Topologically localized insulators

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UNIVERSITAS TURICENSIS MDCCC XXXIII

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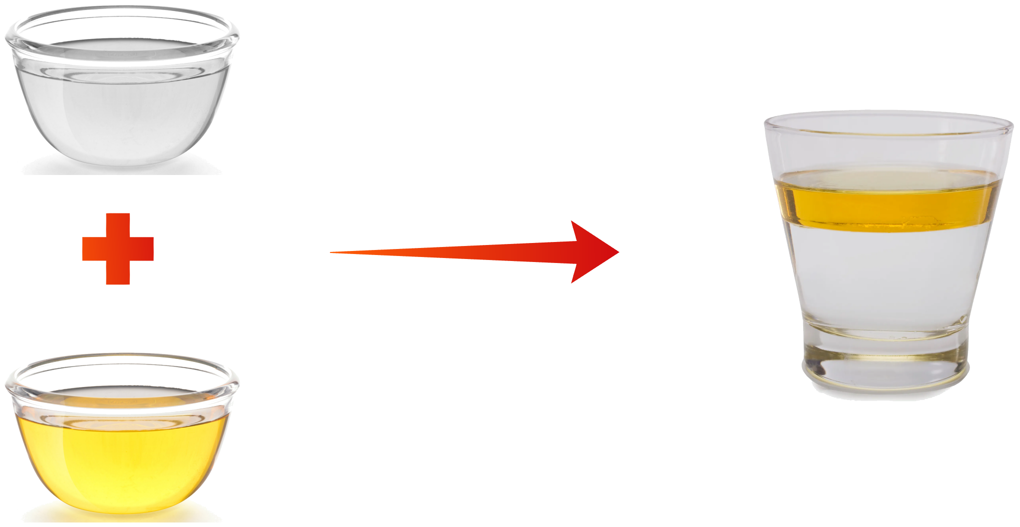
New Recipes

Topologically localized insulators (TLI)

Stuff you need:

- Anderson localization
- Topology



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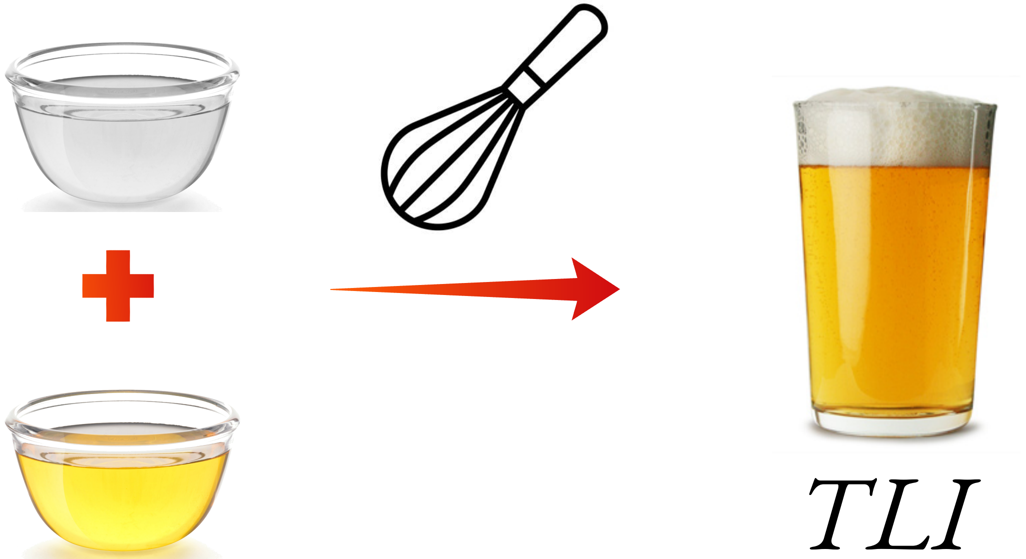
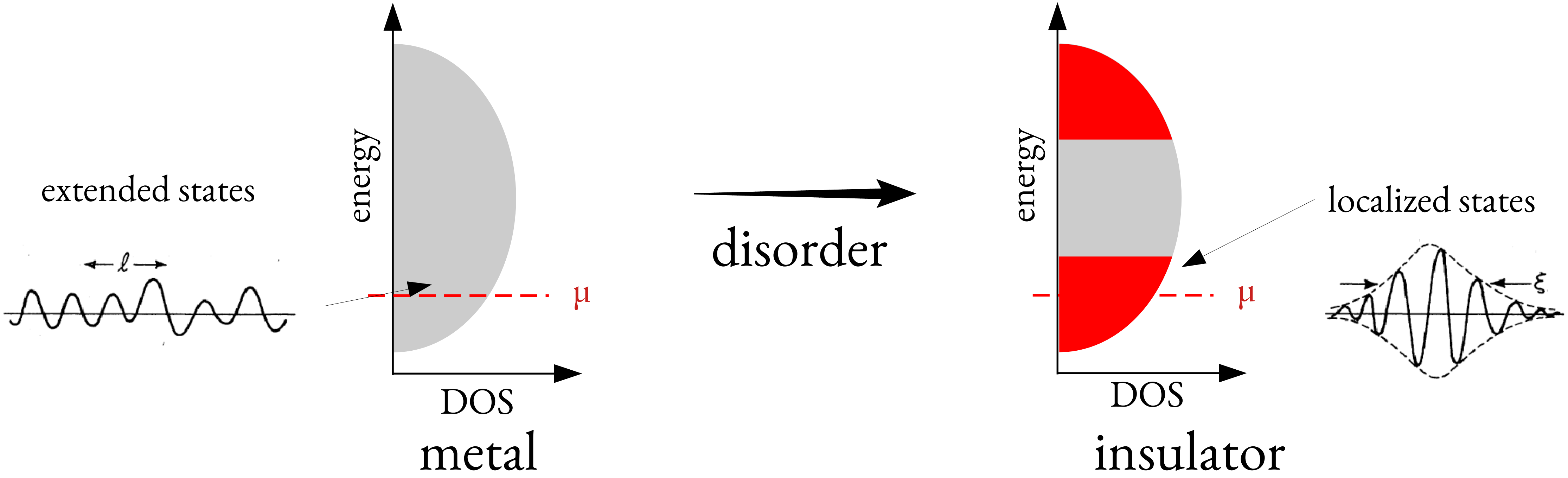
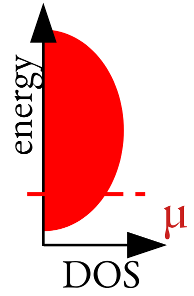


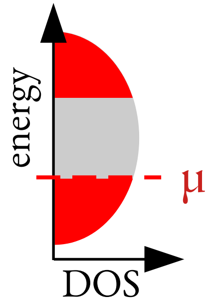
Stuff you need to do: ...



- in dimensions d=1 and 2, localized for arbitrary weak disorder (exceptions in d=2 in presence of topology or...)



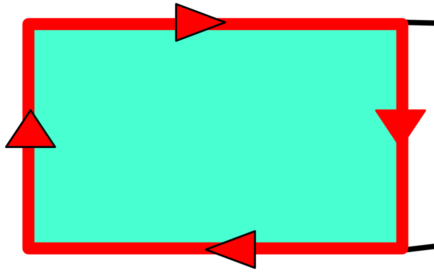
- d=3: disorder induced metal-insulator transition



Anderson (1958)

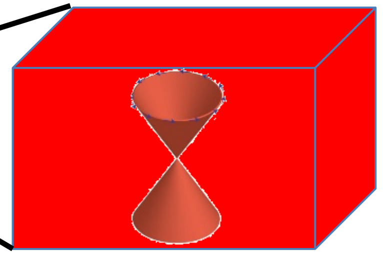
class	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	$d=0$	$d=1$	$d=2$	$d=3$
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	+	0	0	$\mathbb{Z}$	0	0	0
BDI	+	−	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0

Tenfold-way

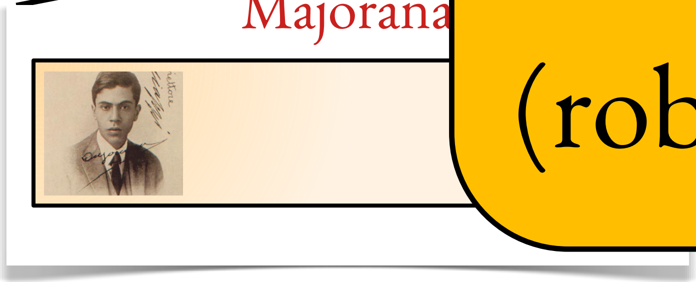


2d Quantum Hall in[sulator]
or Chern insula[tor]

Topological quantum material
↕
(robustly) quantized obeservable

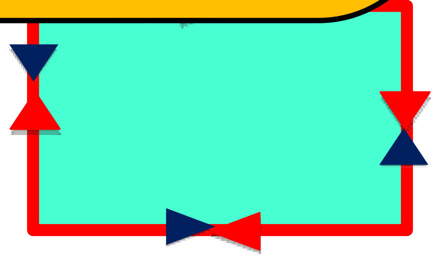


topological insulator
(Bi_2Se_3)



1d Kitaev chain
(InAs poximitized with SC)

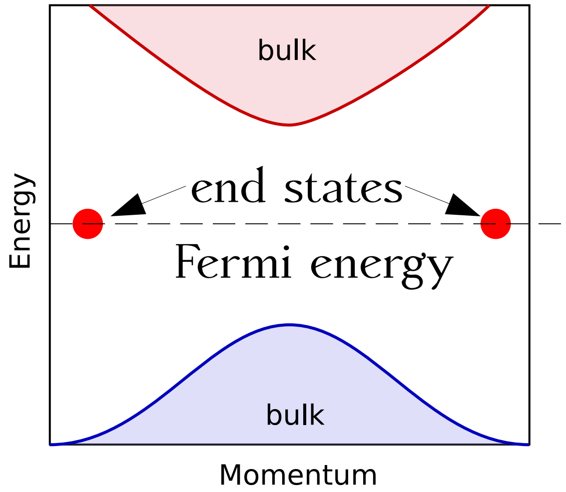
2d chiral superconductor
(UTe_2)



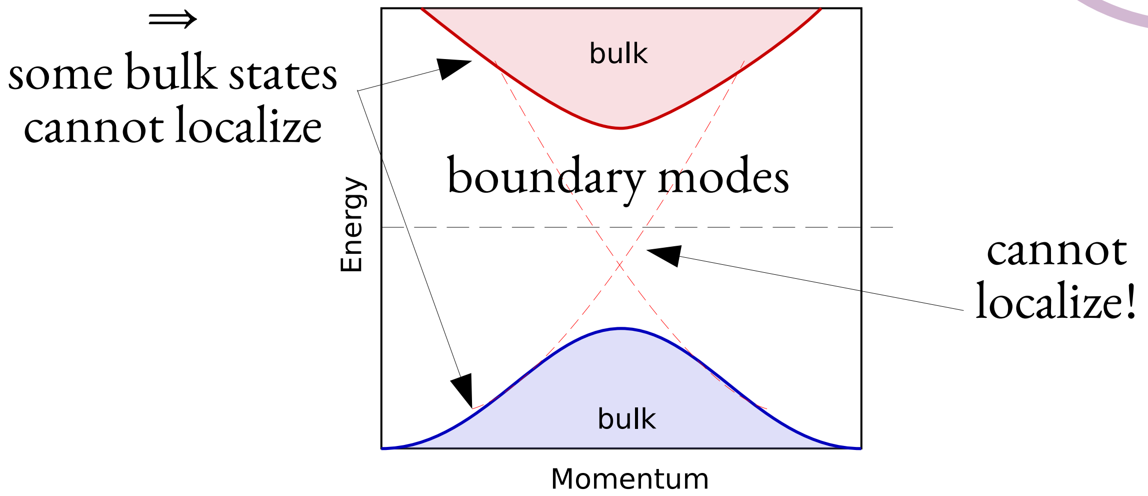
2d Quantum Spin Hall insulator
(WTe_2, HgTe wells)

Schnyder, Ryu, Furusaki, Ludwig (2008)
Kitaev (2009)

Type A

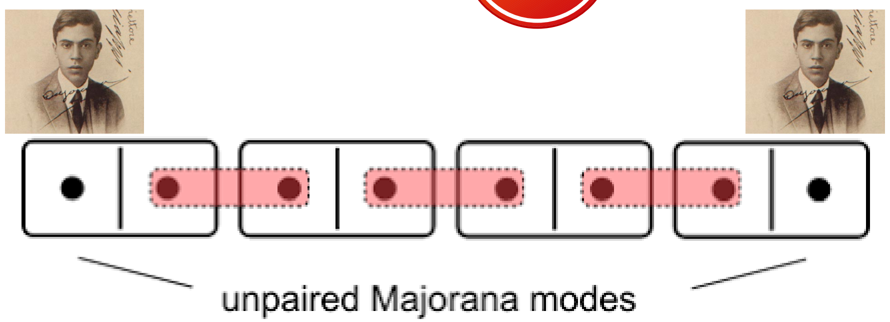


Type B

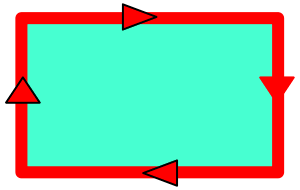


...the title of the talk says «insulators»!

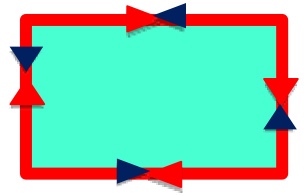
(only for superconductors)



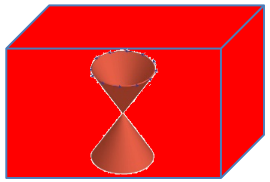
(insulators & superconductors)



Quantum Hall insulator

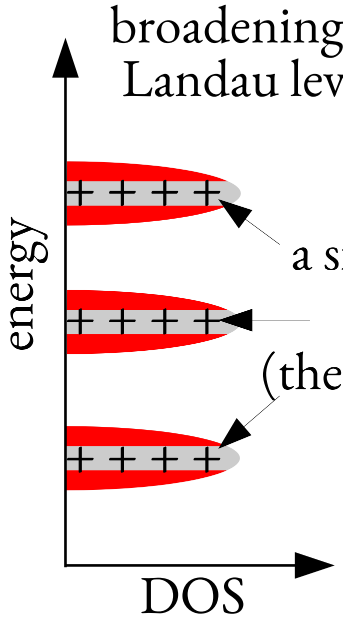


Quantum Spin Hall insulator

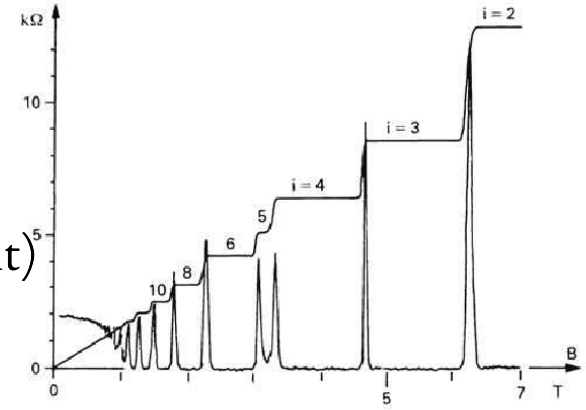


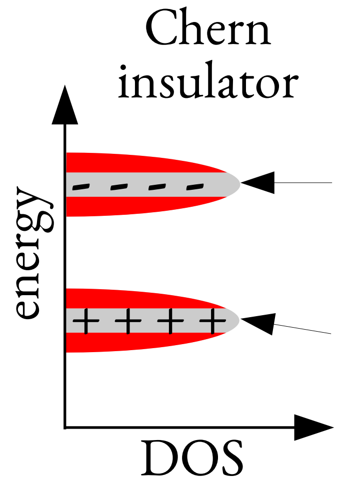
3d topological insulator

Quantum Hall/Chern insulator



a single energy with σ_{xy}=e²/h
(thermodynamic limit)

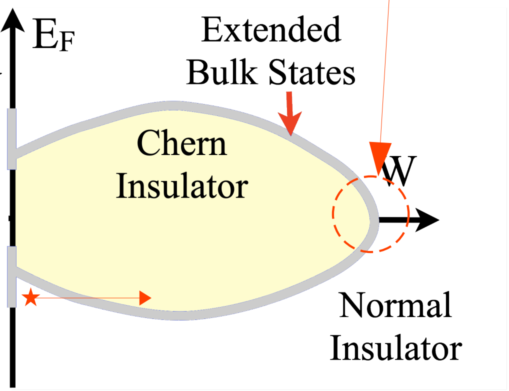




a single energy with σ_{xy}=-e²/h

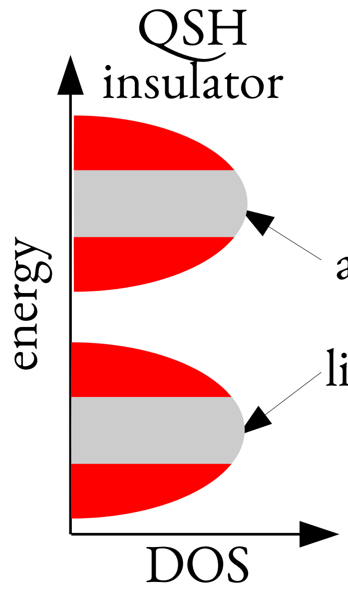
a single energy with σ_{xy}=e²/h

«pair annhilation»



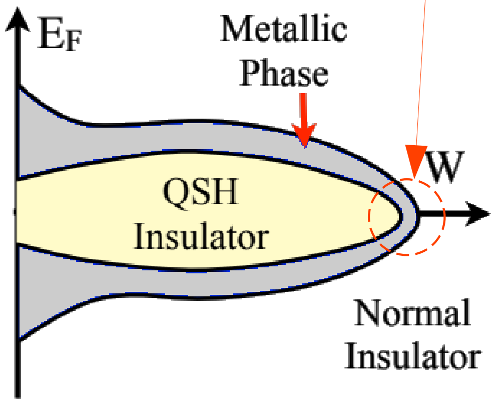
"Topological Anderson Insulator"

Quantum Spin Hall insulator

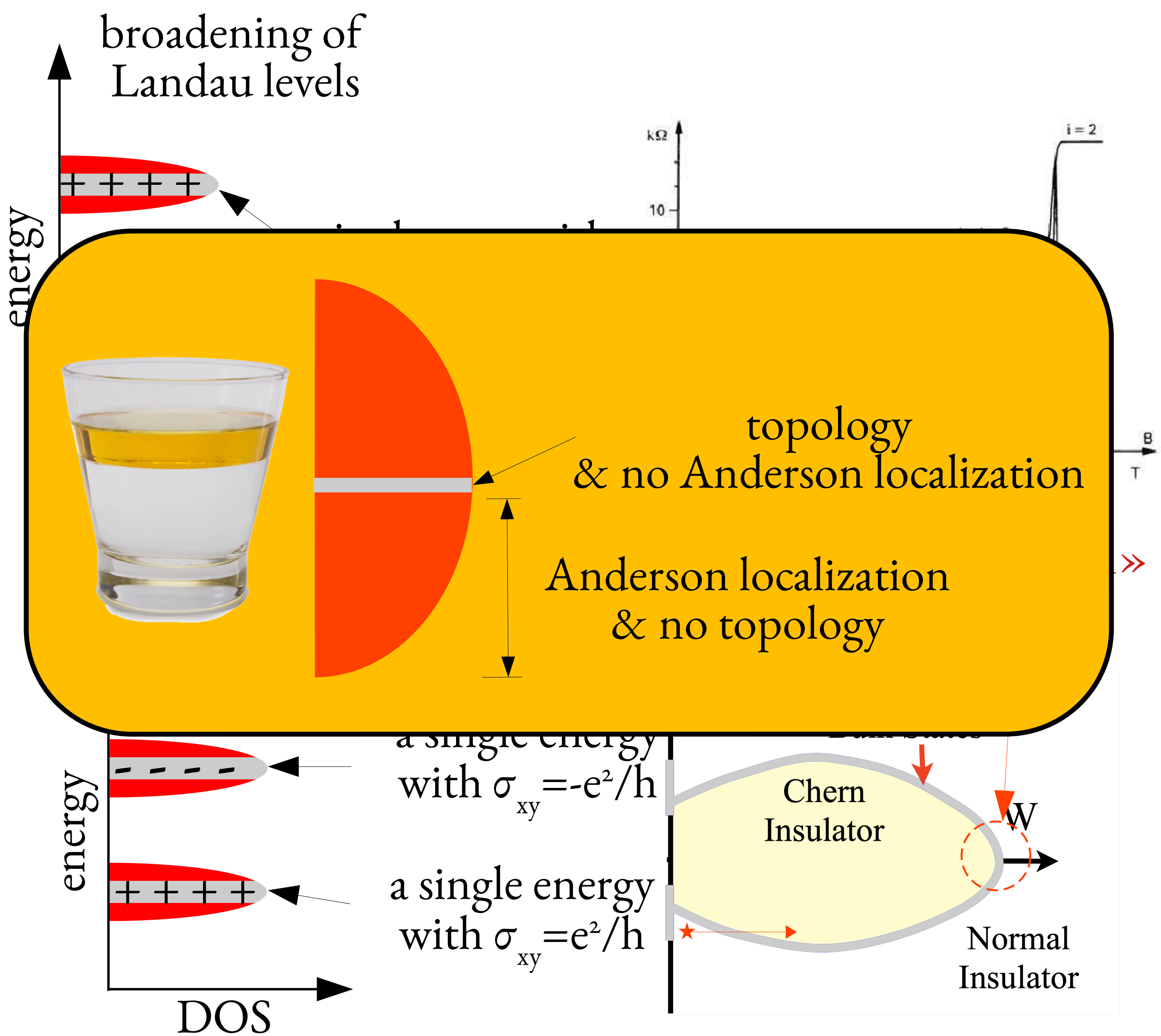


a finite energy window of delocalized states (in thermodynamic) limit with quantized Spin Hall conductance

«pair annhilation»

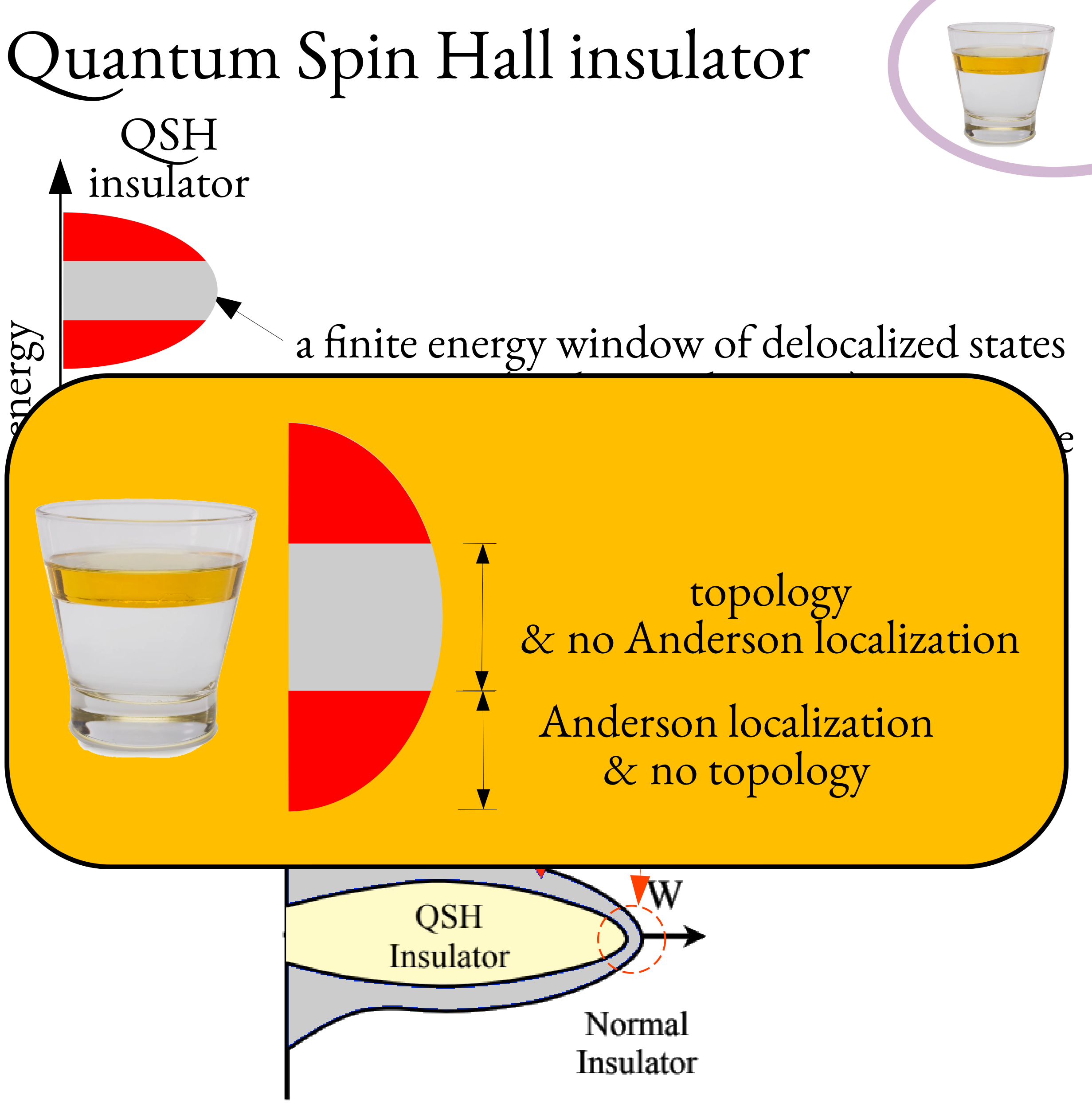


Quantum Hall/Chern insulator



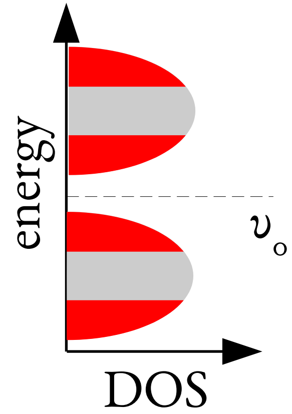
"Topological Anderson Insulator"

Quantum Spin Hall insulator



Tenfold-way

class	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	$d=0$	$d=1$	$d=2$	$d=3$
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	+	0	0	$\mathbb{Z}$	0	0	0
BDI	+	−	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0

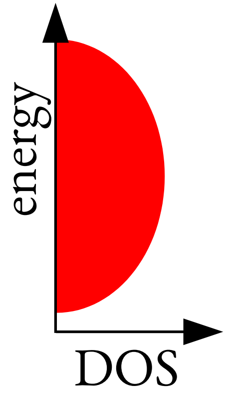


Applies to insulators, i.e., $\sigma_{xx}=0$ (for fixed filling $\upsilon=\upsilon_0$)

Phase transition: $\sigma_{xx}\neq 0$ (for fixed filling $\upsilon=\upsilon_0$)

Topologically localized insulators

class	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	$d=0$	$d=1$	$d=2$	$d=3$
A	0	0	0	?	?	?	$\mathbb{Z}$
AIII	0	0	1	?	?	?	?
AI	+	0	0	?	?	?	?
BDI	+	−	1	?	?	?	?

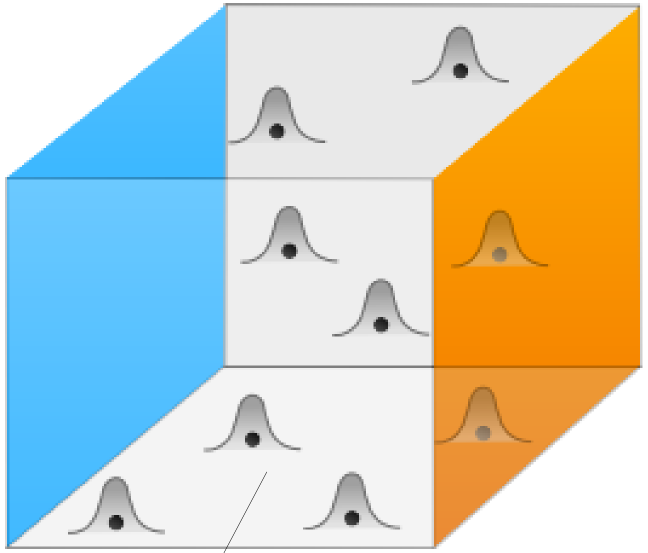


Anderson localization & topology

Applies to fully localized insulators, i.e., $\sigma_{xx}=0$ (for **all** υ)

Phase transition: $\sigma_{xx}\neq 0$ (for **any** υ)

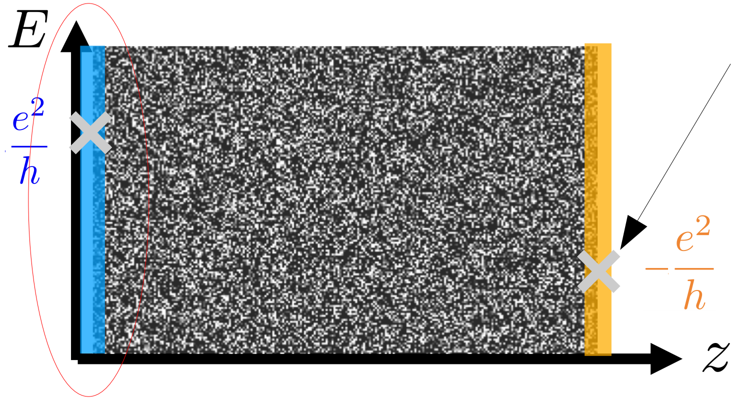
Topologically localized insulators



$$\vec{P} = \alpha_{\text{ME}} \vec{B}$$

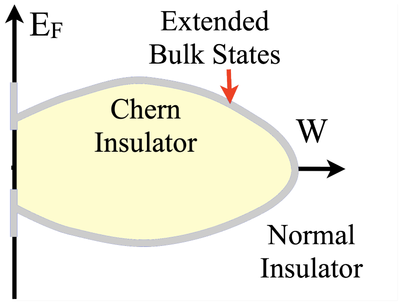
Bulk α_{ME} quantized to an integer value

OBC in z-direction



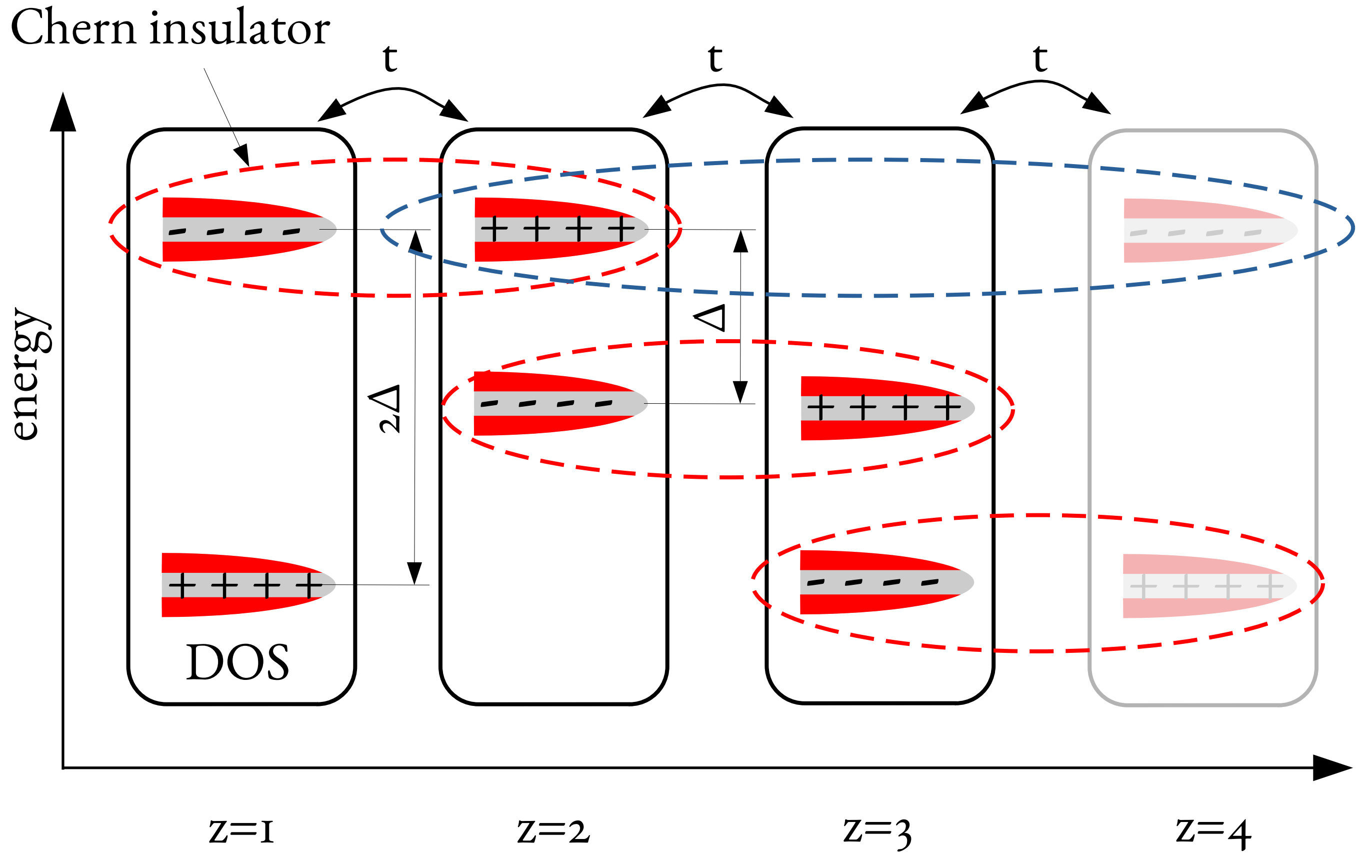
delocalized states at a single energy (per surface)

«half» of Chern insulator



⇒no full Anderson localization for an arbitrary W!

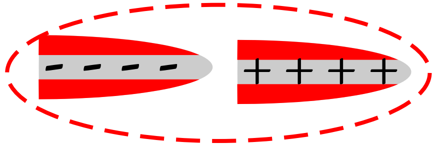
	Tenfold way	TLI
Bulk	Not fully localized	Fully localized
Boundary	Delocalized	Not fully localized



Interlayer coupling t:

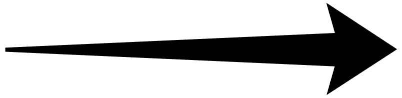
- on-resonant coupling ✓
- off-resonant coupling ✗

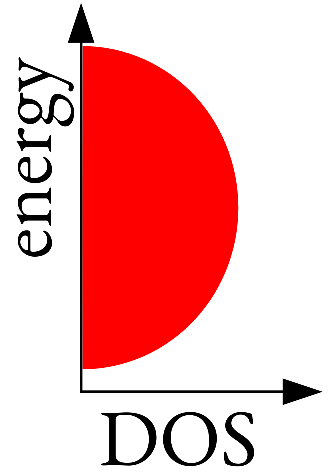
1)

«dimers»  :

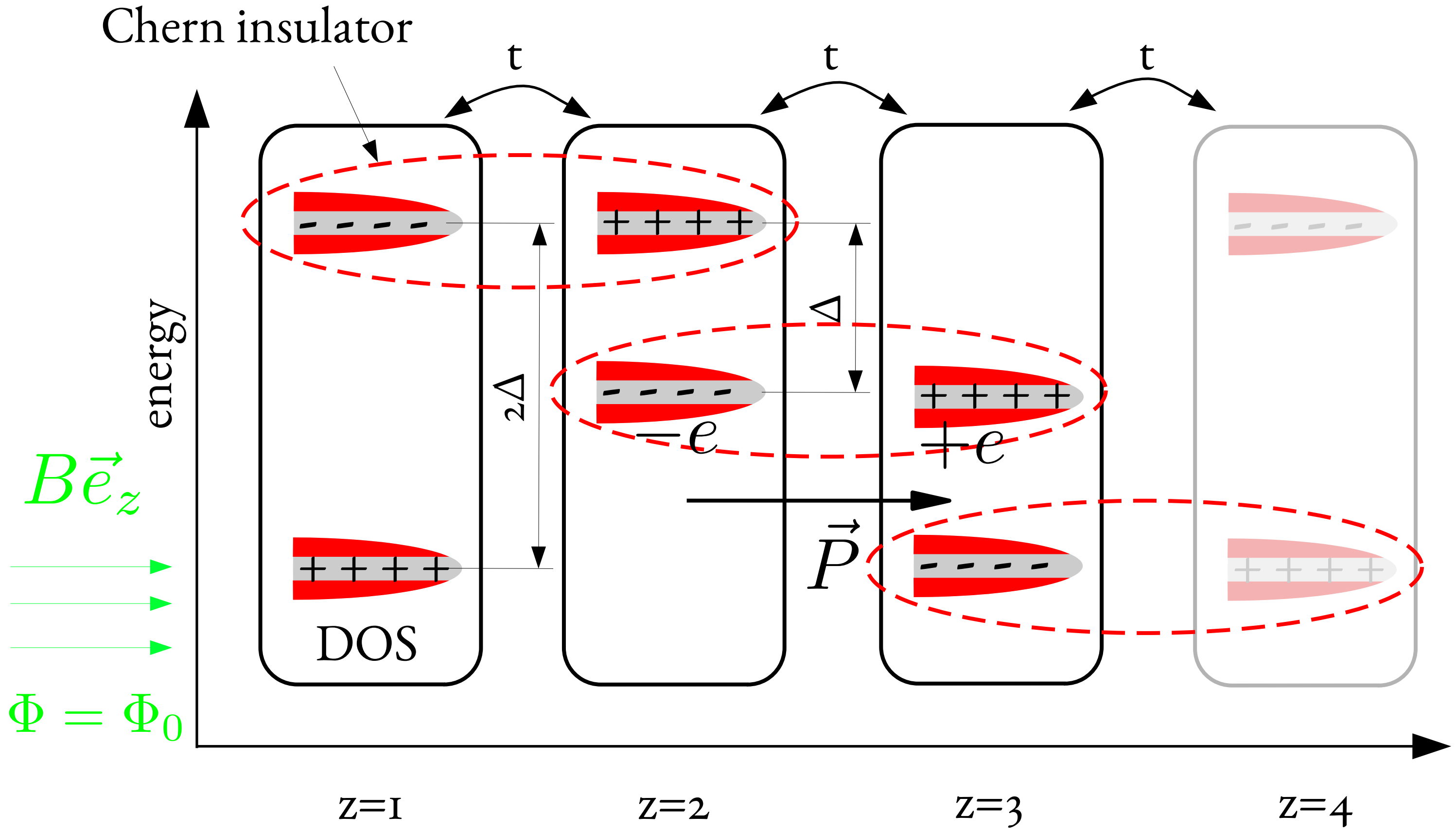
- two-dimensional
- zero net Hall conductance $\sigma_{xy}=0$

pair-annihilation





Bulk is fully localized for arbitrarily weak disorder W!

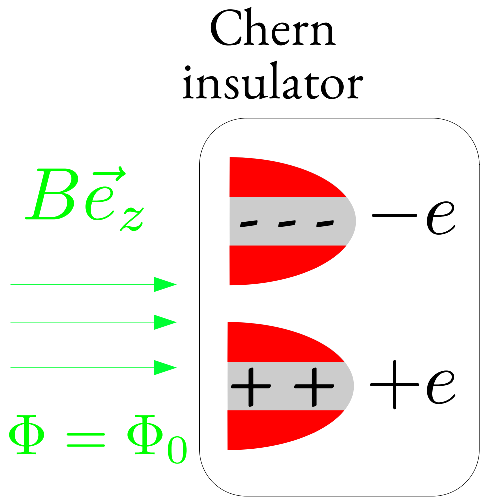


Interlayer coupling t:

- on-resonant coupling ✓
- off-resonant coupling ✗

2) Quantized bulk magnetolectric polarizability!

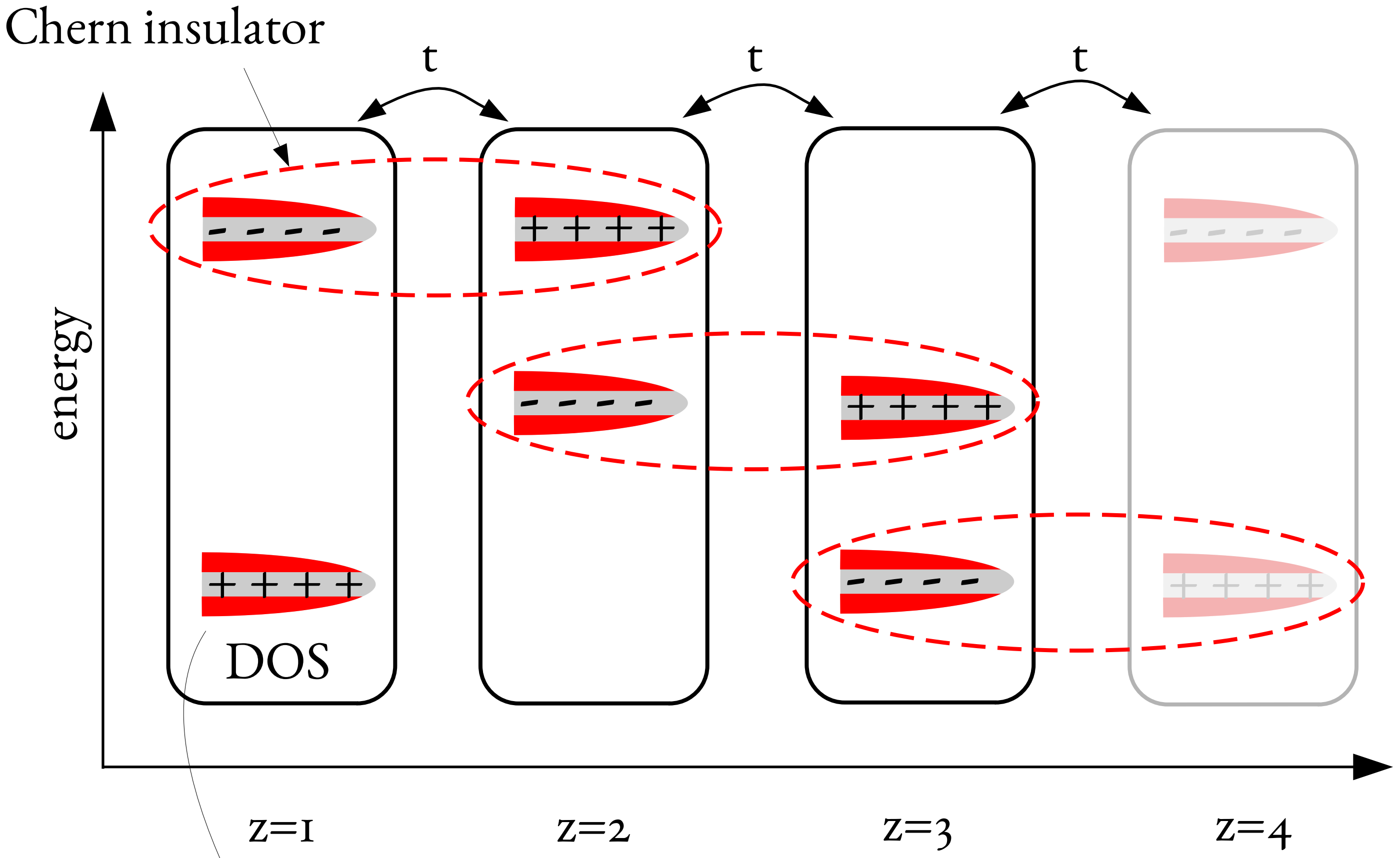
$$\vec{P} = \hat{\alpha}_{\mathrm{ME}} \vec{B}$$



Streda formula:

$$\sigma_{xy} = -e \left(\frac{\partial n}{\partial B} \right)_{\mu}$$

$$\hat{\alpha}_{\mathrm{ME}} = \begin{pmatrix} \alpha_{xx} & \alpha_{xy} & 0 \\ \alpha_{yx} & \alpha_{yy} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

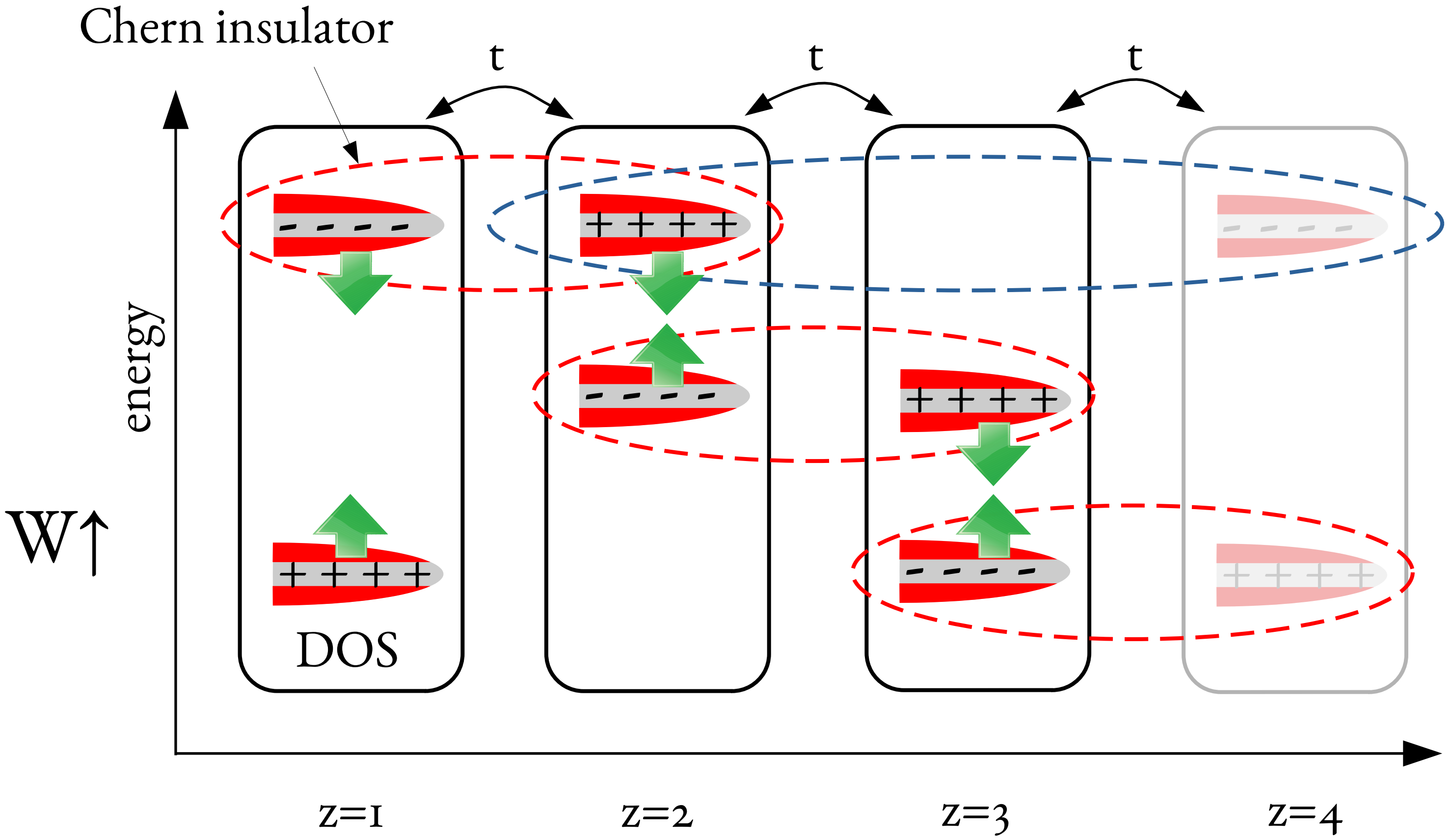


Interlayer coupling t:

- on-resonant coupling ✓
- off-resonant coupling ✗

Remove bulk!

3) Delocalized states at a single energy with quantized surface Hall conductance: $\sigma_{xy}^{\text{surf}} = e^2/h$

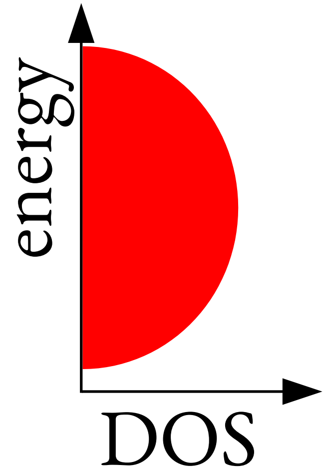


Interlayer coupling t:

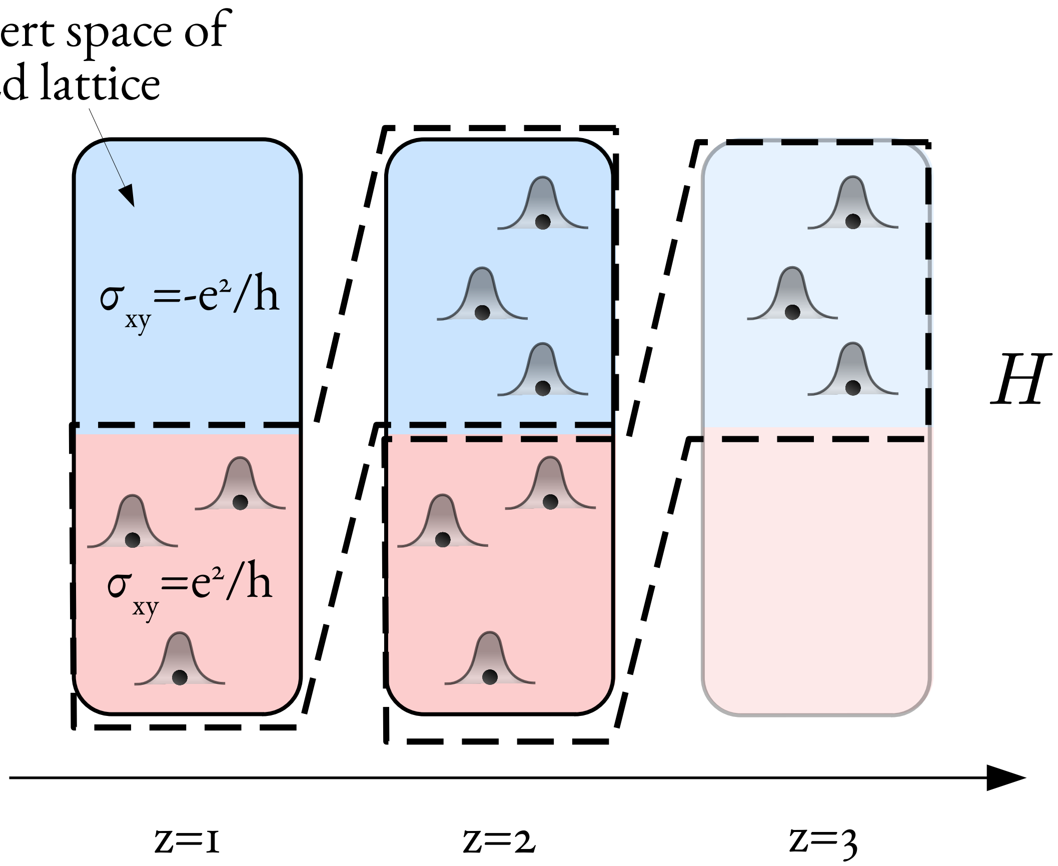
- on-resonant coupling
- off-resonant coupling

Numerically difficult problem

1)



Bulk is fully localized only for finite disorder W!



P_z^+ projector onto

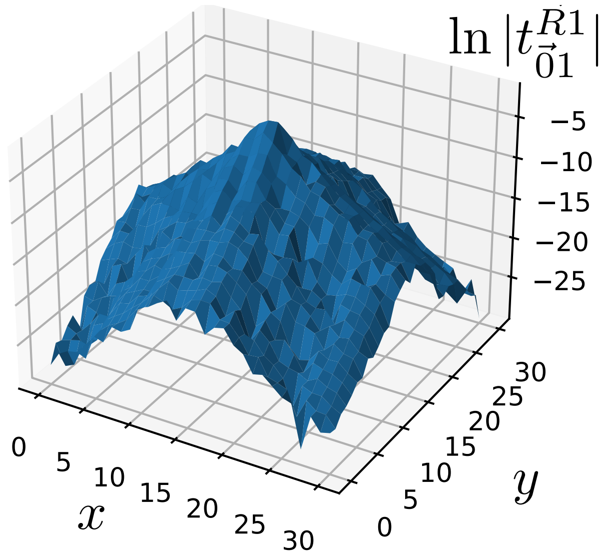
P_z^- projector onto

disorder
(onsite + interlayer hopping)

$$H = \sum_z (P_z^+ + P_{z+1}^-) H_W (P_z^+ + P_{z+1}^-)$$

$$H_W = \sum_{\vec{R},\alpha} W_{\vec{R}\alpha} |g_{\vec{R}\alpha}\rangle \langle g_{\vec{R}\alpha}|$$

with $|g_{\vec{R}\alpha}\rangle = |\vec{R}\alpha\rangle + |(\vec{R}+\hat{e}_z)\alpha\rangle$



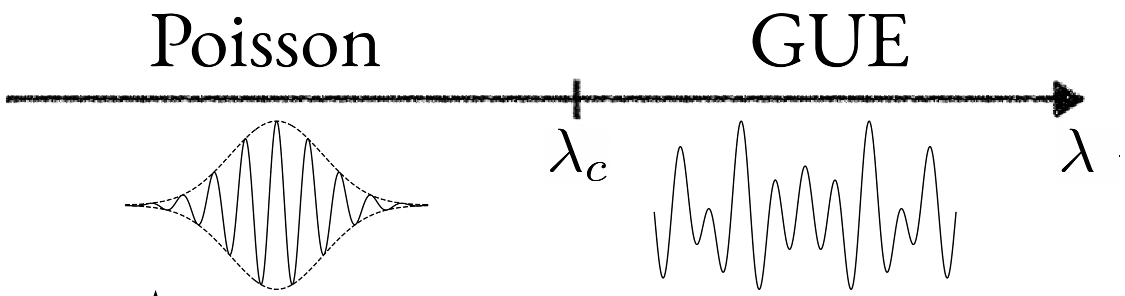
TLI to metal transition: $H_V(\lambda) = H + \lambda V$ → n.n. hopping

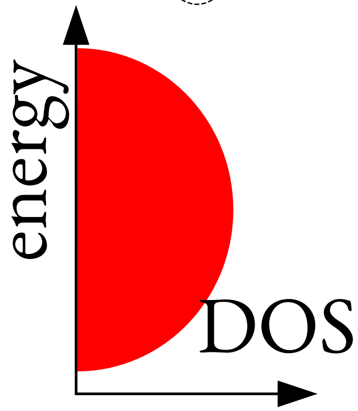
Level spacing statistics (localization/delocalization):

$$s_n = E_{n+1} - E_n$$

→ Level spacing ratio

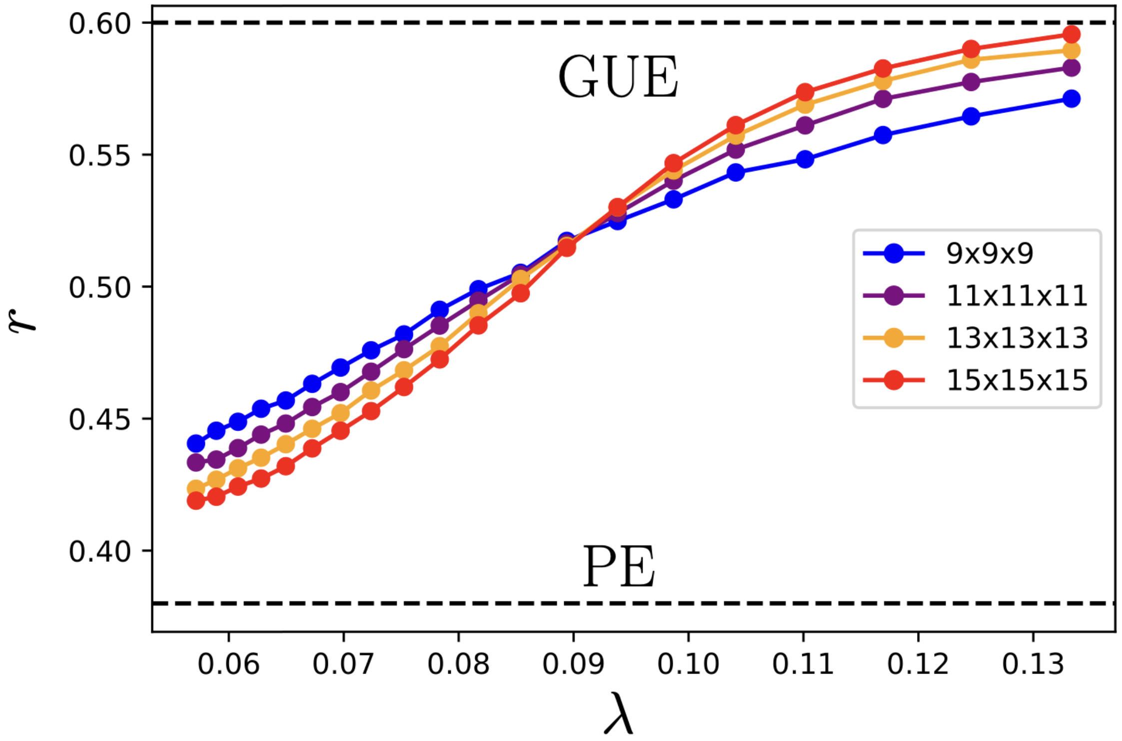
$$r_n = \min\{s_n, s_{n+1}\} / \max\{s_n, s_{n+1}\}$$





1)

$$\lambda_c \approx 0.088 \pm 0.002$$



Bulk & boundary topological invariants $\alpha_{\mathrm{ME}}, \sigma_{xy}^{\mathrm{surf}}$

Fully localized phase: eigenstates $|\psi_n\rangle$ can be labeled by lattice vectors $|\psi_{\mathbf{R}\alpha}\rangle$

Unitary: $U|\mathbf{R}\alpha\rangle=|\psi_{\mathbf{R}\alpha}\rangle$

2) bulk topological invariant is the third winding number of U, $\alpha_{\mathrm{ME}}=\upsilon[U]$

$$\nu[U] = \frac{i\pi}{3}\frac{1}{N_x N_y N_z}\epsilon^{ijk}\mathrm{tr}\left(U^{-1}[\hat{X}_i, U]U^{-1}[\hat{X}_j, U]U^{-1}[\hat{X}_k, U]\right) \in \mathbb{Z} \quad \text{if} \quad \langle\vec{R}'\alpha'|\,U\,|\vec{R}\alpha\rangle \sim e^{-\gamma|\vec{R}-\vec{R}'|}$$

✔ if fully localized

3) boundary topological invariant is given by Chern number, $\sigma^{\mathrm{surf}}_{xy}=\mathrm{Ch}[P]e^2/h$

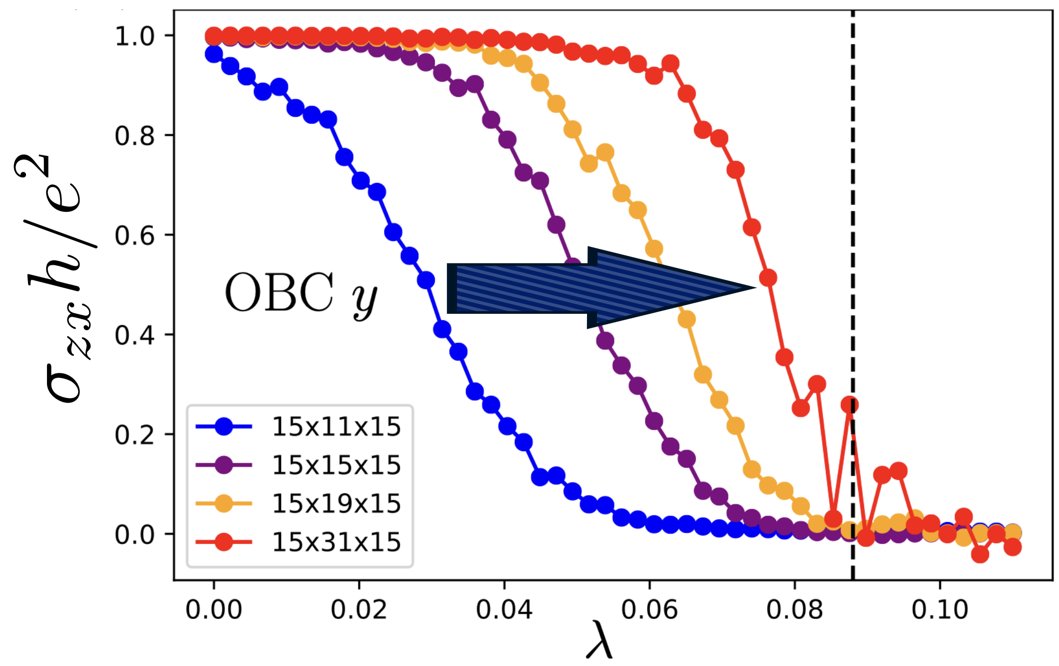
$$\mathrm{Ch}[\mathcal{P}] = \frac{2\pi i}{N_x N_y}\mathrm{Tr}\left(\mathcal{P}\left[\left[\hat{X}_1, \mathcal{P}\right], \left[\hat{X}_2, \mathcal{P}\right]\right]\right)$$

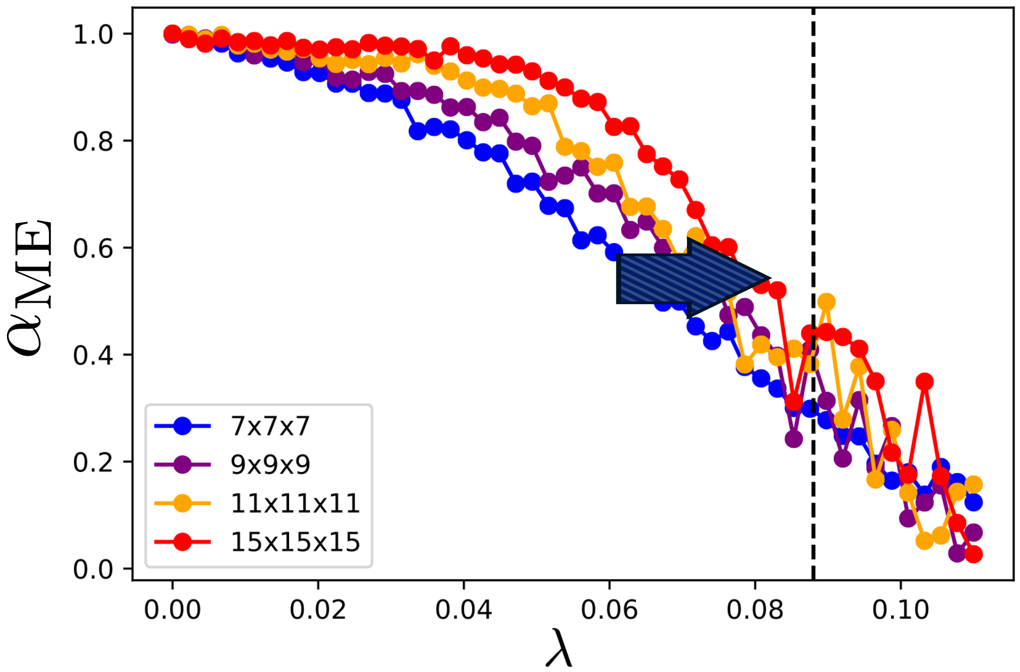
$\mathcal{P}$ projector onto d.o.f. on the surface

E. Prodan, J. Phys. A: Math. Theor. 44, 113001 (2011)

TLI to metal transition: $H_V(\lambda) = H + \lambda V$

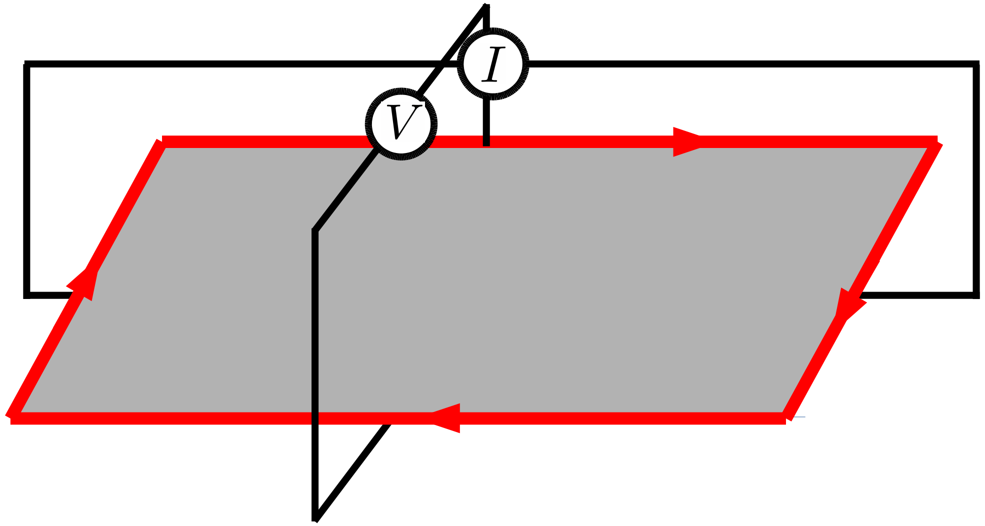
n.n. hopping





How to measure Hall conductance?

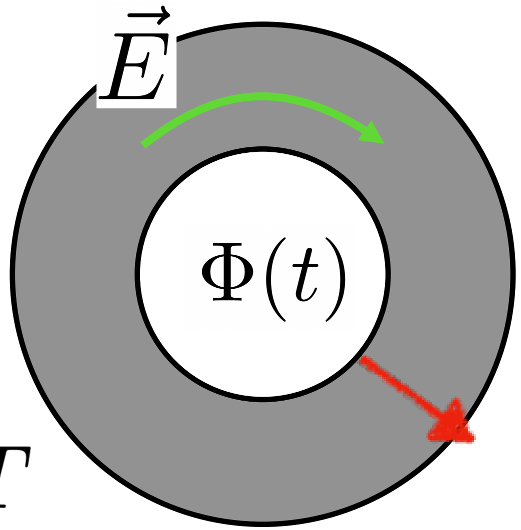
Four-terminal measurement



Only edges contribute to the transport

Corbino disk (Laughlin's pump)

Adiabatic flux insertion:

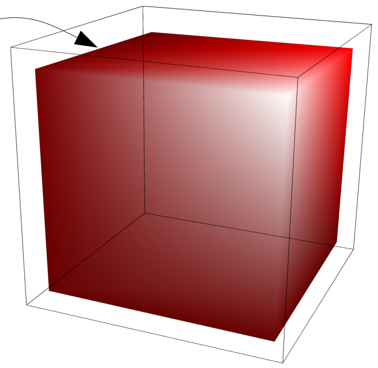


$$\Phi(t) = \Phi_0 t/T$$

Quantized charge transfer (via bulk)

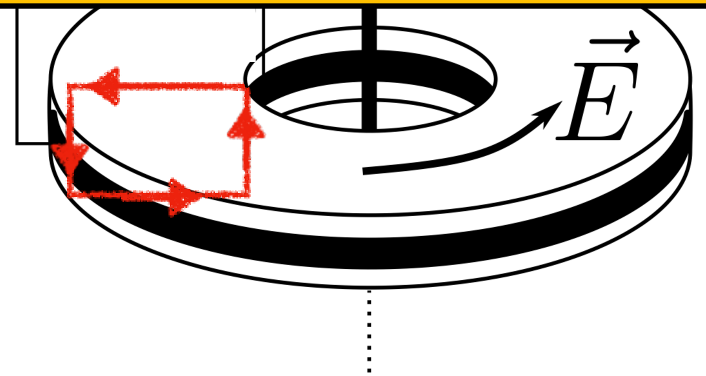
$$\Delta Q = \sigma_H \frac{h}{e} = ne$$

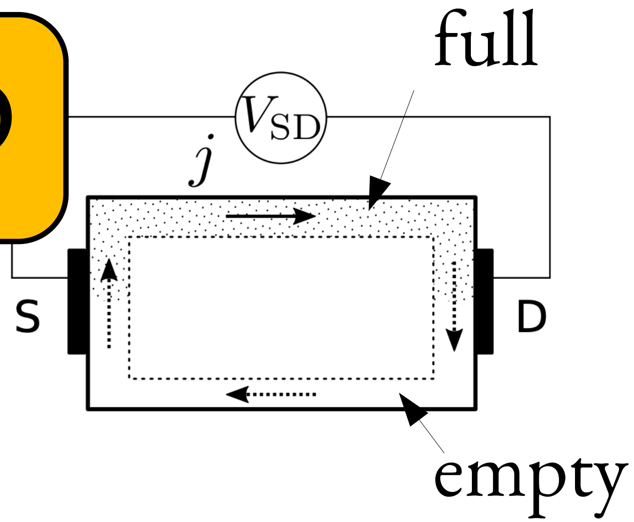
TLI: σ^{surf}_{xy} is guaranteed to be quantized only for fully filled boundary



Adiabatic flux insertion $\Phi(t) = \Phi_0 t/T$

Charge transfer of $e^*\sigma^{surf}_{xy}$ from S to D

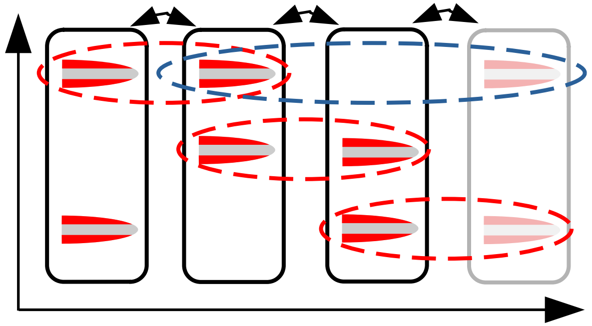




steady-state charge distribution

Outlook

1) Better understanding of the model



2) Field theory for TLI? Critical exponents for TLI to metal transition?

3)

class	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	$d=0$	$d=1$	$d=2$	$d=3$
A	0	0	0	?	?	?	$\mathbb{Z}$
AIII	0	0	1	?	?	?	?
AI	+	0	0	?	?	?	?
BDI	+	−	1	?	?	?	?

tenfold-way

TLIs