Realization of a four-dimensional atomic Hall system

arXiv:2210.06322

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Workshop: Topological Phases Beyond 2D
October 20, 2022

Outline

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1. 2D Hall effect in atomic gases
   - State of the art
   - Quantum Hall ribbon
   - Quantum Hall cylinder

2. Realization of a four-dimensional atomic Hall system
   - State of the art: 4D Hall physics with a 2D charge pump
   - Description of our system
   - 2D Hall responses
   - Velocity distribution and edge modes
   - Cyclotron orbits
   - Reconstructing the second Chern number
   - Direct observation of a 4D Hall non-linear response

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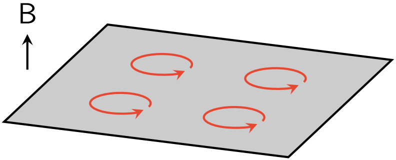
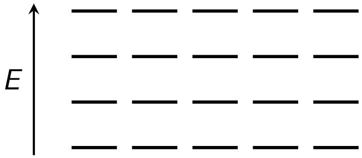
Topological systems

The archetype of topological systems: a **2D quantum Hall insulator**



classical orbits



Landau quantum levels
flat bands & non-trivial topology

A whole zoo of topological systems (topological insulators, superconductors) depending on discrete symmetry class and dimension

Altland, & Zirnbauer, PRB 1997

Topological systems in dimensions $D > 3$ accessible in engineered systems based on **synthetic dimensions**.

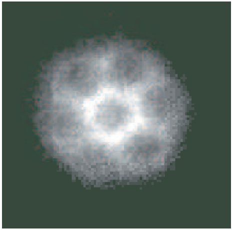
This talk: realization of a 4D quantum Hall system

Simulating an orbital magnetic field with ultracold atoms

Mimicking the Aharonov-Bohm geometrical phase

Rotation

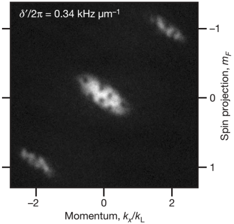
Sagnac phase



Madison et al, PRL 1999

Light dressing

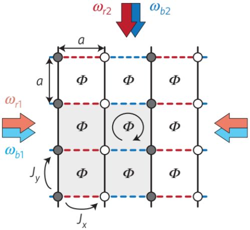
Berry phase



Lin et al, Nature 2009

Shaken lattices

Peierls phase

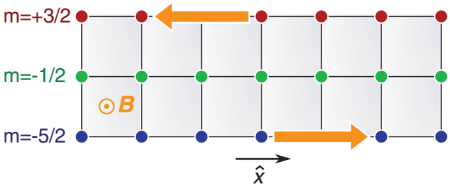


Aidelsburger, PRL 2013

Jotzu et al, Nature 2014

A new tool: synthetic dimensions

Encoding a dimension in a spin degree of freedom.
Magnetic projection m (with $-J \leq m \leq J$) acts as a coordinate.



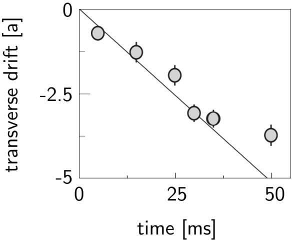
first realizations with 3 states

Mancini et al, Science (2015) and Stuhl et al, Science (2015)

Assets of this method

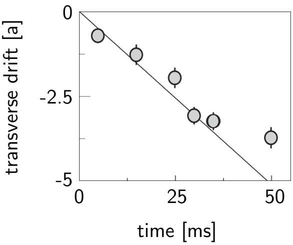
- simple realization of the magnetic field:
  light-induced spin transitions
- sharp edges

- Quantization of transverse response in large & smooth atomic ensembles



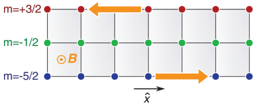
Aidelsburger et al, Nature Phys. (2015)

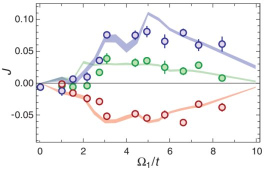
- Quantization of transverse response in large & smooth atomic ensembles



Aidelsburger et al, Nature Phys. (2015)

- Chiral edge modes in very small samples (no notion of a bulk)





Mancini et al, Science (2015)

Stuhl et al, Science (2015)

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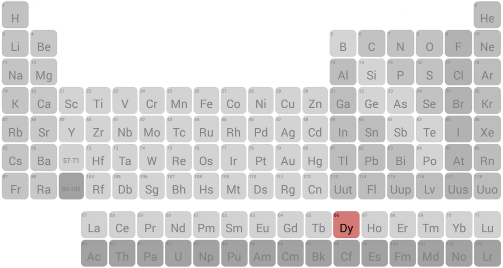
Encoding a large synthetic dimension with Dy atoms



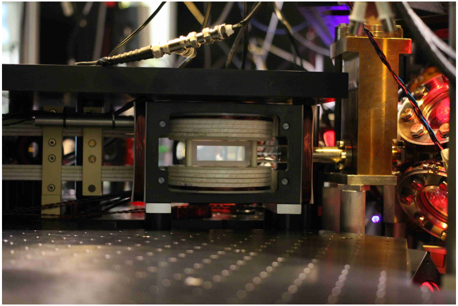
Filled $6s^2$ shell

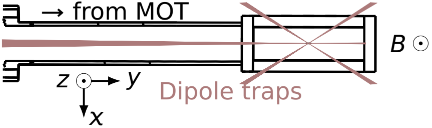
Submerged open $4f^{10}$ shell

Electronic spin $J = 8$

Magnetic projection states m $(-J \leq m \leq J)$ encode a synthetic dimension with $2J + 1 = 17$ sites.

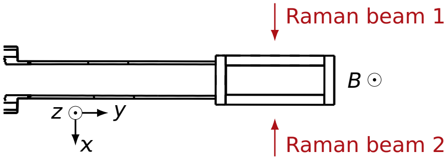
Our system of ultracold Dy atoms

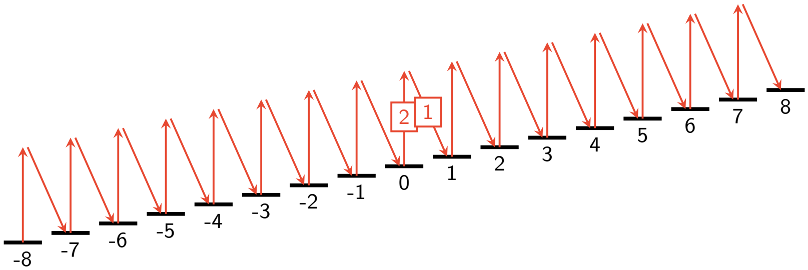




10^5 atoms held in optical tweezers, cooled down to $T = 0.5\,\mu K$.

Realization of a quantum Hall ribbon: spin dynamics





Transitions $m \to m+1$ together with momentum kick $Mv \to Mv - 2\hbar k$

conservation of momentum $p = Mv + 2\hbar k m$.

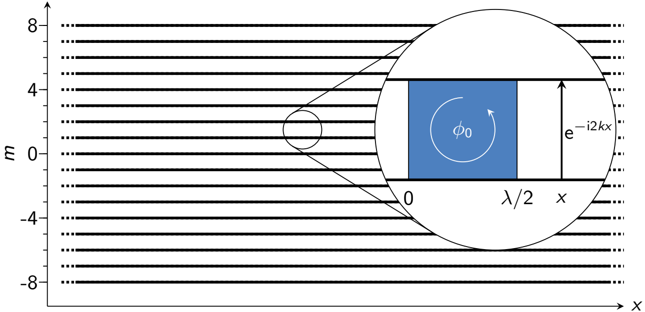
Single-particle Hamiltonian

$$H \simeq \frac{Mv^2}{2} - \hbar\Omega(J_+ \mathrm{e}^{-2\mathrm{i}kx} + \mathrm{hc})$$

Peierls phase for the hopping $m \to m+1$

$$\phi = -2\mathrm{i}kx = \int_m^{m+1} \mathrm{d}m\, A_m = A_m,$$

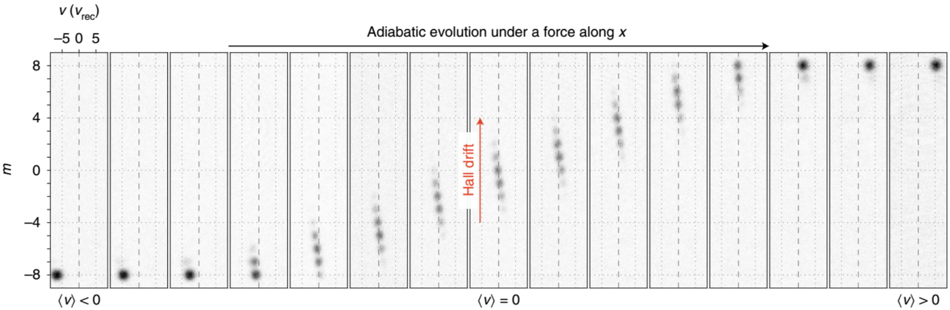
i.e. a magnetic field in the xm plane $B = -\partial_x A_m = 2k$.



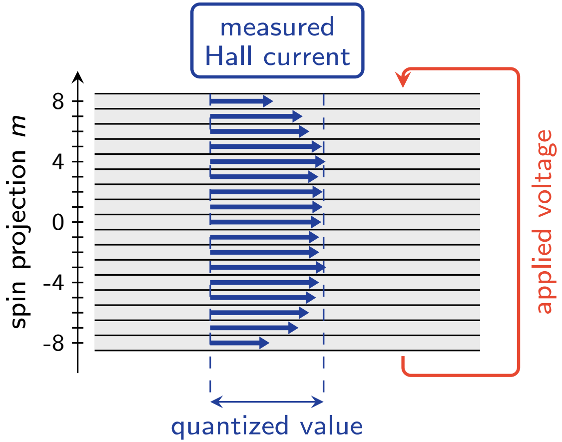
Observables

- absorption image after free expansion $\rightarrow$ velocity distribution
- separation of m sublevels using magnetic field gradient

Spin-velocity distributions through the ground band



Measuring a quantized Hall response



Homogeneous and quantized Hall response in the bulk

T. Chalopin et al, Nat. Phys. (2020)

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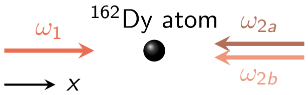
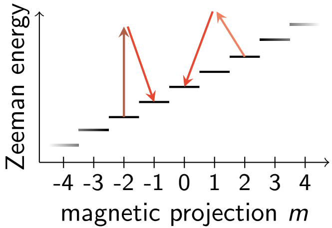
Emergence of a cyclic synthetic dimension

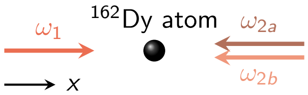


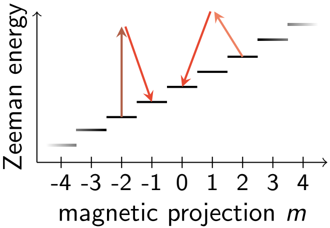


Combination of two Raman transitions

- $m \to m + 1$ together with $Mv \to Mv - 2\hbar k$
- $m \to m - 2$ together with $Mv \to Mv - 2\hbar k$

We lose the conservation of momentum $p = Mv + 2\hbar km$.





Both transitions satisfy

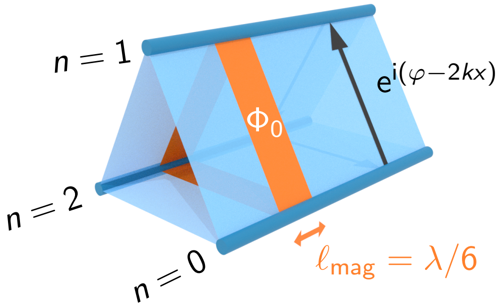
$$n \to n+1 \text{ together with } Mv \to Mv - 2\hbar k$$

for the cyclic dimension

$$n = m \pmod 3.$$

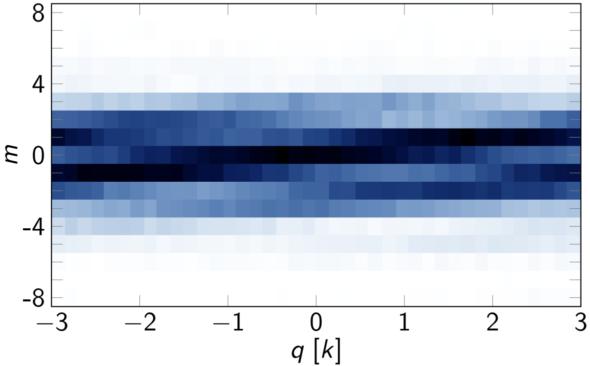
Conservation of the quasi-momentum $q = Mv + 2\hbar kn \pmod{6\hbar k}$

Emergence of a Hall cylinder



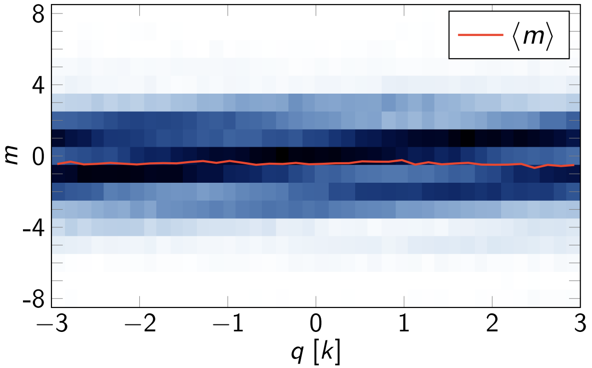
Topological charge pump in a Bloch oscillation

Evolution of spin projection probabilities



A. Fabre et al, Phys. Rev. Lett. (2022)

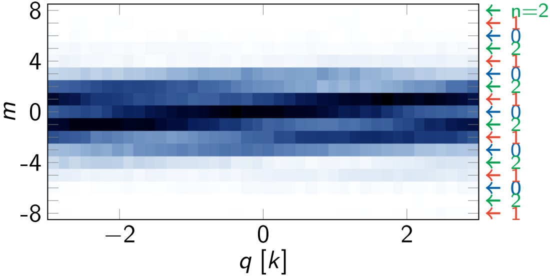
Topological charge pump in a Bloch oscillation



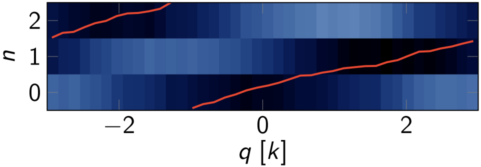
No drift of the mean $\langle m \rangle$

A. Fabre et al, Phys. Rev. Lett. (2022)

Topological charge pump in a Bloch oscillation



regrouping to infer the *n*-projection probabilities



Quantized increase $\Delta n = 3$ for each Bloch oscillation cycle.

A. Fabre et al, Phys. Rev. Lett. (2022)

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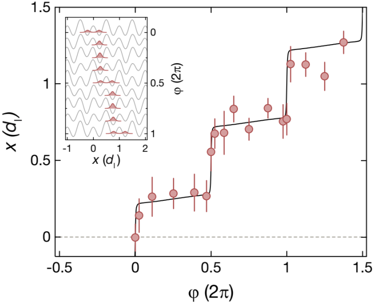
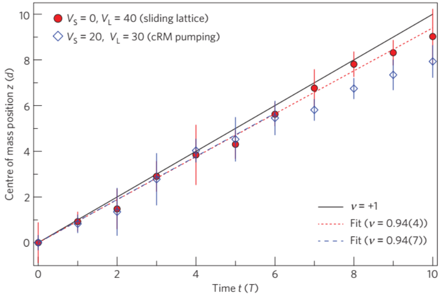
Outline

adiabatic & time-periodic deformation
of a quantum lattice system
↓
quantized charge pump

Thouless, PRB 1983

Realizations in cold atomic systems



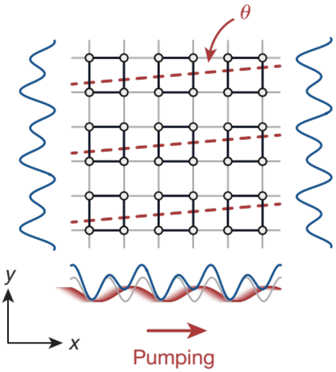


Lohse et al, Nature Phys. 2016

Nakajima et al, Nature Phys. 2016

4D Hall physics with a 2D charge pump

A two-dimensional (super-)lattice system



Cyclic deformation of the superlattice drives quantized charge pump described by a second Chern number.

Lohse et al, Nature 2018

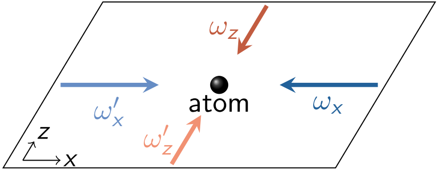
Outline

Definition of the four dimensions

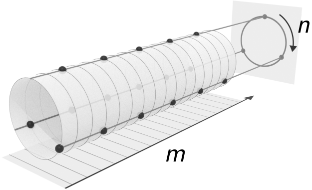
Implementation inspired from previous proposals

Kraus et al, Phys. Rev. Lett. 2013
Price et al, Phys. Rev. Lett. 2015

atom motion
spatial dimensions x,z

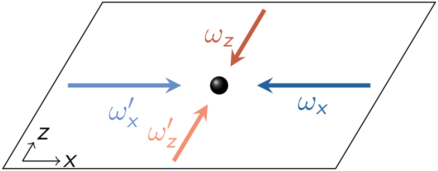


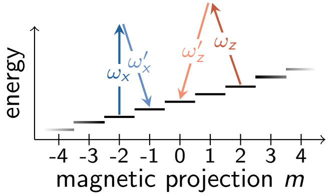
spin $J = 8$
synthetic dimensions n,m



Fabre et al, PRA 2022

Geometry of Raman transitions





Two types of Raman transitions

transition	Δm	Δn	$M\Delta\mathbf{v}$
x	1	1	$-2\hbar k\hat{\mathbf{x}}$
z	-2	1	$-2\hbar k\hat{\mathbf{z}}$

Non-trivial cycles $m \xrightarrow{z} m+2 \xrightarrow{x} m+1 \xrightarrow{x} m$ imparting a velocity kick

$$\mathbf{K} = 2k(2\hat{\mathbf{x}} + \hat{\mathbf{z}})$$

Conservation of the quasi-momentum $\mathbf{P} = M\mathbf{v} + 2km\hat{\mathbf{x}} \pmod{\mathbf{K}}$

Magnetic field in 4D space

$$H \simeq \frac{Mv^2}{2} - t_x \left(\frac{J_+}{J} e^{-2ikx} + \text{hc} \right) - t_z \left(\frac{J_-^2}{J^2} e^{-2ikz} + \text{hc} \right)$$

Peierls phases

$$\phi_x = -2kx = A_n + A_m,$$
$$\phi_z = -2kz = A_n - 2A_m,$$

hence the vector potential

$$\mathbf{A} = \frac{1}{3}(0, 0, 2\phi_x + \phi_z, \phi_x - \phi_z)_{x,z,n,m}$$

and the magnetic field tensor

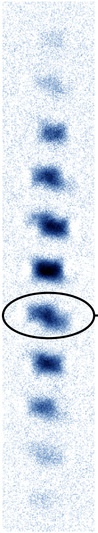
$$\mathbf{B} = \frac{2k}{3} \begin{pmatrix} 0 & 0 & -2 & -1 \\ 0 & 0 & -1 & 1 \\ 2 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}.$$

Experimental sequence

1. Ultracold gas of 10^5 atoms at $T = 0.2\,\mu\text{K}$ polarized in $m = -J$

2. Switch on Raman couplings (off resonant)

3. Ramp detunings adiabatically in frame moving with lattice interference: **inertial force** $\Rightarrow$ control of quasi-momentum **P**



4. Measurement of velocity and spin distributions after time-of-flight

↕ separation of m levels with a B field gradient

→ xz velocity distribution for each m level

Outline

Adiabatic spin pumping

We apply a force in the xz plane and expect a geometrical drift in spin space.

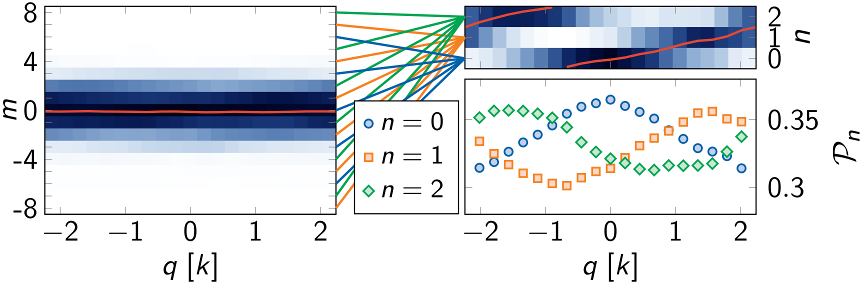
transition	Δm	Δn	$M\Delta\mathbf{v}$
x	1	1	$-2\hbar k\hat{\mathbf{x}}$
z	-2	1	$-2\hbar k\hat{\mathbf{z}}$

Conjugated directions

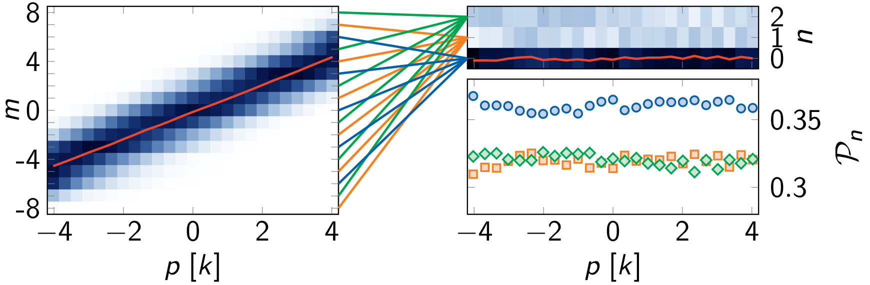
$$n \leftrightarrow \hat{\boldsymbol{\nu}} \equiv \frac{2\hat{\mathbf{x}} + \hat{\mathbf{z}}}{\sqrt{5}}$$

$$m \leftrightarrow \hat{\boldsymbol{\mu}} \equiv \frac{\hat{\mathbf{x}} - \hat{\mathbf{z}}}{\sqrt{2}}$$

Geometrical pumping along n



Geometrical pumping along m



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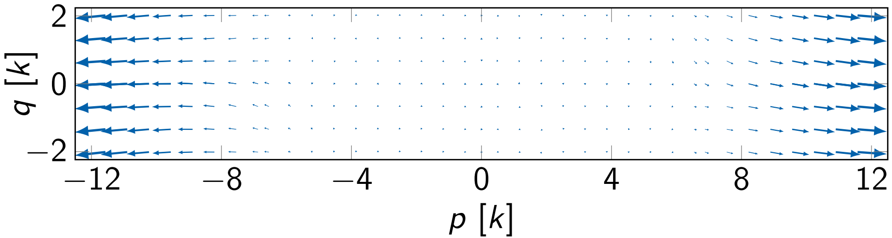
Velocity distribution and edge modes

We decompose the momentum as

$$\mathbf{P} = p\hat{\boldsymbol{\xi}} + q\hat{\boldsymbol{\nu}} \pmod{K\hat{\boldsymbol{\nu}}}.$$

We extract for each momentum state the mean atom velocity.



Arrow length $\propto ||\mathbf{v}||$, with max length $\equiv 5v_\mathrm{r}$

- velocity remains very small in the bulk
- on edges $m = \pm J$, ballistic motion along $\pm\hat{\boldsymbol{\xi}}$, frozen motion along other directions

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Rotations in four-dimensional space and cyclotron motion

Parametrization of rotations

- **Dimension 2**: center and angle
- **Dimension 3**: axis and angle
- **Dimension 4**: 2 invariant orthogonal planes, and two rotation angles (one for each plane)

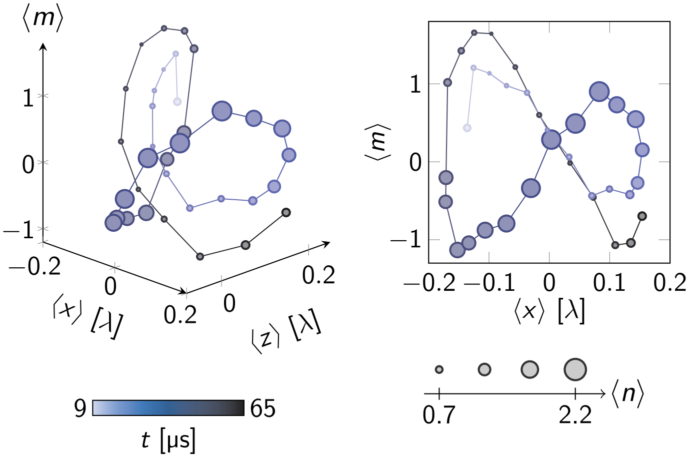
$\Rightarrow$ charged-particle motion in a magnetic field involves two frequencies.

For our system

- Raman process x leads to a rotation in the plane $(\hat{\mathbf{x}}, \hat{\mathbf{m}} + \hat{\mathbf{n}})$ of frequency ω_x
- Raman process z leads to a rotation in the plane $(\hat{\mathbf{z}}, -2\hat{\mathbf{m}} + \hat{\mathbf{n}})$ of frequency ω_z

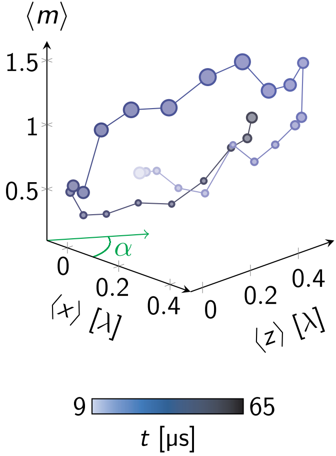
Cyclotron orbit for $\omega_z/\omega_x = 2$

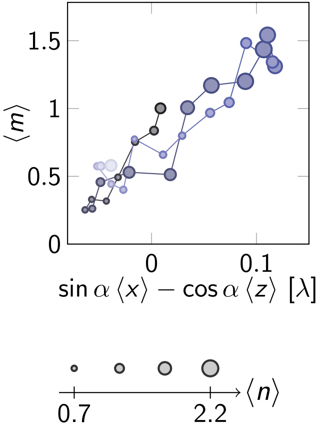
We kick the atoms to drive a cyclotron motion of the center of mass.



Quasi-closed Lissajous orbit (non planar)

Cyclotron orbit for $\omega_z/\omega_x = 1$





Consistent with a planar circular orbit

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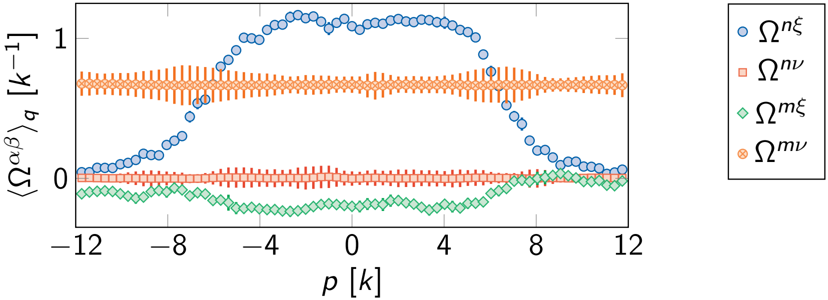
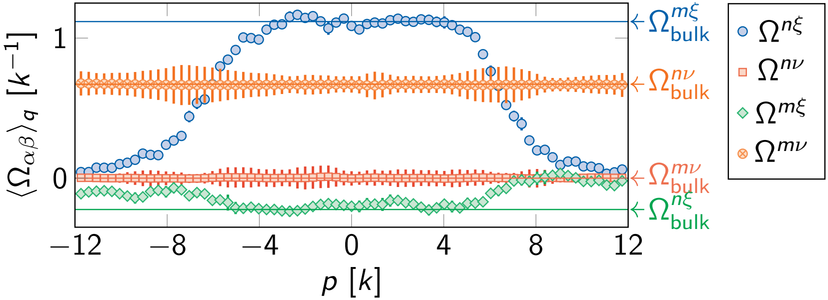
Variations of the mean velocity with momentum give access to Berry curvatures, e.g.

$$\Omega^{\nu m} = -\frac{\sqrt{5}}{2k}\frac{\partial\langle v^{\nu}\rangle}{\partial q}$$



In the bulk and in the absence of dispersion, the Berry curvature is uniform with

$$\mathbf{\Omega}_{\text{bulk}} = \mathsf{B}^{-1} = \frac{1}{2k}\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -2 \\ -1 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \end{pmatrix}$$



Reconstructing the local second Chern marker

In uniform systems, the second Chern number is given by the band integral of Berry curvature products

$$\mathcal{C}_2 = \frac{1}{8\pi^2} \int \rho_2 \, \mathrm{d}^4 q,$$

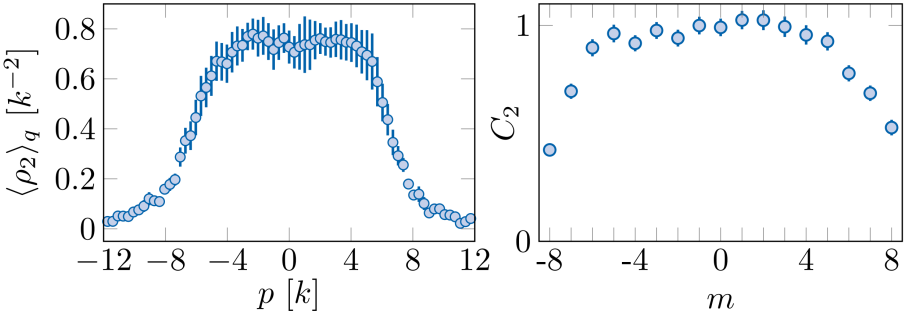
where $\rho_2 = \epsilon_{\mu\nu\delta\gamma} \Omega^{\mu\nu} \Omega^{\delta\gamma}$ is the second Chern character.
In our finite system with edge, we expect a quantized non-linear response *in the bulk only*.

We define a local second Chern marker

$$C_2(m) = \frac{1}{3} \int \rho_2(p, q) \Pi_m(p, q) \mathrm{d}p \mathrm{d}q$$

by weighting the second Chern character with the projection probability Π_m.

Quantized local second Chern marker in the bulk



The local second Chern marker is close to $\mathcal{C}_2 = 1$ in the bulk $-5 \leq m \leq 5$.

Outline

The second Chern number quantizes a non-linear response

We expect a quantized non-linear response to both perturbative electric field f_ν and magnetic field $b_{\alpha\beta}$

$$j^\mu_{\mathrm{NL}} = \frac{\mathcal{C}_2}{4\pi^2}\epsilon^{\mu\alpha\beta\nu} f_\nu b_{\alpha\beta}.$$

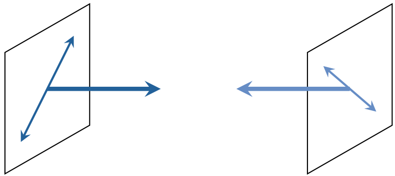
In other words, a magnetic perturbation $b_{\alpha\beta}$ induces a Hall conductivity $\propto \mathcal{C}_2 b_{\alpha\beta}$ in the orthogonal plane.

In our system, we implement a **magnetic perturbation b_{nm}** in the nm plane

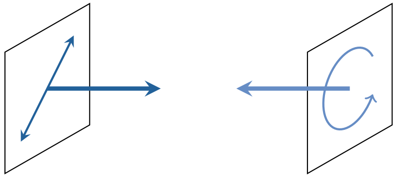
$\Rightarrow$ appearance of a **Hall effect in the xz plane**

Implementation of the magnetic perturbation b_{nm}

We play with the polarizations of one x Raman beam.



linear polarizations $\Rightarrow$ spin hopping algebra J_+



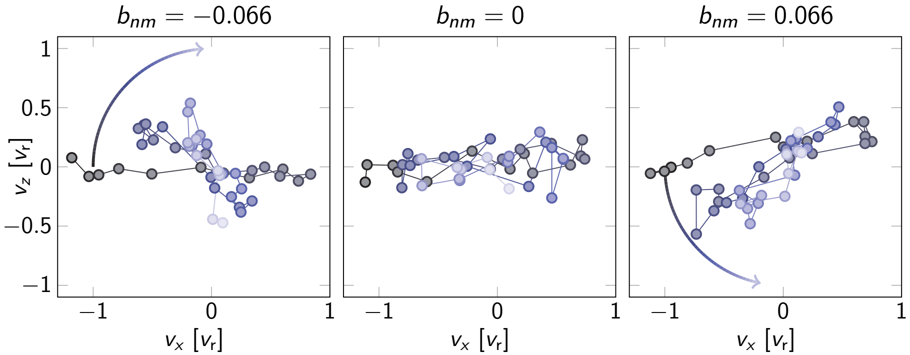
$$J_+ + \mathrm{i}\epsilon\{J_+, J_z\} \simeq J_+ \mathrm{e}^{\mathrm{i}m\epsilon}$$

n-hopping acquires a complex phase $\propto m \Rightarrow b_{nm}$ field

Foucault pendulum precession

We study the modification of cyclotron dynamics induced by the b_{nm} field.

We use $\omega_x = \omega_z$, i.e. isotropic harmonic trapping in the xz plane.



The measured precession rates match well the expected values, governed by the second Chern character ρ_2.

Outlook

Interacting many-body systems in a 4D quantum Hall structure.
Connection with quantum gravity and Yang-Mills theories?

Zhang & Hu, Science 2001
Barns-Graham et al, J. High Energ. Phys. 2018

Requirements:

- characterization of interactions between components of the spin $J = 8$
- control of the interaction range (spatial separation of m levels)

Extension to other high-dimensional topological systems

- Weyl semi-metals in 5D
- Quantum Hall systems in 6D

Lian & Zhang, Phys. Rev. B 2016
Petrides at al, Phys. Rev. B 2018
Lee at al, Phys. Rev. B 2018

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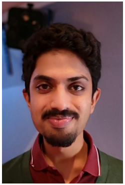

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Thank you for your attention!