\theta)\sin kR + i\, S^{(o)}(\theta)\cos kR\right|^2$$

where the functions $S^{(e)}(\theta)$, $S^{(o)}(\theta)$ are defined as the sums $\sum(2\ell+1)P_\ell(\cos\theta)$ extended over the even and odd values of ℓ, respectively, from 0 to $[kR]$. For small angles θ, and large values of kR, one may again disregard the difference between $S^{(e)}$ and $S^{(o)}$ and take them both equal to I/2 the sum $S(\theta)$ extended over all values of ℓ. Then

$$\sigma(\theta) \approx \frac{1}{4k^2}S^2(\theta) = \frac{1}{4k^2}\left\{\sum_{\ell=0}^{[kR]}(2\ell+1)P_\ell(\cos\theta)\right\}^2.$$

Now, for small θ, the function $S(\theta)$ can be put into a simple closed form, first indicated by Wergeland.* One has

$$P_\ell(\cos\theta) = \sum_{n=0}^{\infty}\frac{1}{n!}\left(-2\sin^2\frac{\theta}{2}\right)^n P_\ell^{(n)}(1)$$

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where Σ_1, Σ_2 are the uncovered parts of Σ in each case. Therefore, to that approximation,

$$a_1 + a_2 = a_0$$

which is the general form of Babinet's principle. In the special case of Fraunhofer diffraction, when a collecting lens is placed behind the scatterer, we have in its local plane (excluding the focus itself) $a_0 = 0$ and, therefore, equality at every point of the intensities $|a_1|^2$, $|a_2|^2$.

* H.Wergeland, Dan.Mat.fys.Medd. 23 (I945) N°I4

with $P_\ell^{|0|}(1) = 1$

$$P_\ell^{|m|}(1) = \frac{(\ell+n)(\ell+n-1)\cdots(\ell-n+1)}{2^n \cdot n!}$$

(as one readily finds by using a recurrence formula for Legendre polynomials and their derivatives). An estimate of the sums over ℓ for large k will be obtained by replacing all factors of the form $(\ell \pm \rho)$ by ℓ. This gives

$$S(\theta) \approx \sum_{n=0}^{\infty} \frac{1}{n!} \left(-2\sin^2\frac{\theta}{2}\right)^n \frac{(kR)^{2n+2}}{2^n \cdot (n+1)!}$$

$$= (kR)^2 \sum_{n=0}^{\infty} \frac{(-1)^n \left(kR\sin\frac{\theta}{2}\right)^{2n}}{n!\,(n+1)!}$$

$$= (kR)^2 \frac{J_1\left(2kR\sin\frac{\theta}{2}\right)}{kR\sin\frac{\theta}{2}}$$

which is Wergeland's formula. The total contribution to the scattering cross-section from a cone of aperture θ_1 is accordingly

$$\sigma_1 = \int_{\text{Cone}(\theta_1)} \sigma(\theta)\, d\Omega = \pi R^2 \int_0^{z_1} \left[\frac{J_1(z)}{z}\right]^2 d(z^2) \qquad (21)$$

$$z_1 = 2kR\sin\frac{\theta_1}{2}$$

The integral in eq.(21) is easily evaluated, using recurrence formulae for Bessel functions and their derivatives: one gets

$$\sigma_1 = \pi R^2 \left[1 - J_0^2(z_1) - J_1^2(z_1)\right] \qquad (22)$$

For z_1, one may take, for instance, the value corresponding to the first zero of the function $S(\theta)$, i.e. $z_1 \approx 3.83$ or $\theta_1 \approx 3.83\,(kR)^{-1}$. The value of the coefficient of πR^2 in eq.(22) is then ≈ 0.84; it is insensitive to the choice of z_1, and actually tends to unity as z_1 increases indefinitely.

* H.Wergeland, Dan Mat.fys Medd.23(1945) N°14.

3.- GREEN'S FUNCTION AND INTEGRAL EQUATION FOR THE SCATTERING PROBLEM.

After this digression let us return to our scattering problem. As a preparation to our main task - the analysis of the resonances - let us first derive, on the general lines of the well-known theory of Sturm - Lionville problems, the integral equation for scattering, which is equivalent to the differential equation-(5). The latter, written as

$$D_r \varphi_\ell \equiv \left[\frac{d^2}{dr^2} + k^2 - \frac{\ell(\ell+1)}{r^2}\right] \varphi_\ell = v(r)\,\psi_\ell \,, \qquad (23)$$

may formally be regarded as an inhomogeneous equation with the "source function" $v(r)\,\varphi_\ell(r)$. The general solution of this equation is obtained by constructing a particular solution and adding to it any solution of the homogeneous equation, i.e. any linear combination of our functions $\hat{j}_\ell(kr)$, $\hat{n}_\ell(kr)$. We want the solution φ_ℓ corresponding to the superposition of the incoming plane wave, represented by its radial component $\hat{j}_\ell(kr)$, and the scattered wave, characterized by the conditions that it vanish at the origin and behave asymptotically as an outgoing wave.

To find this scattered wave, we first construct the Green's function satisfying the same boundary conditions. The Green's function $G_\ell(r,r')$ is, therefore, a linear combination of the solutions of the homogeneous equation $\hat{j}_\ell(kr)$ and $\hat{h}_\ell(kr)$, which satisfy the first and second condition, respectively. It must be such that the solution representing the scattered wave is connected with the source function $F(r)$ by

$$\psi_\ell^{(sc)}(r) = \int_0^\infty G_\ell(r,r')\,F(r')\,dr'$$

This means that $D_r G_\ell (r, r')$ must behave like the distribution $\delta (r-r')$ (i.e.that $G_1(r,r')$ is the solution for a point source at r'). Again, by a simple application of Green's theorem, this behaviour implies that the first derivative $dG_1(r, r')/ dr.$ is discontinuous at r=r' like the step function E(r-r'), defined as

$$E(x) = \begin{cases} 1 & \text{for } x > 0 , \\ 0 & \text{for } x < 0 . \end{cases} \qquad (24)$$

Now, for any two solutions u, v of $D_r u = 0$, one has $uv'-u'v =$ const. eq.(II) is an example of this, for which the constant is just ik. It is clear, therefore, that the required behaviour of $G_\ell (r, r')$ will be secured by putting

$$G_\ell(r,r') = \frac{1}{ik}\left[\hat{h}_\ell^{(1)}(kr)\,\hat{j}_\ell(kr')E(r-r') + \hat{h}_\ell^{(1)}(kr')\,\hat{j}_\ell(kr)E(r'-r)\right] \qquad (25)$$

We may now write down the required solution of eq.(23) in the form

$$\varphi_\ell(r) = I\,\hat{j}_\ell(kr) + \int_0^\infty G_\ell(r,r')\,v(r')\,\varphi_\ell(r')\,dr', \qquad (26)$$

which is the integral equation of the scattering problem. It gets a somewhat simpler form if we specify the normalization of $\varphi_\ell(r)$, and thereby, fix the constant I. It is convenient to normalise $\varphi_\ell(r)$ by the condition independent of K that for $r \to 0$ it reduces to $k^{-(\ell+1)}\hat{j}_\ell(kr)$, i.e.

$$\lim_{r\to 0} \frac{\varphi_\ell(r)}{r^{(\ell+1)}} = \frac{1}{1\cdot 3 \cdots (2\ell+1)} , \qquad (27)$$

This gives

$$\frac{1}{k^{\ell+1}} = I + \frac{1}{ik}\int_0^\infty \hat{h}_\ell^{(1)}(kr')\,v(r')\,\varphi_\ell(r')\,dr'$$

and eq.(26) may be rewritten as

$$\varphi_\ell(r) = \frac{1}{k^{\ell+1}} \hat{f}_\ell(kr) + \int_0^r Q_\ell(r,r') v(r') \varphi_\ell(r') dr' , \qquad (28)$$

with

$$Q_\ell(r,r') = \frac{1}{ik} \left[\hat{h}^{(1)}_\ell(kr) \hat{f}_\ell(kr') - \hat{h}^{(1)}_\ell(kr') \hat{f}_\ell(kr) \right] \qquad (29)$$

$$= \frac{1}{2ik} \left[\hat{h}^{(1)}_\ell(kr) \hat{h}^{(2)}_\ell(kr') - \hat{h}^{(1)}_\ell(kr') \hat{h}^{(2)}_\ell(kr) \right]$$

since

$$\hat{f}_\ell(x) = \frac{1}{2} \left[\hat{h}^{(1)}_\ell(x) + \hat{h}^{(2)}_\ell(x) \right] , \qquad (30)$$

Using the expression (28) for $\varphi_\ell(r)$, one readily derives from the conditions (I2) the following formula for the scattering amplitude:

$$-\frac{a_\ell}{I} = \frac{ik^\ell \int_0^{r_0} \hat{f}_\ell(kr') v(r') \varphi_\ell(r') dr'}{1 + ik^\ell \int_0^{r_0} \hat{h}^{(1)}_\ell(kr') v(r') \varphi_\ell(r') dr'} \qquad (3I)$$

The formulae(28),(3I) might form the basis of an iteration procedure for the explicit computation of the scattering cross-sections; in our discussion, however, they will be helpful in showing explicitly how the scattering amplitude depends on the spatial extension of the scattering potential.

Before we leave this aspect of the problem, let us observe that also the bound states of the system can be determined by an integral equation similar to eq.(26). These states correspond to negative values of the energy E, i.e. to purely imaginary values of k ($= i\kappa$). The eigenfunctions $\Phi_\ell(r, \kappa)$ satisfy, by the same argument as above, the integral equation

$$\Phi_\ell(r,\varkappa) = V\int_0^\infty \Gamma_\ell(r,r')\,w(r')\,\Phi_\ell(r'\varkappa)\,dr' \qquad (32)$$

in which the potential v(r) has been written as the product Vw(r) of a suitably normalized function w(r) with a"strength parameter" V; the Green's function $\Gamma_\ell(r,r')$ is obtained from $G_\ell(r,r')$, eq.(25), by the substitution $k = i\varkappa$. The boundary condition at infinity, that the eigenfunctions vanish, is satisfied provided that is specified to be non-negative. The function $\Gamma_\ell(r,r')$ is then negative definite; on the other hand , if the potential has the same sign for all r, we may always take w(r) positive, the attractive or repulsive character of the force being indicated by the sign of V.

According to the theory of integral equations of Fredholm type, the equation(32), for any fixed $\varkappa$, has in general an infinite sequence of eigensolutions, belonging to eigenvalues V_i (i = 1, 2,....) of the potential strength V, which are necessarily negative (or zero). In other words, we have a bound state of energy $-\varkappa^2$ only if the potential is attractive and has a definite strength $V_i(\varkappa)$ (i = I, 2,.....). This way of looking at the eigenvalue problem is of practical interest when one knows, empirically, the values of the binding energy, but not the strength of the potential: a famous example is that of the ground state of the deuteron.

If the $V_i(\varkappa)$ have been determined for all $\varkappa$, one may reverse the question and ask for the eigenvalues of $\varkappa$ belonging to a given potential strength V. In this respect, an essential difference arises according as the potential has a finite or infinite range. In the first case, there corresponds to the limiting value $\varkappa = 0$ a sequence of distinct eigenvalues $V_i(0)$, the so-called"critical strengths". In a graphical representation, the curves $V_i(\varkappa)$ start from a sequence of points on the V-axis and, obviously, increase monotonically with increasing $\varkappa$.

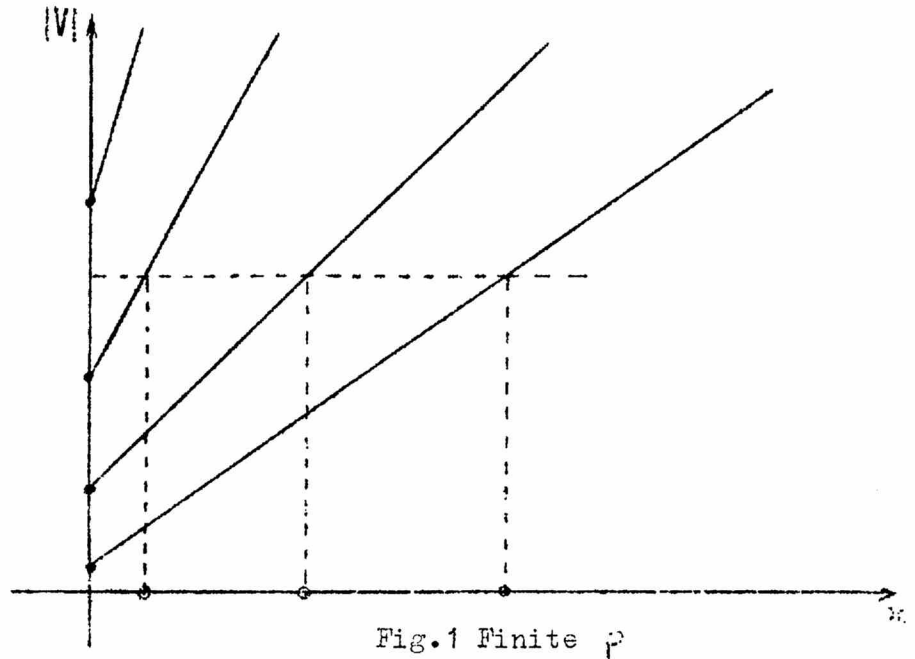

Fig.1 Finite ρ

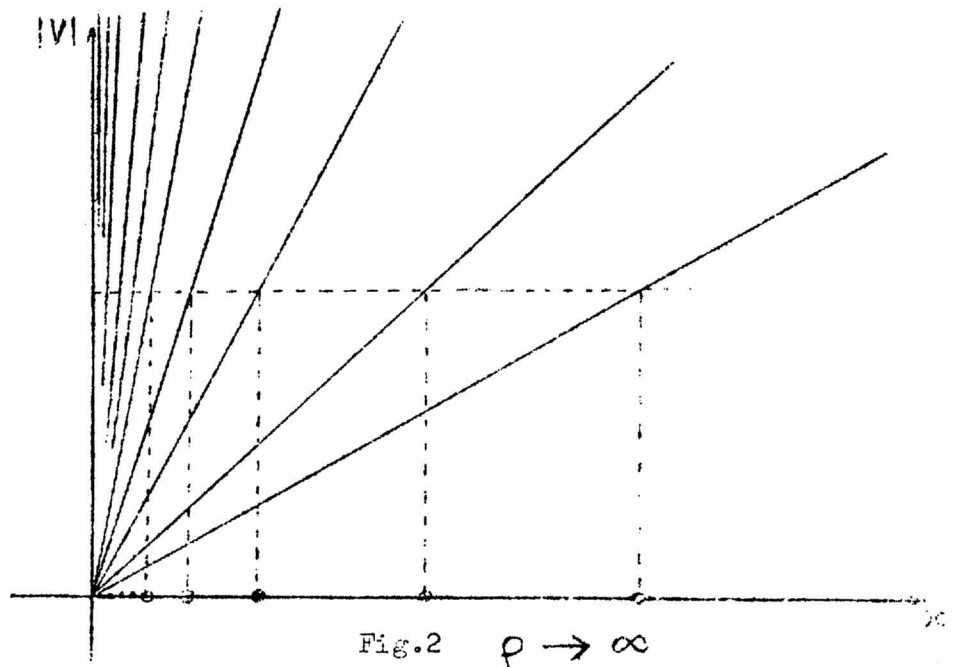

Fig.2 $\rho \to \infty$

A parallel to the $\varkappa$ - axis, corresponding to a given strength V, will accordingly only cut a finite number of $V_i(\varkappa)$-curves: thus, to any potential strength V there is only a finite number of bound states. To study the influence of increasing range ρ of the potential, the only interesting case to consider is that of a potential $w(r/\rho)$ varying as $1/r$ for small r (a potential finite at r = 0 would just be flattened out when $\rho \to \infty$). From simple considerations of dimensions, it is clear from the form of eq.(23) or (32) that in the case just stated the critical strengths vary as $1/\rho$: when the range becomes infinite, they all collapse to the value 0. All curves $V_i(\varkappa)$ accordingly start from the origin, and every parallel to the $\varkappa$ -axis outs them all: for every strength there is an infinite sequence of bound states.

The situation is well illustrated by the case of the Hulthén potential

$$w\left(\frac{r}{\rho}\right) = \frac{1}{\rho} \cdot \frac{e^{-\frac{r}{\rho}}}{1 - e^{-\frac{r}{\rho}}}$$

for which one finds *

$$2\varkappa_n = \frac{|V|}{n} - \frac{n}{\rho}$$

The figure (page 39) shows how the case of finite ρ goes over, when $\rho \to \infty$, into the "Balmer formula".

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* See L.Rosenfeld, Nuclear Forces (Amsterdam,1948) 5.22.

4.- CHARACTERIZATION OF THE RESONANCES.

The occurrence of a maximum of the scattering cross-section for a certain value of the energy of the incident particle is an indication that at this energy there is a state of the system of the scatterer plus parricle which-while not being stationary like the true bound states-partekes to some extent of the character of the latter: the particle impinging with this energy has an appreciable probability, before being scattered, of remaining quasibound within the range of the (attractive) potential during a time much longer than that required for just moving across the scattering region. The ultimate aim of our analysis is to derive a "dispersion formula" for the scattering cross-section, in which the contributions from the various quasi-stationary or virtual states of the system appear explicitly and exhibit the expected resonance behaviour.

For a first orientation it is natural to follow the analogy of optical dispersion, although it must be realized at the outset that the smallness of the electromagnetic coupling confers to the optical scattering problem a simplicity which is entirely lacking in the nuclear case. The virtual states which produce the optical resonances are the "excited states" of the atomic scatterer in which the impinging photon has been absorbed. Owing to the smallness of the interaction between the atomic scatterer and the radiation field, these excited states of the atom may be treated, to a first approximation, as stationary, and then define a complete orthonormal set of eigenfunctions $|n\rangle$, with eigenvalues E_n, which form a basis for the description of the scatterer in the absence of free photons. Moreover, the calculation of the scattered amplitude may be carried out by perturbation methods. It is thus found, in the usual way [*], that the scattering amplitude $a(\theta)$, is proportional to the matrix element, between initial and final state, of the operator

$$H_1 + \sum_n \frac{H_1|n\rangle\langle n|H_1}{E - E_n} ,$$

[*] See P.Dirac, The principles of quantum mechanics,(Oxford,1930) 59.60 (2nd Ed.(1935) 54.55. 3rd.Ed.

where H_1 is the interaction operator, and $E = E_o + \varepsilon_\omega$ is the total energy of the scattering state, given by the sum of the initial energy E_o of the scatterer and the energy ε_ω of the impinging photon. A more refined approximation, when E is very close to one of the excited states E_n, leads to the replacement, in this formula, of E_n by the complex quantity $E_n - I/2\,\Gamma_n$, with $\Gamma_n > o$ (we disregard the shift of the real part). Thus account is taken of the finite life-time of the excited states: the eigenfunction $|n\rangle$ is multiplied by a factor

$$\exp\left[-\frac{i}{\hbar}\frac{\hbar^2}{2M}\left(E_n - \frac{1}{2}i\Gamma_n\right)t\right]$$

decaying exponentially in the course of time.

The transposition of this theory to our case of scattering by a short range potential of the nuclear type is by no means straightforward. In view of the large interactions involved, we must expect the widths of the resonances to be much larger, in relation to the resonance energies, than in the optical case: the whole framework of optical dispersion theory collapses, and to begin with, the very definition of the virtual states raises a delicate problem.

We may observe that the bound states appear as solutions of an extension of the general eq.(26) to purely imaginary values of the parameter k with the added condition I = 0, which means that in these states there is no free particle. The above consirations about the character of the virtual states suggest a further extension of the interpretation of the scattering equation (26) to include solutions corresponding to complex values of the parameter k; and the same condition I = 0 seems quite appropriate to characterize the virtual states as well as the true bound states.

In its explicit form, given by eq.(I2) the equation I = 0 appears as a boundary condition imposed on the logarithmic derivative of the solution:

$$\frac{\varphi_\ell'(r_0; k_n)}{\varphi_\ell(r_0; k_n)} = \frac{\hat{h}_\ell^{(1)\prime}(k_n r_0)}{\hat{h}_\ell^{(1)}(k_n r_0)} \qquad (33)$$

This determines a set of solutions $\varphi_\ell(r; k_n)$ corresponding to complex values of k, $k_n = \alpha_n + i\beta_n$, and therefore also, in general, to complex values of

$$k_n^2 = \alpha_n^2 - \beta_n^2 + 2i\alpha_n\beta_n$$

In fact, the only stationary states of the system, i.e. those for which k^2 is real, are those already mentioned, which correspond to k either real (scattering states, $I \neq 0$) or purely imaginary (bound states).

Since the wave equation (23) only depends on k^2, its solutions are even functions of the variable k. The solutions of eq.(26), with the normalization (27), have this property, since the equation then takes the equivalent form (28), which is invariant for a change of sign of k. The eigenfunctions of the bound states, solutions of eq. (32), are likewise independent of the sign of χ , since the equation, combined with the condition $I = 0$, may be put into the same form as eq.-(28). Moreover, the non-stationary states occur in pairs corresponding to values k_n and k_n^* of k, since the solution

$$[\varphi_\ell(r; k_n)]^* = \varphi_\ell(r; -k_n^*)$$

also satisfies the condition (33), as is easily verified by using simple properties of the Hankel functions. As a result, one finds that the imaginery part β_n of k_n (and $-k_n^*$) is necessarily <u>negative</u>(provided that the real part α_n is $\neq 0$). For, by a familiar procedure, starting from

$$\int_0^{r_0} [\varphi_\ell D_r^* \varphi_\ell^* - \varphi_\ell^* D_r \varphi_\ell]\, dr = 0$$

and taking account of boundary condition (33), one gets:

$$(k_n^{*2} - k_n^2)\int_0^{r_0} |\varphi_\ell|^2 dr = |\varphi_\ell(r_0; k_n)|^2 \left[\frac{\hat{h}_\ell^{(1)'}(k_n r_0)}{\hat{h}_\ell^{(1)}(k_n r_0)} - \frac{\hat{h}_\ell^{(1)'}(-k_n^* r_0)}{\hat{h}_\ell^{(1)}(-k_n^* r_0)}\right]$$

and one can show that the factor between brackets has the form

$$i(k_n + k_n^*)M_\ell - (k_n^{*2} - k_n^2)N_\ell$$

with positive coefficients M_ℓ, N_ℓ.

The "eigenvalues" of k defined by the condition $I = 0$ thus consist of the set of pairs of complex values $\pm\alpha_n - i\beta_n$ with $\beta_n < 0$, and a number, finite as we have seen, of purely imaginary values iK_m ($K_m > 0$). The non-stationary states with a positive real part of the eigenvalue k_n decay in time at the rate $\frac{\Gamma_n}{2}\frac{\hbar}{M}$, with $\Gamma_n = 4\alpha_n|\beta_n|$: their "eigenfunctions" accordingly behave at large distance like outgoing waves of <u>increasing</u> amplitude $e^{i\alpha_n r_\ell |\beta_n| r}$. The associated solutions, with eigenvalues $-k_n^*$, have exactly the same behaviour, since their time factor is the complex conjugate of the original one. The exponential increase of the amplitude with distance, which contrasts so strikingly with the behaviour of the bound state, seems at first sight surprising, but is easily understood. This is an old paradox, well known from Gamow's theory of α-radioactivity, where it was encountered for the first time; since particles are continually emitted from the centre and move away with the radial velocity $v = \alpha_n \hbar/M$, the density at any distance r

...

will be determined at any time t by the value which the central density had at time $t - \frac{r}{v}$; and this was larger than that at time t by factor $\exp.\left[\frac{\Gamma_n \hbar}{2M} \cdot \frac{r}{v}\right] = \exp\left(2|\beta_n| r\right)$.

The set of virtual states defined by the condition $I = 0$ has the drawback, however, that the corresponding eigenfunctions are not orthogonal in the interval $(0, r_o)$; in fact, one finds in the usual way

$$(k_m^2 - k_n^2)\int_0^{r_o} \varphi_n \varphi_m \, dr = \varphi_n(r_o)\,\varphi_m(r_o)\left[\frac{\varphi_n'}{\varphi_n} - \frac{\varphi_m'}{\varphi_m}\right]_{r=r_o}$$

(We have put for brevity $\varphi_n \equiv \varphi_l(r; k_n)$.) The factor within square brackets does not vanish because the boundary condition (33) gives to the logarithmic derivative φ_n'/φ_n at $r = r_o$ a value depending on k_n. This remark suggests a possible remedy: would it be possible to modify the boundary condition in such a way that it fixes the logarithmic derivative independently of k_n, without altering too much the essential features of the virtual states?

The answer proposed by Kapur and Peierls* consists in taking, in the right-hand side of the eq.(33) the Hankel functions for a constant argument kr_o instead of $k_n r_o$, k being a real parameter: of course, when the corresponding set of virtual states will be applied to the solution of the scattering problem, the parameter k will be identified with the wave number of the incident wave. The virtual states defined by the Kapur-Peierls form an orthogonal set, at least in the sence that $\int_0^{r_o} \varphi_m \varphi_n \, dr$, rather than the usual $\int_0^{r_o} \varphi_m^* \varphi_n \, dr$, vanishes for $k_m \neq k_n$. No attempt has ever been made to investigate under what conditions this set may also be complete: it will simply be assumed that it has this essential property. The price to pay for this advantage,

* P.L.Kapur and R.Peierls, Proc.Roy Soc. A 166(1938) 277.

however, is that the definition of the set of virtual states determining the resonances now depends on the ernergy of the impinging particle: each scattering state has its own set of virtual states, whose energies and widths will vary with the energy of the scattering process. It is worth while, nevertheless, to examine Kapur and Peierl's treatment in some detail, so as to assess more prec isely its advantages and shortcomings. We shall then come back to the more satisfactory boundary condition (33) and develop on its basis an appropriate method of discussion of the resonance properties.

CHAPTER IV. SCATTERING OF PARTICLES BYSHORT RANGE POTENTIAL

(continued)

I. - DISPERSION THEORY OF KAPUR AND PEIERLS.

[An exposition of the paper by these authors quoted above. This theory has the following shortcomings:

(a) it gives a simple dispersion formula only for narrow levels of low energy,

(b) the position of the resonances, as well as the magnitude of their contribution, compared with that ascribed to resonance scattering, depends sensitively on the choice of the limit r_0 of the scattering region.]

2. - DISPERSION THEORY OF HUMBLET.

J.Humblet, Mémoires in -8° de la Soc.Roy. des Sciences de Liège, I2 (I952) N° 4

[This theory is based on boundary condition (33). The resonance behaviour is obtained as a Mittag-Leffler expansion of the scattering amplitude. The drawbacks (a) and (b) are eliminated. The contribution from potential scattering is much reduced. The interference between it and the resonances is incorporated in the latter, where it appears in the asymmetrical form of the numerators of the resonance terms.]

PART II- ELEMENTARY PARTICLES

CHAPTER I. FERMIONS AND BOSONS.

I.- INTRODUCTION. Quantum theory has brought a new conception of matter. The old dualism of matter and force is eliminated, since it is recognised that every " agent " can occur under two aspects: as "particle", in which it is a source of other agents and acted upon by them, and as "field" transmitting force between other agents. The electromagnetic agent is known classically as field created by charged particles and acting upon them; in its particle aspect, as photon, it is the source of pair fields and is acted upon by such fields. Electrons are observed in the classical limit as charged particles. They appear in their field aspect either as pair fields or in producing chemical forces.

The two aspects are complementarity, i.e. mutually exclusive : if an agent acts as a field in some situation, it cannot be isolated as particle without destroying the situation. From this complementarity, there follows an important relation between the mass of the particle and the range of the force transmitted by the associated field: the range is of the order of the "Compton wave-length" $\hbar/Mc$, where M is the mass. This relation led Yukawa to the prediction of the existence of mesons as the particle aspect of the nuclear field.

All the agents are coupled together; the fundamental problem is thus again twofold: we must try to determine the characteristics of the "elementary" agents and define their mutual couplings.

The concept "elementary" at the present stage is used in the same pragmatic sense as by the chemists. The eventual reduction of the number of elementary agents is a matter to be decided by experience. Speculative"principles" of structure may inspire new experiments; but they may just as well lead us astray.

Lavoisier's"principle" of structure for acids prevented for some time the recognition of "dephlogisticated muriatic acid " as the element chlorine. The early principle of structure according to which nuclei are composed of protons and electrons created the "paradox of the ^{14}N spin", which could only be solved, after the discovery of the neutron, by discarding the principle and replacing it by a better founded one.

2.- FERMIONS AND BOSONS. The most important distinction enabling a classification of the elementary agents is that between fermions and bosons, i.e. agents obeying the exclusion principle and those forming entirely symmetrical configurations.

Intermediate configurations are by no means excluded: they occur for instance in atomic and molecular systems when the spin- orbit coupling may be neglected. There is no decisive argument to exclude such configurations at the level of "elementary" agents. One argument against their occurrence is that they contain an essential degeneracy. The symmetrical and the antisymmetrical configuration are the only non degenerate ones.

Boson fields , as observable [*] are quantized in the canonical way. It is then possible to define non-hermitian creation and annihilation operators in terms of the field variables, and they are found to obey commutation rules of the type

$$[a_i , a_k]_- = [a_i^+ , a_k^+]_- = 0 \quad ; \quad [a_i^+ , a_k]_- = \delta_{ik}$$

with the minus sign.

For Fermion fields, the quantization is not canonical, since the field variables are not observables. Formally, one may imitate the procedure developed for Bosons, with the difference that the commutators are(arbitrarily replaced by anticommutators. The operators a^+ , a are then seen to have the properties of creation and annihilation operators in harmony with the exclusion

[*] Charged bosons are described by pairs of harmitian variables.

principle. This is often called "second quantization" because the Fermion field itself is (rightly) regarded as the product of a first (legitimate) quantization of classical dynamics. This is unneccessarily confusing. The introduction of the operators a, a^+ is not a quantization, but just a definition of convenient operators related to the number variables $N = a^+ a$ in a way compatible with the exclusion principle. The latter is independent of the rules of quantization.

The construction of the operators a, a^+ is easily affected by the technique of dichotomic variables. For this technique, see Nuclear Forces 4.I. Every time there is a distinction between two alternatives, a "dichotomy", it can be expressed by a specific dichotomic variable. This entails the introduction of a "dichotomic space", characterized by 3 matrices σ_1 , σ_2 , σ_3 , such that the most general dichotomic variable in this space is $\vec{\sigma} \cdot \vec{n}$, where $\vec{n}$ is an arbitrary unit vector. The group of unitary transformations of dichotomic space is isomorphous with the rotation group in 3 dimensional space.

Thus, owing to the exclusion principle, the number variable N for any state of the Fermion field is associated with a dichotomic variable ν_2 :

$$N = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{2}\left(1 - \nu_2\right)$$

The annihilation operator

$$a \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix} = \begin{pmatrix} \psi_1 \\ 0 \end{pmatrix}$$

is thus

$$a = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \frac{1}{2}\left(\nu_1 + i\,\nu_2\right)$$

and the creation operator accordingly

$$a^+ = \frac{1}{2}\left(\nu_1 - i\,\nu_2\right)$$

One verifies immediately the anticommutation rule

$$a\,a^+ + a^+ a = 1$$

as well as $a^+ a = \frac{1}{2}\left(1 + i\,\nu_1\,\nu_2\right) = \frac{1}{2}\left(1 - \nu_3\right) = N$

For a set of Fermions states, enumerated in a definite order, one builds up direct products of operators a, 1 and ν_3, the last being the same as $(-1)^N$ thus:

$$\underline{a_i} = \nu_3 \times \nu_3 \times \ldots\ldots \times a_i \times 1 \times 1 \times \ldots\ldots$$

$$\underline{a_i^+} = \nu_3 \times \nu_3 \times \ldots\ldots \times a_i^+ \times 1 \times 1 \times \ldots\ldots$$

We have $\nu_3\, a_i = a_i$ and $a_i\, \nu_3 = -a_i$

One can then check all anticommutation rules.

3. - Fermion families. The question arises, whether the operators belonging to different Fermion fields should be assumed to anticommute or to commute. In fact, one may even

ask the question when considering different states of the same Fermion field. The question was raised with respect to electrons with spin up and spin down: in this case (assuming only direct, non relativistic interactions of the type $\sum_{ijkl} a_i^\dagger a_k^\dagger a_l a_j V_{(ik,jl)}$) one can show that it is immaterial which alternative one chooses: the exclusion principle may be formulated for the two"species" separately, or for the one species differenciated by the spin variable, with . exactly the same result. If the operators pertaining to the two states. a_1 , a_2 are assumed to anticommute, then the equivalent operators

$$b_1 = a_1$$

$$b_2 = (-1)^{\sum N_i} a_2$$

(the sum pertains to the possibility of a set of states defined by other variables)

will commute: $[b_1, b_2]_- = 0$ .This transformation was given by Klein.

However, it has been pointed out by Umezawa, Podolanski and Oneda (Proc.Phys.Soc.A.68 (1955) 503) that the possibility of such an indifferent choice is severely limited. In general the Klein transformation introduces a non-local element in the interaction: if one requires that the interaction shall be local, the choice between commutator or anticommutator can be unequivocally decided. The argument is based on the very simple remark that in any interaction term containing a product of creation or annihilation operators, only an even number of such operators anticommuting with a given field operator can occur.

From the examination of known processes involving various types of elementary agents, it is possible, by application of this principle, to draw the following conclusions:

1°. The neutral agents, π°, θ°, τ° and the photon, commute with any field and are accordingly bosons.

2°. The pairs (π^+, π^-), (θ^+, θ^-), (τ^+, τ^-) have all the same commutation rules with any field. Hence the charged mesons are either all fermions or all bosons.

3°. Each of the groups $(\Lambda^\pm P)$, $(\Lambda^\circ N)$ has the same commutation rules with any field. Thus, if one assumes $[P,P]_+$ and $[N,N]_+$, one has also $[\Lambda^\circ,\Lambda^\circ]_+$, $[\Lambda^\pm,\Lambda^\pm]_+$, $[P,\Lambda^\pm]_+$ and $[N,\Lambda^\circ]_+$ (the symbol means "anti-commutation rules").

4°. All baryons have the same commutation rules with the charged pions. Thus

either $[P,N]_+$ and $[PN,\pi^\pm]_-$

or $[P,N]_-$ and $[PN,\pi^\pm]_+$

5°. For the leptons, one finds that $\mu^\pm$ and $e^\pm$ have the same commutation rules with any field. Therefore, since the $e^\pm$ anticommute, so do the $\mu^\pm$ and $[\mu^\pm,e^\pm]_+$

The alternative 4° cannot be resolved without a further assumption. Kinoshita proposes the assumption that bosons commute with all fields: this removes the last alternative and leads to the conclusion that all baryons anticommute. Umezawa et al. introduce the concept of fermion family: a family is a set of anticommuting fields, commuting with all other fields. The assumption of the existence of families among the fermions is somewhat more restrictive than Kinoshita's assumption (because it imposes commutativity also on the fermions of different families). If the baryons form a family, one again gets the first

alternative. Since the only known interaction between baryons and leptons (the β - interaction) involves even numbers of each type, one cannot decide whether or not the leptons form a distinct family.

CHAPTER II - TRANSFORMATION PROPERTIES OF SPINOR FIELDS.

1. - Introduction.

Invariance requirements with respect to certain groups of transformation are one of the most powerful helps in setting up tentative expressions for the interaction Lagrangians between various types of fields, in order to confront their consequences with experience and so reduce the number of possibilities, and ultimately, perhaps, arrive at the actual expression for the interaction.

Boson fields are represented by usual observables, which have tensor properties with respect to the (restricted) Lorentz group and have an even or odd parity with respect to space reflexions. They need no special discussion, except as regards their isobaric properties, on which we shall comment later. The fermion fields, on the other hand, are represented by spinors and it is necessary to specify their transformation properties; this will now be done by the technique of dichotomic variables. This method has the drawback of not being "manifestly" covariant, but it allows one to write down (in a special representation, it is true) explicit expressions for the transformation operators, which can be found quite easily by simple inspection.

2.- ... CONTINUOUS TRANSFORMATIONS.

The linearisation of the energy-momentum relation

$$E^2 = \vec{p}^2 + m^2 \qquad (c = 1)$$

is effected by two dichotomic variables σ, ρ . This is based on the product relation

$$\vec{\sigma}(\vec{n})\,\vec{\sigma}(\vec{n}') = \vec{n}.\vec{n}' + i\,\vec{\sigma}(\vec{n}\wedge\vec{n}')$$

Thus,

$$\vec{p}^{\,2} = (\vec{p}.\vec{\sigma})^2$$

and a further linearisation is effected by another variable ρ ; here one has only two of the 3 components ρ_1, ρ_2, ρ_3 appearing explicitly. The usual choice is

$$E = \rho_1\,\vec{\sigma}.\vec{p} + \rho_3\,m$$

The variable σ_3 discriminates between the two " spin" states $\rho 3$ between the two signs of the mass.

For a continuous group, the infinitesimal transformation dq' of the variable q' , defined as

$$Q' = q' - dq'$$

(transformation of the frame of reference)corresponds for the state vectors to a unitary operator

$$D_{dq'} = 1 - \frac{i}{\hbar}\,dq'.p$$

The operator p must be hermitian (and therefore D unitary) in order to ensure invariance of the scalar product.
Alternatively, one may leave the state vectors unchanged and apply the transformation to the quantal variables. Then, the new variable $A^{(1)}$ is given in terms of the old A by

$$A^{(1)} = D^{-1}AD = A + \frac{i}{\hbar}\,dq'[p,A]$$

This shows (1) that the operator p is canonically conjugate to q (since one must have $q'_{qp} = q_{qp} + q'$)

(2) that if the Hamiltonian is invariant for the transformations in question, p is an integral.

The finite transformation is expressed by

$$D(q') = e^{-\frac{i}{\hbar} q'p} = e^{-q'\frac{\partial}{\partial q'}}$$

The relation between invariance for a continuous group and conservation law, being expressible in terms of Poisson brackets, remains valid in the classical limit so long as it concerns observables with classical correspondence. We have full correspondance with classical theory. The physically important cases are:

group	conservation law
space translation	momentum
time translation	energy
space rotation	angular momentum
Galilean relativity(translation in momentum space)	centre of mass

A non-classical example is that of the electromagnetic gauge-group, which, when the gauge transformation is combined with a transformation of the phase of the (complex) charged field, yields the law of conservation of electric charge.

The rotation in spin space around axis 1 by an angle ϑ is effected by the transformation

$$D = e^{-\frac{1}{2} i \vartheta \sigma_1}$$

from which it is apparent that the spin contributes an angular momentum $\frac{\hbar}{2}\vec{\sigma}$. The argument leading to this result is typical for all the following:

First apply to all tensor variables the usual transformation $e^{-\frac{i}{\hbar}\theta L_x}$ (L_x = x - component of "orbital"angular momentum).

This transforms $\vec{\sigma}, \vec{p}$ into $\vec{\sigma}^{(0)}, \vec{p}^{(0)}$,where $\vec{\sigma}^{(0)}$ is a linear combination of the fixed matrices σ_1 , σ_2 , σ_3 . We must now introduce a transformation in spin space to restore the $\vec{\sigma}^{(0)}$ to the fixed form $\vec{\sigma}$:

$$D\,\vec{\sigma}^{(0)}D^{-1} = \vec{\sigma}\,.$$

3. DISCONTINUOUS TRANSFORMATIONS.

The discontinuous transformations such as spatial reflexion or charge conjugation are entirely unclassical in character. In classical physics, one may exclude them because they would imply transitions of a type which cannot be described in classical terms. Thus, the transition from levogyre to dextrogyre configuration of a molecule involves a "tunnel effect" through a potential barrier; the creation of an electron pair involves a transition from negative to positive mass over a "gap", i.e. a region in which the energy is not defined classically. Hence, the associated conservation laws concern things with no classical correspondence, such as "parity" or " number of particles minus antiparticles".

The spinor transformation for space reflexion is

$$D = \varepsilon\,\rho_3$$

with a phase factor ϵ .On repeated reflexion, one may either restore the original spinor or its opposite: $\epsilon^2 = \pm 1$. Hence, there are two classes of spinors according as $\epsilon = 1$ or i (the double sign is immaterial, since the two signs occur in ρ_3). The transformation ρ_3 means that if one analyses the 4-component wave-function into two two-component spinors ξ , η belonging to $\rho_3' = 1$ and -1 , respectively, these two spinors behave in opposite ways under space reflexion:

$$\xi^{(1)} = \epsilon \xi \ , \quad \eta^{(1)} = -\epsilon \eta \ .$$

The two kinds of spinors are irreducible to each other.

To discuss time reflexion, one usually considers the reflexion in all 4 coordinates. Then, the unitary transformation ρ_1 will do, provided one also changes the sign of the charge: this is the Schwinger or strong reflexion. Cancelling space reflexion one gets

$$D_5 = i \rho_2 \epsilon$$

One may avoid the change of sign of the charge, but at the price of sacrificing unitarity (Wigner) by passing to the <u>complex conjugate</u> (or transposed) equation. Without space reflexion, the Wigner transformation is

$$\chi = \epsilon \sigma_2 \psi^*$$

The matrix σ_2 is in our representation the "B matrix" of Pauli, which transforms in any representation $\vec{\sigma}^*$ into $-\vec{\sigma}$.

The anticonjugation is effected by

$$\varphi = \epsilon' \rho_2 \sigma_2 \psi^*$$

Hence, it is obvious that the product of any two of SR, WR and C is equivalent to the third. The Lüders theorem states that S R invariance is implied by Lorentz invariance (and local character) of a Lagrangian involving all types of fermions and bosons. Hence , one has always invariance for combined P (space reflexion), T (time reversal) and C(anticonjugation).

4.- ISOBARIC SPIN.

Besides the charge symmetry expressed by anticonjugation invariance, one has more extended charge invariance properties when one includes neutral particles (which may or may not have distinct anticonjugates). The distinction between charged and neutral particles of a certain type is effected by a dichotomic variable T called isobaric spin. For a stable system of nucleons, we have from the first only to expect conservation of a 3-component $\sum \tau_3$: this, combined with the constancy of the total number of nucleons, expresses the constancy of the number of protons and is thus, in this case, identical with the conservation of charge. The invariance for rotations around the 3-axis inisobaric space implies for the nuclear forces, the property known as " charge symmetry":the forces between protons are the same as those between neutrons. This is less than the "charge independence", which also asserts equality of the proton-neutron forces with the others:"charge independence" is a consequence of the wider requirement of invariance for <u>all</u> rotations in isobaric space. We then have as a further invariant the isobaric spin quantum number,defined by $T(T+1) = (\frac{1}{2} \sum \vec{\tau})^2$. There is strong evidence, in some light nuclei, for the existence of states forming an isobaric triplet T = 1, with binding energies very close to each other.

The concept of isobaric spin may be extended in such a way that the conservation law applies to interactions involving other types of particles than the nucleons. The Λ_0 - baryon , e.g., existing only in the neutral state, will be assigned the isobaric spin 0. The Σ -baryon and the π -meson,on the other

hand, exist in three charged states , which are conveniently treated as corresponding to isobaric spin 1.

One may ask whether reflexions in isobaric space should also be considered. D'Espagnat and Prentki have shown that this leads to a further quantum number equivalent to Gell-Mann's strangeness. One has to characterize each particle by the factor by which it becomes multiplied on inversion in isobaric space. For the isobosons ($T = 0, 1$) this can only be ± 1 ; for the isofermions, it is necessary to consider the two irreducible possibilities $\pm \varepsilon$ and it is most convenient to take $\varepsilon = i$. Thus, we have a factor i^U , with $U = \pm 1$ for the isofermions (of Ist. and 2nd.kind), and $U = 0$ for the isobosons; $U = \pm 2$, which would indicate odd parity, does not occur: it is perhaps significant that in contrast with the other cases odd parity would not be unambiguously defined by U. For anti - isofermions, the isoparity is - U. The invariance of ΣU expresses the conservation of isofermions (minus anti-isofermions) .

The electric charge Q is connected with isobaric spin and isoparity by the remarkable relation

$$Q = T_3 \pm \frac{1}{2} U$$

This allows a geometrical interpretation of the operator Consider the transformation

$$e^{i\pi Q} = e^{i\pi T_3} e^{i\frac{\pi}{2} U}$$

	T	U
Ξ	I/2	- 1
Σ	1	0
Λ_0	0	0
N	I/2	1
π	1	0
K	I/2	1

it represents a reflexion, followed by a rotation by π around the 3-axis, i.e. a reflexion by the plane perpendicular to the 3-axis. This mirroring has the effect of multiplying the field variable by the factor $(1)^Q$.

5. - THE NEUTRINO.

The equation for the neutrino

$$(\rho_1 \vec{\sigma} . \vec{p} - E)\,\psi = 0$$

can be split into two two-component equations, corresponding to the eigenvalues ± 1 of ρ_1 . This can be done, without changing the representation, by applying the projection operators

$$P = \tfrac{1}{2}(1 + \rho_1) \quad , \quad P' = \tfrac{1}{2}(1 - \rho_1)$$

(One has $P^2 = P$, $P'^2 = P'$, $PP' = 0$, $P + P' = 1$).

Put $\varphi = P\psi$ $\varphi' = P'\psi$

One has

$$\left[\vec{\sigma} . \vec{p} - E\right]\varphi = 0 \qquad \left[\vec{\sigma} . (-\vec{p}) - E\right]\varphi' = 0$$

They represent the two types of longitudinal polarisation. They go over into each other by space reflexion and by anticonjugation.

J.Serpe (Physica 18 (1952), 295) has shown that the restriction to one type of longitudinal neutrino, say φ, is equivalent to imposing a Majorana representation. To define the latter, it is convenient to use a representation in terms of annihilation and creation operators

$$\varphi = \sum a_{\vec{p}} u_{\vec{p}}$$

(the $u_{\vec{p}}$ is what we had hitherto called φ). One must then carefully define, for any $\varphi \equiv \begin{pmatrix} \cdot \\ \cdot \end{pmatrix}$ the transposed $\tilde{\varphi} = (\cdot\ \cdot)$, the complex conjugate $\varphi^* = \begin{pmatrix} \cdot^* \\ \cdot^* \end{pmatrix}$ and the adjoint $\varphi^+ (\cdot^+ \cdot^+)$. With these definitions, split φ into its real and imaginary parts:

$$\varphi = \frac{1}{\sqrt{2}} (\varphi_R + i \varphi_I)$$

and build up the 4-component spinor

$$\Phi = \begin{pmatrix} \varphi_R \\ \varphi_I \end{pmatrix}$$

It satisfies the equation

$$(\vec{\Sigma} \cdot \vec{p} - E)\, \Phi = 0 \qquad \vec{\Sigma} = (\sigma_1, -\rho_2 \sigma_2, \sigma_3)$$

If we introduce the real operators $\vec{\pi}, \varepsilon$ by $\vec{\pi} = i\vec{p}, \varepsilon = -iE$, we have an equation

$$(\vec{\Sigma} \cdot \vec{\pi} + \varepsilon)\, \Phi = 0$$

which is entirely real.

Any electromagnetic coupling, say a magnetic one $\mu \vec{\sigma} . \vec{H}$ $\mu \vec{\sigma} . \vec{H}$, becomes in the new equation $-i \rho_2 \mu \vec{\Sigma} . \vec{H}$, i.e. also real.

This scheme is invariant for space reflexions. The transformation operator is

$$\varphi^{(1)} = i \xi \sigma_2 \varphi^*$$

with an arbitrary phase factor $\xi = e^{i\alpha}$ .This gives for Φ :

$$\Phi^{(1)} = i \sigma_2 \rho_3 e^{i/2 \alpha} \Phi$$

$$\vec{\Sigma}^{(1)} = - \vec{\Sigma}$$

Time reflexion and anticonjugation are effected by operating in the same way (with independent phase factors)on the spinor variables.

It is therefore a matter of taste to say that the neutrino carries a "chirality". One has an equivalent description by a Majorana representation, which is invariant for both P and C: the lack of invariance is then shifted to the interactions.

S T R A N G E P A R T I C L E S

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CHAPTER I

FUNDAMENTAL INTERACTION LAGRANGIAN

Assuming with Gell-Mann and Nishijima that

(1) $N = \begin{pmatrix} p \\ n \end{pmatrix}$ $\quad \Xi = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}$ $\quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$ are iso-spinors,

$\underset{\sim}{\Sigma} = (\Sigma_1, \Sigma_2, \Sigma_3)$ and $\underset{\sim}{\Pi} = (\Pi_1, \Pi_2, \Pi_3)$ $\quad \Sigma^+ = \frac{\Sigma_1 - i\Sigma_2}{\sqrt{2}}$

are iso-vectors and Λ is iso-scalar; and assuming

(2) charge - independence

(3) charge and nucleon-number conservation

(4) field interactions

there are 8 possible coupling terms

$$\mathcal{L} = \left(g_1 \bar{N} \underset{\sim}{\tau} N + g_4 \bar{\Xi} \underset{\sim}{\tau} \Xi \right) \cdot \underset{\sim}{\Pi}$$
$$+ g_2 \left(\bar{\Lambda} \underline{\Sigma} \cdot \underline{\Pi} + h.c. \right)$$
$$+ g_3 \, i \bar{\underline{\Sigma}} \wedge \underline{\Sigma} \cdot \underline{\Pi}$$
$$+ \bar{N} \left(g_5 \Lambda + g_6 \underline{\tau} \cdot \underline{\Sigma} \right) K + h.c.$$
$$+ \bar{\Xi} \left(g_7 \Lambda + g_8 \underline{\tau} \cdot \underline{\Sigma} \right) K' + h.c.$$

$K' = -i \tau_2 K^* = \begin{pmatrix} -\bar{K}^0 \\ K^- \end{pmatrix}$

Considering all the difficulty we had in fixing the one coupling constant g_1, it seems highly implausible that without further theoretical guidance we shall fix the 8 constants from experimental comparison alone in a finite time.

Theoretical guidance is provided by the remark that the masses of hyperons are not wildly different. If as a first approximation we assume their means are equal, the free Lagrangian possesses a number of symmetries. These same symmetries also exist for the interaction Lagrangian provided some of the coupling constants are equal, and it seems plausible to infer that the coupling constants concerned are equal to the approximation that the relevant masses are equal. A number of such symmetries have been pointed out by various authors; we list them here and then later try to find a deeper basis for them.

In all this it is being assumed that

N, Σ, Λ, Ξ are $\left(\frac{1}{2}+\right)$ particles

(1) (Salam-Polkinghorne -Il Nuovo Cim ..)

To the approximation that $m_N = m_\Xi$ $\left(\begin{matrix} N \leftrightarrow \Xi \\ K \leftrightarrow K' \end{matrix}\right)$

We may infer

$$g_1 = g_4 \,, \quad g_5 = g_6 \,, \quad g_7 = g_8$$

(2) If $m_\Lambda = m_\Sigma$ The free Lagrangian is invariant for

$$\Lambda \longleftrightarrow \underset{\sim}{\Upsilon} \cdot \underset{\sim}{\Sigma}$$

The interaction Lagrangian is invariant if

$$g_5 = g_7 \,, \quad g_6 = g_8 \qquad g_3 = 0$$

(D'Espagnat-, Prentki, Salam, Nuc.Phys..)

(3) Write

$$\frac{1}{\sqrt{2}}(\tau\cdot\Sigma+\Lambda^0)=\begin{pmatrix}\Sigma^0 & \Sigma^+\\ \Sigma^- & Y^0\end{pmatrix}=\begin{pmatrix}\Sigma_2 & \Sigma_1\end{pmatrix}$$

where $\Sigma^0=\frac{1}{\sqrt{2}}(\Lambda^0+\Sigma^0)$ $\qquad$ $Y^0=\frac{1}{\sqrt{2}}(\Lambda^0-\Sigma^0)$

Assume

$$m_\Lambda = m_\Sigma$$

The free Lagrangian is

$$\mathcal{L}_f=\left(\bar{N}\mathcal{D}_N N+\bar{\Sigma}_1\mathcal{D}_{\Sigma_1}\Sigma_1+\bar{\Sigma}_2\mathcal{D}_{\Sigma_2}\Sigma_2+\bar{\Xi}\mathcal{D}_\Xi\Xi\right)$$

Now (1)

$$\bar{\Sigma}_1\tau\cdot\pi\Sigma_1+\bar{\Sigma}_2\tau\cdot\bar{\pi}\Sigma_2=\bar{\Lambda}^0\underline{\Sigma}\cdot\underline{\pi}+hc \qquad \text{Gell-Mann, Schwinger,}$$
$$-i\,\bar{\underline{\Sigma}}\wedge\underline{\Sigma}\cdot\pi$$

Also(2)

$$\bar{N}(\Lambda+\underline{\tau}\cdot\underline{\Sigma})K_\cdot+\bar{\Xi}(\Lambda+\tau\cdot\Sigma)K'+hc=$$

$$(\bar{N}\ \bar{\Xi})\begin{pmatrix}K^0 & K^+\\ K^- & -\bar{K}^0\end{pmatrix}\begin{pmatrix}\Sigma_1\\ \Sigma_2\end{pmatrix}+hc$$

Tiomno

To the approximation that $m_N=m_\Lambda=m_\Sigma=m_\Xi$ the free Lagrangian is invariant for the substitutions

$$N\rightarrow\Sigma_1$$
$$\Xi\rightarrow\Sigma_2 \qquad K^0\rightarrow+\bar{K}^0$$

Likewise the interaction is invariant if $G=g_1=g_2=-g_3=g_4$

and $$g_5=g_6=g_7=g_8=G'$$

It would appear from this that there is the possibility of a very considerable symmetry in the strong-interaction Lagrangian and to a very good approximation, there may just be 2 coupling constants G and G'.

The substitutions we have considered are linear transformations. It is possible to give a deeper significance to these linear transformations and consider these as rotations in a higher dimensional symmetry space. Attempts in this direction using [4] spaces have been made by Salam & Polkinghorne and Schwinger. The most general attempt is that of Tiomno who considers rotations in a [7] space

Write

$$\Psi_1 = \begin{pmatrix} N \\ \Xi \end{pmatrix}$$

$$\Psi_2 = \begin{pmatrix} \Sigma_1 \\ \Sigma_2 \end{pmatrix}$$

$$\Psi = \begin{pmatrix} N \\ \Xi \\ \Sigma_1 \\ \Sigma_2 \end{pmatrix}$$

Ψ is an 8-component spinor, appropriate to a [7] space.

Write

$$\Gamma_i = \begin{pmatrix} 0 & \tau_i \\ \tau_i & 0 \end{pmatrix}_{i=1,2,3} \qquad \Gamma_4 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\Gamma_\alpha = \begin{pmatrix} \tau_\alpha & 0 \\ 0 & -\tau_\alpha \end{pmatrix} \quad \alpha = 5,6,7$$

where $\underset{\sim}{\tau}$ are usual 2 x 2 Pauli matrices.

These 7 Γ matrices anti-commute and form a Dirac set.

Also write

$$A_\mu = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad B_\mu = \begin{pmatrix} 0 \\ 0 \\ 0 \\ K_1 \\ K_2 \\ K_3 \\ K_4 \end{pmatrix}$$

$$K^+ = K_1 - i K_2$$

$$K^0 = K_3 - i K_4$$

Then

$$\mathcal{L}_{int} = G\,\bar{\Psi}\,T_\mu\,\Psi\,A_\mu + G'\,\bar{\Psi}\,T_\mu\,\Psi\,B_\mu$$

To see that the part of the Lagrangian involving G' can be written in the form above, not that

$$G'\left(\bar{N}\ \ \bar{\Xi}\right)\begin{pmatrix} K^0 & K^+ \\ K^- & -K^0 \end{pmatrix}\begin{pmatrix} \Sigma_1 \\ \Sigma_2 \end{pmatrix} + h.c$$

$$= G'\,\bar{\Psi}_1\left(\underline{K}\cdot\underline{\tau} - iK_4\right)\Psi_2 + \bar{\Psi}_2\left(\underline{K}\cdot\underline{\tau} + iK_4\right)\Psi_1$$

$$= G'\,\bar{\Psi}\,T_\mu\,\Psi\,B_\mu$$

Also note that $G = g_1 = -g_2 = +g_3 = g_4$

unlike the case where only substitutions were being considered and where $G = g_1 = g_2 = g_3 = g_4$.

Tiomno has also considered the approximation $m_\pi = m_K$; so that vectors A_μ and B_μ can be combined into a single vector with 7-components

and thus $G = G' = G$,

$$K_\mu = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ K_1 \\ K_2 \\ K_3 \\ K_4 \end{pmatrix}$$

write

$$\mathcal{L}_{int} = G_1\,\bar{\Psi}\,T_\mu\,K_\mu\,\Psi$$

He goes to define the quantum-numbers that arise when the rotational aspect of the [7] space is fully exploited. For this reference may be made to his paper in Nuovo-Cimento -(June 1957)- and Rochester Conference Report-(April 1957)-.

Although Tiomno's full theory with the assumption $m_\pi = m_K$ is very attractive, we shall not adopt this attitude. The attitude I prefer is that of the paper of d'Espagnat, Prentki and Salam: to the approximation that hyperon mass-differences can be neglected, there appears a high degree of symmetry in Lint leading one to infer that there are possibily two fundamental parameters G and G' in the theory, the actual coupling constants $g_1, \ldots, g_4$ and $g_5, \ldots, g_8$ differing from G and G' to a small extent.

Gell-Mann and Schwinger have started a different point of view. They wish to assume that π - couplings (presumably stranger than K -couplings) represent a higher degree of symmetry and thus $g_1 = g_2 = -g_3 = g_4 = G$, exactly. The K -couplings g_5, g_6, g_7, g_8 are unequal and these break the higher symmetry represented by π interactions.

To sustain this point of view one should first try to decide if present experiments give the result $G > G'$ (more precisely $G > g_5, g_6, g_7, g_8$). Gell-Mann has tried to make a case for this assumption based on the observation that $K\Sigma, \Lambda$ production in $\pi - N$ collisions and in $\gamma - p$ collisions seems to be smaller than π -production. In support of this view may also be adduced evidenee from high-energy jets (10^5 Bev etc).
At these high energies hyperon-masses are irrelevant and since experiments seem to show very much larger π production compared to K production , it may be inferred that $G > G'$.

Against this is the K-N scattering data which although in a preliminary stage still seems to give about as high K-N scattering as π-N scattering. Since in lowest order perturbation calculations

$$\frac{\sigma\,(K \text{ production})}{\sigma\,(\pi \text{ production})} \approx \frac{G'^2}{G^2}$$

and

$$\frac{\sigma\,(K\,N \text{ scattering})}{\sigma\,(\pi\,N \text{ scattering})} \approx \frac{G'^4}{G^4}$$

it seems hard to reconcile $\frac{G'^4}{G^4} \approx 1$ with $\frac{G'^2}{G^2} \approx \frac{1}{50}$.
It is possible that the experiments on production of strange particles are in not too good a shape.

In all this work we have considered only 3 field interactions. There is always the possibility of 4-field (renormalizable) interactions like $\lambda_1 (\bar{K} K)(\underset{\sim}{\pi}^2)$

If $\lambda > G'$, then in the lowest order

$$\frac{\sigma(KN \text{ scattering})}{\sigma(\pi N \text{ scattering})} \approx \lambda^2$$

and in this case the suspected experimental small K-production and large K- scattering may find an explanation in terms of a term like the above in the Lagrangian.

CHAPTER II

CHARGE-INDEPENDENCE-:

In all theoretical thinking about strong interactions it has been assumed that charge independence holds. There are perhaps two direct pieces of evidence

(1) charge-independence hypothesis has found experimental confirmation in pion-nucleon scattering and the deviations from strict charge-independence seem attributable to electro-magnetism. If the strange particles are strongly coupled to pions and nucleons, the π-N scattering would proceed through intermediate states involving strange particles and thus we expect their interactions to be charge independent also.

(2) The mass of Σ^0 is close to the masses of Σ^+ and Σ^- . This is strong evidence for charge-independence (as against charge-symmetry). It is possible to obtain a number of relations (equalities or inequalities) between various reactions on the basis of assumed charge-independence . A fairly complete list has been given by D.Feldman (Phys.Rev.103, 254,1956). The procedure for obtaining these is standard. Consider initial and final states $|i|$ and $|f|$ each involving, for simplicity, two particles.

$$|i\rangle = |t_1, t_2, t_1^z, t_2^z$$

in an obvious notation where t_1 , t_1^z give isotopic-characterization of particle 1 and t_2 , t_2^z for particle 2 etc..

From charge independence $T_f = T_i$, $T_f^z = T_i^z$

where $T_i^z = t_1^z + t_2^z$

Write the Clebsch-Gordon coefficient

$$(T, T^z, \; t_1, t_2 \,|\, t_1, t_2, t_1^z, t_2^z) \quad \text{as} \quad (T/i)$$

Now it is well known that

(i) $\sum_{T,T_3} |T, T^3, t_1, t_2\rangle\langle T, T^3, t_1, t_2| = 1$ (completeness-relation)

(ii) $(T', T'^3 | S | T, T^3) = \delta(T-T')\,\delta(T'^3 - T^3)\,\alpha(T)$,

where S is the scattering-matrix. Thus

$$(f|S|i) = \sum_T \langle f|T\rangle(T|i)\,\alpha(T)$$

Consider as an example

$|i_1\rangle = |\pi^- + p\rangle \qquad |i_2\rangle = |\pi^+ + p\rangle$

$|f_1\rangle = |\Sigma^- + K^+\rangle \qquad |f_2\rangle = |K^+ + \Sigma^+\rangle$

$|f_1'\rangle = |\Sigma^0 + K^0\rangle$

$\langle \tfrac{1}{2}|i_1\rangle = -\sqrt{\tfrac{2}{3}} = \langle f_1|\tfrac{1}{2}\rangle \qquad \langle f_1'|\tfrac{1}{2}\rangle = \sqrt{\tfrac{1}{3}}$

$\langle \tfrac{3}{2}|i_1\rangle = \sqrt{\tfrac{1}{3}} = \langle f_1|\tfrac{3}{2}\rangle \qquad \langle f_1'|\tfrac{3}{2}\rangle = \sqrt{\tfrac{2}{3}}$

$(\tfrac{3}{2}|i_2) = 1 = (f_2|\,^3/_2)$

Therefore

$(f_1|S|i_1) = +\tfrac{2}{3}\alpha(\tfrac{1}{2}) + \tfrac{1}{3}\alpha(\tfrac{3}{2})$

$(f_1'|S|i_1) = -\sqrt{\tfrac{2}{3}}\,\alpha(\tfrac{1}{2}) + \sqrt{\tfrac{2}{3}}\,\alpha(\tfrac{3}{2})$

$(f_2|S|i_2) = \alpha(\tfrac{3}{2})$

From

$$a = \sqrt{|(f_1|S|i_1)|^2} = \frac{1}{3}\left|+2\alpha\left(\tfrac{1}{2}\right)+\alpha\left(\tfrac{3}{2}\right)\right|$$

$$b = \sqrt{2\,|(f_1'|S|i_1)|^2} = \frac{1}{3}\left|-2\alpha\left(\tfrac{1}{2}\right)+2\alpha\left(\tfrac{3}{2}\right)\right|$$

$$c = \sqrt{|(f_2|S|i_2)|^2} = \frac{1}{3}\left|3\alpha\left(\tfrac{3}{2}\right)\right|$$

We deduce that square rotes of (differential and total) cross-sections a, b, c, satisfy triangular inequalities.

$$a+b-c \geq 0\,,\quad b+c-a \geq 0\,,\quad c+a-b \geq 0\,.$$

Feldman lists a number of reactions where equalities (rather than inequalities) can be obtained. For example for

a	$K^- p$	$\longrightarrow \Sigma^+ \pi^-$	$a + c + e = 2(b+d)$
b		$\longrightarrow \Sigma^0 \pi^0$	
c		$\longrightarrow \Sigma^- \pi^+$	
d	$K^- n$	$\longrightarrow \Sigma^0 \pi^-$	$a + c + d = 2(b+c)$
e		$\longrightarrow \Sigma^- \pi^0$	

He.. remarks that all cases where equalities are obtained are special cases of Watson's lemma, which is stated thus-:
For collisions involving two $T = \frac{1}{2}$ particles, if the scatterers is with equal probability a proton and a neutron, the number of charged π's (or charged Σ's) produced is twice the number of neutral π's or Σ's .

A simple plausibility proof for the case above may be given.
The initial state:

$$\frac{K^- p - \bar{K}^0 n}{\sqrt{2}} \qquad T = 0$$

constitutes an "unpolarized beam" in insotopic-space, as also do the set of states

$$\left.\begin{array}{c} \bar{K}^0 p \\ \dfrac{K^- p + \bar{K}^0 n}{\sqrt{2}} \\ K^- n \end{array}\right\} \quad T = 1$$

taken with equal probability. This means that respective sums of matrix elements for π^+, π^0, π^- production arising from the 4 initial states $K^- p$, $K^- n$, $\bar{K}^0 p$, $\bar{K}^0 n$ are equal. Since $K^- p$ and $\bar{K}^0 n$ as well as $K^- n$, $\bar{K}^0 p$ are charge-symmetric states, particularization to $K^- p$, $K^0 n$ processes gives the milder result that sums of matrix-elements involving π^+ or π^- mesons is twice those for π^0 mesons.

K^- absorption in deutrons or He^4 would provide the best way of checking Feldman's relations. The difficulty of course as always is the identification of events involving two neutral particles.

In more sophisticated theories (4-dimensional isotopic space), for example in Salam-Polkinghorne theory where the particles are labelled with two or more quantum-numbers (I^+, I^- in S-P case) and $|\underset{\sim}{I}^+|$ and $|\underset{\sim}{I}^-|$ conservation holds ($|\underset{\sim}{I}^-|$ to the extent that $\Lambda_1 \equiv$ mass can be neglected, i.e. at energies far above the threshold),

$$(f|S|i) = \sum_{I^+, I^-} (f|I^+, I^-)(I^+, I^-|i) \quad \alpha (I^+, I^-)$$

and the procedure is entirely analogous to the case discussed above. Feldman has given relations holding in such cases as well.

Glaser in a recentpre-print (Charge Independence and Hyperon Production, Brown, Glaser, Meyer, Perl, Velde and Cronin,) has considered the experimental angular distributions at 1-1 Bev. (π^- energy) for

(a) $\pi^- + p \longrightarrow \Sigma^- + K^+$

(b) $\pi^- + p \longrightarrow \Sigma^0 + K^0$

(c) $\pi^+ + p \longrightarrow \Sigma^+ + K^+$

As is well known the charged hyperons tend to go forward in c.m. system and neutral ones go backward. Glaser maintains (on the basis of rather a small number of events, 15 Σ^0's)that the triangular in equality

$$\sqrt{\frac{d\sigma(\Sigma^-)}{d\Omega}} + \sqrt{\frac{d\sigma(\Sigma^+)}{d\Omega}} - \sqrt{\frac{2\,d\sigma(\Sigma^0)}{d\Omega}} \geq 0$$

is not satisfied. If this is true, this presents indeed a most serious situation and clearer experimental check on this point is strongly indicated.

CHAPTER III

STRANGE PARTICLE DECAYS

Strange particle decays fall into 2 classes-:

class 1 -: where all final state particles have strong interactions and a definite value of isotopic spin ($K \longrightarrow \pi + \pi$ etc)

class 2 -: Only some of the final state particles carry isotopic spin ($K \longrightarrow \mu + \nu$, $K \longrightarrow \pi + \mu + \nu$ etc).

In all cases however it appears that

$$T_f^{3} - T_i^{3} = -\frac{1}{2}$$

This observation has been made into a principle. Decays involving $|\Delta T^{3}| > \frac{1}{2}$ may exist but are much slower than those involving $|\Delta T^{3}| = \frac{1}{2}$. This gives one way of understanding why the Ξ particle does not decay into a $N + \pi$.

Theoretical work has also considered extending $|\Delta T^{3}| = \frac{1}{2}$ rule to total isotopic-spin i.e. not only $|\Delta T^{3}| = \frac{1}{2}$, but also $\Delta |\underset{\sim}{T}| = \frac{1}{2}$. This is an ad-hoc rule with no theoretical basis. It has been considered by a number of authors (Gatto, Wentzel, Takeda, Gell-Mann, Dalitz and others). It provides some preductions which can be checked against experiment.

The processes to consider as-:

(1) Branching ratios $\dfrac{\Lambda^0 \longrightarrow p + \pi^-}{\Lambda^0 \longrightarrow N \pi^0}$

(2) Branching ratios $\dfrac{\Sigma^+ \longrightarrow p \pi^0}{\Sigma^+ \longrightarrow n \pi^+}$

(3) Decay-probabilities $\dfrac{[\Sigma^+]}{[\Sigma^-]}$

(4) Decay probabilities $\dfrac{[\Xi^-]}{[\Xi^0]}$

---

(5) Branching ratios $\dfrac{\tau^+ \longrightarrow \pi^+ \pi^- \pi^0}{\tau^+ \longrightarrow \pi^0 \pi^0 \pi^+}$

(6) Branching ratios $\dfrac{\theta_1^0 \longrightarrow \pi^+ \pi^-}{\theta_1^0 \longrightarrow \pi^0 \pi^0}$

(7) Decay probabilities $\dfrac{K^+ \longrightarrow \pi^+ \pi^0}{\theta_1^0 \longrightarrow \pi^+ \pi^-}$

There are many ways to state the $\Delta|T| = \frac{1}{2}$ rule precisely.

Statement (1) The decay Lagrangian should transform as spinor in isotopic-space.

Statement (2) Equivalently, in any decay, a particle of $T = I/2$ $T_z = I/2$ is emitted while charge-independence strictly holds. This particle carries no energy and momentum and has been called a spurion(s)

Consider process (1). This may be pictured as the reaction

$$\bar{s} + \Lambda \longrightarrow \begin{matrix} p + \pi^- \\ n + \pi^0 \end{matrix}$$

where $\bar{s}$ is the anti-spurion absorbed by Λ^0 and charge-independence holds.

Using the methods of the last lecture

$$(f|S|i) = \alpha(\tfrac{1}{2})\,(f|\tfrac{1}{2}) + \sqrt{\tfrac{2}{3}}\;\alpha(\tfrac{1}{2})$$

$$(f'|S|i) = \alpha(\tfrac{1}{2})\,(f'|\tfrac{1}{2}) = \sqrt{\tfrac{1}{3}}\;\alpha(\tfrac{1}{2})$$

where $|f) = |p\pi^-)$ $\quad |f') = |n\pi^0)$.

Thus
$$\frac{\Lambda^0 \longrightarrow p + \pi^-}{\Lambda^0 \longrightarrow n + \pi^0} = \frac{2}{\Lambda}.$$

<u>This seems to be experimentally verified</u>

consider (2) and (3)

$$\bar{s} + \Sigma^+ \longrightarrow n + \pi^+$$
$$\longrightarrow p + \pi^0$$
$$\bar{s} + \Sigma^- \longrightarrow n + \pi^-$$

This is exactly analogous to the case considered in the last lecture

$$\left.\begin{array}{lll} p + \pi^- & \longrightarrow & K^- + \Sigma^+ \\ & \longrightarrow & K^0 + \Sigma^0 \\ p + \pi^+ & \longrightarrow & K^+ + \Sigma^+ \end{array}\right)$$

so that

$$(\Sigma^+ \to n\pi^+)^{\frac{1}{2}} : (2\{\Sigma^+ \to p\pi^0\})^{\frac{1}{2}} : (\Sigma^- \to n\pi^-)^{\frac{1}{2}}$$
$$= \left|\alpha(\tfrac{1}{2}) + \tfrac{1}{2}\alpha(\tfrac{3}{2})\right| : \left|-\alpha(\tfrac{1}{2}) + \alpha(\tfrac{3}{2})\right| : \left|\tfrac{3}{2}\alpha(\tfrac{3}{2})\right|$$

Clearly a Δ-lar inequality exists between

$(\Sigma^+ \to n\pi^+)^{1/2}$, $(2\{\Sigma^+ \to p\pi^0\})^{1/2}$ and $(\Sigma^- \to n\pi^-)^{1/2}$

This seems to be checked experimentally . To obtain more definite results we must make further assumptions about the decay-process.

(i) Assume T (time-reversal) invariance of the S-matrix .

(ii) Also for simplicity assume P-invariance and Σ to be (I/2 -)particle so that the final pion-nucleon state is an S-state. We shall relax assumption (ii) shortly lent for illustrative purposes it is convenient. Then Takeda has shown that

$$\alpha(T) \approx i\, a(T) e^{2i\delta(T)}$$

where $\delta(T)$ is the S-wave pion-nucleon scattering phase-shift in the final state and $a(T)$ is real. At the energies concerned in Σ-decay, the S-phase shifts are small and thus $\alpha(T) \approx i\, a(T)$, ($a$ real)

If Σ is (I/2 +) and parity is conserved, $\delta(T)$ refer to P-phase shifts but even these are small ($< 15^\circ$)and can be neglected. Thus with T and P conservation we have the result

$$\Sigma^+ \to n\pi^+ \qquad \alpha = a^2\left(\tfrac{1}{2}\right) + \tfrac{1}{4} a^2\left(\tfrac{3}{2}\right) + a\left(\tfrac{1}{2}\right) a\left(\tfrac{3}{2}\right)$$

$$\to p\pi^0 \qquad \alpha = \frac{a^2}{2}\left(\tfrac{1}{2}\right) + \frac{a^2}{2}\left(\tfrac{3}{2}\right) - a\left(\tfrac{1}{2}\right) a\left(\tfrac{3}{2}\right)$$

$$\Sigma^- \to n\pi^- \qquad \alpha = \frac{9}{4} a^2\left(\tfrac{3}{2}\right)$$

From the known experimental results $\dfrac{\Sigma^+ \to n\pi^+}{\Sigma^+ \to p\pi^0} \sim 1$ and $\dfrac{\omega(\Sigma^+)}{\omega(\Sigma^-)} \sim 2$, it is clear that there is a definite discrepancy and no numbers $a(I/2)$, $a(3/2)$ exist to give the two experimental results.

Feld (Pre-print) has recently pointed out that there is no reason to assume parity-conservation in these decays, so that irrespective of whether Σ is (I/2 +) or (I/2 -) particle, the final nucleon-pion state should contain both S and P phases. The number of amplitudes describing the decay is doubled ($\alpha_s(T), \alpha_p(T)$) rather than merely $\alpha(T)$)).

Write

$$\alpha_s\left(\tfrac{1}{2}\right) = a\, e^{i\delta_1}$$
$$\tfrac{1}{2}\alpha_s\left(\tfrac{3}{2}\right) = b\, e^{i\delta_3}$$
$$\alpha_p\left(\tfrac{1}{2}\right) = a\,\xi_1\, e^{i(\delta_{11}+\eta)}$$
$$\tfrac{1}{2}\alpha_p\left(\tfrac{3}{2}\right) = b\,\xi_2\, e^{i(\delta_{31}+\eta)}$$

a, b are arbitrary pure imaginary constants. ξ_1, ξ_2 are real constants which measure the degree of parity non-conservation in the decay and η is a phase factor whose magnitude gives the degree of break-down of time-reversal invariance in the decay. The introduction of η is necessary since on account of the PCT theorem, P-violation implies that either or both of T and C must be violated. In particular if PC is conserved $\eta = 0$, if PT is conserved $\eta = \pi/2$, and if only PCT remains $0 < \eta < \pi/2$.

To obtain the decay-rates one must integrate over all angles. After this integration there remain no terms corresponding to interference between s and p-states. It turns out that the decay rates are independent of η as well.

Setting $\delta_1 = \delta_3 = \delta_{11} = \delta_{33} = 0$, obtain

$$\omega(\Sigma^-) = 3\, b^2 (1 + \xi_2^2)$$
$$\omega(\Sigma^+ \to p\pi^\circ) = \tfrac{1}{3} a^2(1+\xi_1^2) + \tfrac{2}{3} b^2(1+\xi_2^2) + \frac{2\sqrt{2}}{3} ab(1+\xi_1\xi_2)$$
$$\omega(\Sigma^+ \to n\pi^+) = \tfrac{2}{3} a^2(1+\xi_1^2) + \tfrac{1}{3} b^2(1+\xi_2^2) - \frac{2\sqrt{2}}{3} ab(1+\xi_1\xi_2)$$

Feld makes the "natural" assumption $\xi_1^2 = \xi_2^2$; it turns out then that the two experimental ratios can be fitted with $\xi_1 = -\xi_2 = \pm .71$ and $a/b = \sqrt{5.0}$. Feld gives reasons for believing that these particular values of ξ_1, and ξ_2 $(\pm .71)$ correspond to maximum parity violation in the decay process concerned.

To calculate (5),(6) and (7) it is perhaps easier not to use the "spurion" but to proceed directly.

Consider process (6) :

From $\Delta|T| = I/2$ rule the final state can only be a $T = 0$ or $T = 1$ state.

If $\underset{\sim}{\pi}(1)$, $\underset{\sim}{\pi}(2)$ are two iso-vectors referring to the two final pions

$T = 0$ corresponds to $\underset{\sim}{\pi}(1) \cdot \underset{\sim}{\pi}(2) = \pi^+(1)\pi^-(2) + \pi^+(2)\pi^-(1) + \pi^0(1)\pi^0(2)$

$T = 2$ corresponds to $\underset{\sim}{\pi}(1) \wedge \underset{\sim}{\pi}(2)$.

Assuming that θ has zero spin, the final-state wave function must be symmetric. Clearly no symmetric wave function exists for $T = 1$ case.

For $T = 0$, the wave function for $\pi^+\pi^-$ — : $\dfrac{\pi^+(1)\pi^-(2) + \pi^+(2)\pi^-(1)}{\sqrt{2}} = \chi_1$

for $\pi^0\pi^0$ — : $\pi^0(1)\,\pi^0(2) \quad = \chi_2$

Reading from ($T = 0$) $\underset{\sim}{\pi}(1) \cdot \underset{\sim}{\pi}(2) = \sqrt{2}\,\chi_1 + \chi_2$

it is clear that $\dfrac{\theta^0 \longrightarrow \pi^+ + \pi^-}{\longrightarrow \pi^0 + \pi^0} = \left(\dfrac{\sqrt{2}}{1}\right)^2 = 2$

The experimental result(with a large error for this ratio is ~ 4 or 5 and seemingly irreconciable with 2. This is the first clear-cut failure of the rule $\Delta|T| = I/2$ we have encountered so far. We, shall come back to this later.

Consider process (5) :

Assuming τ^+ spin zero, Dalitz has given arguments(based on barrier penetration factors, see Steinberger's lectures)to show that the final pion wave-function should be totally symmetric.

From $\Delta|T| = I/2$ rule, the final state must have

T = 1. With 3 iso-vectors

$\underset{\sim}{\pi}(1)$, $\underset{\sim}{\pi}(2)$, $\underset{\sim}{\pi}(3)$ a symmetric T = 1 state corresponds to

$$\underset{\sim}{\pi}(1)\left[\underset{\sim}{\pi}(2).\underset{\sim}{\pi}(3)\right] + \underset{\sim}{\pi}(2)\left[\underset{\sim}{\pi}(3).\underset{\sim}{\pi}(1)\right] + \underset{\sim}{\pi}(3)\left[\underset{\sim}{\pi}(1).\underset{\sim}{\pi}(2)\right]$$

For τ^+ decay, it gives

$$\frac{1}{\sqrt{2}}\sum \underset{\sim}{\pi}^+(1)\left[\pi^+(2)\pi^-(3) + \pi^-(2)\pi^+(3) + \pi^0(2)\pi^0(3)\right]$$

$$= \left[\sqrt{2}\,\pi^+(1)\,\pi^+(2)\,\pi^-(3) + \ldots + \ldots\right] + \left[\frac{\pi^0(1)\,\pi^0(2)\,\pi^+(3) + \ldots + \ldots}{\sqrt{2}}\right]$$

Clearly $$\frac{\tau^+ \longrightarrow \pi^+ + \pi^+ + \pi^-}{\longrightarrow \pi^0 + \pi^0 + \pi^+} = \left(\frac{2}{1}\right)^2 = 4$$

The experimental result is nearer 3 and this is accounted for by the smaller phase-space volume available for the $\pi^+\pi^+\pi^-$ state as against $\pi^0\,\pi^0\,\pi^+$, the smaller volume of phase-space arising from the fact that π^+ is heavier than π^0.

Consider process (7) -:

From $\Delta|T| = I/2$ rule spin zero K^+ can not decay at all into two pions. This is because there does not exist a symmetric final state of two pions with T = 1. Clearly a small admixture of $\Delta|T| = 3/2$ is necessary in order that K^+ decay at all into two pions. This is at once a great point in favour of the $\Delta|T| = I/2$ rule (in so far as K^+ is very much longer lived than K° and no other "natural" explanation of this exists) and also an ambarrassment on account of the arbitrariness introduced into the situation. It was already clear from lack of agreement of

$$\frac{\theta^0_1 \longrightarrow \pi^+ + \pi^-}{\longrightarrow \pi^0 + \pi^0}$$

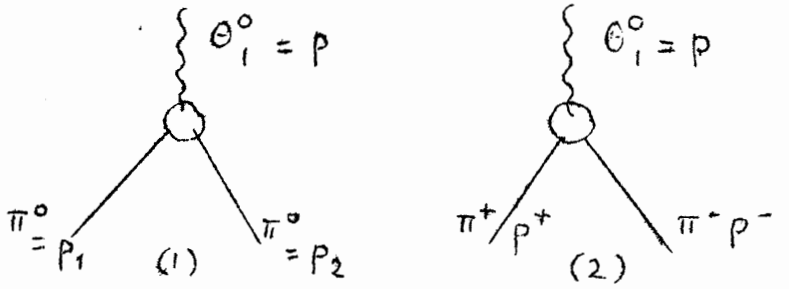
ratio with experiment

that some $\Delta|T| = 3/2$ admixture would be necessary.

It was hoped that the same admixture might also give the correct ratio

$$\frac{K^+ \longrightarrow \pi^+ + \pi^\circ}{\theta_1^\circ \longrightarrow 2\pi}$$

Calculations were made by M.Gell-Mann (Il Nuovo Cim.5, 758, (I957) and R.H.Dalitz (Proc.Phys.Soc.69, 527, (I957)) to see if this is the case , but their result is that to fit both (5) and (6) not only is it necessary to have $\Delta|T| = 3/2$ but also $\Delta|T| = 5/2$ admixture.
That at all events the $\Delta|T| = I/2$ rule must be an approximate rule is of course clear from the fact that electro-magnetic interactions would always violate strict conservation laws which refer to total isotopic spin. It is commonly stated that the violation of $\Delta|T| = I/2$ rule(and consequently the extent of $\Delta|T| = 3/2$ admixture) would be proportional to $\alpha \simeq \frac{1}{137}$ in the matrix elements. This is fallacious for the following reason. Electromagnetic interaction is supposed to produce the π^+, π° mass difference, which is considerable and this mass difference can have a large effect on the matrix-elements of for-example θ_1° decay to π^+, π^- or $2\pi^\circ$'s.(We are not discussing the phase-space factors here but the matrix elements themselves)



The matrix-element for process(1) is

$$M_o = M(p^2, p_1^2, p_2^2) \qquad p = p_1 + p_2$$

For process (2) it is

$$M_c = M(p^2, p_+^2, p_-^2) \qquad p = p_+ + p_-$$

Here p, p_1, p_2 etc.are 4-momenta

so that $p^2 = m_\theta^2$

$$p_1^2 = p_2^2 = m_{\pi^0}^2 \qquad p_+^2 = p_-^2 = m_{\pi^+}^2$$

If $m_{\pi^0} = m_{\pi^+}$, $M_0 = M_c$

and since $\theta^0 \to \pi^+ + \pi^- = 2 M c^2$ (2 comes from symmetrization)

$$\to \pi^0 + \pi^0 = M_0^2$$

We would obtain the result stated before (i.e. $\theta^0 \dfrac{\pi^+ + \pi^-}{\pi^0 + \pi^0} = 2$)

However $m_{\pi^0} \neq m_{\pi^+}$. Thus

$$M_0 = M(p^2, p_1^2, p_2^2)$$

$$= M(p^2, p_+^2, p_-^2) + (p_1^2 - p_+^2)\frac{\partial M}{\partial p_+^2} + (p_2^2 - p_-^2)\frac{\partial M}{\partial p_-^2} + \dots$$

If for an order of magnitude argument we assume $\dfrac{\partial M}{\partial p_+^2} \sim \dfrac{M}{p_+^2}$, we obtain $M_0 \approx M_c\left[\dfrac{1 - 2(m_{\pi^0}^2 - m_{\pi^+}^2)}{m_{\pi^+}^2}\right] \approx \dfrac{6}{7} M_c$

so that $\theta^0 \longrightarrow \dfrac{\pi^+ + \pi^-}{\pi^0 + \pi^0} \approx 2.7$

We still have not reached the experimental value (~4) but the fact remains that electro-magnetic effects can make very sizable differences.

Summing up, $\Delta|T| = 1/2$ rule has a number of things in its favour; it fits Λ^0 decay ratio, it can with parity-violation give an account of Σ decays, it serves usefully for giving τ decays ratios and it gives an explanation of the long-life of charged K compared to K^0. It fails for θ_1^0 decay but it is possible that the discrepancy can be resolved with better experiments and a better calculation of electro-magnetic violation of $\Delta|T| = 1/2$ rule.
The rule of course has no deep theoretical basis.

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STRANGE PARTICLES

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STRANGE PARTICLES

I.- INTRODUCTION

The cross-section for strange particle production is about a few per cent of the geometrical cross section. Therefore, every time a sufficiently energetic π^- posses across a proton, it has an appreciable chance → about 1/10 - of producing a strange particle. If the production process was just, for example,

$$\pi^- + p \longrightarrow \Lambda^\circ,$$

we should expect the Λ° to decay via the inverse process in about

$$10 \times \frac{\hbar}{Mc^2} \simeq 10^{-21} \text{ secs.}$$

In fact, all the lifetimes are 10^{-10} sec. or longer (except perhaps the Σ°).

This is the dilemma from which all theories of strange particles start.

2.- HIGH SPIN THEORIES.-

These are now dead, but it is good to mention them to understand why one is forced to the later theories.

If the matrix elements for processes involving strange particles increased rapidly with energy, then it would be explained why production, which happens at high energies, was copious, whereas decay, at relatively low energies, was slow. It was assumed that strange particles had high spin, about 7/2 $\hbar$. They would then have to be produced either in pairs or in states of large orbital angular momentum. In the decay, the high orbital angular momentum would provide a centrifugal barrier, which had the necessary energy dependence.

The objections to this theory were :

a) if the strange particle decayed in or near nucelar matter, some of the

energetic pions in the nucleons' clouds could provide the necessary angular momentum. Thus the lifetime would decrease with the density of matter. This is not observed.

b) whilst this theory predicts production in pairs, as is observed, it does not predict production in certain kinds of pair but not others. This will be discussed later.

c) many other detailed predictions of the later theories make one certain that they are on the right lines.

3.- THEORY OF GELL-MANN AND NISHIJIMA

The strange particles are observed to occur in groups with nearly equal masses but different charge (Σ^+ and Σ^-, K^+, K^0 and K^-). This suggests extending the theory of isotopic spin to include strange particles, and to do so in such a way that their slow decay is connected with the existence of approximate selection rules.

The important idea is the following : for nucleons and pions, we have the relations

$$Q = T_3 + 1/2\ B \qquad (1)$$

T_3 is a component of isotopic spin, Q is charge and B is baryon (i.e. heavy particle) numbers. Since Q and B are supposed to be conserved in all interactions, the conservation of T_3 is guaranteed by the above equation, and gives us nothing new. After the particles have been assigned to isotopic multiplets, the entire content of the theory of isotopic spin is that T is conserved. The new idea is that equation (1) should not hold for strange particles ; so that conservation of T_3 becomes something new, and we have another quantum number at our disposal.

The assignments of T and T_3 are shown in table I. Interactions are then divided into three classes, as shown in table II. They are associated with typical coupling constants, the number of quantum numbers conserved decreases with the size of the coupling constant.

./...

	MASS (Mev)	LIFETIMES (Secs)	Tz	S	d'Espagnat Prentki type	Racah iso parity
$e^{\pm}$	0.510976	∞	-	-		
$\mu^{\pm}$	$105.70 \pm .06$	$2.22 \pm .02 \times 10^{-6}$	-	-	-	-
$\pi^{\pm}$	$139.63 \pm .06$	$2.56 \pm .05 \times 10^{-8}$	± 1	0	iso-pseudo vector	1
π°	135.0 ± 0.2	$< 0.4 \times 10^{-15}$	0	0		
$K^{\pm}$	494 ± 0.2	$1.24 \pm .02 \times 10^{-8}$	$\pm 1/2$	± 1	iso-spinor anti-iso-spinor	$\pm i$
K°	493 + 5	K_1° $0.95 \pm .08 \times 10^{-10}$ K_2° $(0.3 < \tau < 10) \times 10^{-7}$	- 1/2	1	iso-spinor	i
P	$938.23 \pm .03$	∞	1/2	0	iso-spinor	i
N	$939.53 \pm .03$	110 ± 222	- 1/2	0		
Λ°	$1114.8 \pm .2$	$2.8 \pm 2 \times 10^{-10}$	0	- 1	iso-scalar	1
Σ°	1189 ± 3	? ($< 10^{-11}$)	0	- 1	iso-pseudo-vector	1
Σ^{+}	$1189.70 \pm .25$	$0.69 \pm .1 \times 10^{-10}$	v1	- 1		
Σ^{-}	$1196.65 \pm .35$	$1.6 \pm .2 \times 10^{-10}$	- 1	- 1		
Ξ^{-}	1321 ± 4	$(1.6 < \tau < 200) \times 10^{-10}$	- 1/2	- 2	anti-iso-spinor	-i
Ξ°	?	?	1/2	- 2		

NOTES

Data from Alvarez (Rochester 1957)

K^{+} mass is from τ - decay

Ξ^{-} only seen is cosmic rays (few events)

Ξ° not yet seen

$\hbar = 6.57 \times 10^{-22}$ Mev x Sec

$Q = T_z + 1/2\ B + 1/2\ S$

$= T_z + 1/2\ U$

$p = i^{U}$

Particle and anti-particle are s shown together, and always have opposite T_z, B, S and U

TABLE II

Interaction class	Coupling constant $g^2/4\pi$	Quantum numbers conserved	Not conserved
Strong	15	Q,B,T,T_3,P	-
Electromagnetic	$\frac{1}{137}$	Q,B,T_3,P	T
Weak	10^{-14}	Q, B	T, T_3, P

The strong interactions are supposed to account for all production processes, with a strength very roughly the same as the pion-nucleon interactions. That electromagnetic interactions should conserve T_3 but not T, is an extrapolation from the known case of pions and nucleons.

Decays are known (neutron and pion), with a coupling constant of about 10^{-14}, in which the secondaries have no isotopic spin assigned to them. Conservation of T_3 in these cases makes no sense. It is therefore a reasonable extrapolation to say that T_3 is not conserved in any weak interaction. Parity, P, is put in for completeness, and also because it is the only known connection of these ideas with physical space-time. Charge conjugation, C, should accompany P.

Although the relation between Q and T_3 is no longer equation (1), there is still a <u>linear relation within any charge multiplet</u>.This fact may be summed up by writing

$$Q = T_3 + 1/2\ B + 1/2\ S \qquad (2)$$

which defines a new quantum member S. S is a convenient number to use in practice, because it is zero for nucleons and pions but non-zero for strange particles. However, its use is exactly equivalent to the use of Q, B and T_3 together.

The first consequence of the theory is the one it was designed for i.e.that strange particles are produced in pairs. This overcomes the initial dilemma. They may only decay though the weak

interactions (except for Σ°., which can decay by electromagnetic interaction like the π°).

The scheme has had the following successes :

a) it predicted the Σ°. This is not, as might be thought, absolutely necessary for any theory using isotopic spin at all. Karplus, and Yang (Phys. Rev. 101, 874) using a finite isotopic group, found the possibility of isotopic multiplets with Q differing by 2.

b) It says that possible associated productions are

$$\pi^- + P \longrightarrow \Lambda + K^\circ$$
$$\longrightarrow \Sigma^- + K^+$$
$$\longrightarrow N + K^+ + K^-$$

but not

$$\pi^- + P \longrightarrow \Lambda + \Lambda$$
$$\longrightarrow \Sigma^+ + K^-$$

Thus double Λ production is prohibited, and has indeed not be seen. Also the threshold for K^- production is above that for K^+, and even then a K^- can only be produced with a K^+. So K^+'s should be fore more abundant than K^-'s, as indeed they are.

c) K^+ and K^- behave very differently on scattering from nuclei. K^+ can only scatter (with or without charge exchange), but K^- can be absorbed in processes like

$$K^- + N \longrightarrow \Lambda + \pi^- + \pi^+$$

with hyperon production. This is observed.

d) K° cannot be the same as $\overline{K^\circ}$ since they have different values of T_3. (Particle and anti-particle must always have equal and opposite values of T_3, to allow them to annihilate). This has very remarkable consequences. If we write, in terms of Hermitian fields,

$$K^\circ = \frac{1}{\sqrt{2}} (K_1^\circ + i K_2^\circ),$$

then K_1° and K_2° may have different properties for weak interactions, and so have different life-times and slightly different masses (i.e. masses differing by $\frac{\hbar}{10^{-10}}$ 10^{-5} sec). Thus K_1 and K_2 may behave as

two different particles.

This sort of behaviour between particle and anti-particle is unique. For charged particles it is forbidden by gauge invariance, for neutral baryons by conservation of B, and for π^0 because $\pi^0 = \overline{\pi^0}$.

This prediction is not independent of (b). In fact, if K^0 were equal to $\overline{K^0}$, the known reaction

$$\pi^- + P \longrightarrow \Lambda^0 + K^0$$

would lead to double Λ production, by using the same formula again with K^0 carried over to the other side.

The theory makes some predictions which have not yet been verified.

a) The existence of Ξ^0. The likely decay $\Xi^0 \longrightarrow \Lambda^0 + \pi^0$ would be hard to see.

b) Charge independence in strong interactions involving strange particles. Charge symmetry has already been roughly checked in hyperfragments.

One feature of the theory is at first sight odd. Bosons and fermions are treated differently. Although

$$\pi^- = \overline{\pi^+}, \qquad \Sigma^- \neq \overline{\Sigma^+}$$

(it can't be for B conservation, and the masses of $\Sigma^\pm$ are now known to be different). Similarly, two fermions with $T = 1/2$ are included, but only one boson. The reason behind these facts is that for fermions an extra quantum member , B, is available. This distinguishes between $\overline{\Sigma^+}$ and Σ^- and between P and Ξ^-, but nothing would distinguish π^- from π^+.

This theory has had great practical successes. However, the important fact that Q is linear related to T_3 within a charge multiplet is only expressed by equation (2). And the number S in this equation is introduced in an arbitrary manner. A number of

attempts have been made to generalize the isotopic space in such a way that a linear relation appeared naturally. Before we can understand these theories, we must briefly consider the theory of the representation of rotation groups in 3 and 4 dimensions.

4.- REPRESENTATIONS OF ROTATION GROUPS.

Our treatment uses infinitesimal generators constantly, and is supposed to make relatively simple that part of the theory which has so far proved relevant to physics. We speak only of the homogeneous groups, that is the groups of rotations without translations.

Take first the Euclidean rotation group in 3 dimensions. That is the set of all operations on a set of orthogonal coordinate axes, which keep them orthogonal and their origin fixed. Such a set forms a group because it obeys the three axioms of a group :

a) If R_1 and R_2 are rotations then the product R_2R_1 is defined as the operation which takes the axes into the position obtained by applying R_1 and then R_2. R_2R_1 is itself a rotation.

b) There is a unit rotation, 1, which makes no charge at all ; and for any rotation R there is an inverse R^{-1} such that

$$R^{-1} R = RR^{-1} = 1.$$

c) $R_1(R_2 R_3) = (R_1 R_2) R_3$. A representation of a group is a set of square matrices, such that with any element of the group is associated one and only one matrix, M_R, and such that, if $R_1 R_2 = R_3$, then $M_{R_1} M_{R_2} = M_{R_3}$. We also consider two-valued representations, where we allow $M_{R_1} M_{R_2} = \pm M_{R_3}$.

A representation is completely reducible if every matrix in it can be written in the form

$$\left(\begin{array}{c|c} & 0 \\ \hline 0 & \end{array}\right)$$

where the non-vanishing components make up two square matrices down the diagonal. For our purposes, we can call such a case : reducible (though there is really a difference between complete reducibility and reducibility). Obviously, then, a reducible representation splits up into two quite independent sets of matrices, and there is no point in considering, them together.

If we have a representation of matrices M, and we replace each M by $M' = T^{-1}MT$ (where T is a non singular square matrix) ; then obviously we have another representation. Two such representations are called equivalent. They are not essentially different.

One therefore seeks all inequivalent, irreducible representations. We will in fact find only those of finite order (that is with finite matrices).

One can convince ourself that any rotation can be made up of a product of a sufficient number of rotations about one or other of the co-ordinate axes separately. Let $R_3(\theta)$ be the rotation though an angle θ about the 3-axis. Define

$$A_3 = \frac{d}{d\theta} A_3(\theta)\Big|_{\theta=0}$$

Then $(1 + A_3\epsilon)$ is the rotation through an infinitesimal angle ϵ about the 3-axis. Since the product of two rotations about the 3-axis is itself such an rotation, $R_3(\theta)$ may be made up of infinitesimal rotations, thus :

$$R_3(\theta) = \lim_{n\to\infty}\left(1 + A_3\frac{\theta}{n}\right)^n = \exp(A_3\theta)$$

(where the exponential is defined by its series expansion). Thus all rotations may be generated by using the infinitesimal generators A_1, A_2, A_3.

The A_i obey certain commutation relations which we want to know. These can be found by abstract methods, but we will use an easier (though less elegant) method. We describe any rotation by the 3 x 3 matrix which gives the co-ordinates of the rotated axes in the co-ordinate system defined by the un-rotated axes. Thus

$$R_3(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Therefore $$A_3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Note that A_3 is <u>not</u> itself a rotation, but the infinitesimal rotation

$$(1 + A_3\epsilon) = \begin{pmatrix} 1 & \epsilon & 0 \\ -\epsilon & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Using similar rotation for A_1 and A_2, we obtain :

$$\left.\begin{aligned} [A_1 , A_2] &= -A_3 \\ [A_2 , A_3] &= -A_1 \\ [A_3 , A_1] &= -A_2 \end{aligned}\right\} \qquad (3)$$

These are the requered commutation relations.

To complete the group we have to include <u>reflections</u>. All reflections can be made up out of rotations and any one reflection. We choose as our basic reflection the one given by the matrix

$$S = \begin{pmatrix} -1 & & \\ & -1 & \\ & & -1 \end{pmatrix}$$

S obeys the additional relations

$$\left.\begin{aligned} [S, A_i] &= 0, \\ S^2 &= 1 \end{aligned}\right\} \qquad (4)$$

Since the entire group can be built up out of the A_i and S, it is sufficient to seek, for representations, four matrices M_i, P, obeying the relations (3) et (4) ; except that for two-valued representations we can allow

$$P^2 = \pm I$$

(where I is the unit matrix).

The matrices $i M_i$ obey the same commutation relations as the components of an angular momentum. In any standard book on quantum mechanics, it is shown that we can make M_3 and $M^2 = M_1^2 + M_2^2 + M_3^2$ simultaneously diagonal, with $M^2 = -\ell(\ell+1)$ and M_3 having eigenvalues $-\ell, -\ell+1 \ldots\ldots \ell-1, \ell$, where ℓ is integral or half-integral. Thus, there is one representation, of dimension $2\ell+1$, for each integral or half integral ℓ. It can, in fact, be proved that all equivalent irreducible representations can be constructed in this way. However, we will be satisfied to be able to construct the simplest cases.

Since P commutes with all the other matrices, it is plausible that, in an irreducible representation, P should be a multiple of the unit matrix (this, in fact, follows from Schur's lemma). We have, then

$P = \pm 1$ for single valued representations (ℓ integral)

$P = \pm 1$ or $\pm i$ for double valued representations (ℓ half integral).

We can now construct the three first representations. For $\ell = 0$, $M_1 = M_2 = M_3 = 0$, and all rotations map into unity. This is the scalar representations for $P = \pm 1$; the pseudo-scalar, for $P = -1$.

For $\ell = 1$, we can take for the M_i the matrices we used to describe the rotations in the first place. This is the vector ($P = -1$) or pseudo-vector ($P = +1$) representation. Vectors are column matrices which transform by the matrix of this representations for any rotation, and thus transform the same way as the co-ordinates $\begin{matrix} x \\ y \\ z \end{matrix}$ of a point when the axes are rotated.

For $\ell = 1/2$, we can take

$$M_i = 1/2\, i\tau_i \qquad P = \pm i, \pm 1$$

where $\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\tau_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Let $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ be a spinor which transforms according to this representation, i.e. which, for any rotation, is multiplied by the matrix corresponding to that representation. Then $\psi^* \psi$ is a scalar, and $\psi^* \tau_i \psi$ is a pseudo vector. The easiest way to see this

to consider the infinitesimal rotations $(1 + \epsilon A_i)$, under which

$$\psi \to \left(1 + \tfrac{1}{2} i \tau_i \epsilon\right) \psi$$

and verify that the above combinations transform in the correct way. The fact that $\psi^* \psi$ is a scalar, means that this representations is unitory.

Consider, now the quantities $\psi^T C \psi'$ and $\psi^T C \tau_i \psi'$, where $C = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}$ and ψ' is another spinor transforming the same way as ψ. (T stands for transpose). One may verify that these are scalar and pseudo-vector, or pseudo-scalar and vector, according as $P = \pm 1$ or $P = \pm i$. (The reason being that $C^{-1} \tau_i^T C = -\tau_i$). It is seen, therefore, that one "advantage" of a spinor representation with $P = \pm i$, is that one can form all four quantities, scalar, pseudo-scalar, vector and pseudo-vector. Whereas, if $P = \pm 1$, one can only form scalar and pseudo-vector.

It must be emphasized, however, that a representation exists quite independently of other covariant quantities that one can form with it. From a purely mathematical point of view, all four representations, $P = \pm 1, \pm i$, exist on an absolutely equal basis. We shall refer to these facts in section 5.

Now let us consider rotations in a 4-dimensional Euclidean space. Here there are six infinitesimal generators, $A_{ij} = -A_{ji}$ corresponding to rotations in each of the co-ordinate planes $(i\ j)$. They must obey commutation relations, of which the following pair are typical.

$$[A_{12}, A_{34}] = 0, \qquad [A_{12}, A_{23}] = +A_{13}$$

Defining $L_i = A_{4i}$ $(i = 1, 2, 3)$, $E_1 = A_{23}$, $E_2 = A_{31} = E_3 = A_{12}$, we obtain

$$[E_1, E_2] = -E_3, \text{ etc.}$$
$$[L_1, L_2] = -E_3, \text{ etc,}$$
$$[L_1, E_2] = -L_3, \text{ etc.}$$

Hence, defining $G_i = \frac{1}{2}(E_i + L_i)$, $H_i = \frac{1}{2}(E_i - L_i)$, $(i = 1, 2, 3)$,

$$[G_1, G_2] = -G_3, \text{ etc,}$$
$$[H_1, H_2] = -H_3, \text{ etc,}$$
$$[G_i, H_i] = 0.$$

Thus the generators break up into two independent sets, each of which has the same commutation relations as the 3-dimensional case. In any representation, therefore, each set is characterized by an integral or half integral spin value, corresponding to ℓ. Every representation is thus characterized by a pair of integers or half integers $\{g, h\}$ and there are two quantum numbers additively conserved, corresponding to G_3 and H_3.

The representation $\{g, h\}$ clearly has $(2g + 1)(2h + 1)$ dimension. This enables as to identify the simplest ones $\{0,0\}$ is a scalar. $\{\frac{1}{2}, 0\}$ and $\{0, \frac{1}{2}\}$ are two different types of two component spinor. $\{1, 0\}$ is a self-dual skew-symmetric tensor. $\{\frac{1}{2}, \frac{1}{2}\}$ is a vector.

Let ξ and η be spinors of type $\{\frac{1}{2}, 0\}$ and $\{0, \frac{1}{2}\}$ respectively. Then $\xi^* \xi$ is a scalar, $\xi^* \sigma_i \xi$ a self-dual skew-symmetric tensor, $\eta^* \eta$ another scalar and $\eta^* \sigma_i \eta$ an anti-self-dual skew-symmetric tensor. The quantity $(\xi^* \sigma_i \eta , i \xi^* \eta)$ is a 4-vector. In all these combinations, ξ^* may be replaced by $\xi^T C$. These facts may be verified as in the 3-dimensional case, but ther may be remembered by means of the rules

$$\{\tfrac{1}{2}, 0\} \times \{\tfrac{1}{2}, 0\} = \{1, 0\} + \{0, 0\}$$
$$\{\tfrac{1}{2}, 0\} \times \{0, \tfrac{1}{2}\} = \{\tfrac{1}{2}, \tfrac{1}{2}\}$$

These rules are special cases of the fact that the quantum numbers g and h combine seperately in the same way as angular momentum in 3-dimensions, i.e. according to

$$\{\ell\} \times \{\ell'\} = \{\ell + \ell'\} + \{\ell + \ell' - 1\} + \ldots \quad (|\ell - \ell'|)$$

We have still to consider reflections. We choose as the basic reflection

$$S = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix}$$

(Note that $\begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$ is not a reflection but a rotation).

The commutation relations for S are more complicated than in 3-dimensions :

$$S A_{4i} + A_{4i} S = 0 \qquad (i = 1, 2, 3)$$

$$S A_{12} - A_{12} S = 0 \quad \text{etc}$$

Therefore

$$S^{-1} G_i S = H_i, \quad S^{-1} H_i S = G_i \qquad (i = 1, 2, 3).$$

It is clear, therefore, that, when reflections are included, the G_i and H_i are no longer independent. $\{1, 0\}$ is no longer a representation, but we have to take the sum $\{1, 0\} + \{0, 1\}$. Similarly, spinors become 4-component, $\{\frac{1}{2}, 0\} + \{0, \frac{1}{2}\}$. We shall not pursue this further here.

Now consider 4-dimensional Lorentz space. The commutation relations are now :

$$\left[A_{12}, A_{23}\right] = A_{13} \text{ etc, but } \left[A_{14}, A_{43}\right] = - A_{13} \text{ etc.}$$

We define L_i and E_i as before, but now put

$$G_i = \tfrac{1}{2} (E_i + iL_i), \qquad H_i = \tfrac{1}{2} (E_i - iL_i).$$

These G_i and H_i obey the same commutation relations as before. The representations are characterized by $\{g, h\}$ as before. An example of a $\{1, 0\}$ is the combination $\underset{\sim}{H} + i \underset{\sim}{E}$ of electromagnetism.

If, again, ξ is a $\{\frac{1}{2}, 0\}$ and η is a $\{0, \frac{1}{2}\}$, one may verify that $\xi^* \xi$ is not a scalar ; so that, in contrast to the Euclidean space, the representations is not unitary. In fact $\xi^* \eta$ and $\xi^T C \xi$ are scalars, and $\eta^* \tau_i \xi$ and $\xi^T C \tau_i \xi$ are self-dual anti-symmetric tensors, and $(\xi^* \tau_i \xi, \xi^* \xi)$ is a 4-vector. These facts are consistent with the rules $\{\frac{1}{2}, 0\} \times \{0, \frac{1}{2}\} = \{\frac{1}{2}, \frac{1}{2}\}$, if it is recognized that ξ^* transforms like η not like ξ.

Space-inversion, S, makes the same changes as in the Euclidean space. As an example, $\underset{\sim}{H} + i \underset{\sim}{E}$ has to be replaced by the six real quantities F_{ij} ($\{1, 0\} + \{0, 1\}$). Similarly, 2-component spinors have to be replaced by 4-component spinors, which can be taken real (Majorana representation) in complete analogy with F_{ij}.

5.- FORMALISM OF D'ESPAGNAT AND PRENTKI.

We now return to the question raised at the end of section 3. First we replace equation (2) by

$$Q = T_3 + I/2\ U$$

where U is an additive quantum number, whose existence is to be "explained". It takes the values $0, \pm 1$, as shown in table 1.

Isotopic spin space is generalized to include reflections. Then as demonstrated at the beginning of section 4, representations with T = 0 or 1 can have parity $p = \pm$ and representation with T = I/2 can have $p = \pm$ i. p. is not an additive quantum number so we define

$$p = i^U = e^{i\frac{\pi}{2}U}$$

Then U is additively conserved mod.4. It is then assumed that the Lagrangian for the strong interactions contains only Yakawa like couplings, with the values of U that have been used, that U cannot change by more than 3 in any elementary interaction, and is therefore conserved.

Thus, if only scalars, pseudo-vectors and spinors with $p = \pm$ i occur, and if they interact through Yykawa-like terms, then the exists a quantum member U which is additively conserved. D'Espagnant and Prentki stress the assumption that only scalars and pseudo-vectors occur; but not that only spinors with $p = \pm$ i.occur. As explained in the last section, these spiners ,although they may have practical advantages, have no priviledged position over those with $p = \pm 1$.

Now consider the operator

$$e^{i\pi Q} = e^{i\pi T_3} \cdot P$$

Since $e^{i\pi T_3}$ is the representative of a rotation through π about the 3-axis, $e^{i\pi Q}$ represents a reflection in the (I2)plane.

Perhaps the best way to state the important point about this scheme is to say that, if $e^{i\pi Q}$ is to be interpreted as the reproduction operator in the (I2)-plane and if Q is to have eigen-vales + 1, 0, then only scalars, pseudo-vectors and spiners with $p = \pm$ i can be allowed. From this point of view, there is a gap for a boson with T = 0. (Phys. Rev.I02, I684 (I956)).

6.- INTRODUCTION TO OTHER FORMALISMS.

Other schemes for the strange particles have been proposed, most of them using, explicitly or implicitly, a four-dimensional space. The purpose of this section, together with section 4, is to provide a background to help in the understanding of the literature on this subject.

The earliest 4-dimensional schemes were invented to provide an interpretation of the quantum number S or U. Pais wrote

$$Q = T_3 + T'_3 + \tfrac{1}{2} B$$

where T_3 and T'_3 are the thirds components of the angular momenta corresponding to the two sets of generators G_i and H_i of section 4. T_3 was identified with ordinary isotopic spin. The trouble with this scheme was that it predicted the existence of doubly-charged particles, and others, which have never been seen.

Salam and Polkinghorne wrote :

$$Q = T_3 + T'_3$$

and used only single valued representations. The trouble with this scheme was that N and Ξ were combined into one 4-vector, with no explanation given of their mass difference.

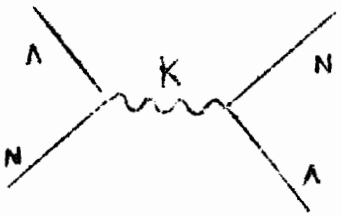
Schwinger then proposed a 4-dimensional scheme, which he has since withdrawn and replaced by another. This is very similar to Gell Mann's new ideas, which we shall describe first. (Schwinger, Phys. Rev. 104, 1164 (1956).

Gell-Mann starts by drawing some conclusions he believes are indicated by experiment. He assumes that all mesons have spin 0, and all baryons have spin $\frac{1}{2}$. The parity of all particles with $S = \pm 1$ is inaccessible. In strong interactions, S is conserved, and so in any parity assignment a factor $(-1)^S$ can be inserted without altering the conclusions. In weak interactions, there is no reason to suppose that parity is conserved. Therefore, Λ^0 can be conventionnally assigned the parity + 1 (relative to P and N). The parity of the Σ's and K's is then a matter for experiment.

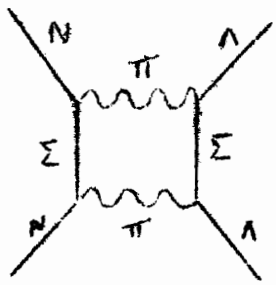
Gell-Mann asserts that photon-production of K-mesons, and, in fact, all strange particle production reactions (particularly high-energy jets) point to a weaker coupling of K-mesons than π-mesons. It is known that photoproduction of pions at threshold affords a direct measure of the pion coupling constant, $g^2_\pi/4\pi \simeq 15$

For K-mesons, the experiments are supposed to give a considerably smaller value, perhaps $g^2_K/4\pi \sim 1$ (The comparison may not be valid unless the K is pseudo-scalar).

Now consider the Λ-nucleon force, as indicated by hyperfragments. One possible mechanism is



If the K coupling is small, this will not be the dominant mechanism. The other possibility is



Provided that the pion couplings here are of the same strength as the pion-nucleon coupling, and provided that the parity of Σ is + 1 this will give a similar potential to the two-pion exchange term in nucleon-nucleon forces. Thus (if we can attach some rough meaning to this way of calculating), the Λ N force will be similar to NN force, but somewhat weaker and of shorter range (since there is no <u>one</u> pion exchange term). This is supposed to be consistant with hyperfragment data.

Thus, Gell-Mann concludes K-mesons are more weakly coupled than pions, and that nucleons, Λ and Σ have the same parity. In opposition to this view there is the fact that K^+ scattering seems to be large, and Marshak's considerations about $\Sigma^\pm$ mass difference (see below).

Arguing from these considerations, Gell-Mann divides the strong couplings into V S (pion couplings) and M S (K couplings). In a generalization of table II, he supposes that V S interactions exhibit greater symmetries than M S ones. Thus, he first supposes the K-meson couplings turned off, and then looks for new symmetries among the remaining couplings.

The simplest possible pion couplings may be written (leaving out γ -matrices)

$$g_1 N^* \underset{\sim}{\tau} N . \underset{\sim}{\pi} - g_2 \Lambda^* \underset{\sim}{\Sigma} . \underset{\sim}{\pi} + c.c.$$
$$- g_3 i \underset{\sim}{\Sigma}^* \wedge \underset{\sim}{\Sigma} . \underset{\sim}{\pi} + g_4 \Xi^* \underset{\sim}{\tau} \Xi . \underset{\sim}{\pi}$$

According to Salam, charge conjugation implies that the four g's may be taken real (Nuc. Phys. 2, 173 (1956)). Define two 2-component quantities

$$\Sigma = \begin{pmatrix} \Sigma^+ \\ \dfrac{\Sigma^0 + \Lambda^0}{\sqrt{2}} \end{pmatrix} \qquad \Sigma' = \begin{pmatrix} \dfrac{\Sigma^0 - \Lambda^0}{\sqrt{2}} \\ \Sigma^- \end{pmatrix}$$

Then, with a suitable choice of the g's, the above interaction Lagrangian may be re-written as

$$g\left(N^* \underset{\sim}{\tau} N + \Sigma^* \underset{\sim}{\tau} \Sigma + \Sigma'^* \underset{\sim}{\tau} \Sigma' + \Xi^* \underset{\sim}{\tau} \Xi \right) . \underset{\sim}{\pi}$$

Gell-Mann therefore assumes that the baryon mass splittings are due to the K-mesons, and that when these are turned off, the four doublets have a common mass and identical interactions with the pions.

At this stage the symmetry between the four baryon doublets is a purely formal one. However it is certainly possible to combine them into two 4-vectors in a four-dimensional space. This has been done, for instance, by Schwinger.

Gell -Mann then deduces certain consequences, by turning on the K-meson interaction and calculating it in second order perturbation theory (but taking into account pion interactions exactly). Let M be the mass of the baryons when K-interactions are switched off. Then, to second order in K-interactions

$$M_N = M + (g^2_{N\Lambda K} + 3\, g^2_{N\Sigma K})\, I$$

$$M_\Xi = M + (g^2_{\Xi\Lambda K} + 3\, g^2_{\Xi\Sigma K})\, I$$

$$M_\Lambda = M + 2(g^2_{N\Lambda K} + g^2_{\Xi\Lambda K})\, I$$

$$M_\Sigma = M + 2(g^2_{N\Sigma K} + g^2_{\Xi K K})\, I$$

where I is the same integral in each case. Thus, whatever the values of the g_K

$$\frac{M_N + M_\Xi}{2} = \frac{M_\Lambda + 3\, M_\Sigma}{4}$$

The two sides of this equation are in fact 190 Mev and 235 Mev above the nucleon mass. Gell-Mann also deduces that, if the NN force is written as

$$V_{NN} = A + B\, \tau_1 \cdot \tau_2$$

then

$$V_{N\Lambda} = A$$

D'Espagnat, Prentki and Salam have given an exhaustive account of all the different possibilities that exist in 4-dimensional spaces. However, if we accept Gell-Mann's starting point, the alternatives can be considerably reduced.

a) It has been argued on very general grounds by Higgs and by Schwinger that only Euclidean spaces can be used ; for otherwise terms appear in the free part of the Lagrangian with the wrong sign.

b) If the pion is to retain its identity it must be a 3-component quantity, and therefore a $\{1, 0\}$ representation. Therefore, reflections cannot be included. As a consequence, there can be only 2-component spinors (not 4-component spinors). (See section 4.)

Schwinger's new scheme appears to be very similar to Gell-Mann's. He seems to assert that, if there are any groups of transformations corresponding to internal degrees of freedom of the particles, then, on general grounds, the simplest groups must be the Euclidean rotation groups in 2, 3, or 4-dimensions. The 2-dimensional rotations are gauge transformations.

Pais has revived his old 4-dimensional scheme. Corresponding to the fact, already mentionned, that the parity of strange particles is inaccessible, he proposed to define parity, P, so that

$$P S + S P = 0$$

He asserts that this enables him to remove the unwanted particles from his scheme.

(A general reference for this section is Rochester Conference Report, 1957).

7.- ELECTROMAGNETIC MASS DIFFERENCES.

Feynman and Speisman have shown that the P - N and π^+, π^0 mass differences can be understood as due to the action of charge and anomalous magnetic moment. Their result is

$$\delta M = M \left[I_0 \mp I_1 \mu + I_2 \mu^2 \right]$$

for positive or negative particles, and

$$\delta M = M I_2 \mu^2$$

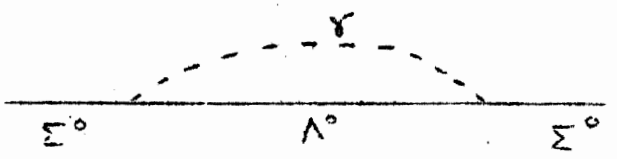
for neutral particles. The I's are integrals whose numerical value depends upon the cut-off used. μ is the anomalous magnetic moment.

Marshak et. al. have applied these methods to the Σ mass differences. For instance, they have the formula

$$M_{\Sigma^-} - M_{\Sigma^+} = \left[I_1 + I_2 (\mu_+ - \mu_-) \right] (\mu_+ + \mu_-)$$

In $(\mu_+ + \mu_-)$ the effects of the pion clouds cancel out (since, when $\Sigma^+ \to \Lambda + \pi^+$, $\Sigma^- \to \Lambda + \pi^-$), therefore it is very difficult to see how the observed large mass difference can be obtained without involving strongly coupled K-mesons.

The above formula for the self-mass of a neutral particle has to be modified to include the graph



for the case of the Σ°. This probably amounts to a factor of about 2.

In the course of his calculations Marshak uses the interesting result

$$\mu_{+} + \mu_{-} = 2\mu_{o}$$

which is a direct consequence of the fact that

$$Q = T_3 + \text{constant}$$

(Marshak et. al. Phys. Rev. 106, 599 (1957)).

8.- LIFE-TIMES AND BRANCHING RATIOS.

The observed lifetimes are given in table I. The following are observed values of branching ratios (1957)

$$\frac{\Lambda^{\circ} \to \pi^{\circ} + N}{\Lambda^{\circ}} = .32 \pm .05$$

$$\frac{K_1^{\circ} \to \pi^{\circ} + \pi^{\circ}}{K_1^{\circ} \to 2\pi} = .14 \pm .06$$

$$\frac{K^{+} \to \pi^{+} + \pi^{\circ} + \pi^{\circ}}{K^{+} \to \pi^{+} + \pi^{-} + \pi^{+}} = .33 \pm .07$$

$$\frac{\Sigma^{+} \to P + \pi^{\circ}}{\Sigma^{+}} \simeq \frac{1}{2}$$

$$K^{+}_{\mu_2} : K^{+}_{\pi_2} : K^{+}_{\pi_3} : K^{+}_{\mu_3} : K^{+}_{e_3} \simeq 60 : 30 : 7 : 3 : 2$$

The first thing to note is that the life-times of Λ, $\Sigma^{\pm}$ and K^{o} are about what would be expected on the basis of a crude phase space calculation using a coupling constant of the order 10^{-15} (the same value that is required for π, μ. N decay).

For decay of a fermion (mass M) into a fermion (mass m) and a boson (mass μ), the estimate is (using the simplest possible direct interaction between the three fields)

$$\tau^{-1} \sim g^2 \frac{E+m}{M} p$$

where p is the momentum and E the energy of the secondary fermion, so that $M = \sqrt{m^2 + p^2} + \sqrt{\mu^2 + p^2}$

For the Λ, $p \simeq 200$ and $\tau^{-1} \sim g^2 400$ Mev

$$\tau \sim \frac{10^{-21}}{400 \times 10^{-15}} \sim 10^{-10}$$

For the $\Sigma^{\pm}$, $p \simeq 300$ which predicts $\tau \sim 10^{-10}$ again.

For the K_1^o decay into two pions, we assume a coupling constant for the three boson interaction of $M_K g$. The estimate is then

$$\tau^{-1} \sim (M_K g)^2 \frac{p}{M_K \omega}$$

where p is the momentum of a pion and ω its energy. We have $\omega = \frac{1}{2} M_K$, $p \simeq 200$; so that again $\tau^{-1} \sim g^2 200$ Mev

Now consider the long life-time of K_2^o. The theory of Gell-Mann certainly predicts the existence of the two particles K_1^o and K_2^o (see section 3). We assume parity is not conserved in any of the weak decays. The situations is then complicated in general (see Lee, Oehme and Yang). However, if C P is conserved and K_2^o is defined to be the eigenstate with eigenvalue - 1, then K_2^o cannot decay into two pions. The 3π decay mode is most easily estimated. It has the same angular momentum states available as in $K^{+}_{\pi_3}$ and so should have a similar partial life-time of about 10^{-7}. According to the $\Delta T = \pm \frac{1}{2}$ (see below), these two partial life-times should in fact be exactly equal.

(Pais and Treiman, Phys. Rev.) This same value can be obtained from a phase-space estimate, assuming a direct four boson interactions. In a non - relativistic approximation for the pions, this gives

$$\tau^{-1} \sim g^2 \frac{(M-3\mu)^2}{M} \simeq g^2\ 10\ \text{Mev}$$

$$\tau \sim 10^{-7}\ \text{sec}$$

The branching ratios in the K_2^0 decay are not known at present, but the 3π mode seems to account for only the smaller part of the decay, the remainder being $K^0_{2\,e_3}$ and K^0_{2,μ_3}. If this is so, the lifetime should be as short as 10^{-8} sec., in poor agreement with experiment.

Next consider the long lifetime of the K^+. The easiest way to explain this is to invent the (approximate) selection rule $\Delta T = \pm \frac{1}{2}$, for weak decays. This implies $\Delta T_3 = \pm \frac{1}{2}$, and therefore that the decay $\Xi^- \to N + \pi^-$ is forbidden, which is consistent with observation. We shall discuss the consequences of this rule below. But first we note that it is necessary also to explain why the $K^+_{\mu 2}$ partial life-time is also long. We make a crude phase-space comparison with $\pi^+ \to \mu^+ + \nu$ decay. For decay of a boson (mass M) into a meson (mass m) and a neutrino, the simplest direct interaction yields ;

$$\tau^{-1} \sim g^2 \frac{p^2}{M^2}$$

where p is the meson momentum, $p = \frac{M^2 - \mu^2}{2M}$.

The two values are

$$\tau_\pi^{-1} \sim \frac{1}{4} g^2$$

$$\tau_{K^+}^{-1} \sim 12\, g^2$$

Thus, on this basis, we would expect

$$\tau_{K^+} \sim \frac{1}{50} \tau_{\pi^+} \sim 10^{-9}$$

which is a little short. If, for instance, the decays proceeded by an intermediate baryon loop, this would be an argument for having K-mesons more weakly coupled to baryons than π-mesons.

Now consider other consequences of the rule $\Delta T = \pm \frac{1}{2}$. If this is assumed, there is, in Λ decay, only one final channel, with $T = \frac{1}{2}$, and the branching ratio must be

$$\frac{\Lambda \to N + \pi^0}{\Lambda \to p + \pi^-} = \frac{1}{2}$$

in excellent agreement with experiment. Similarly, in $K^0_{1\pi_2}$ decay the decay channel must be $T = 0$ (since T must be even), and so

$$\frac{K_1^0 \to 2\pi^0}{K_1^0 \to \pi^+ + \pi^-} = \frac{1}{2}$$

This is not in good agreement with experiment. However, the selection rule can only be approximate in any case. It has been shown that the observed value of $\frac{\tau_{K^+}}{\tau_{K^0_1}}$ and the observed value of the above branching ratio, agree if we allow an admixture of about 10 % of both $\Delta T = \pm 3/2$ and $\Delta T = \pm 5/2$. This is rather a large admixture, and the selection rule looses much of its attractiveness.

For $K^+_{\pi 3}$ decays the rule makes no definite predictions, since a state of $T = 1$ can be made up from three pions in an infinity of ways. However, if the spatial wave-function is completely symmetric in the three pions (as is suggested by the distribution of points in the Dalitz diagram), there is the unique prediction

$$\frac{K^+ \to \pi^+ + \pi^0 + \pi^0}{K^+ \to \pi^+ + \pi^- + \pi^+} = \frac{1}{4}$$

Taking account of the different quantities $(M - 3\mu)^2$ and $(M - \mu - 2\mu_0)^2$ appearing in the phase space this ratio is raised to 1/3, in agreement with experiment. (Dalitz, Proc. Phys. Soc. A 69, 527, (1956)).

In the case of $\Sigma^\pm$ there are two final channels, $T = \frac{1}{2}$ and 3/2. There are also two measurable quantities, the ratio of the life-times and the branching ratio in Σ^+ decay. The analysis is complicated by the interference between the two channels, which means we have to know the relative phases of the amplitudes. Takeda has shown that the

amplitudes are of the form $a e^{i\delta}$, where a is real and δ is the phase shift for the scattering of pion on nucleon in the relevant final state. These phases are known, and actually, at the energy of decay, are negligible (since T = 3/2 states are not involved). (See Takeda, Phys.Rev.101, 1547 (1956)).

Using this method of analysis, Feld has shown that, assuming parity conservation, the two measured quantities cannot be fitted without a very large violation of $\Delta T = \pm I/2$ (about I/3). However, if parity is not conserved, both S. and P waves are possible for the decay particles; and so more a's are at our disposal. Feld obtains a fit with maximum parity non-conservation, and $(1+\gamma_5)$ appearing with $\Delta T = +I/2$ and $(1-\gamma_5)$ with $\Delta T = -I/2$ (or the other way round). (See Feld, Phys. Rev., to be published). This may possibly be thought to have some connection with the ideas of Pais, mentioned at the end of section 5.

The lifetime of Σ^0 estimated from an interaction

$$e\left(\bar{\Sigma}^0 \sigma_{\mu\nu} \Lambda\right) F_{\mu\nu}$$ is

$$\tau^{-1} \sim e^2 \text{ Mev} \quad \text{or} \quad \tau \sim 10^{-20} \text{ sec}$$

This beyond the reach of present experimental technique, but the lifetime of π^0 estimated in the same way is compatible with experiment.

We make no discussion of the $K^+_{\mu 3}$ and $K^+_{e 3}$ decay modes. Most, if not all, of these are $K^+ \to \mu^+ + \nu + \pi^0$ e^+ Decays into $e^\pm + \nu$ and into $\mu^\pm + e^\mp$, which are a priori possible, have not been seen. The decays $\pi^+ \to e^+ + \nu$ and $\pi^+ \to e^+ + \nu + \gamma$ are also known to have at most very small probabilities. The latter is particularly puzzling as it would be expected to follow from a tenser term in neutron beta-decay. (Treiman and Wyld, Phys. Rev.101, 155, (1956)).

The following theorems of Lee, Oehme and Yang are relevant to strange particles. Using only TCP theorem (i.e. not assuming T.C.P. separately.):

a) Masses of particle and anti-particle are equal ;

b) Lifetimes of particle and anti-particle are equal to second order in weak interaction coupling-constant.

Using C P or T :

(c) the probabilities of, for instance

$$K_2^0 \rightarrow e^+ + \nu + \pi^- \quad \text{and} \quad K_2^0 \rightarrow e^- + \bar{\nu} + \pi^+$$

are equal. However, I believe this is not a very good test of T, since with the lifetime of K_2^0 being so much longer than K_1^0, these probabilities would have to be nearly equal anyway.