

The Effective Theory of Inflation and Dark Matter in the Standard Model of the Universe

H. J. de Vega

LPTHE, CNRS/Université Paris VI

Pennsylvania State University, University Park, March 31, 2010

The History of the Universe

It is a history of **EXPANSION** and **cooling down**.

EXPANSION: the space **itself** expands with the time.

$$ds^2 = dt^2 - a^2(t) d\vec{x}^2 \quad , \quad a(t) = \text{scale factor.}$$

FRW: Homogeneous, isotropic and spatially **flat** geometry.

Cooling: temperature decreases as $1/a(t)$: $T(t) \sim 1/a(t)$.

The Universe underwent a succession of phase transitions towards the less symmetric phases.

Wavelengths **redshift** as $a(t)$: $\lambda(t) = a(t) \frac{\lambda(t_0)}{a(t_0)}$

Redshift z : $z + 1 = \frac{a(\text{today})}{a(t)}$, $a(\text{today}) \equiv 1$

The deeper you go in the past, the larger is the redshift and the smaller is $a(t)$.

Standard Cosmological Model: Λ CDM

Λ CDM = Cold Dark Matter + Cosmological Constant

- Begins by the **inflationary** era. Slow-Roll inflation explains horizon and flatness.
- Gravity is described by Einstein's General Relativity.
- Particle Physics described by the Standard Model of Particle Physics: $SU(3) \otimes SU(2) \otimes U(1) =$ qcd+electroweak model.
- CDM: dark matter is **cold** (non-relativistic) during the matter dominated era where structure formation happens. DM is outside the SM of particle physics.
- Dark energy described by the cosmological constant Λ .

Standard Cosmological Model: Λ CDM

Λ CDM = Cold Dark Matter + Cosmological Constant
begins by the Inflationary Era. **Explains** the Observations:

- Seven years WMAP data and further CMB data
- Light Elements Abundances
- Large Scale Structures (LSS) Observations. BAO.
- Acceleration of the Universe expansion:
Supernova Luminosity/Distance and Radio Galaxies.
- Gravitational Lensing Observations
- Lyman α Forest Observations
- Hubble Constant (H_0) Measurements
- Properties of Clusters of Galaxies
- Measurements of the Age of the Universe

Standard Cosmological Model: Concordance Model

$ds^2 = dt^2 - a^2(t) d\vec{x}^2$: spatially **flat** geometry.

The Universe starts by an **INFLATIONARY ERA**.

Inflation = Accelerated Expansion: $\frac{d^2 a}{dt^2} > 0$.

During inflation the universe expands by at least sixty e-folds: $e^{62} \simeq 10^{27}$. Inflation **lasts** $\simeq 10^{-36}$ sec and ends by $z \sim 10^{29}$ followed by a **radiation** dominated era.

Energy scale when inflation starts $\sim 10^{16}$ GeV (\Leftarrow CMB anisotropies) which **coincides** with the GUT scale.

Matter can be effectively described during inflation by a Scalar Field $\phi(t, \mathbf{x})$: the **Inflaton**.

Lagrangian: $\mathcal{L} = a^3(t) \left[\frac{\dot{\phi}^2}{2} - \frac{(\nabla\phi)^2}{2 a^2(t)} - V(\phi) \right]$.

Friedmann eq.: $H^2(t) = \frac{1}{3 M_{Pl}^2} \left[\frac{\dot{\phi}^2}{2} + V(\phi) \right]$, $H(t) \equiv \dot{a}(t)/a(t)$

Inflation Evolution Equations

Evolution equation for the Inflaton:

$$\ddot{\phi} + 3 H(t) \dot{\phi} - \frac{1}{a^2(t)} \nabla^2 \phi + V'(\phi) = 0, \quad H(t) \equiv \frac{\dot{a}(t)}{a(t)} = \text{Hubble.}$$

$$\text{energy density} = \rho = \frac{1}{2} \left[\dot{\phi}^2 + \frac{1}{a^2(t)} (\nabla \phi)^2 \right] + V(\phi)$$

$$\text{pressure} = p = \frac{1}{2} \left[\dot{\phi}^2 - \frac{1}{3 a^2(t)} (\nabla \phi)^2 \right] - V(\phi)$$

The scale factor grows exponentially during inflation and **suppresses** spatial gradient terms.

The inflaton field is therefore **homogeneous**: $\phi = \phi(t)$.

$$\ddot{\phi} + 3 H(t) \dot{\phi} + V'(\phi) = 0 \quad (1)$$

The Einstein equations reduce to a **single** equation: the Friedmann equation:

$$H^2(t) = \frac{1}{3 M_{Pl}^2} \rho = \frac{1}{3 M_{Pl}^2} \left[\frac{\dot{\phi}^2}{2} + V(\phi) \right] \quad (2)$$

Physics during Inflation

- **Out of equilibrium** evolution in a fastly expanding geometry. Vacuum energy **DOMINATED** (De Sitter) universe $a(t) \simeq e^{Ht}$.
- **Explosive** particle production due to spinodal or parametric **instabilities**. Quantum non-linear phenomena eventually **shut-off** the instabilities and **stop** inflation. Radiation dominated era follows: $a(t) = \sqrt{t}$.
- Huge redshift classicalizes the dynamics: an **assembly** of (superhorizon) quantum modes behave as a classical and homogeneous inflaton field. Inflaton slow-roll.

D. Boyanovsky, C. Destri, H. J. de Vega, N. G. Sánchez, The Effective Theory of Inflation in the Standard Model of the Universe and the CMB+LSS data analysis (**review article**), arXiv:0901.0549, Int.J.Mod.Phys.A 24, 3669-3864 (2009).

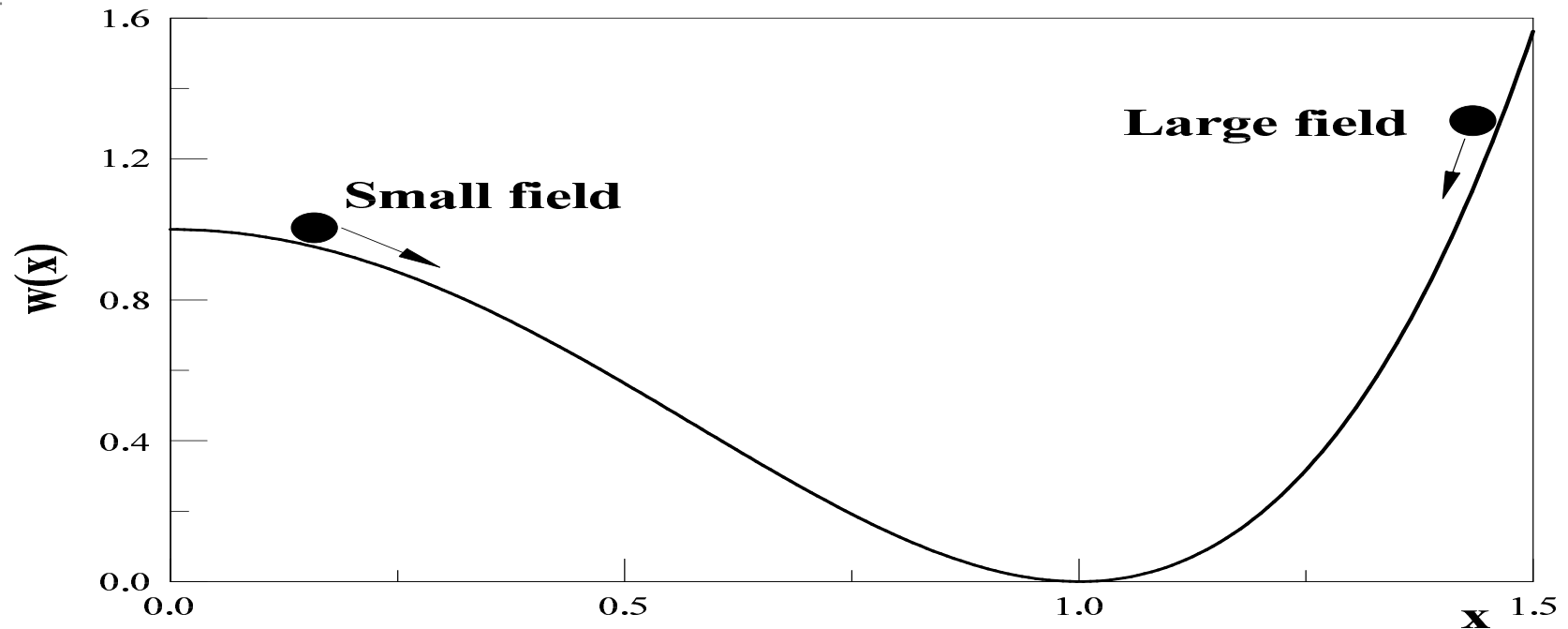
The Theory of Inflation

The inflaton is an **effective** field in the Ginsburg-Landau sense.

Relevant effective theories in physics:

- Ginsburg-Landau theory of superconductivity. It is an effective theory for Cooper pairs in the microscopic BCS theory of superconductivity.
- The $O(4)$ sigma model for pions, the sigma and photons at energies $\lesssim 1$ GeV. The microscopic theory is QCD: quarks and gluons. $\pi \simeq \bar{q}q$, $\sigma \simeq \bar{q}q$.
- The theory of second order phase transitions à la Landau-Kadanoff-Wilson... (ferromagnetic, antiferromagnetic, liquid-gas, Helium 3 and 4, ...)
- Fermi Theory of Weak Interactions (current-current).

Slow Roll Inflation



The field evolves towards the minimum of the potential.

$V(\text{Min}) = V'(\text{Min}) = 0$: inflation **ends** after a finite number of efolds. **Slow-roll is needed** to produce enough efolds of inflation (≥ 62) to explain the part of today's entropy in the universe having a primordial origin.

\implies the slope of the potential $V(\phi)$ must be **small**.

Slow-roll evolution of the Inflaton

During slow-roll the inflaton derivatives are **small** and the evolution equations (1) and (2) can be approximated by:

$$3 H(t) \dot{\phi} + V'(\phi) = 0 \quad , \quad H^2(t) = \frac{V(\phi)}{3M_{Pl}^2}$$

These first order equations can be solved in closed form as:

$$M_{Pl}^2 N[\phi] = - \int_{\phi}^{\phi_{end}} V(\varphi) \frac{d\varphi}{dV} d\varphi \quad .$$

$N[\phi]$ = the number of e-folds since the field ϕ **exits** the horizon till the end of inflation. $N \sim 60$.

ϕ_{end} = absolute minimum of $V(\phi)$.

Therefore, $\phi^2 =$ **scales** as $N M_{Pl}^2$. We define:

$\chi \equiv \frac{\phi}{\sqrt{N} M_{Pl}}$ **dimensionless** and **slow** field.

Universal form of the slow-roll inflaton potential:

$V(\phi) = N M^4 w(\chi)$, M = energy scale of inflation.

Primordial Power Spectrum

Adiabatic Scalar Perturbations: $P(k) = |\Delta_{k ad}^{(S)}|^2 k^{n_s-1}$.

To dominant order in slow-roll:

$$|\Delta_{k ad}^{(S)}|^2 = \frac{N^2}{12\pi^2} \left(\frac{M}{M_{Pl}}\right)^4 \frac{w^3(\chi)}{w'^2(\chi)} .$$

Hence, for **all** slow-roll inflation models:

$$|\Delta_{k ad}^{(S)}| \sim \frac{N}{2\pi\sqrt{3}} \left(\frac{M}{M_{Pl}}\right)^2$$

The WMAP7 result: $|\Delta_{k ad}^{(S)}| = (0.494 \pm 0.01) \times 10^{-4}$

determines the scale of inflation M (using $N \simeq 60$)

$$\left(\frac{M}{M_{Pl}}\right)^2 = 0.85 \times 10^{-5} \longrightarrow M = 0.70 \times 10^{16} \text{ GeV}$$

The inflation energy scale **turns to be** the grand unification energy scale !! We find the scale of inflation **without** knowing the tensor/scalar ratio r !!

The scale M is independent of the shape of $w(\chi)$.

spectral index n_s and the ratio r

$r \equiv$ ratio of tensor to scalar fluctuations.

tensor fluctuations = primordial **gravitons**.

$$n_s - 1 = -\frac{3}{N} \left[\frac{w'(\chi)}{w(\chi)} \right]^2 + \frac{2}{N} \frac{w''(\chi)}{w(\chi)}, \quad r = \frac{8}{N} \left[\frac{w'(\chi)}{w(\chi)} \right]^2$$

$$\frac{dn_s}{d \ln k} = -\frac{2}{N^2} \frac{w'(\chi) w'''(\chi)}{w^2(\chi)} - \frac{6}{N^2} \frac{[w'(\chi)]^4}{w^4(\chi)} + \frac{8}{N^2} \frac{[w'(\chi)]^2 w''(\chi)}{w^3(\chi)},$$

χ is the inflaton field at horizon exit.

$n_s - 1$ and r are **always** of order $1/N \sim 0.02$ (model indep.)

Running of n_s of order $1/N^2 \sim 0.0003$ (model independent).

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,
Phys. Rev. D 73, 023008 (2006), astro-ph/0507595.

Ginsburg-Landau Approach

Ginsburg-Landau potentials:

polynomials in the field starting by a constant term.

Linear terms can always be eliminated by a **constant** shift of the inflaton field.

The quadratic term can have a positive or a negative sign:

$$\begin{cases} w''(0) > 0 \rightarrow \text{single well potential} \rightarrow \text{large field (chaotic) inflation} \\ w''(0) < 0 \rightarrow \text{double well potential} \rightarrow \text{small field (new) inflation} \end{cases}$$

The inflaton potential must be **bounded** from below \implies **highest** order term must be **even** with a **positive** coefficient.

Renormalizability \implies degree of the inflaton potential ≤ 4 .

The theory of inflation is an **effective** theory \implies higher degree potentials are **acceptable**

Fourth order Ginsburg-Landau inflationary models

$$w(\chi) = w_o \pm \frac{\chi^2}{2} + G_3 \chi^3 + G_4 \chi^4 \quad , \quad G_3 = \mathcal{O}(1) = G_4$$

$$V(\phi) = N M^4 w \left(\frac{\phi}{\sqrt{N} M_{Pl}} \right) = V_o \pm \frac{m^2}{2} \phi^2 + g \phi^3 + \lambda \phi^4 .$$

$$m = \frac{M^2}{M_{Pl}} \quad , \quad g = \frac{m}{\sqrt{N}} \left(\frac{M}{M_{Pl}} \right)^2 G_3 \quad , \quad \lambda = \frac{G_4}{N} \left(\frac{M}{M_{Pl}} \right)^4$$

Notice that

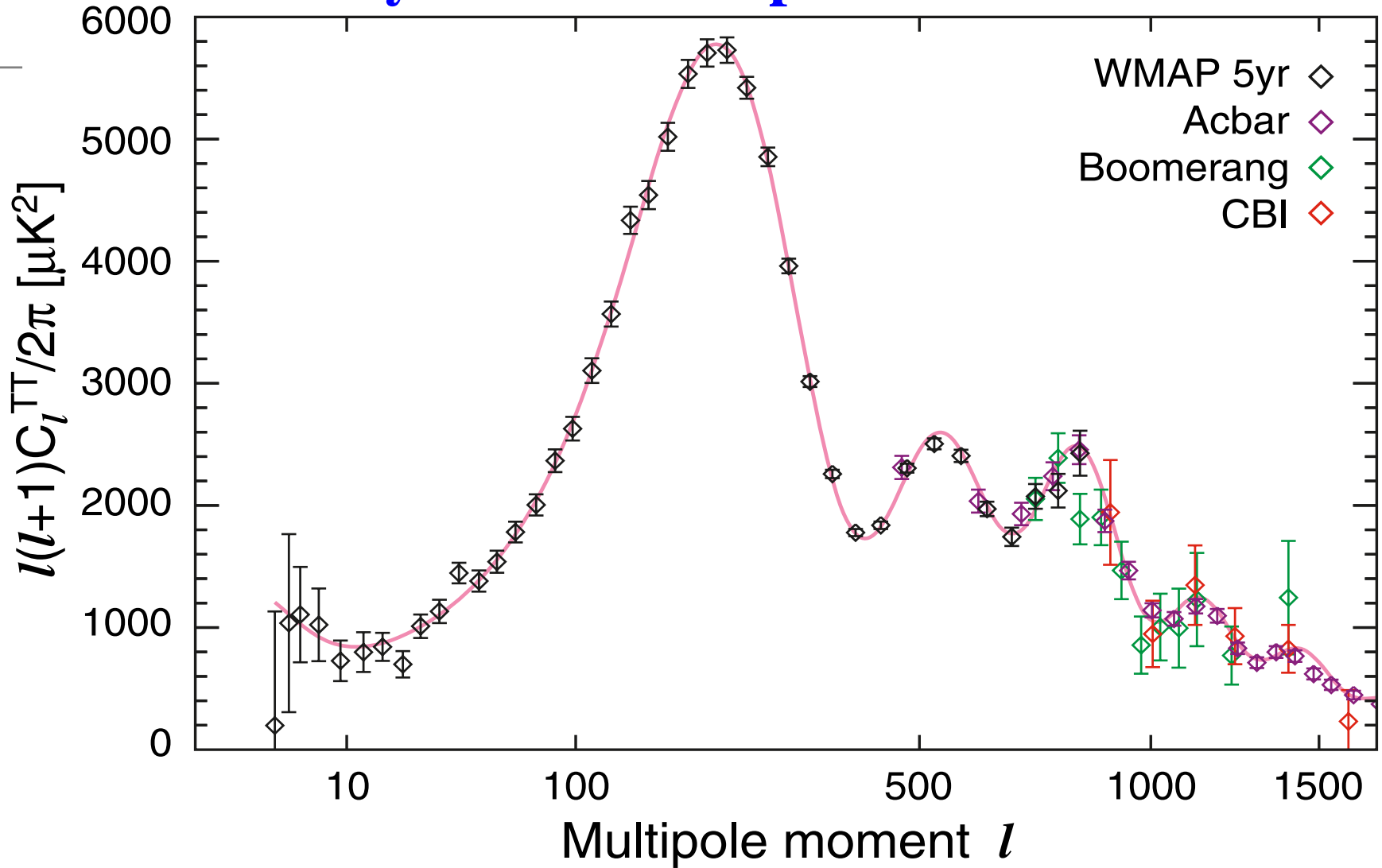
$$\left(\frac{M}{M_{Pl}} \right)^2 \simeq 10^{-5} \quad , \quad \left(\frac{M}{M_{Pl}} \right)^4 \simeq 10^{-10} \quad , \quad N \simeq 60 .$$

- Small couplings arise **naturally** as ratio of two energy scales: inflation and Planck.
- The inflaton is a **light** particle:

$$m = \frac{M^2}{M_{Pl}} \simeq 0.003 M \quad , \quad m = 2.5 \times 10^{13} \text{ GeV}$$

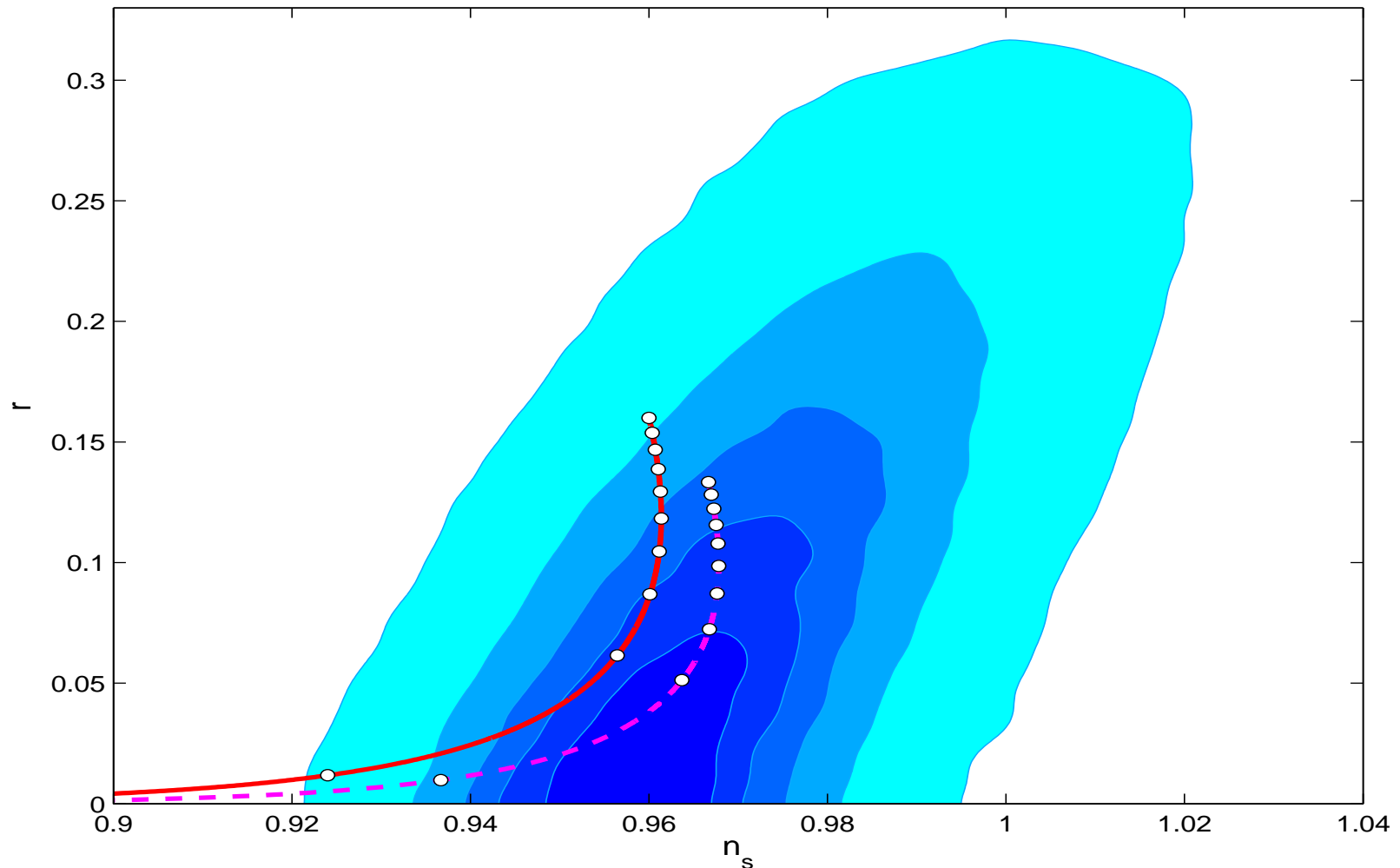
$$H \sim \sqrt{N} m \simeq 2 \times 10^{14} \text{ GeV}.$$

WMAP 5 years data set plus other CMB data



Theory and observations **nicely agree** except for the lowest multipoles: **the quadrupole suppression**.

MCMC Results for the double-well inflaton potential



Solid line for $N = 50$ and dashed line for $N = 60$

White dots: $z = 0.01 + 0.11 * n$, $n = 0, 1, \dots, 9$,

y increases from the uppermost dot $y = 0$, $z = 1$.

MCMC Results for double-well inflaton potential

Bounds: $r > 0.023$ (95% CL) , $r > 0.046$ (68% CL)

Most probable values: $n_s \simeq 0.964$, $r \simeq 0.051$ \leftarrow **measurable!!**

The most probable double-well inflaton potential has a moderate nonlinearity with the quartic coupling $y \simeq 1.26 \dots$

The $\chi \rightarrow -\chi$ symmetry is here spontaneously broken since the absolute minimum of the potential is at $\chi \neq 0$

$$w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y} \right)^2$$

MCMC analysis calls for $w''(\chi) < 0$ at horizon exit

\implies double well potential **favoured**.

C. Destri, H. J. de Vega, N. Sanchez, MCMC analysis of WMAP3 data points to broken symmetry inflaton potentials and provides a lower bound on the tensor to scalar ratio, Phys. Rev. D77, 043509 (2008), astro-ph/0703417.

Similar results from WMAP5 data.

Acbar08 data slightly increases $n_s < 1$ and r .

Quantum Fluctuations During Inflation and after

The Universe is homogeneous and isotropic after inflation thanks to the fast and **gigantic** expansion stretching lengths by a factor $e^{62} \simeq 10^{27}$. By the end of inflation: $T \sim 10^{14}$ GeV.

Quantum fluctuations around the classical inflaton and FRW geometry were of course **present**.

These inflationary quantum fluctuations are the **seeds** of the structure formation and of the CMB anisotropies today: galaxies, clusters, stars, planets, ...

That is, our present universe **was built** out of inflationary quantum fluctuations. CMB anisotropies spectrum:

$$3 \times 10^{-32} \text{cm} < \lambda_{\text{begin inflation}} < 3 \times 10^{-28} \text{cm}$$

$$M_{\text{Planck}} \gtrsim 10^{18} \text{ GeV} > \lambda_{\text{begin inflation}}^{-1} > 10^{14} \text{ GeV.}$$

total redshift since inflation begins till today = 10^{56} :

$$0.1 \text{ Mpc} < \lambda_{\text{today}} < 1 \text{ Gpc}, \quad 1 \text{ pc} = 3 \times 10^{18} \text{ cm} = 200000 \text{ AU}$$

Universe expansion classicalizes the physics: **decoherence**

Dark Matter

DM must be **non-relativistic** by structure formation ($z < 30$) in order to reproduce the observed small structures at $\sim 2 - 3$ kpc.

DM particles can decouple being **ultrarelativistic** (UR) at $T_d \gg m$ or non-relativistic $T_d \ll m$.

Consider particles that decouple **at or out** of LTE (LTE = local thermal equilibrium).

Distribution function: $F_d[p_c]$ **freezes out** at decoupling.

p_c = comoving momentum.

$P_f(t) = p_c/a(t)$ = Physical momentum,

Velocity fluctuations:

$$y = P_f(t)/T_d(t) = p_c/T_d$$

$$\langle \vec{V}^2(t) \rangle = \left\langle \frac{\vec{P}_f^2(t)}{m^2} \right\rangle = \left[\frac{T_d}{m a(t)} \right]^2 \frac{\int_0^\infty y^4 F_d(y) dy}{\int_0^\infty y^2 F_d(y) dy} .$$

Dark Matter density and DM velocity dispersion

Energy Density: $\rho_{DM}(t) = \frac{m g}{2\pi^2} \frac{T_d^3}{a^3(t)} \int_0^\infty y^2 F_d(y) dy$,

g : # of internal degrees of freedom of the DM particle,
 $1 \leq g \leq 4$. For $z \lesssim 30 \Rightarrow$ DM particles are non-relativistic:

Using entropy conservation: $T_d = \left(\frac{2}{g_d}\right)^{\frac{1}{3}} T_{cmb}$,

g_d = effective # of UR degrees of freedom at decoupling,
 $T_{cmb} = 0.2348 \cdot 10^{-3}$ eV, and

$$\rho_{DM}(\text{today}) = \frac{m g}{\pi^2 g_d} T_{cmb}^3 \int_0^\infty y^2 F_d(y) dy = 1.107 \frac{\text{keV}}{\text{cm}^3} \quad (1)$$

We obtain for the **primordial** velocity dispersion:

$$\sigma_{prim}(z) = \sqrt{\frac{1}{3} \langle \vec{V}^2 \rangle(z)} = 0.05124 \frac{1+z}{g_d^{\frac{1}{3}}} \left[\frac{\int_0^\infty y^4 F_d(y) dy}{\int_0^\infty y^2 F_d(y) dy} \right]^{\frac{1}{2}} \frac{\text{keV}}{m} \frac{\text{km}}{\text{s}}$$

Goal: determine m and g_d . We need **TWO constraints**.

The Phase-space density ρ/σ^3 and its decrease factor Z

The phase-space density ρ/σ^3 is **invariant** under the cosmological expansion and can **only decrease** under self-gravity interactions (gravitational clustering).

The phase-space density **today** follows observing dwarf spheroidal satellite galaxies of the Milky Way (dSphs)

$$\frac{\rho_s}{\sigma_s^3} \sim 5 \times 10^3 \frac{\text{keV}/\text{cm}^3}{(\text{km}/\text{s})^3} = (0.18 \text{ keV})^4 \quad \text{Gilmore et al. 07 and 08.}$$

During structure formation ($z \lesssim 30$), ρ/σ^3 **decreases** by a factor that we call Z .

$$\frac{\rho_s}{\sigma_s^3} = \frac{1}{Z} \frac{\rho_{prim}}{\sigma_{prim}^3} \quad (2)$$

The spherical model and N -body simulations indicate: $10000 > Z > 1$. Z is galaxy dependent.

Constraints: **First** $\rho_{DM}(\text{today})$, **Second** $\rho/\sigma^3(\text{today}) = \rho_s/\sigma_s^3$

Mass Estimates for DM particles

Combining the previous expressions lead to **general formulas** for m and g_d :

$$m = 0.2504 \text{ keV} \left(\frac{Z}{g}\right)^{\frac{1}{4}} \left[\int_0^\infty y^4 F_d(y) dy \right]^{\frac{3}{8}} \left[\int_0^\infty y^2 F_d(y) dy \right]^{-\frac{3}{8}},$$

$$g_d = 35.96 Z^{\frac{1}{4}} g^{\frac{3}{4}} \left[\int_0^\infty y^4 F_d(y) dy \int_0^\infty y^2 F_d(y) dy \right]^{\frac{3}{8}}$$

These formulas yield for relics decoupling **UR at LTE**:

$$m = \left(\frac{Z}{g}\right)^{\frac{1}{4}} \text{ keV} \begin{cases} 0.568 \\ 0.484 \end{cases}, \quad g_d = g^{\frac{3}{4}} Z^{\frac{1}{4}} \begin{cases} 155 \text{ Fermions} \\ 180 \text{ Bosons} \end{cases}.$$

Since $g = 1 - 4$, we see that $g_d \gtrsim 100 \Rightarrow T_d \gtrsim 100 \text{ GeV}$.

$1 < Z^{\frac{1}{4}} < 10$ for $1 < Z < 10000$. **Example:** for DM Majorana fermions ($g = 2$) $m \simeq 0.85 \text{ keV}$.

Out of thermal equilibrium decoupling

Results for m and g_d on the **same** scales for DM particles decoupling UR **out of thermal equilibrium**.

Particle physics candidates for UR decoupling in the keV scale: sterile neutrinos, gravitinos, ...

Relics decoupling non-relativistic:

similar bounds: $\text{keV} \lesssim m \lesssim \text{MeV}$

D. Boyanovsky, H. J. de Vega, N. Sanchez,
Phys. Rev. D 77, 043518 (2008), arXiv:0710.5180.

H. J. de Vega, N. G. Sanchez, arXiv:0901.0922 to appear in
Mon. Not. Royal Astronomical Society and 0907.0006

All direct searches of DM particles look for $m \gtrsim 1 \text{ GeV}$.

Our results explain **why** nothing has been found ...

e^+ and \bar{p} excess in cosmic rays explained by astrophysics:

P.L. Biermann, et al., PRL 103:061101 (2009).

P. Blasi, P. D. Serpico, PRL 103:051104 and 081103 (2009).

Galaxies

Physical variables in galaxies:

- a) **Nonuniversal** quantities: mass, size, luminosity, fraction of DM, DM core radius r_0 , central DM density ρ_0 , ...
- b) **Universal** quantities: surface density $\mu_0 \equiv r_0 \rho_0$ and DM density profiles.

The galaxy variables are related by **universal** empirical relations. Only one **free** variable.

Universal DM density profile in Galaxies:

$$\rho(r) = \rho_0 F\left(\frac{r}{r_0}\right), \quad F(0) = 1, \quad x \equiv \frac{r}{r_0}, \quad r_0 = \text{DM core radius.}$$

Empirical cored profiles: $F_{Burkert}(x) = \frac{1}{(1+x)(1+x^2)}$.

Long distance tail reproduce galaxy rotation curves.

Cored profiles **do reproduce** the astronomical observations.

The constant surface density in DM and luminous galaxies

The Surface density for dark matter (DM) halos and for luminous matter galaxies defined as: $\mu_{0D} \equiv r_0 \rho_0$,

r_0 = halo core radius, ρ_0 = central density for DM galaxies

$$\mu_{0D} \simeq 120 \frac{M_\odot}{\text{pc}^2} = 5500 (\text{MeV})^3 = (17.6 \text{ MeV})^3$$

5 kpc < r_0 < 100 kpc. For luminous galaxies $\rho_0 = \rho(r_0)$.

Donato et al. 09, Gentile et al. 09

Universal value for μ_{0D} : **independent** of galaxy luminosity for a large number of galactic systems (spirals, dwarf irregular and spheroidals, elliptics) spanning over 14 magnitudes in luminosity and of different Hubble types.

Similar values $\mu_{0D} \simeq 80 \frac{M_\odot}{\text{pc}^2}$ in interstellar molecular clouds of size r_0 of different type and composition over scales $0.001 \text{ pc} < r_0 < 100 \text{ pc}$ (Larson laws, 1981).

DM surface density from linear Boltzmann-Vlasov eq

The distribution function of the decoupled DM particles:

$$f(\vec{x}, \vec{p}; t) = g f_0(p) + F_1(\vec{x}, \vec{p}; t)$$

$f_0(p)$ = thermal equilibrium function at temperature T_d .

We evolve the distribution function $F_1(\vec{x}, \vec{p}; t)$ according to the linearized Boltzmann-Vlasov equation since the end of inflation where the **primordial inflationary** fluctuations are:

$$|\phi_k| = \sqrt{2} \pi |\Delta_0| \left(\frac{k}{k_0} \right)^{n_s/2-2} \quad \text{where}$$

$$|\Delta_0| \simeq 4.94 \cdot 10^{-5}, \quad n_s \simeq 0.964, \quad k_0 = 2 \text{ Gpc}^{-1}.$$

The linear approximation **turns to improve** for larger galaxies $r_0 > 70 \text{ kpc}$ (i. e. more diluted).

Therefore, universal quantities **can be reproduced** by the linear approximation.

The distribution function Today

We obtain solving the linearized Boltzmann-Vlasov since the end of inflation:

$$\rho_{lin}(r) = \rho_{lin}(0) F(r/r_{lin})$$

Characteristic scale for the density profile decrease:

$$r_{lin} \equiv \frac{\sqrt{2}}{k_{fs}} = 58.1 \left(\frac{100}{Z}\right)^{\frac{1}{3}} \text{ kpc} \sim \text{free streaming length.}$$

Recall,

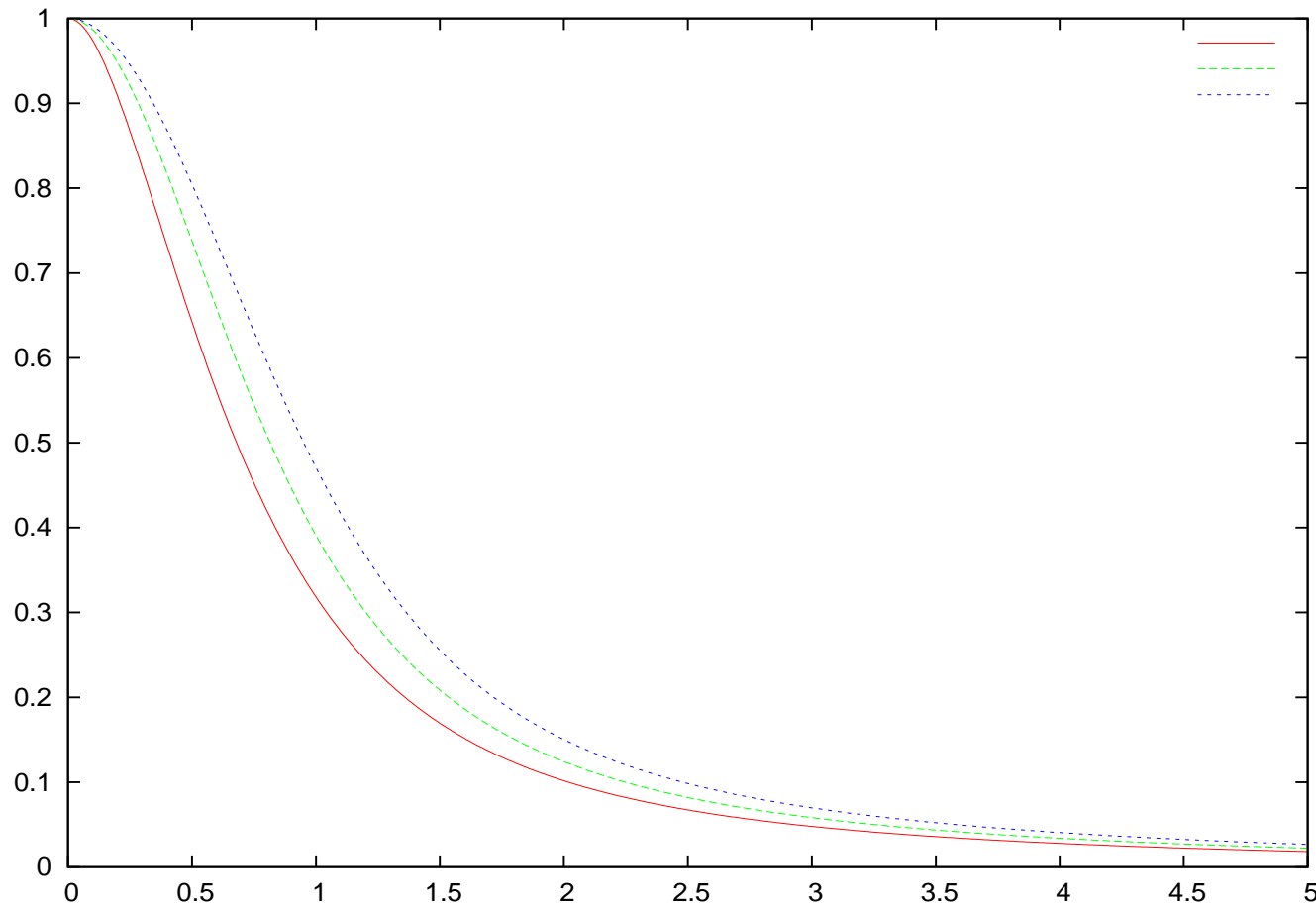
$$m \simeq Z^{\frac{1}{4}} \text{ keV for UR decoupling}$$

$$\text{and } m \simeq Z^{\frac{1}{3}} \text{ keV for NR decoupling.}$$

H. J. de Vega, N. G. Sanchez,

On the constant surface density in dark matter galaxies and interstellar molecular clouds, arXiv:0907.0006

Density profiles in the linear approximation



Profiles $\rho_{lin}(r)/\rho_{lin}(0)$ vs. $x \equiv r/r_{lin}$. These are **universal** profiles as functions of x . r_{lin} **depends** on the galaxy.

Fermions and **Bosons** decoupling ultrarelativistically and particles decoupling non-relativistically (**Maxwell-Boltzmann** statistics)

Density profiles in the linear approximation

Particle Statistics	$\mu_{0D} = r_{lin} \rho_{lin}(0) , n_s/6 = 0.16$
Bose-Einstein	$(18.9 \text{ Mev})^3 (Z/100)^{0.16}$
Fermi-Dirac	$(17.7 \text{ Mev})^3 (Z/100)^{0.16}$
Maxwell-Boltzmann	$(16.7 \text{ Mev})^3 (Z/100)^{0.16}$

Observed value: $\mu_{0D} \simeq (17.6 \text{ Mev})^3 \Rightarrow Z \sim 10 - 1000$

The linear profiles obtained are **cored** at the scale r_{lin}

$\rho_{lin}(r)$ **scales** with the **primordial spectral index** n_s :

$$\rho_{lin}(r) \stackrel{r \gg r_{lin}}{=} r^{-1-n_s/2} = r^{-1.482} ,$$

in agreement with the universal empirical behaviour

$r^{-1.6 \pm 0.4}$: M. G. Walker et al. (2009) (observations), I. M.

Vass et al. (2009) (simulations).

The agreement between the linear theory and the

observations is **remarkable**.

Summary: keV scale DM particles

- Reproduce the phase-space density observed in dwarf satellite galaxies and spiral galaxies
- Provide **cored** universal galaxy profiles in agreement with observations
- Reproduce the universal surface density of DM dominated galaxies.
- Alleviate the satellite problem which appears when wimps are used (Avila-Reese et al. 2000, Götz & Sommer-Larsen 2002)
- Alleviate the voids problem which appears when wimps are used (Tikhonov et al. 2009).
- Explain why direct searches of wimps are so far unsuccessful.

Dark Energy

$76 \pm 5\%$ of the **present** energy of the Universe is Dark !

Current observed value:

$$\rho_\Lambda = \Omega_\Lambda \rho_c = (2.39 \text{ meV})^4, \quad 1 \text{ meV} = 10^{-3} \text{ eV}.$$

Equation of state $p_\Lambda = -\rho_\Lambda$ within observational errors.

Quantum zero point energy. Renormalized value is finite.

Bosons (fermions) give positive (negative) contributions.

Mass of the lightest particles $\sim 1 \text{ meV}$ is in the right scale.

Spontaneous symmetry breaking of continuous symmetries produces massless scalars as Goldstone bosons. A small symmetry breaking provide light scalars: axions, majorons...

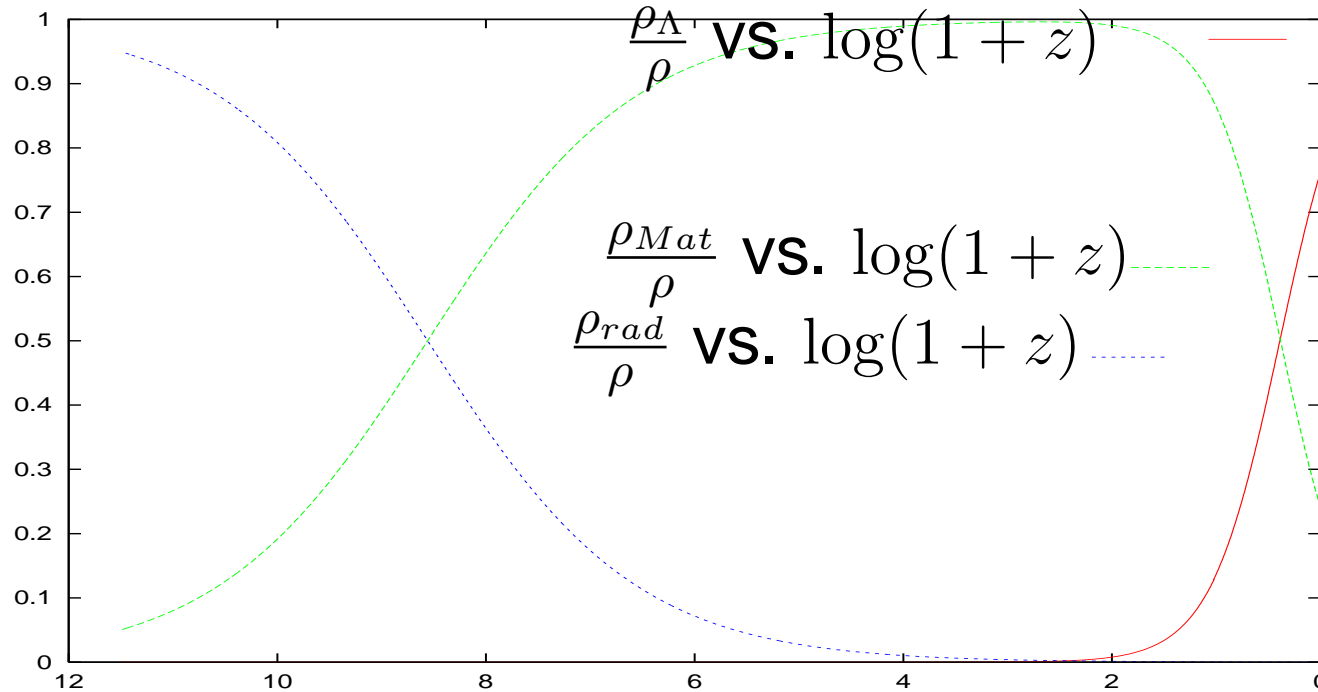
Observational Axion window $10^{-3} \text{ meV} \lesssim M_{\text{axion}} \lesssim 10 \text{ meV}$.

Dark energy **can be** a cosmological zero point effect. (As the Casimir effect in Minkowski with non-trivial boundaries).

We need to learn the **physics of light particles** ($< 1 \text{ MeV}$),

also to understand dark matter !!

The Universe is made of radiation, matter and dark energy



End of inflation: $z \sim 10^{29}$, $T_{reh} \lesssim 10^{16}$ GeV, $t \sim 10^{-36}$ sec.

E-W phase transition: $z \sim 10^{15}$, $T_{EW} \sim 100$ GeV, $t \sim 10^{-11}$ s.

QCD conf. transition: $z \sim 10^{12}$, $T_{QCD} \sim 170$ MeV, $t \sim 10^{-5}$ s.

BBN: $z \sim 10^9$, $T \simeq 0.1$ MeV, $t \sim 20$ sec.

Rad-Mat equality: $z \simeq 3200$, $T \simeq 0.7$ eV, $t \sim 57000$ yr.

CMB last scattering: $z \simeq 1100$, $T \simeq 0.25$ eV, $t \sim 370000$ yr.

Mat-DE equality: $z \simeq 0.47$, $T \simeq 0.345$ meV, $t \sim 8.9$ Gyr.

Today: $z = 0$, $T = 2.725$ K = 0.2348 meV, $t = 13.72$ Gyr.

Summary and Conclusions

- We formulate inflation as an **effective** field theory in the Ginsburg-Landau spirit with energy scale $M \sim M_{GUT} \sim 10^{16} \text{ GeV} \ll M_{Pl}$. Inflaton mass **small**: $m \sim H/\sqrt{N} \sim M^2/M_{Pl} \ll M$. Infrared regime !!
- For all slow-roll models $n_s - 1$ and r are $1/N$, $N \sim 60$.
- MCMC analysis of WMAP+LSS data **plus** this theory input indicates a spontaneously broken inflaton potential: $w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y} \right)^2$, $y \simeq 1.26$.
- Lower Bounds: $r > 0.023$ (95% CL), $r > 0.046$ (68% CL). The most probable values are **$r \simeq 0.051$** (\Leftarrow measurable !!) $n_s \simeq 0.964$.
- CMB quadrupole suppression may be explained by the effect of **fast-roll inflation** provided the today's horizon size modes exited by the end of fast-roll inflation.

Summary and Conclusions 2

- Model independent analysis of dark matter points to a particle mass at the **keV** scale. T_d may be > 100 GeV. DM is cold.
 - Universal Surface density in DM galaxies [$\mu_{0D} \simeq (18 \text{ MeV})^3$] explained by keV mass scale DM. Density profile scales and decreases for intermediate scales with the **spectral index** n_s : $\rho(r) \sim r^{-1-n_s/2}$.
 - Quantum (loop) corrections in the effective theory of inflation are of the order $(H/M_{Pl})^2 \sim 10^{-9}$. Same order of magnitude as loop graviton corrections.
- D. Boyanovsky, H. J. de Vega, N. G. Sanchez, Quantum corrections to the inflaton potential and the power spectra from superhorizon modes and trace anomalies, PRD72, 103006 (2005), Quantum corrections to slow roll inflation and new scaling of superhorizon fluctuations. Nucl. Phys. B 747, 25 (2006), astro-ph/0503669.

Future Perspectives

The **Golden Age** of Cosmology and Astrophysics continues.

A wealth of data from Planck, Atacama Cosmology Tel and further experiments are coming.

Galaxy and Star formation. DM properties from galaxy observations. Better upper bounds on DM cross-sections.

DM in planets and the earth. Flyby and Pioneer anomalies?

The **Dark Ages**...Reionisation...the 21cm line...

Nature of **Dark Energy**? 76% of the energy of the universe.

Nature of **Dark Matter**? 83% of the matter in the universe.

Light DM particles are **strongly** favoured $m_{DM} \sim \text{keV}$.

Sterile neutrinos? Some **unknown light** particle ??

Need to learn about the **physics of light particles** ($< 1 \text{ MeV}$).

COSMIC HISTORY AND CMB QUADRUPOLE SUPPRESSION

DAWN OF TIME
?

Planck time: $t \sim 10^{-44}$ sec

$t \sim 10^{-39}$ sec



inflation

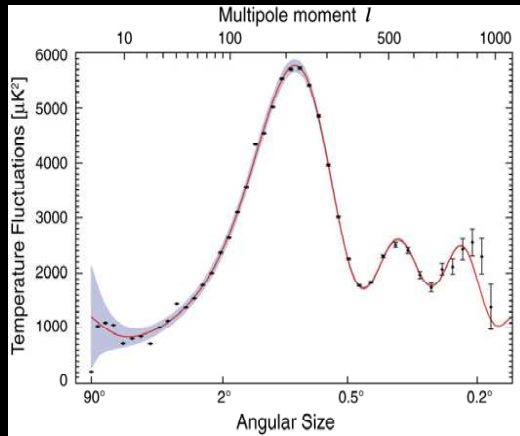
Fast roll inflation produces the CMB quadrupole suppression

Fast roll inflation

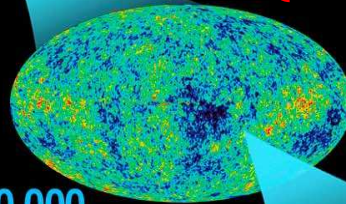
10^{-39} sec $\lesssim t \lesssim 10^{-38}$ sec

Slow roll inflation

10^{-38} sec $\lesssim t \lesssim 10^{-36}$ sec



380,000 years



13.7 billion years



The Universe is our ultimate physics laboratory

THANK YOU VERY MUCH
FOR YOUR ATTENTION!!

Forthcoming Chalonge Conferences

Ecole Internationale Daniel Chalonge

<http://chalonge.obspm.fr/>

a) 14th Paris Cosmology Colloquium 2010

THE STANDARD MODEL OF THE UNIVERSE: THEORY AND OBSERVATIONS

Observatoire de Paris, Paris campus,

Thursday 22 , Friday 23 and Saturday 24 July 2010

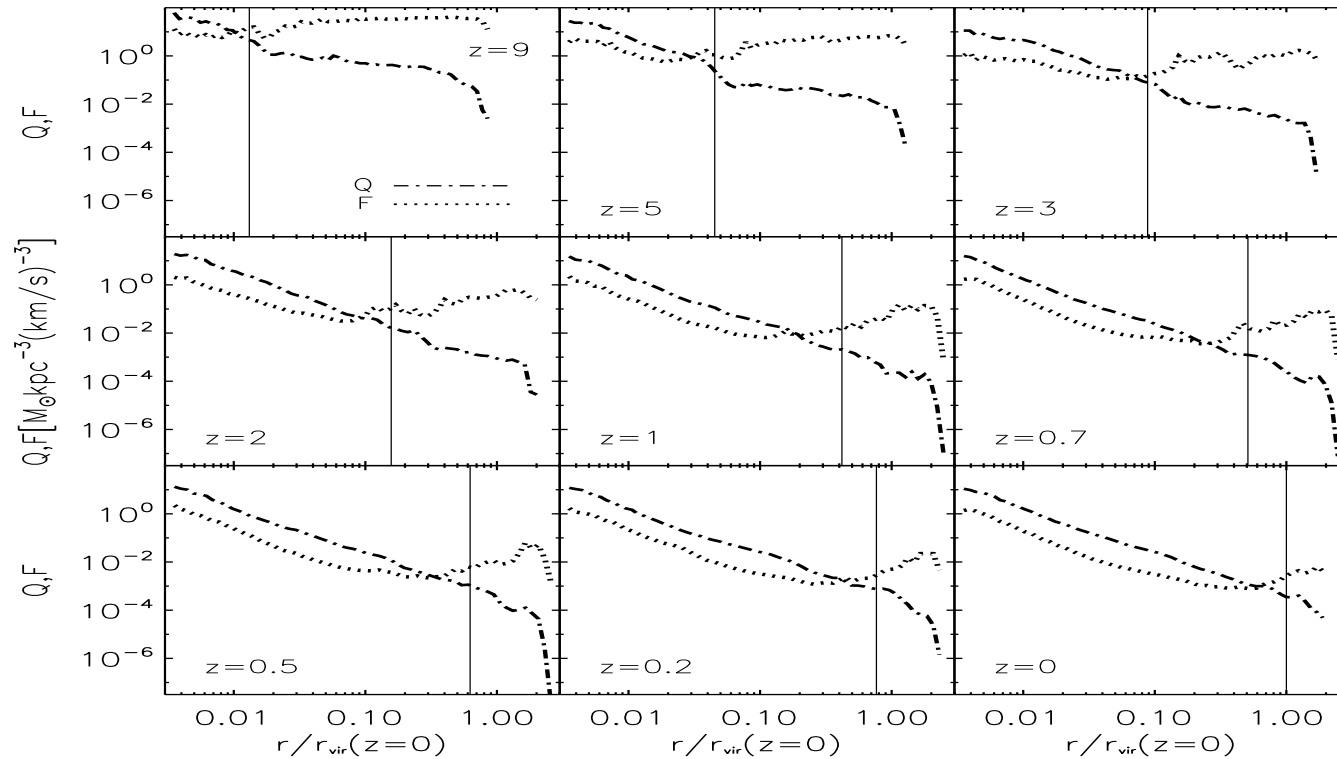
b) Dark Matter Workshop Meudon 2010

DARK MATTER IN THE UNIVERSE AND UNIVERSAL PROPERTIES OF GALAXIES: THEORY AND OBSERVATIONS

Observatoire de Paris, Château de Meudon,

8, 9, 10 and 11 June 2010

ρ/σ^3 vs. r for different z from Λ CDM simulations



Phase-space density $Q \equiv \rho/\sigma^3$ vs. $r/r_{vir}(z=0)$ **dot-dashed line** for different redshifts: $0 \leq z \leq 9$.

We see that from $z = 9$ to $z = 0$ the r -average of ρ/σ^3 decreases by a factor $Z \sim 10$.

I. M. Vass et al. MNRAS, 395, 1225 (2009).

Relics decoupling non-relativistic

$$F_d^{NR}(p_c) = \frac{2^{\frac{5}{2}} \pi^{\frac{7}{2}}}{45} g_d Y_\infty \left(\frac{T_d}{m}\right)^{\frac{3}{2}} e^{-\frac{p_c^2}{2mT_d}} = \frac{2^{\frac{5}{2}} \pi^{\frac{7}{2}}}{45} \frac{g_d Y_\infty}{x^{\frac{3}{2}}} e^{-\frac{y^2}{2x}}$$

$Y(t) = n(t)/s(t)$, $n(t)$ number of DM particles per unit volume, $s(t)$ entropy per unit volume, $x \equiv m/T_d$, $T_d < m$.

$$Y_\infty = \frac{1}{\pi} \sqrt{\frac{45}{8}} \frac{1}{\sqrt{g_d T_d} \sigma_0 M_{Pl}} \text{ late time limit of Boltzmann.}$$

σ_0 : thermally averaged total annihilation cross-section times the velocity.

From our previous **general equations** for m and g_d :

$$m = \frac{45}{4 \pi^2} \frac{\Omega_{DM} \rho_c}{g T_\gamma^3 Y_\infty} = \frac{0.748}{g Y_\infty} \text{ eV} \quad \text{and} \quad m^{\frac{5}{2}} T_d^{\frac{3}{2}} = \frac{45}{2 \pi^2} \frac{1}{g g_d Y_\infty} Z \frac{\rho_s}{\sigma_s^3}$$

$$\text{Finally:} \quad \sqrt{m T_d} = 1.47 \left(\frac{Z}{g_d}\right)^{\frac{1}{3}} \text{ keV}$$

We used ρ_{DM} today **and** the decrease of the phase space density by a factor Z .

Relics decoupling non-relativistic 2

Allowed ranges for m and T_d .

$m > T_d > b$ eV where $b > 1$ or $b \gg 1$ for DM decoupling in the RD era

$$\left(\frac{Z}{g_d}\right)^{\frac{1}{3}} 1.47 \text{ keV} < m < \frac{2.16}{b} \text{ MeV} \left(\frac{Z}{g_d}\right)^{\frac{2}{3}}$$

$g_d \simeq 3$ for $1 \text{ eV} < T_d < 100 \text{ keV}$ and $1 < Z < 10^3$

$$1.02 \text{ keV} < m < \frac{104}{b} \text{ MeV} \quad , \quad T_d < 10.2 \text{ keV}.$$

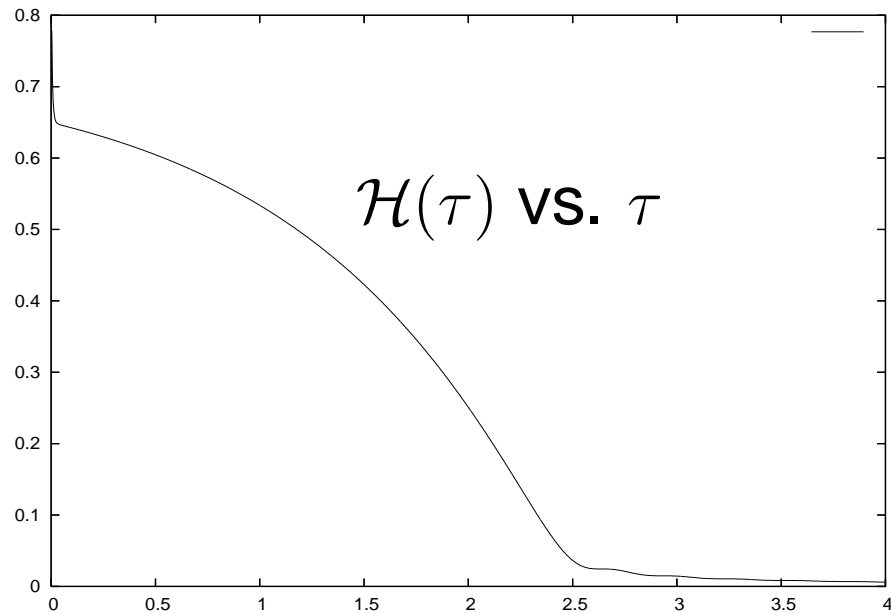
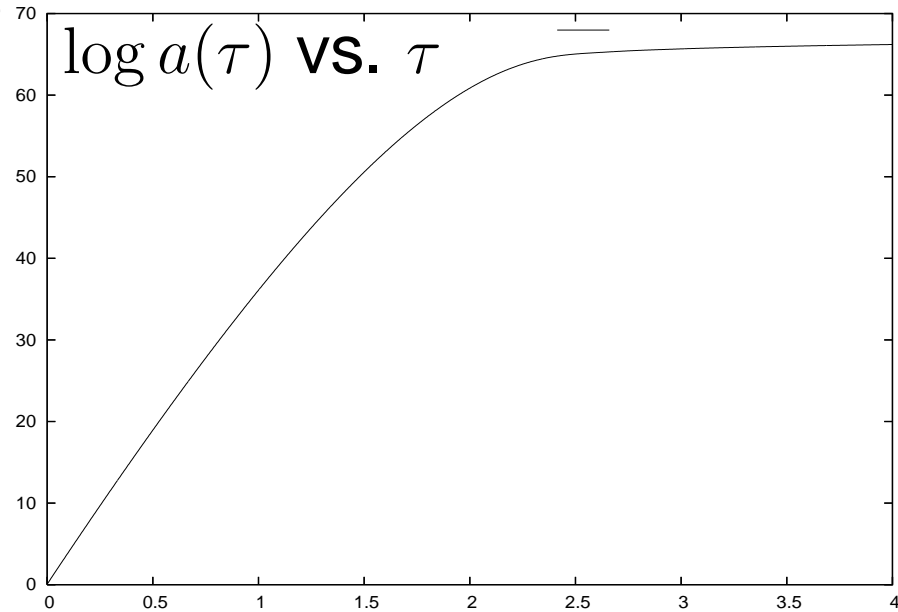
Only using ρ_{DM} today (**ignoring** the phase space density information) gives **one** equation with **three** unknowns:

m , T_d and σ_0 ,

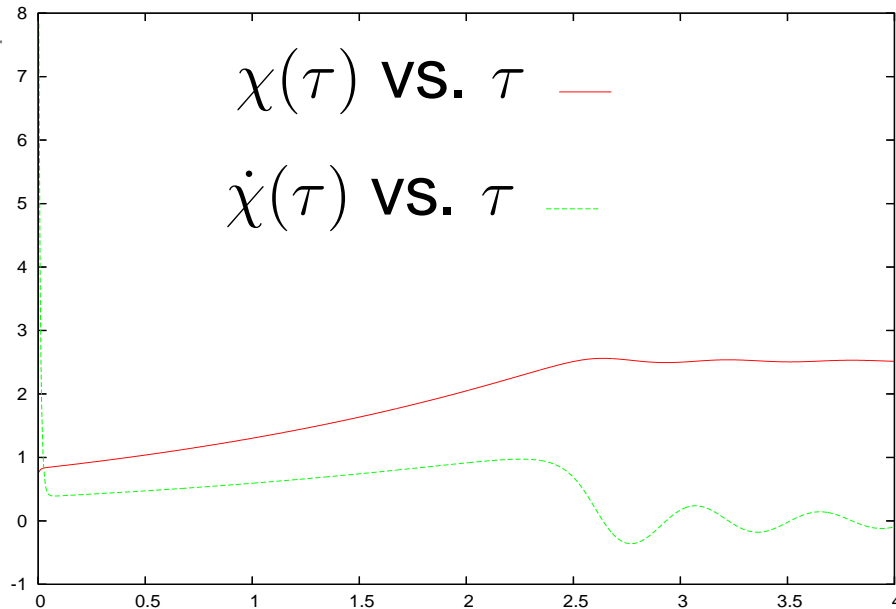
$$\sigma_0 = 0.16 \text{ pbarn} \frac{g}{\sqrt{g_d}} \frac{m}{T_d} \quad \text{http://pdg.lbl.gov}$$

WIMPS with $m = 100 \text{ GeV}$ and $T_d = 5 \text{ GeV}$ require $Z \sim 10^{23}$.

Exact Inflaton Dynamics: $w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y} \right)^2$



Exact Inflaton Dynamics: $w(\chi) = \frac{y}{32}(\chi^2 - \frac{8}{y})^2$



In these plots $y = 1.26$ and $\chi_{min} = \sqrt{\frac{8}{y}} = 2.52$.

We choose $\chi(0) = 0.73587$, $\frac{1}{2N} \dot{\chi}(0)^2 = w(\chi(0))$,
 $\implies \dot{\chi}(0) = 12.624$ which ensure $N_{tot} \simeq 66$.

We have here neglected **spatial** gradient terms:

$\frac{(\nabla\phi)^2}{2a^2(t)}$ since $a(t)$ **grows** exponentially during inflation.

Higher Order Inflaton Potentials

Till here we considered **fourth degree** inflaton potentials.
Can higher order terms **modify** the physical results and the observable predictions?

We systematically study the effects produced by higher order terms ($n > 4$) in the inflationary potential on the observables n_s and r .

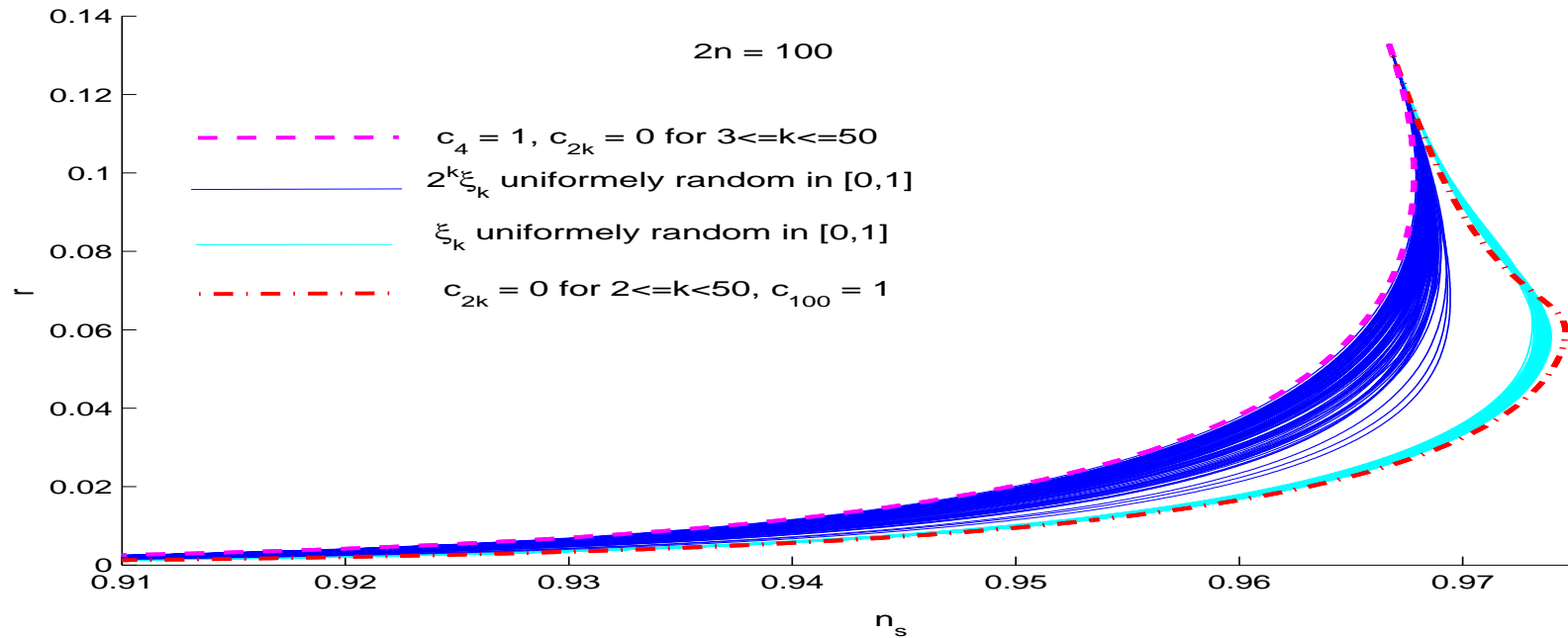
All coefficients in the potential w become **order one** using the field χ within the Ginsburg-Landau approach:

$$w(\chi) = c_0 - \frac{1}{2} \chi^2 + \sum_{n=2}^{\infty} \frac{c_n}{n} \chi^{2n} \quad , \quad c_n = \mathcal{O}(1) .$$

All $r = r(n_s)$ curves for double-well potentials of arbitrary high order fall **inside** a universal banana-shaped region \mathcal{B} . Moreover, the $r = r(n_s)$ curves for double-well potentials even for arbitrary positive higher order terms lie inside the banana region \mathcal{B} .

C. Destri, H. J. de Vega, N. G. Sanchez, arXiv:0906.4102.

The 100th degree polynomial inflaton potential



$$w(\chi) = \frac{4}{y} - \frac{1}{2} \chi^2 + \frac{4}{y} \sum_{k=2}^n \frac{c_{2k}}{k} \left(\frac{y^k}{8^k} \chi^{2k} - 1 \right)$$

The coefficients c_{2k} were extracted at random.

The lower border of the banana-shaped region is given by the potential:

$$w(\chi) = \frac{4}{y} - \frac{1}{2} \chi^2 + \frac{4}{n y} \left(\frac{y^n}{8^n} \chi^{2n} - 1 \right) \text{ with } n = 50.$$

The inflaton potential from a fermion condensate

Inflaton coupled to Dirac fermions Ψ during inflation:

$$\mathcal{L} = \bar{\Psi} [i \gamma^\mu \mathcal{D}_\mu - m_f - g_Y \phi] \Psi$$

g_Y = Yukawa coupling, γ^μ = curved space-time γ -matrices.

Hubble parameter $H = \text{constant}$. Effective potential \equiv fermions energy for a constant inflaton ϕ during inflation.

Dynamically generated inflaton potential:

$$V_f(\phi) = V_0 + \frac{1}{2} \mu^2 \phi^2 + \frac{1}{4} \lambda \phi^4 + H^4 Q \left(g_Y \frac{\phi}{H} \right), \text{ where}$$

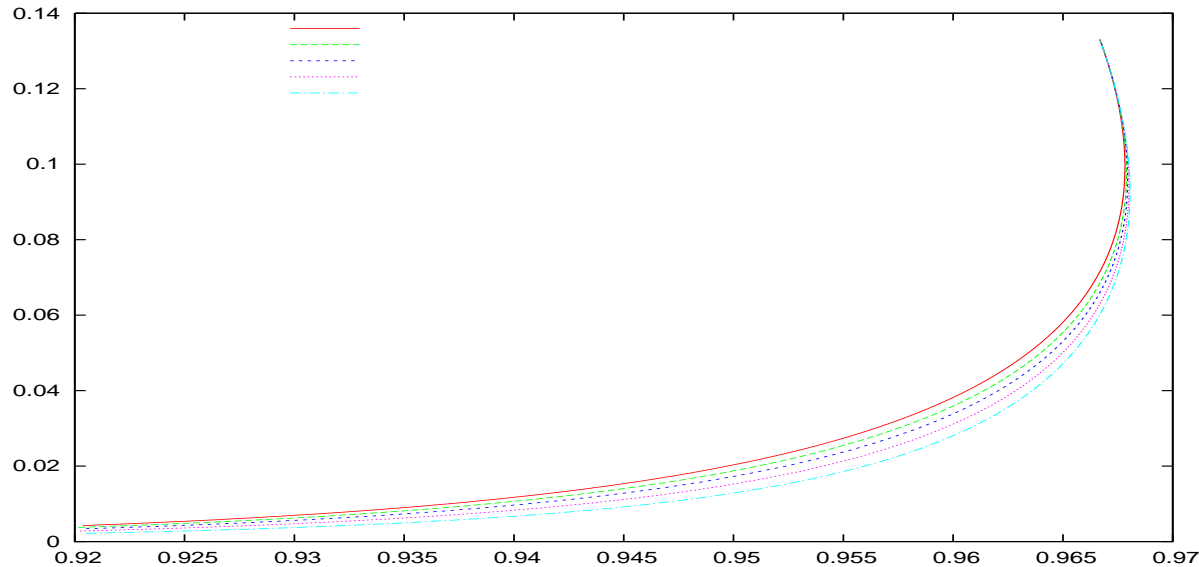
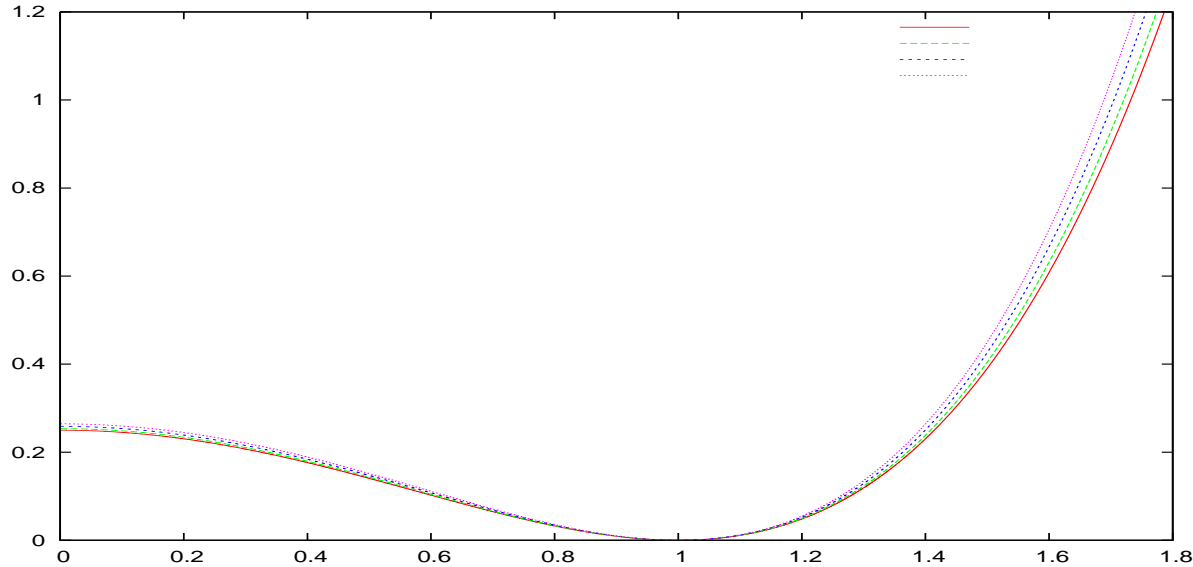
$\mu^2 = -m^2 < 0$ mass squared, $\lambda = \text{quartic coupling}$,

$$Q(x) = \frac{x^2}{8\pi^2} \left\{ (1+x^2) [\gamma + \text{Re} \psi(1+ix)] - \zeta(3) x^2 \right\} = \\ = \frac{x^4}{8\pi^2} \left[(1+x^2) \sum_{n=1}^{\infty} \frac{1}{n(n^2+x^2)} - \zeta(3) \right], \quad x \equiv g_Y \frac{\phi}{H}$$

$$Q(x) \stackrel{x \rightarrow \infty}{\simeq} \frac{x^4}{8\pi^2} \left[\log x + \gamma - \zeta(3) + \mathcal{O}\left(\frac{1}{x}\right) \right]$$

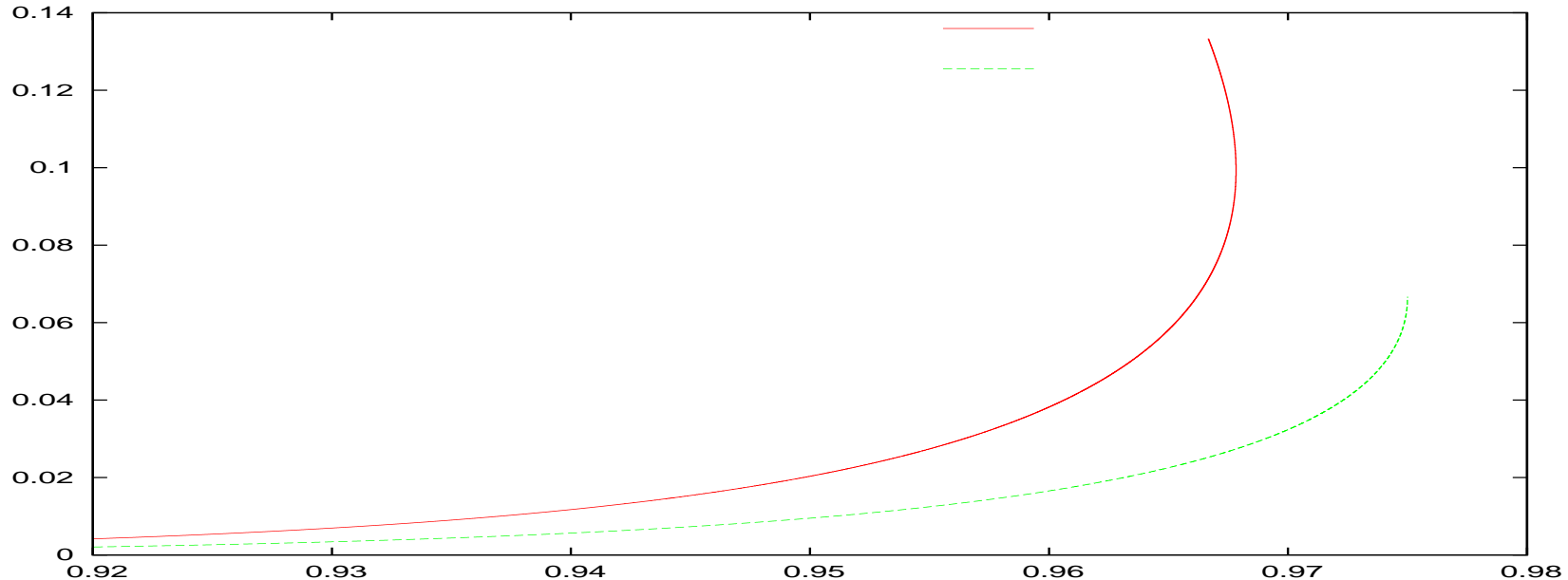
Minkowski limit (Coleman-Weinberg potential)

Effective fermionic inflaton potential and r vs. n_s



$y V(\phi)/[8 N M^4]$ vs. ϕ/ϕ_{min} for $0 < g_Y < 500 H/\phi_{min}$
 r vs. n_s for $0 < g_Y < 800 H/\phi_{min}$

The universal banana region



We find that all $r = r(n_s)$ curves for double-well inflaton potentials in the Ginsburg-Landau spirit fall **inside** the **universal** banana region \mathcal{B} .

The lower border of \mathcal{B} corresponds to the limiting potential:

$$w(\chi) = \frac{4}{y} - \frac{1}{2} \chi^2 \quad \text{for } \chi < \sqrt{\frac{8}{y}} \quad , \quad w(\chi) = +\infty \quad \text{for } \chi > \sqrt{\frac{8}{y}}$$

This gives a **lower bound** for r for **all** potentials in the Ginsburg-Landau class: $r > 0.021$ for the current best value of the spectral index $n_s = 0.964$.

The Energy Scale of Inflation

Grand Unification Idea (GUT)

- Renormalization group running of electromagnetic, weak and strong couplings shows that they **all meet** at $E_{GUT} \simeq 2 \times 10^{16}$ GeV
- Neutrino masses are explained by the **see-saw** mechanism: $m_\nu \sim \frac{M_{\text{Fermi}}^2}{M_R}$ with $M_R \sim 10^{16}$ GeV.
- Inflation energy scale: $M \simeq 10^{16}$ GeV.

Conclusion: the GUT energy scale appears in at least **three** independent ways.

Moreover, moduli potentials: $V_{\text{moduli}} = M_{\text{SUSY}}^4 v \left(\frac{\phi}{M_{\text{Pl}}} \right)$
resemble inflation potentials provided $M_{\text{SUSY}} \sim 10^{16}$ GeV.
First observation of SUSY in nature??

The number of e-folds in Slow-roll

The number of e-folds $N[\chi]$ since the field χ exits the horizon till the end of inflation is:

$$N[\chi] = N \int_{\chi_{end}}^{\chi} \frac{w(\chi)}{w'(\chi)} d\chi. \text{ We choose then } N = N[\chi].$$

The spontaneously broken symmetric potential:

$$w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y} \right)^2$$

produces inflation with $0 < \sqrt{y} \chi_{initial} \ll 1$ and $\chi_{end} = \sqrt{\frac{8}{y}}$.

This is **small field** inflation.

From the above integral: $y = z - 1 - \log z$

where $z \equiv y \chi^2 / 8$ and we have $0 < y < \infty$ for $1 > z > 0$.

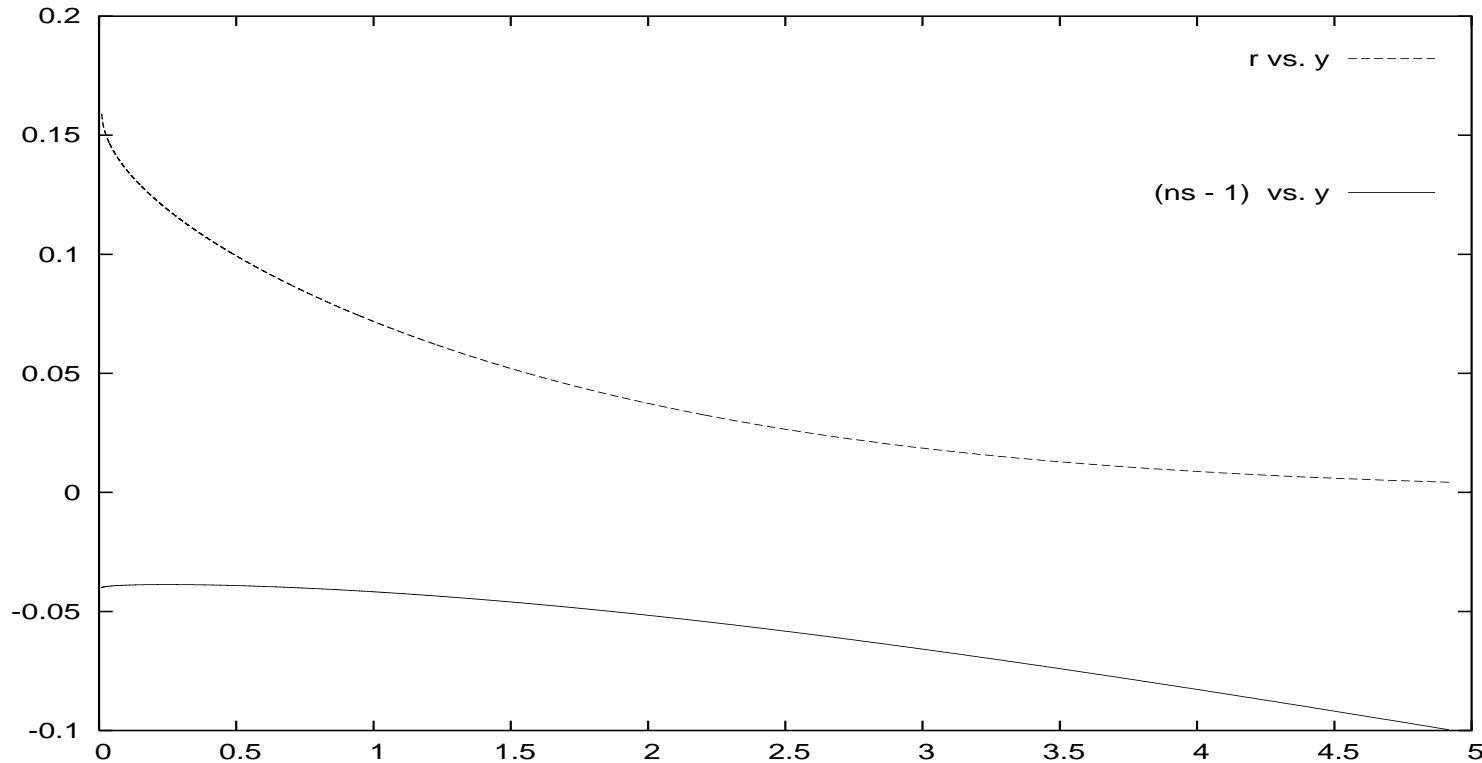
Spectral index n_s and the ratio r as functions of y :

$$n_s = 1 - \frac{y}{N} \frac{3z+1}{(z-1)^2}, \quad r = \frac{16y}{N} \frac{z}{(z-1)^2}$$

Binomial New Inflation: ($y = \text{coupling}$).

r decreases monotonically with y :

(strong coupling) $0 < r < \frac{8}{N} = 0.16$ (zero coupling).

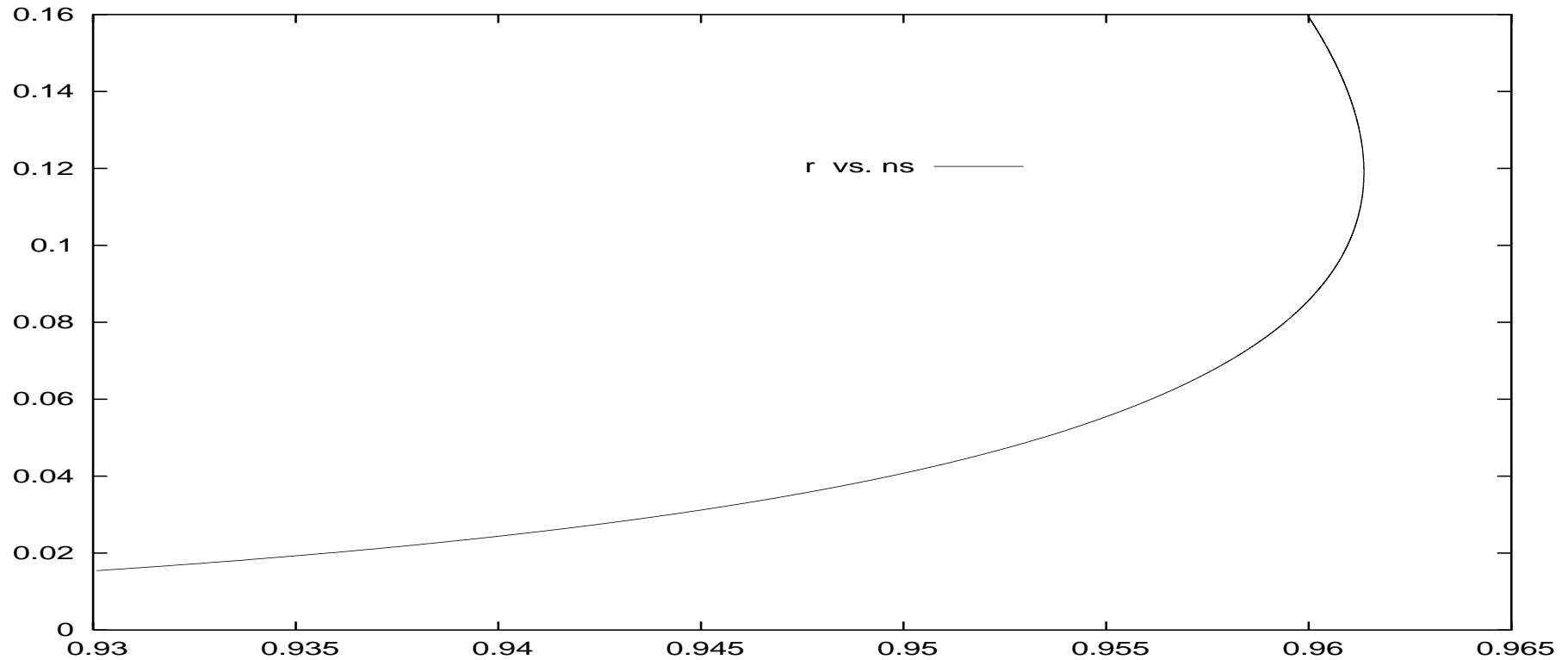


n_s first grows with y , reaches a **maximum value**

$n_{s,maximum} = 0.96139 \dots$ at $y = 0.2387 \dots$ and then n_s

decreases monotonically with y .

Binomial New Inflation



$$r = \frac{\delta}{N} = 0.16 \text{ and } n_s = 1 - \frac{2}{N} = 0.96 \text{ at } y = 0.$$

r is a **double valued** function of n_s .

Quadrupole suppression and Fast-roll Inflation

The observed CMB-quadrupole (COBE, WMAP5) is **six times** smaller than the Λ CDM model value.

In the best Λ CDM fit the probability that the quadrupole is as low or lower than the observed value is 3%.

It is hence relevant to find a **cosmological** explanation of the quadrupole suppression.

Generically, the classical evolution of the inflaton has a brief **fast-roll stage** that precedes the slow-roll regime.

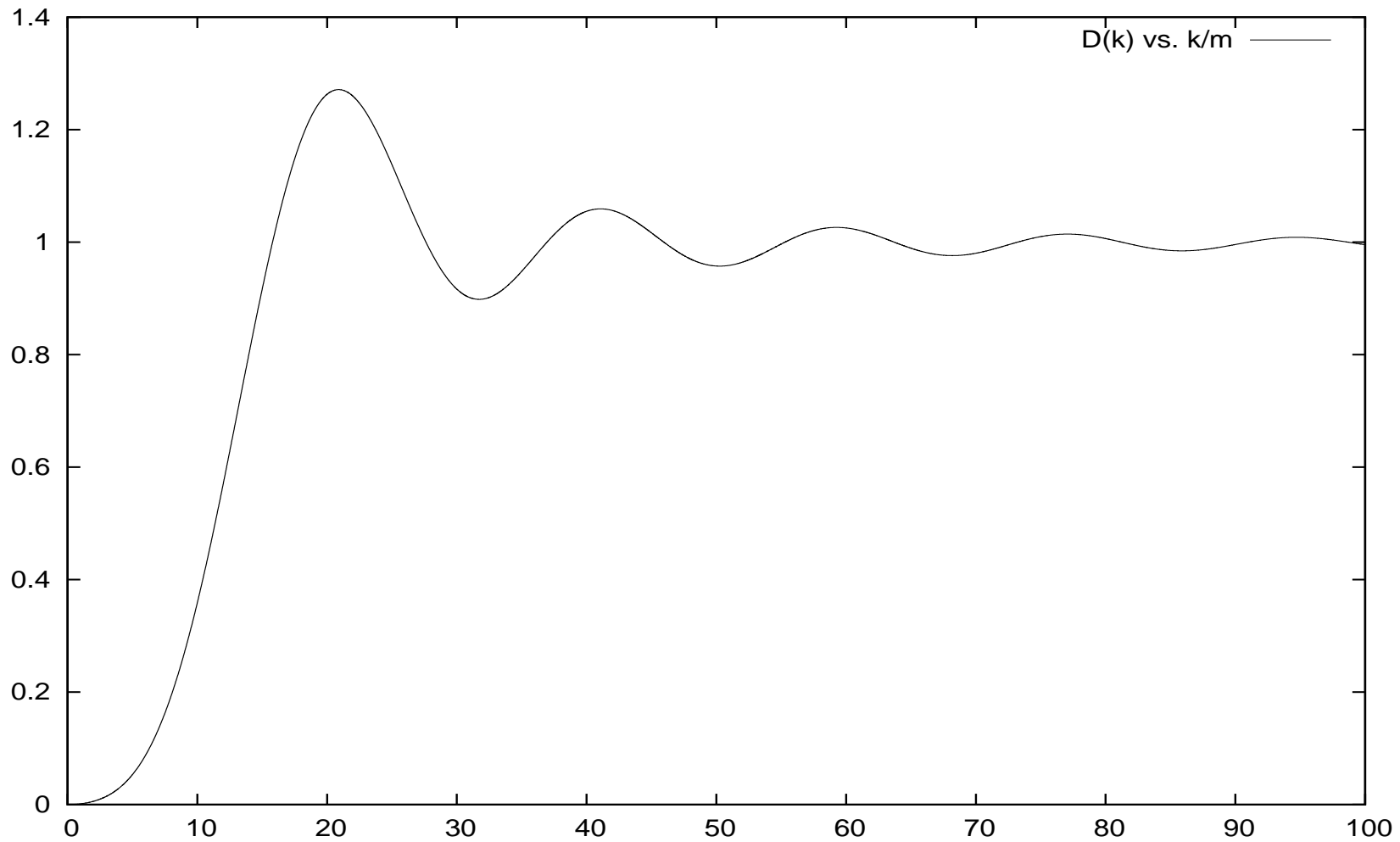
In case the quadrupole CMB mode leaves the horizon during fast-roll, before slow-roll starts, we find that the quadrupole mode gets **suppressed**.

$$P(k) = |\Delta_{k ad}^{(S)}|^2 (k/k_0)^{n_s-1} [1 + D(k)]$$

The transfer function $D(k)$ **changes** the primordial power.

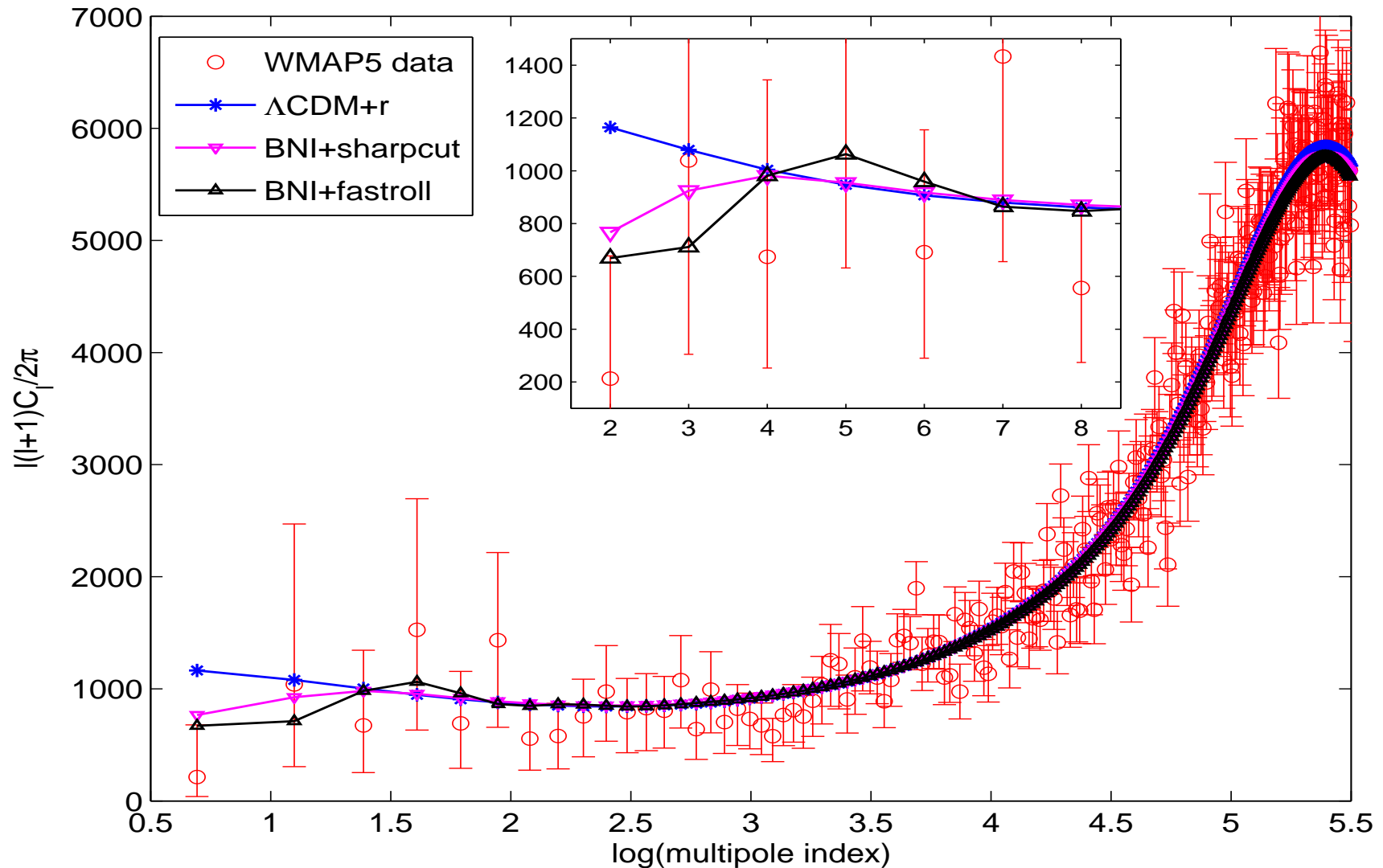
$$1 + D(0) = 0, \quad D(\infty) = 0$$

The Fast-Roll Transfer Function

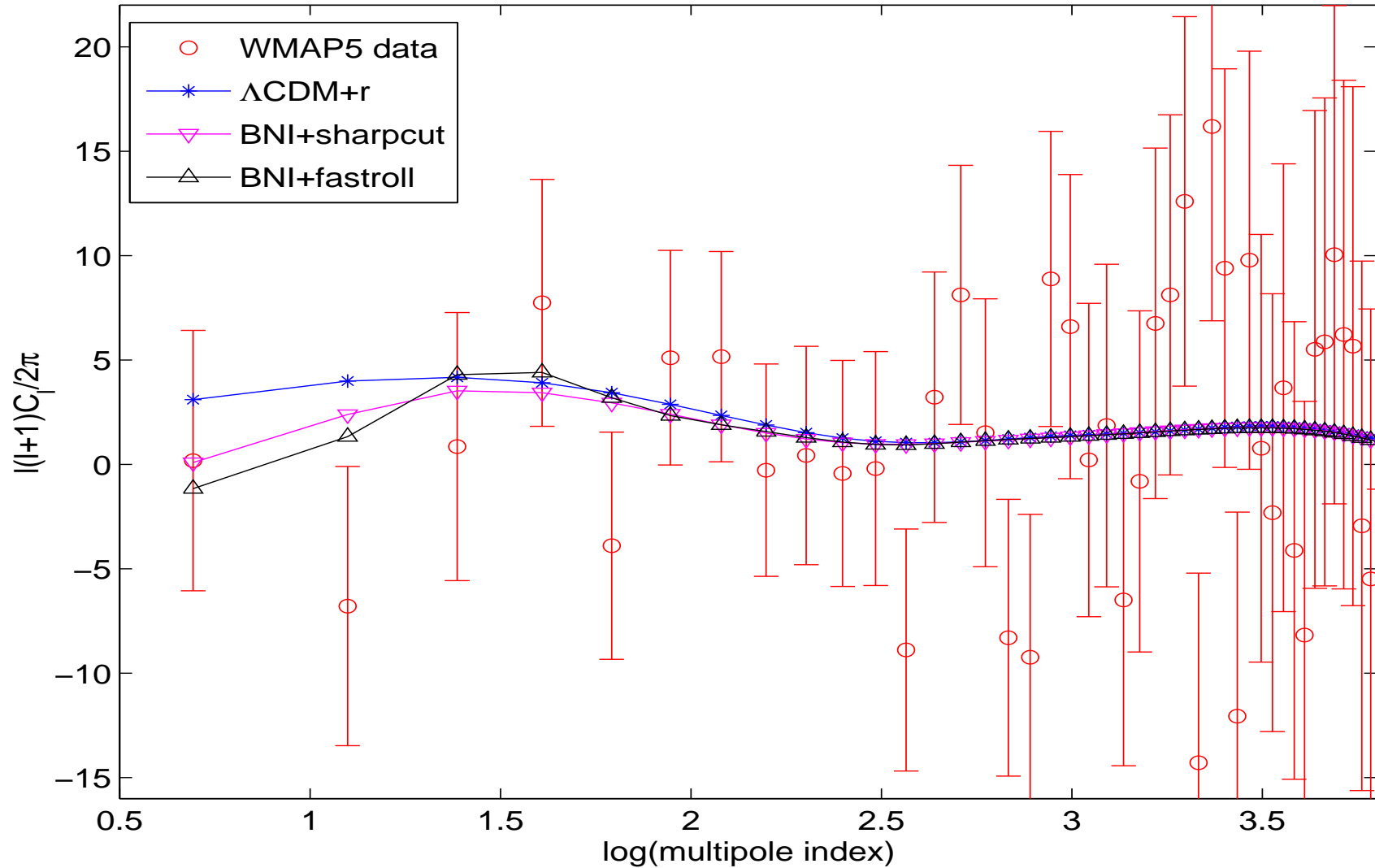


$k_Q = 11.5 m$, $k_{fastroll \rightarrow slowroll} = 14 m$, $k_{pivot} = 96.7 m$,
 $m = 1.21 \cdot 10^{13} \text{ GeV}$, $k_Q^{today} = 0.238 \text{ Gpc}^{-1} \implies$ redshift at the
beginning of inflation $= 0.9 \times 10^{56} \simeq e^{129}$.

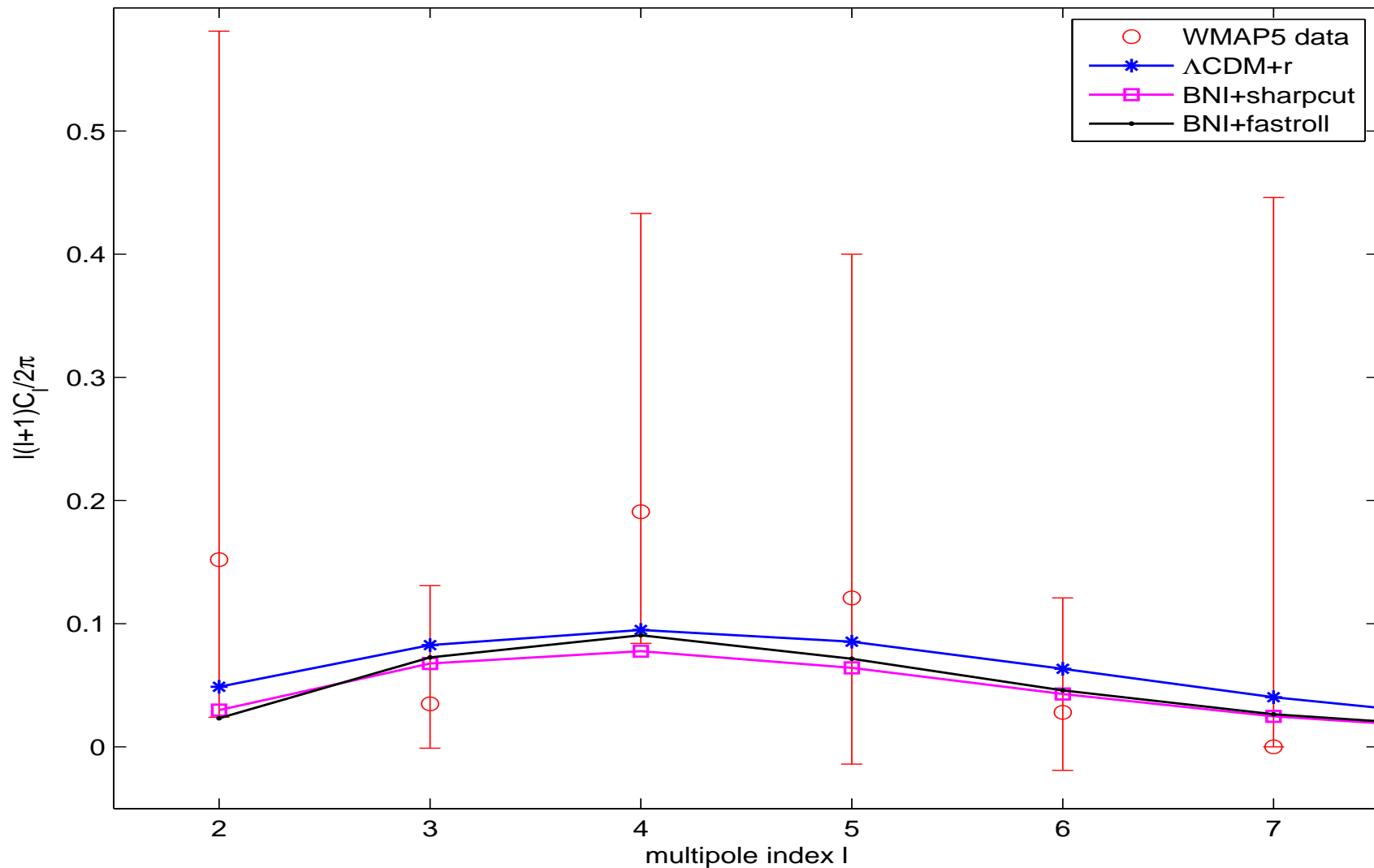
Comparison, with the experimental WMAP5 data of the theoretical C_ℓ^{TT} multipoles



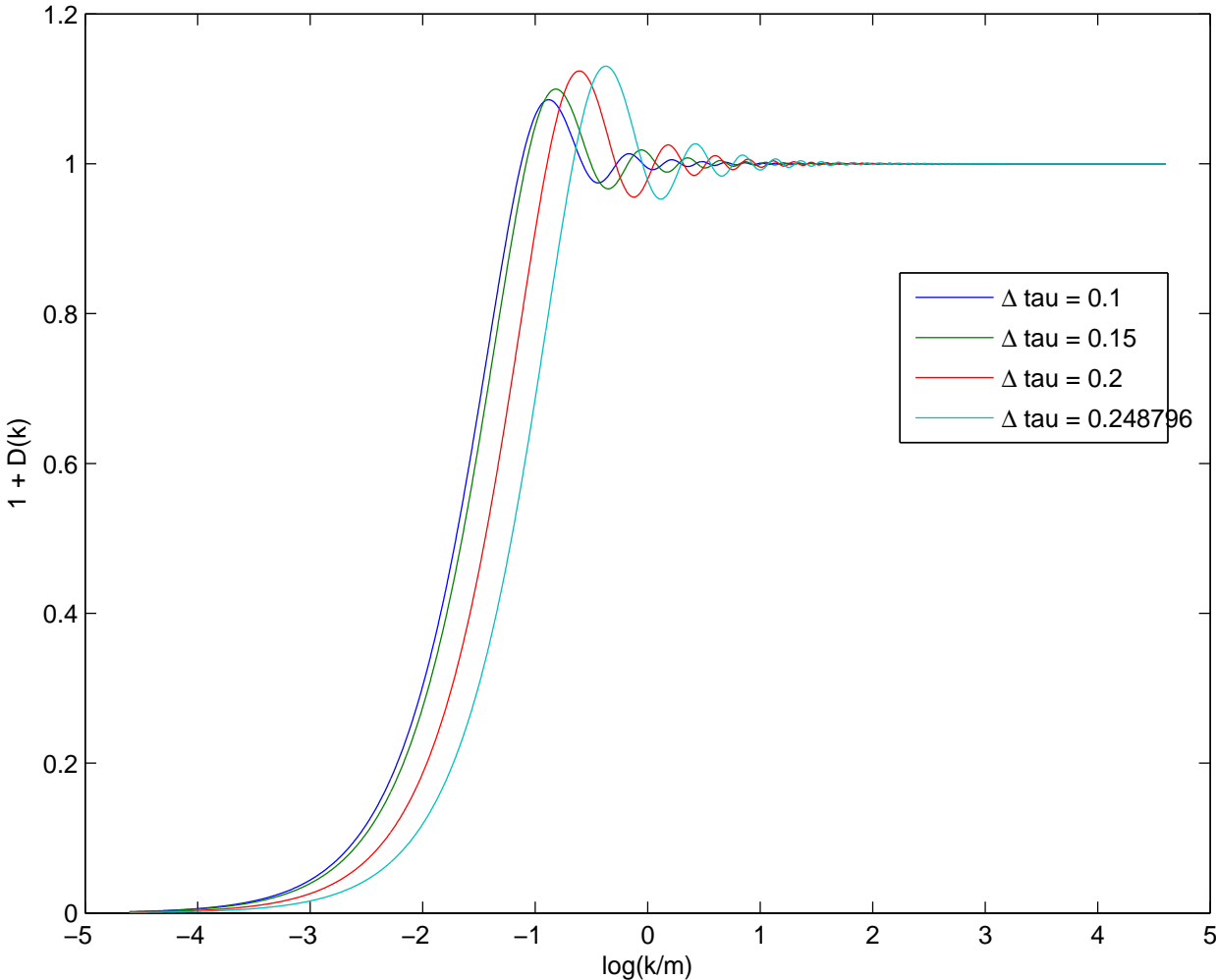
Comparison, with the experimental WMAP5 data of the theoretical C_l^{TE} multipoles



Comparison, with the experimental WMAP-5 data of the theoretical C_l^{EE} multipoles

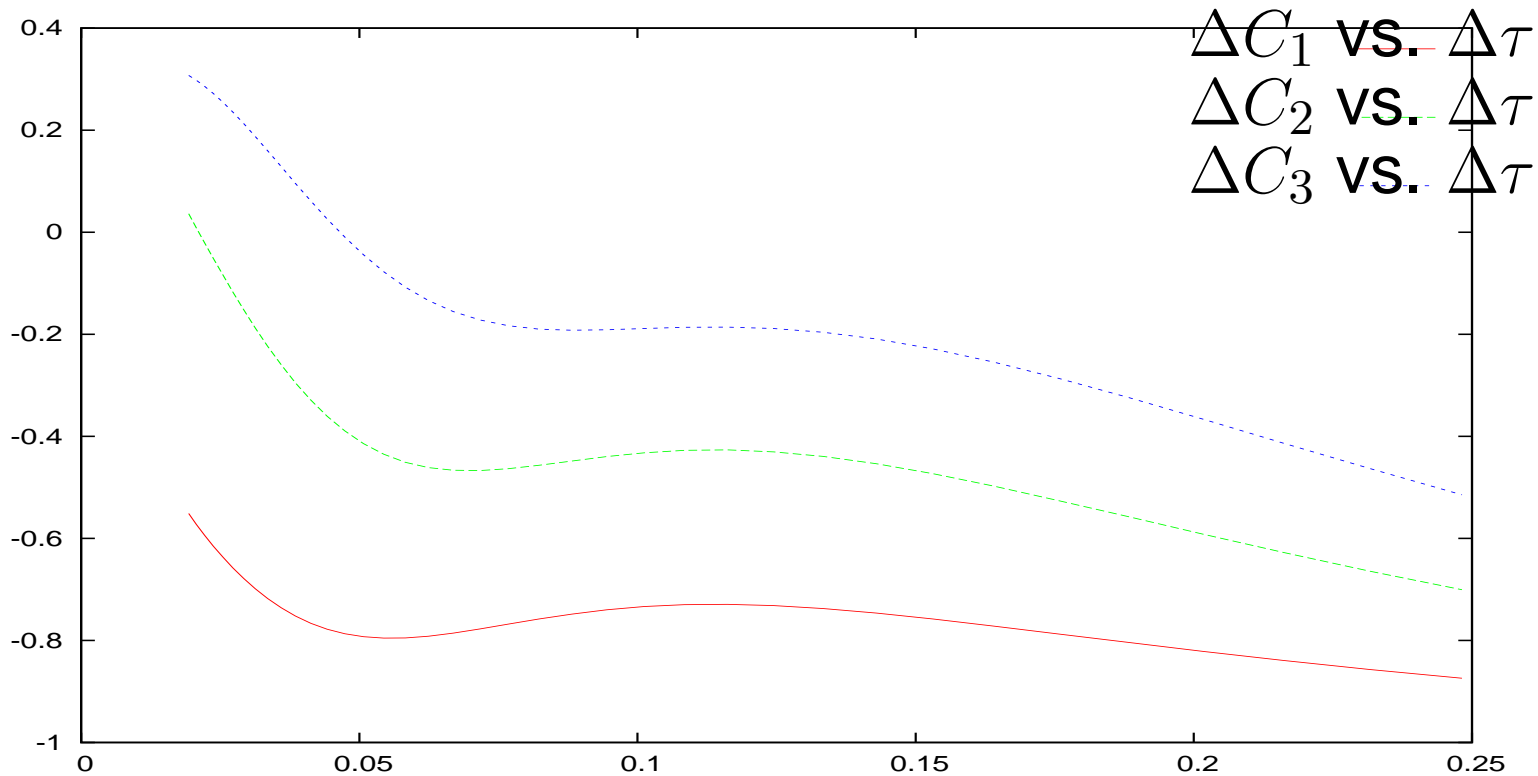


Transfer Function for different initial times of fluctuations



Transfer function $1 + D(k)$ for different initial times of fluctuations: $\Delta\tau$ from the beginning of fast-roll. BD initial conditions. $\Delta\tau = 0.25$: beginning of slow-roll.

$\Delta C_\ell^{\text{TT}}$ vs. initial time of fluctuations



Changes on the dipole, quadrupole and octupole amplitudes according to the starting time $\Delta\tau$ chosen for the fluctuations from the beginning of fast-roll. BD initial conditions. $\Delta\tau = 0.25$: beginning of slow-roll.

Loop Quantum Corrections to Slow-Roll Inflation

$$\phi(\vec{x}, t) = \Phi_0(t) + \varphi(\vec{x}, t), \quad \Phi_0(t) \equiv \langle \phi(\vec{x}, t) \rangle, \quad \langle \varphi(\vec{x}, t) \rangle = 0$$

$$\varphi(\vec{x}, t) = \frac{1}{a(\eta)} \int \frac{d^3 k}{(2\pi)^3} \left[a_{\vec{k}} \chi_k(\eta) e^{i\vec{k}\cdot\vec{x}} + \text{h.c.} \right],$$

$a_{\vec{k}}^\dagger, a_{\vec{k}}$ are creation/annihilation operators,

$\chi_k(\eta)$ are mode functions. $\eta = \text{conformal time}$.

To one loop order the equation of motion for the inflaton is

$$\ddot{\Phi}_0(t) + 3H \dot{\Phi}_0(t) + V'(\Phi_0) + g(\Phi_0) \langle [\varphi(\mathbf{x}, t)]^2 \rangle = 0$$

where $g(\Phi_0) = \frac{1}{2} V'''(\Phi_0)$.

The mode functions obey:

$$\chi_k''(\eta) + \left[k^2 + M^2(\Phi_0) a^2(\eta) - \frac{a''(\eta)}{a(\eta)} \right] \chi_k(\eta) = 0$$

where $M^2(\Phi_0) = V''(\Phi_0) = 3 H_0^2 \eta_V + \mathcal{O}(1/N^2)$

Quantum Corrections to the Friedmann Equation

The mode functions equations for slow-roll become,

$$\chi_k''(\eta) + \left[k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2} \right] \chi_k(\eta) = 0 \quad , \quad \nu = \frac{3}{2} + \epsilon_V - \eta_V + \mathcal{O}(1/N^2).$$

The scale factor during slow roll is $a(\eta) = -\frac{1}{H_0 \eta (1 - \epsilon_V)}$.

Scale invariant case: $\nu = \frac{3}{2}$. $\Delta \equiv \frac{3}{2} - \nu = \eta_V - \epsilon_V$ controls the departure from scale invariance.

Explicit solutions in slow-roll:

$$\chi_k(\eta) = \frac{1}{2} \sqrt{-\pi\eta} i^{\nu + \frac{1}{2}} H_\nu^{(1)}(-k\eta), \quad H_\nu^{(1)}(z) = \text{Hankel function}$$

Quantum fluctuations: $\langle [\varphi(\mathbf{x}, t)]^2 \rangle = \frac{1}{a^2(\eta)} \int \frac{d^3k}{(2\pi)^3} |\chi_k(\eta)|^2$

$$\frac{1}{2} \langle [\varphi(\mathbf{x}, t)]^2 \rangle = \left(\frac{H_0}{4\pi} \right)^2 \left[\Lambda_p^2 + \ln \Lambda_p^2 + \frac{1}{\Delta} + 2\gamma - 4 + \mathcal{O}(\Delta) \right]$$

UV cutoff $\Lambda_p = \text{physical cutoff}/H$, $\frac{1}{\Delta} = \text{infrared pole}$.

$\langle \dot{\varphi}^2 \rangle$, $\langle (\nabla\varphi)^2 \rangle$ are **infrared finite**

Quantum Corrections to the Inflaton Potential

Upon UV renormalization the Friedmann equation results

$$H^2 = \frac{1}{3M_{Pl}^2} \left[\frac{1}{2} \dot{\Phi}_0^2 + V_R(\Phi_0) + \left(\frac{H_0}{4\pi}\right)^2 \frac{V_R''(\Phi_0)}{\Delta} + \mathcal{O}\left(\frac{1}{N}\right) \right]$$

Quantum corrections are **proportional** to $\left(\frac{H}{M_{Pl}}\right)^2 \sim 10^{-9}$!!

The Friedmann equation gives for the effective potential:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \left(\frac{H_0}{4\pi}\right)^2 \frac{V_R''(\Phi_0)}{\Delta}$$

$$V_{eff}(\Phi_0) = V_R(\Phi_0) \left[1 + \left(\frac{H_0}{4\pi M_{Pl}}\right)^2 \frac{\eta_V}{\eta_V - \epsilon_V} \right]$$

in terms of slow-roll parameters

Very **DIFFERENT** from the one-loop effective potential in **Minkowski** space-time:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \frac{[V_R''(\Phi_0)]^2}{64\pi^2} \ln \frac{V_R''(\Phi_0)}{M^2}$$

Quantum Fluctuations:

Scalar Curvature, Tensor, Fermion, Light Scalar.

All these quantum fluctuations **contribute** to the inflaton potential **and** to the primordial power spectra.

In de Sitter space-time: $\langle T_{\mu\nu} \rangle = \frac{1}{4} g_{\mu\nu} \langle T^\alpha_\alpha \rangle$

Hence, $V_{eff} = V_R + \langle T^0_0 \rangle = V_R + \frac{1}{4} \langle T^\alpha_\alpha \rangle$

Sub-horizon (Ultraviolet) contributions appear through the **trace anomaly** and only depend on the spin of the particle. Superhorizon (Infrared) contributions are of the order N^0 and can be expressed in terms of the **slow-roll parameters**.

$$V_{eff}(\Phi_0) = V(\Phi_0) \left[1 + \frac{H_0^2}{3 (4\pi)^2 M_{Pl}^2} \left(\frac{\eta_v - 4\epsilon_v}{\eta_v - 3\epsilon_v} + \frac{3\eta_\sigma}{\eta_\sigma - \epsilon_v} + \mathcal{T} \right) \right]$$

$\mathcal{T} = \mathcal{T}_\Phi + \mathcal{T}_s + \mathcal{T}_t + \mathcal{T}_F = -\frac{2903}{20}$ is the total trace anomaly.

$$\mathcal{T}_\Phi = \mathcal{T}_s = -\frac{29}{30}, \quad \mathcal{T}_t = -\frac{717}{5}, \quad \mathcal{T}_F = \frac{11}{60}$$

→ the **graviton** (t) dominates.

Corrections to the Primordial Scalar and Tensor Power

$$\begin{aligned} |\Delta_{k,eff}^{(S)}|^2 &= |\Delta_k^{(S)}|^2 \left\{ 1 + \right. \\ &\quad \left. + \frac{2}{3} \left(\frac{H_0}{4\pi M_{Pl}} \right)^2 \left[1 + \frac{\frac{3}{8} r (n_s - 1) + 2 \frac{dn_s}{d \ln k}}{(n_s - 1)^2} + \frac{2903}{40} \right] \right\} \\ |\Delta_{k,eff}^{(T)}|^2 &= |\Delta_k^{(T)}|^2 \left\{ 1 - \frac{1}{3} \left(\frac{H_0}{4\pi M_{Pl}} \right)^2 \left[-1 + \frac{1}{8} \frac{r}{n_s - 1} + \frac{2903}{20} \right] \right\}. \end{aligned}$$

The anomaly contribution $-\frac{2903}{20} = -145.15$ **DOMINATES** as long as the number of fermions less than 783.

The scalar curvature fluctuations $|\Delta_k^{(S)}|^2$ are **ENHANCED** and the tensor fluctuations $|\Delta_k^{(T)}|^2$ **REDUCED**.

However, $\left(\frac{H}{M_{Pl}} \right)^2 \sim 10^{-9}$.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

Linear results for μ_{0D} and the profile vs. observations

Since the surface density $r_0 \rho(0)$ should be **universal**, we can **identify** $r_{lin} \rho_{lin}(0)$ from a spherically symmetric solution of the **linearized** Boltzmann-Vlasov equation.

The comparison of our theoretical values for μ_{0D} and the observational value indicates that $Z \sim 10 - 1000$. Recalling the DM particle mass:

$$m = 0.568 \left(\frac{Z}{g} \right)^{\frac{1}{4}} \text{ keV for Fermions.}$$

This implies that the DM particle mass is in the **keV range**.

Remarks:

1) For larger scales nonlinear effects from small k should give the customary r^{-3} tail in the density profile.

2) The linear approximation describe the limit of **very large galaxies** with typical inner size $r_{lin} \sim 100$ kpc.