$\mathcal{N} = 1$ heterotic vacua in four dimensions
Warped resolved orbifoldized conifolds in heterotic sugra
The worldsheet CFT for the conifold solutions
$T^2$ fibrations
Conclusions

Heterotic Torsional Backgrounds, from Supergravity to CFT

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work in progress
Introduction

Heterotic compactifications

- $\mathcal{N} = 1$ heterotic compactifications correspond to solvable perturbative worldsheet CFT only at very special points: toroidal orbifolds, Gepner models, free fermions,...
- Usually the geometrical interpretation is lost – although the topological data matches geometrical Calabi-Yau compactifications

Smooth compactifications

- Instead one can consider in supergravity a CY compactification
- One specifies a gauge bundle $V$ on the compactification manifold. Needs to satisfy the modified Bianchi identity for the NSNS two-form
- Satisfied automatically for the standard embedding of the $SU(3)$ spin connection in the gauge connection however not interesting
- For more general gauge bundles, torsion usually is needed fluxes appear naturally even without trying to stabilize moduli
Heterotic Flux compactifications (torsional)

- Less understood than type II models, as *not* conformally CY, not even Kähler manifolds appear
- One needs therefore to deal with explicit examples (few are known in the compact case)
- However, perturbative heterotic compatifications are potentially accessible to worldsheet CFT methods because (i) no R-R flux and (ii) the dilaton is not stabilized perturbatively

Local models of flux compactifications

- Simpler descriptions can be found near smoothed singularities (as Klebanov-Strassler throats in type IIB)\(\rightarrow\) local models
- In heterotic, we find two examples with a solvable worldsheet CFT: a \(T^2\) fibration over Eguchi-Hanson and a resolved conifold
- As in type II one may have interesting physics ’localized’ in these throats due to the strong warping \(\rightarrow\) SUSY breaking?
Outline

1. $\mathcal{N} = 1$ heterotic vacua in four dimensions
2. Warped resolved orbifoldized conifolds in heterotic sugra
3. The worldsheet CFT for the conifold solutions
4. $T^2$ fibrations
5. Conclusions
Manifolds with $SU(3)$ structure and $\mathcal{N} = 1$ heterotic vacua

- $\mathcal{N} = 1$ compactification with torsion: 6d manifold admitting a covariantly constant spinor w.r.t. spin connection $\omega - \frac{1}{2} \mathcal{H}$
- Defines an $SU(3)$ structure, characterized by the $SU(3)$-invariant real 2-form $J$ and complex 3-form $\Omega$
- $(J, \Omega)$ give a complex structure and a metric
- Susy conditions: calibrations for wrapped 5-branes
  \[
  \begin{cases}
  d(e^{-2\Phi} \Omega) = 0 \\
  d(e^{-2\Phi} J \wedge J) = 0 \\
  d(e^{-2\Phi} J) - e^{-2\Phi} \ast_6 \mathcal{H} = 0
  \end{cases}
  \]
  non Kähler manifolds
- Gauge bundle has to satisfy Hermitean Yang-Mills equations:
  \[
  \mathcal{F}^{\bar{a}\bar{b}} J_{\bar{a}\bar{b}} = \mathcal{F}^{ab} = \mathcal{F}^{\bar{a}\bar{b}} = 0
  \]
- Massive spinorial reps of the gauge group $\Rightarrow c_1(V) \in H^2(2\mathbb{Z})$

Bianchi identity

\[
d\mathcal{H} = \alpha' [\text{tr} R(\omega -) \wedge R(\omega -) - \text{Tr}_v F \wedge F] \text{ in forms}
\]

★ Non-linear constraint: $R(\omega -)$ constructed w. connection $\omega - \frac{1}{2} \mathcal{H}$
Warped Conifold I: supergravity ansatz

- Local model of heterotic $\mathcal{N} = 1$ compactification in 4d
  - conifold singularity
  - hypersurface $z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0$ in $\mathbb{C}^4$

- Singular cone over a $T^{1,1} \sim (SU(2) \times SU(2))/U(1)$ base:
  - $S^1$ fibration over $S^2 \times S^2 \Rightarrow S^3 \times S^2$ topology

- Singularity regularized with a finite $S^2$ (blow-up) or a finite $S^3$ (deformation)
  - the latter case is used in type IIB (Klebanov-Strassler) as the compact 3-cycle can support RR 3-form flux corresponding to fractional D3-branes

- In heterotic needs instead a 2 or 4-cycle (by Hodge duality) to support magnetic flux.

- We will consider the latter, which is topologically possible for the orbifold $\text{conifold}/\mathbb{Z}_2$
Heterotic supergravity ansatz

\[ ds^2 = dx^\mu dx_\mu + \frac{3}{2} H(r) \left[ \frac{dr^2}{f^2(r)} + \frac{r^2}{6} (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2 + d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2) \right. \\
\left. + \frac{r^2}{9} f(r)^2 (d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2)^2 \right] \]

\[ \mathcal{H}_{[3]} = \frac{\alpha' k}{6} g_1(r)^2 (\Omega_1 + \Omega_2) \wedge (d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2) \]

- The \( S^1 \) fiber is deformed by the squashing factor \( f(r) \leq 1 \)
  \( \Rightarrow \) non-Kähler warped conifold with torsion
- Unlike in type II the \( \mathbb{R}^{3,1} \) space-time is unwarped (in string frame)
- We will find below a 'bolt' for some \( r = a (f(a) = 0) \), as in Eguchi-Hanson space
  \( \Rightarrow \) conical singularity removed by the orbifold \( \psi \sim \psi + 2\pi \)
Susy, Bianchi and HYM

- The **SUSY calibration conditions** give the system
  \[
  \begin{align*}
  f^2 H' &= -2\alpha' k g_1^2 / r^3 \\
  r^3 H f f' + 3r^2 H (f^2 - 1) + \alpha' k g_1^2 &= 0
  \end{align*}
  \]

- The **HYM equations** are satisfied with a line bundle (\(\vec{p}, \vec{q}\) gives embedding in the Cartan \(\vec{T}\)):

  \[
  A = \frac{1}{2} \vec{p} \cdot \vec{T} (\cos \theta_1 d\phi_1 - \cos \theta_2 d\phi_2) + \vec{q} \cdot \vec{T} \left( \frac{a}{r} \right)^4 (d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2)
  \]

- Second magnetic field (\(\vec{q}\)) responsible for the resolution of the conifold singularity  
  \(\Rightarrow\) free parameter \(a\) (blow-up parameter)

- In the large charges limit (\(\vec{p}^2, \vec{q}^2 \gg 1\)) the \(R^2\) term in **Bianchi identity** is negligible (checked 'on-shell')

- \(d\mathcal{H} = \alpha' \text{Tr} F^2\)
  \(\Rightarrow\) \(g_1^2(r) = \frac{3}{4} (1 - (\frac{a}{r})^4)\) and \(k = \vec{p}^2 = 4\vec{q}^2\)
Numerical solution

- One can solve numerically the susy equations for $f(r)$ and $H(r)$.
- Smooth and weakly coupled everywhere.
- For $r \to \infty$ one finds the usual Ricci-flat conifold and constant dilaton, however with non-zero NSNS and magnetic charges.
- For $r \to a$, the $S^1$ fiber degenerates (bolt).

One finds a non-Ricci flat conifold resolved by a 4-cycle.

*Conformal factor $H(r)$*

*Resolution function $f(r)$*

*both numerical (blue) and near-horizon (purple) solutions are plotted*
Analytical solution in the double-scaling limit

- String coupling diverges in the blow-down limit (point-like instanton)
  - One can define a double-scaling limit \( g_s \to 0 \) with \( g_s \alpha'/a^2 \) fixed
- It isolates the near-bolt region \( a^2 \leq r^2 \ll \alpha'k \) from the asymptotically Ricci-flat region
- In this regime, one can solve analytically the susy equations

**Double-scaling limit of the warped orbifoldized deformed conifold**

\[
\begin{align*}
\text{ds}^2 &= \frac{\alpha' k}{2r^2} \left[ \frac{dr^2}{1 - \frac{a^8}{r^8}} + \frac{r^2}{8} \left( \text{d}\theta_2^2 + \sin^2 \theta_1 \text{d}\phi_1^2 + \text{d}\theta_2^2 + \sin^2 \theta_2 \text{d}\phi_2^2 \right) \\
&\quad + \frac{r^2}{16} \left( 1 - \frac{a^8}{r^8} \right) \left( \text{d}\psi + \cos \theta_1 \text{d}\phi_1 + \cos \theta_2 \text{d}\phi_2 \right)^2 \right] \\
\text{e}^{2\Phi} &= \text{e}^{2\Phi_0} \frac{(\alpha'k)^2}{r^4} \\
\mathcal{A} &= \left[ \frac{1}{2} \left( \cos \theta_1 \text{d}\phi_1 - \cos \theta_2 \text{d}\phi_2 \right) \vec{p} + \left( \frac{a}{r} \right)^4 \left( \text{d}\psi + \cos \theta_1 \text{d}\phi_1 + \cos \theta_2 \text{d}\phi_2 \right) \vec{q} \right] \cdot \vec{H} \\
\mathcal{B}_{[2]} &= \frac{\alpha' k}{8} \left( \cos \theta_1 \text{d}\phi_1 + \cos \theta_2 \text{d}\phi_2 \right) \wedge \text{d}\psi
\end{align*}
\]

- Smooth weakly coupled background without sources (as \( r \geq a \))
In the blow-down limit one gets a linear dilaton $\times T^{1,1}$

- Non-Einstein $[SU(2)^2]/U(1)$ coset w. torsion and & line bundle

Obtained in the worldsheet CFT as a $U(1)_L \backslash SU(2)_k \times SU(2)_k$

asymmetrically gauged $\mathcal{N} = (1, 0)$ WZW model (left action)

- Classical 'anomaly' of the gauging cancelled by an action in the $SO(32)_1$ or $(\hat{E}_8 \times \hat{E}_8)_1$ anti-holomorphic CFT

- Specified by a 16-dim vector of charges $\vec{p}$, with $k = \bar{p}^2$

- in space-time, Abelian magnetic field
The heterotic orbifoldized warped conifold resolved by a 4-cycle has a worldsheet CFT \textit{also} in the double scaling limit.

Asymmetically gauged WZW model:

\[
\frac{SL(2)_{k/2} \times U(1)}{U(1)_L \times U(1)_R} \quad SU(2)_k \times SU(2)_k
\]

\(\Rightarrow\) Needs gauging in \(\hat{G}_1\) specified by \(\bar{q}\), with \(k = \bar{p}^2 = 4\bar{q}^2 - 4\)

One can extract from the gauged \textit{wzw} model the background fields to all orders in \(\alpha'\) \(\Rightarrow\) exact solution to Bianchi!

Left \(\mathcal{N} = 2\) SCA \(\Rightarrow\) \(\mathcal{N} = 1\) 4d SUSY in spacetime as expected

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Spectrum (from one-loop partition function)

- Discrete \(SL(2)\) representations (localized bound states)
  \(\Rightarrow\) massive \(U(1)\) gauge field along \(\bar{q} \cdot \bar{T}\) and set of massless 4d chiral multiplets localized at the bolt

- Unbroken gauge bosons and graviton \(\Rightarrow\) non-normalizable massless modes and continuum of massive modes
Worldsheet instantons and Liouville potentials

- So far, the $\mathbb{Z}_2$ orbifold of $T^{1,1}$ put 'by hand' in the CFT is there an analogue of no-conical singularity condition of sugra?
- Condition on the bundle $c_1(V) \in H^2(2\mathbb{Z})$ role in worldsheet CFT?
- $\frac{SL(2)}{U(1)}$ corrected by worldsheet non-perturbative effects (Fateev, Zamolodchikov)

Liouville-like potentials for Abelian bundles

- In our heterotic coset, the dynamically generated $\mathcal{N} = (2,0)$ Liouville potential reads $\mu_L \int (\psi^\rho + \psi^3) e^{-\frac{\sqrt{q^2-4}}{2} (\rho + iY^3_L)} - i\frac{q}{2} \cdot \tilde{X}_R + c.c.$
- ($\bar{\partial}X^n_R = \bar{\psi}^{2n-1} \psi^{2n}$ is the bosonized Cartan)
- Belongs to the twisted sector of the $\mathbb{Z}_2$ orbifold
- Orbifold and (right) GSO invariant only if $c_1(V) \in H^2(2\mathbb{Z})$:
  Explicitely $\sum_\ell (p_\ell \pm q_\ell) \equiv 0$ mod 4
One can get also type IIA/B or heterotic local models of $T^2 \rightarrow \mathcal{M}_6 \rightarrow K3$ with torsion

Fibered EH solution in the double scaling limit

\[ ds^2 = \frac{U_2}{T_2} \left| dx^1 + q_1 A_1 + T(dx^2 + q_2 A_2) \right|^2 + \frac{\alpha' k}{r^2} \left[ \frac{dr^2}{1 - \frac{a^4}{r^4}} + \sigma_1^2 + \sigma_2^2 + \left( 1 - \frac{a^4}{r^4} \right) \sigma_3^2 \right] \]

- $A_{1,2} \sim \sqrt{k} \frac{\sigma_3 a^2}{2r^2}$
- Bianchi identity $\Rightarrow k = \frac{4U_2}{T_2} |q_1 + Tq_2|^2$
- The worldsheet theory only well defined for rational $T^2$ CFT: $T, U$ valued in the same imaginary quadratic number field: $K = \mathbb{Q}(\sqrt{D}) \sim \mathbb{Q} + \mathbb{Q}\sqrt{D}$ w. $D = b^2 - 4ac < 0 \& \ gcd(a, b, c) = 1$
  $\Rightarrow$ Neat worldsheet understanding of moduli stabilization
Conclusions

- We studied local models of flux heterotic compactifications to four dimensions, with line bundles → new resolved warped orbifoldized conifold solutions and torsional $T^2$ fibrations over EH.
- One can define double scaling limits in order to isolate the throat regions (similar to KS throats in IIB flux compactifications).
- Remarkably these throats admit weakly coupled solvable worldsheet CFT descriptions, valid even for large curvatures.
- Susy breaking possible (work in progress).

Future directions

- Computing all $\alpha'$ corrections using the gauged WZW model.
- Holographic understanding → confining $\mathcal{N} = 1$ theories, $\mathcal{N} = 2$ theories with compact Coulomb bramches,...
- Precise embedding of local models in heterotic flux compactifications → $T^2$ fibrations at Gepner points.
- Description of non-Abelian bundles.
SUSY breaking in heterotic throats: generalities

- Is it possible to break SUSY at tree-level in the smooth throat solutions using a \textit{normalizable} deformation (cf. debate about KS)?
- In all models, the worldsheet CFT predicts a discrete spectrum of normalized modes near the tip
  - The \textit{Banks and Dixon theorem} (SUSY breaking for an internal $\mathcal{N} = 2$ CFT with compact target space) applies
  - So no 'small' susy breaking by normalizable deformations is possible (hard breaking by orbifold still possible)
- Continuum of delta-fct normalizable modes? $\Rightarrow$ all massive (non-chiral operators of $\mathcal{N} = 2$)
- Remains one possibility: use \textit{non-normalizable operators}, that correspond to non-unitary representations of the $\mathcal{N} = 2$ algebra.
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Example with \( T^2 \) fibration over EH

**Only** normalizable two-form of EH is the \((1, 1)\) a.s.d.

\[ \text{non-normalizable connection unavoidable} \]

SUSY-breaking \( T^2 \) fibration over EH (with a square torus)

\[
\begin{align*}
\text{ds}^2 &= \frac{k}{r^2} \left[ \frac{dr^2}{1 - \frac{a^4}{r^4}} + r^2 \left( \sigma_1^2 + \sigma_2^2 + \frac{(1 - \frac{a^4}{r^4})(1 - \lambda^2)}{1 - \lambda^2 \frac{a^4}{r^4}} \sigma_3^2 \right) \right] + \frac{U}{T^2} |dx^1 + A_1 + T(dx^2 + A_2)|^2 \\
\text{with } A_1 &= \lambda \frac{\sqrt{k}}{R_1} \frac{1 - \frac{a^4}{r^4}}{1 - \lambda^2 \frac{a^4}{r^4}} \sigma_3 \text{ and } A_2 = \frac{\sqrt{k}}{R_2} \frac{a^2}{r^2} \frac{1 - \lambda^2 \frac{a^4}{r^4}}{1 - \lambda^2 \frac{a^4}{r^4}} \sigma_3
\end{align*}
\]

- Torus moduli become also base-dependent:
  \[
  T = \frac{R_2}{R_1} (\lambda \frac{a^2}{r^2} + i \sqrt{1 - \lambda^2 \frac{a^2}{r^2}}) \text{ and } U = R_1 R_2 (\lambda \frac{a^2}{r^2} + i \sqrt{1 - \lambda^2 \frac{a^2}{r^2}})
  \]

- Constraint in the CFT: \( \sqrt{k R_1} \lambda \in \mathbb{Z} \)
  \[ \Rightarrow \text{not a continuous deformation for given } T^2 \text{ moduli} \]

- Worldsheet CFT still solvable \( \Rightarrow \text{no tachyons found} \)