Edge modes and counting statistics in the 2D and 4D Quantum Hall effects

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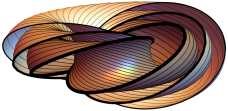
Topological Quantum Phases of Matter Beyond Two Dimensions, Jussieu 2022

[Estienne & JMS, Phys. Rev. B 101, 2019]
[Estienne, Oblak & JMS, Scipost Physics 11, 2021]
[Estienne, JMS & Witczak-Krempa, Nat. Comm. 13, 2022]



Slides + Chalk

Outline

1. Simple 2d Integer Quantum Hall wavefunctions
2. Simple 4d Integer Quantum Hall wavefunctions
3. Counting statistics and related problems

Hamiltonian in a magnetic field + trapping potential

$$H = \frac{1}{2}(\mathbf{p} - \mathbf{A})^2 + \frac{k}{2}(x^2 + y^2)$$

Symmetric gauge

$$\mathbf{A} = \frac{B}{2}\begin{pmatrix} -y \\ x \end{pmatrix}$$

leads –after lowest Landau level projection– to singleparticle wave functions

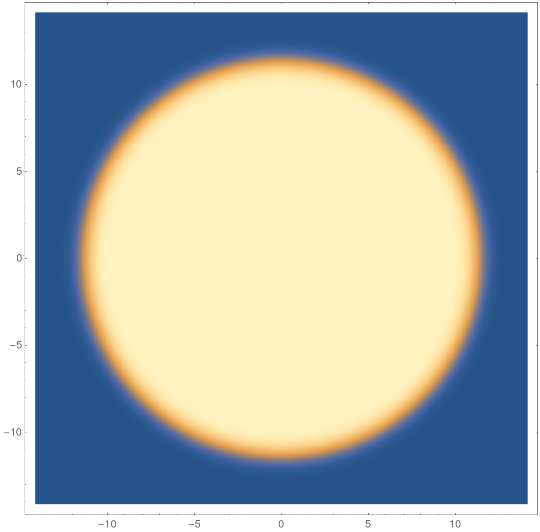
$$\phi_m(z) = \frac{z^m}{\sqrt{\pi m!}} e^{-|z^2|/2}$$

with single particle energies $\epsilon_m = \hbar\omega m$ with $\omega = \frac{k}{B}$.

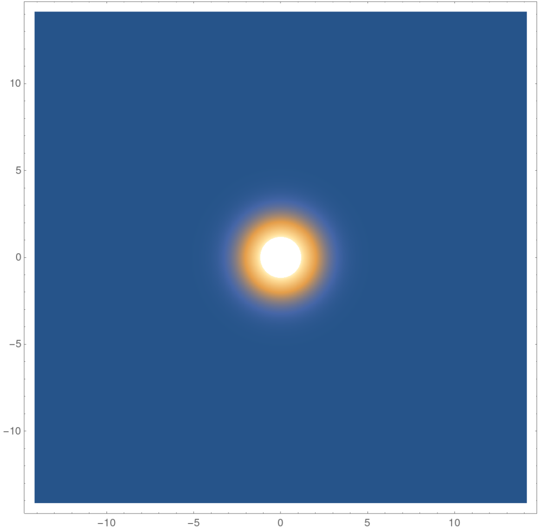
Two-point function/correlation kernel

$$
\begin{aligned}
K_N(z, z') &= \sum_{m=0}^{N-1} \phi_m^*(z)\phi_m(z') \\
&= \sum_{m=0}^{N-1} \frac{(z^* z')^m}{\pi m!} e^{-(|z|^2+|z'|^2)/2}
\end{aligned}
$$

Density profile for large but finite N



Two-point function (correlation kernel)



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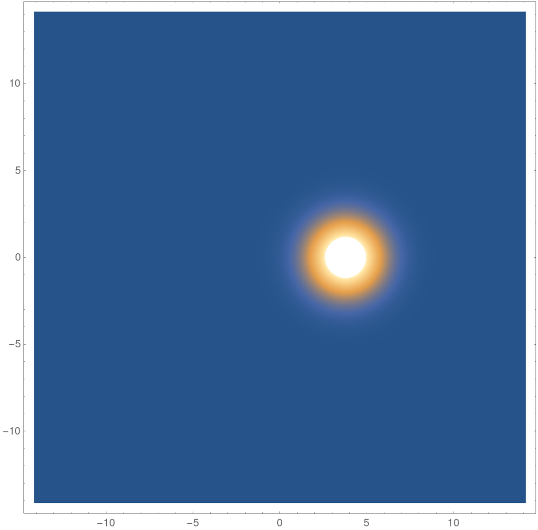
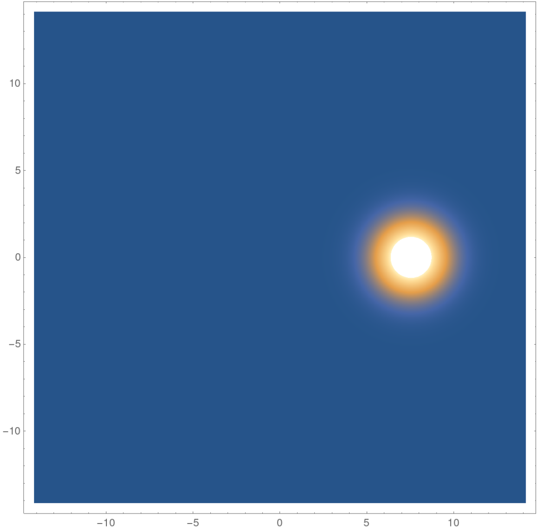


Two-point function (correlation kernel)



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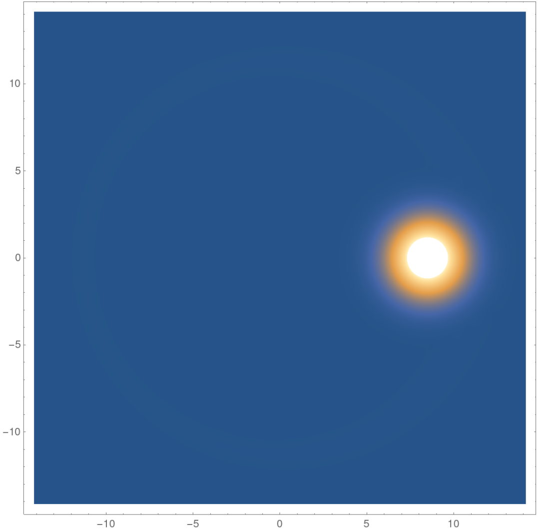
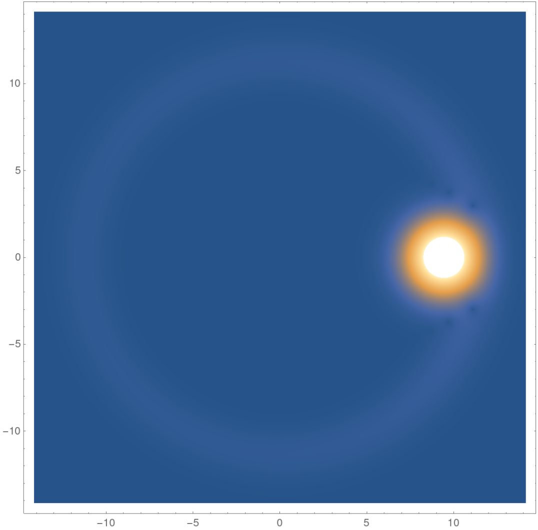
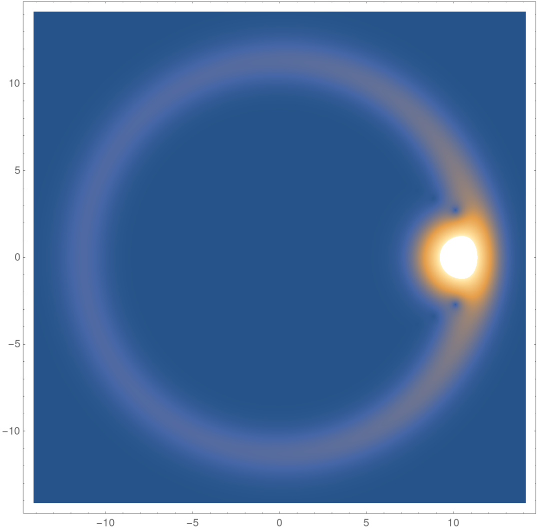


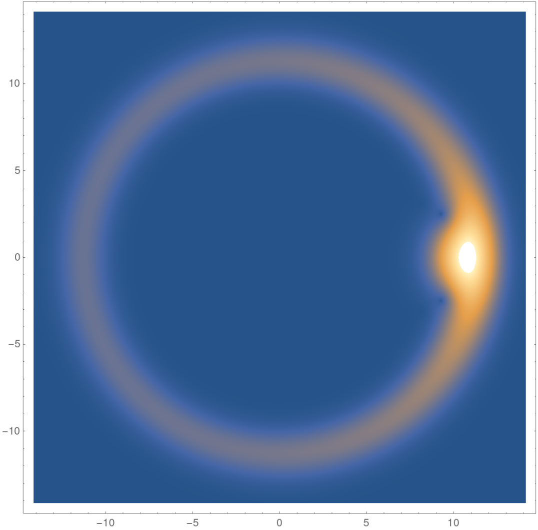
Two-point function (correlation kernel)



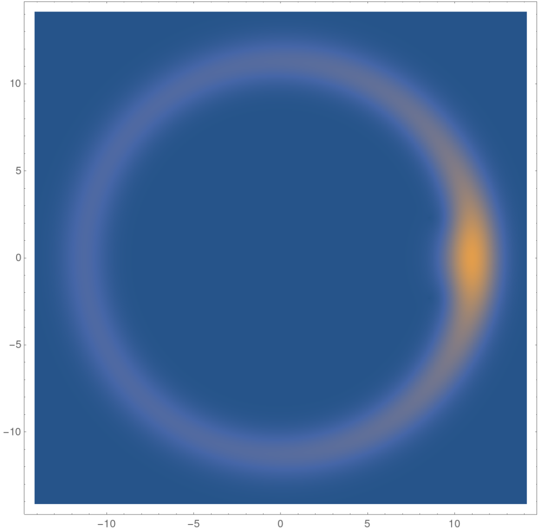
Two-point function (correlation kernel)



Two-point function (correlation kernel)



Two-point function (correlation kernel)



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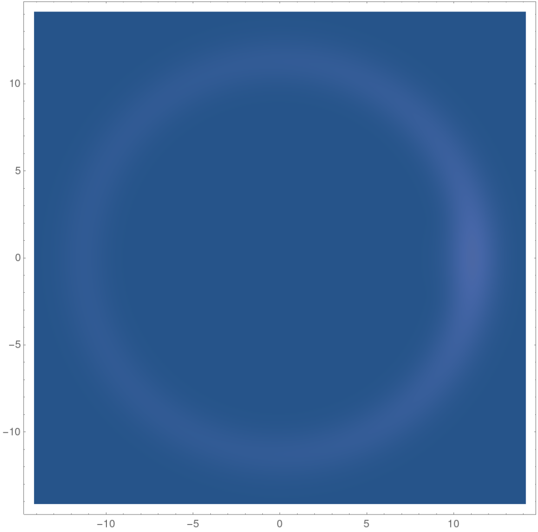


4d Quantum Hall effect

$(x, y, u, v) \in \mathbb{R}^4$. Simplest generalization of the previous model:

$$H = (\mathbf{p} - \mathbf{A})^2 + \frac{k}{2}\left(x^2 + y^2\right) + \frac{k'}{2}\left(u^2 + v^2\right)$$

with vector potential

$$\mathbf{A} = \frac{B}{2}\begin{pmatrix} -y \\ x \\ -v \\ u \end{pmatrix}$$

leads to ($z = x + iy$, $w = u + iv$ up to some units)

$$\phi_{m,n}(z, w) = \frac{z^m w^n}{\sqrt{\pi^2 m! n!}} e^{-(|z|^2 + |w|^2)/2}$$

with energies $\epsilon_{m,n} = \hbar(\omega m + \omega' n)$ with $\omega = \frac{k}{B}$, $\omega' = \frac{k'}{B}$

Generalizations of QHE to 4d predict anisotropic edge modes

[Zhang & Hu, Science 2001]

[Karabali & Nair, Nucl. Phys. B 2002]

[Helvang & Polschinski, CRP 2003]

Possible experimental realisations by engineering synthetic dimensions (quasicrystals, internal states, photonics,. . .)

[Kraus, Ringel & Zilberberg, PRL 2013]

[Price, Zilberberg, Ozawa, Carusotto & Goldman, PRL 2015]

[Lohse, Schweizer, Price, Zilberberg & Bloch Nature 2018]

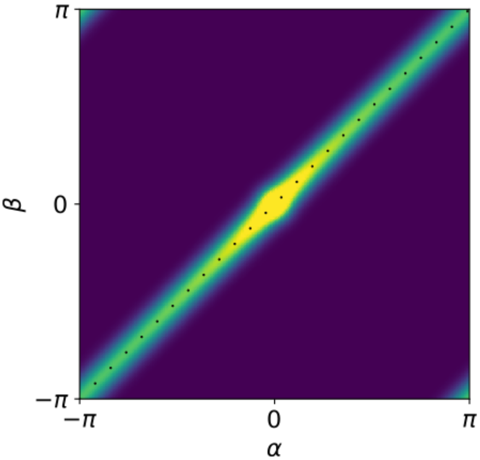
[Bouhiron *et al.*, arXiv:2210.06322] (see previous talk by Nascimbene)

. . .

$$K_N(z,w|z',w') = \sum_{m+\Delta n<N} \frac{(z^*z')^m (w^*w')^n}{\pi^2 m!n!} e^{-(|z|^2+|w|^2+|z'|^2+|w'^2|)/2}$$

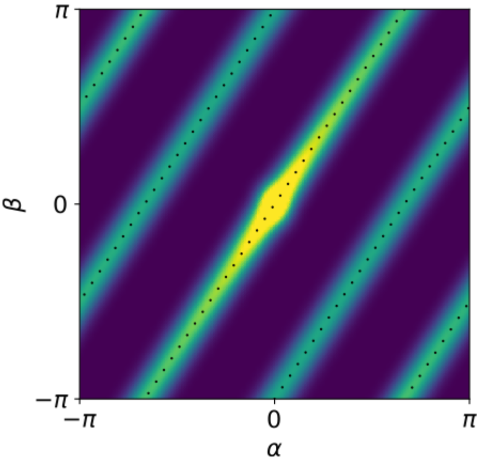
$$\Delta = \frac{\omega'}{\omega}$$

Edge modes on the torus



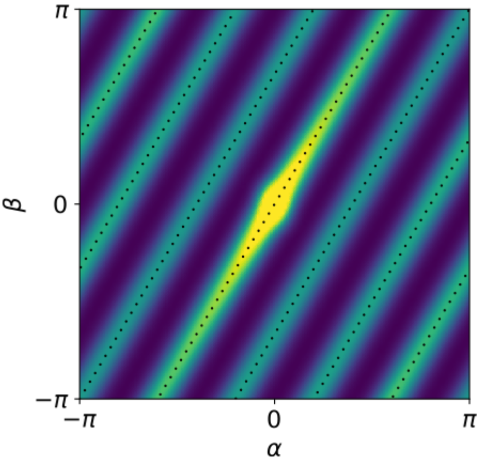
$\Delta = 1$

Edge modes on the torus



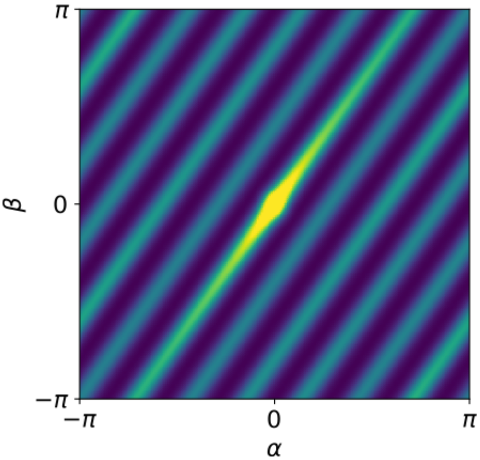
$$\Delta = 3/2$$

Edge modes on the torus



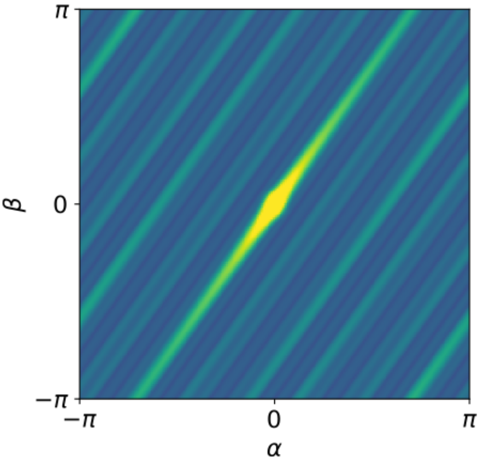
$$\Delta = 5/3$$

Edge modes on the torus



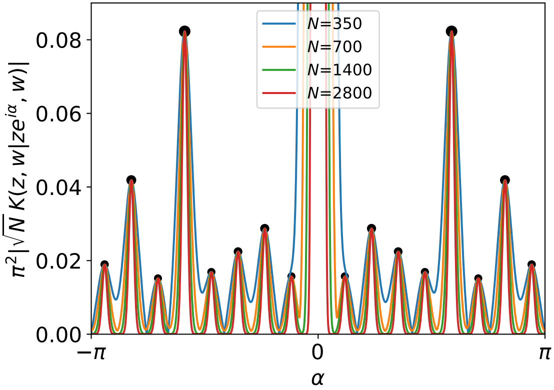
$$\Delta = 7/5$$

Edge modes on the torus



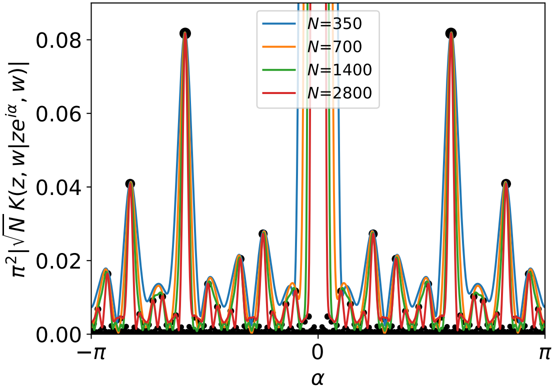
$\Delta = 41/29$

1d slices at $\beta = 0$



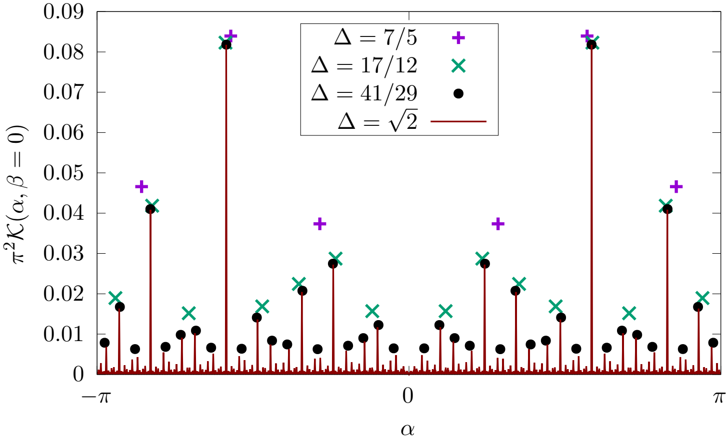
$$\Delta = 17/12$$

1d slices at $\beta = 0$



$$\Delta = \sqrt{2}$$

1d slices at $\beta = 0$



$$\Delta = \sqrt{2}$$

Counting statistics and related problems

Thank you!