Chiral orbital order of ultracold bosons without higher bands

Nathan Goldman

Marco Di Liberto and NG, arXiv:2111.13572 (+ new results)

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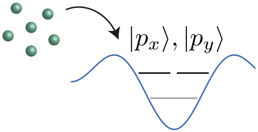

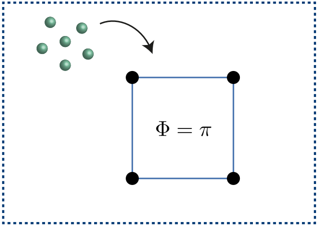

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I. Brief reminder : p band physics and breaking of time-reversal symmetry

II. The pi-flux plaquette building block

III. Collective mode on the plaquette

IV. From the building block to an extended lattice (BBH model)

V. Another extended lattice : towards topological superfluids (in progress)

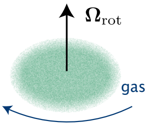
VI. Strong interactions and chiral Mott phases (in progress)

VII. Going beyond 2D (a few thoughts to trigger curiosity ...)

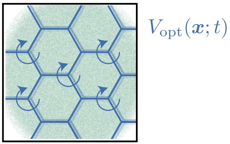
I. Brief reminder : p band physics and breaking of time-reversal symmetry

- **Breaking time-reversal symmetry ... by driving !**

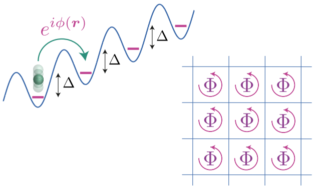
Rotation



Circular shaking



Complex hopping engineering



Review : Aidelsburger, Nascimbene & NG, *Comptes Rendus Phys.* '18

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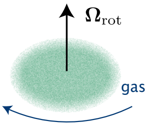

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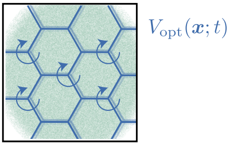

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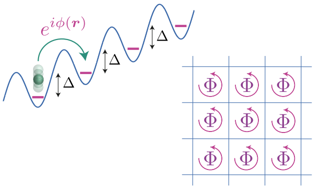

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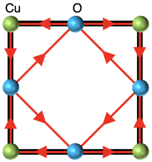
- **Breaking time-reversal symmetry *without driving*?**

$\longrightarrow$ combine **orbital** degrees of freedom and **interactions**

Hidden phase in cuprates
(Varma model ‘97)



from Mielke et al., Nature 2022

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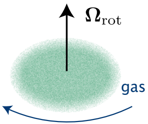

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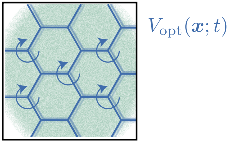

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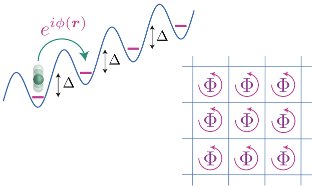

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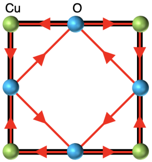
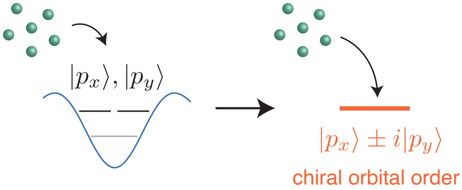


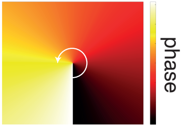
from Mielke et al., Nature 2022

In cold atoms : load into **higher orbital Bloch bands** of an optical lattice

Review : X. Li and W. V. Liu, *Rep. Prog. Phys.* '16

• Loading interacting bosonic atoms into p bands :





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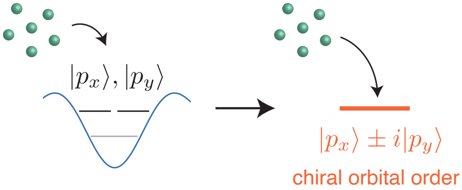

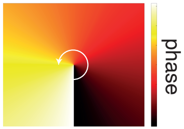

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- **Why ?** The Hubbard (on-site) interactions become

$$\hat{H}_{\text{int}} = \frac{U}{2} \sum_{\boldsymbol{r}} \left(\hat{n}_{\boldsymbol{r}}^2 - \frac{1}{3} \hat{L}_{z,\boldsymbol{r}}^2 \right), \qquad U > 0 \text{ (repulsive)}$$

where $\hat{L}_{z,\boldsymbol{r}} = -i \left(\hat{p}_{x,\boldsymbol{r}}^{\dagger} \hat{p}_{y,\boldsymbol{r}} - \hat{p}_{y,\boldsymbol{r}}^{\dagger} \hat{p}_{x,\boldsymbol{r}} \right)$: **orbital angular momentum**

$\Longrightarrow$ ground state maximizes $|L_z|$ on each site, **breaking TRS** (locally) !

Review : Congjun Wu, *Mod. Phys. Lett. B* '09

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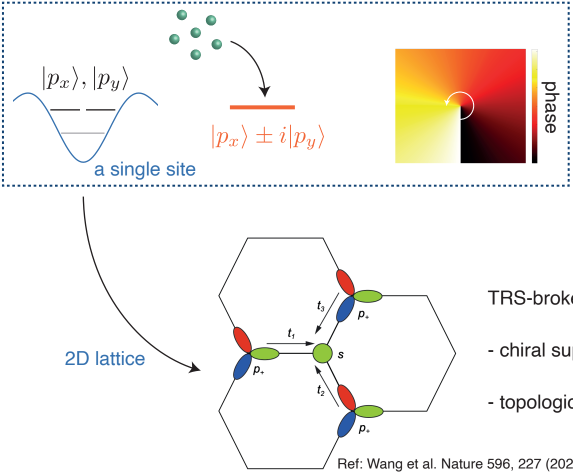


Ref: Wang et al. Nature 596, 227 (2021)

TRS-broken phases (local/global) :

- chiral superfluids, chiral Mott insulators

- topological superfluids

Review : X. Li and W. V. Liu, *Rep. Prog. Phys.* '16

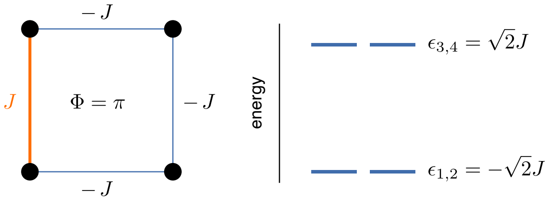
Limitation of this p-band approach :

- Loading into higher Bloch bands $\rightarrow$ Limited lifetime !

Here : a novel route towards **chiral orbital order** that **does not rely on higher bands**

II. The pi-flux plaquette building block

- **Ingredient 1** : a square plaquette with π flux



TRS ✓

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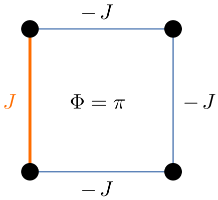

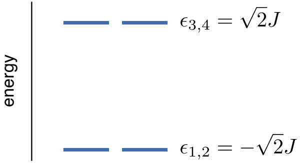

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- **Ingredient 2** : N bosons with Hubbard (on-site) interactions

$$\hat{H}_{\text{int}} = \frac{U}{2}\sum_{i=1}^{4} \hat{n}_i(\hat{n}_i - 1), \qquad U > 0, \quad g \equiv UN \ll J$$

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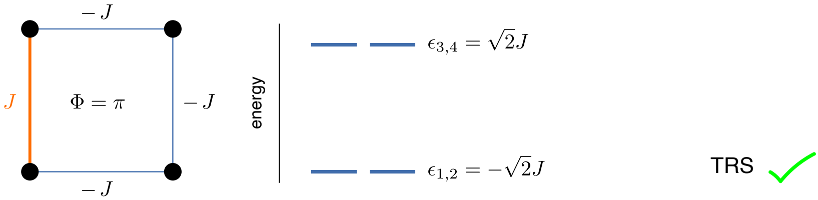

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- **Trick** : Project onto the low-energy orbitals $\{|d_1\rangle, |d_2\rangle\}$

$$\hat{H}_{\text{eff}} = -\left(\sqrt{2}J + \frac{U}{8} + \mu\right)\hat{n} + \frac{3U}{16}\hat{n}^2 - \frac{U}{16}\hat{L}_z^2$$

where $\hat{n} = \hat{d}_1^\dagger \hat{d}_1 + \hat{d}_2^\dagger \hat{d}_2$ is the number operator in the subspace,

and $\hat{L}_z = i(\hat{d}_1^\dagger \hat{d}_2 - \hat{d}_2^\dagger \hat{d}_1)$ is the "orbital angular momentum"

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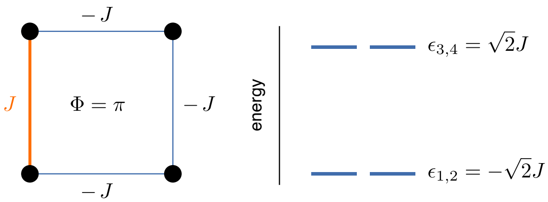

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$\longrightarrow$ reminiscent of interacting bosons in p-bands $\Longrightarrow$ **chiral orbital order** !

- The **ground-state** : We note that $[\hat{H}_{\text{eff}}, \hat{L}_z] = 0$

$\Longrightarrow$ we use eigenstates $\hat{L}_z|\pm\rangle = (\pm 1)\,|\pm\rangle$, where $|\pm\rangle = (|d_1\rangle \pm i|d_2\rangle)/\sqrt{2}$

A generic many-body eigenstate of $\hat{H}_{\text{eff}}$ reads

$$|n_+, n_-\rangle = \frac{1}{\sqrt{n_+!\,n_-!}}(\hat{d}_+^\dagger)^{n_+}(\hat{d}_-^\dagger)^{n_-}|0\rangle, \qquad n_+ + n_- = N$$

with eigenenergy

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$\Longrightarrow$ **two degenerate ground states** ($n_+ = N$ or $n_- = N$)

$$|\psi_{\text{GS}}\rangle_+ \sim \left(\hat{d}_+^\dagger\right)^N |0\rangle, \qquad |\psi_{\text{GS}}\rangle_- \sim \left(\hat{d}_-^\dagger\right)^N |0\rangle$$

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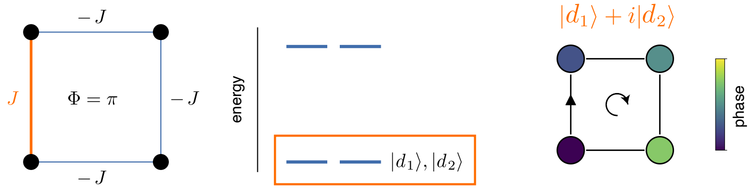
$\Longrightarrow$ **spontaneous TRS breaking**

- **In real space** : the complex orbitals show a **vortex** structure **on the plaquette**



- The two degenerate many-body ground-states :

$$|\psi_{\mathrm{GS}}\rangle_{\pm} \sim \left(\hat{d}_1^\dagger \pm i\hat{d}_2^\dagger\right)^N |0\rangle \quad \Longrightarrow \textbf{chiral orbital order}$$

$\Longrightarrow$ two degenerate solutions with opposite superfluid currents

III. Collective mode on the plaquette

- A generic many-body eigenstate of $\hat{H}_{\text{eff}}$ reads

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- Take one ground state $|\psi_{\text{GS}}\rangle_+$ and move one particle from $|+\rangle \longrightarrow |-\rangle$

$$\text{Energy cost : } E_{\text{exc}} = \frac{UN}{4} - \frac{U}{4}$$

$$= \frac{g}{4} \text{ in the mean-field limit } (N \to \infty, g = UN \ll J)$$

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- More insight from a hydrodynamic approach ?

- We introduce two mean-field variables ($g \equiv UN \ll J$) :

$$\langle \hat{d}_1 \rangle = \sqrt{\rho_1}\,, \quad \langle \hat{d}_2 \rangle = e^{i\theta}\sqrt{\rho_2}\,, \qquad \rho_1 + \rho_2 = N$$

- **Ground state** corresponds to $\rho_{1,2} = N/2$ and relative phase $\theta = \pm\pi/2$

- **Collective mode** : We study the dynamics of fluctuations

$$\rho_1 = N/2 + \delta\rho, \quad \rho_2 = N/2 - \delta\rho, \quad \theta = \pi/2 + \delta\theta \qquad (\delta\rho, \delta\theta \text{ small})$$

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Solving the equations of motion for the variables ($\delta\rho, \delta\theta$) yield the proper mode :

$$\delta\theta = \mathcal{A}\cos\omega_0 t\,, \quad \delta\rho = -(\mathcal{A}N/2)\sin\omega_0 t\,, \quad \text{with frequency } \omega_0 = g/4 = UN/4$$

$\Longrightarrow$ oscillation of relative phase and population with $\pi/2$ phase difference

$\Longrightarrow$ a **gapped collective mode** above the ground state

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- **Exciting the mode ?**

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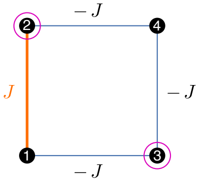
$$\hat{V}(t) = \frac{f(t)}{2}(\hat{d}_1^\dagger \hat{d}_1 - \hat{d}_2^\dagger \hat{d}_2) \approx f(t)(\hat{b}_3^\dagger \hat{b}_3 - \hat{b}_2^\dagger \hat{b}_2)$$



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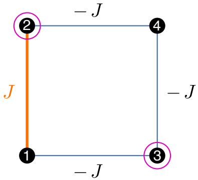

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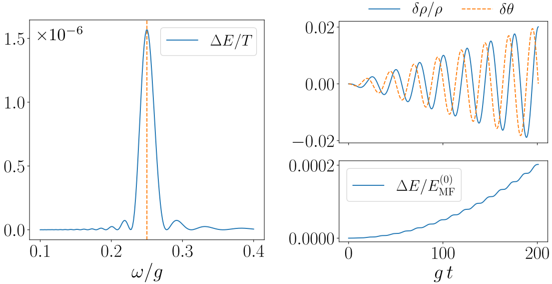
- **Numerical analysis** within linear response ($V_0 = 10^{-4} g$)



- **Real space picture** :

  Exciting the gapped mode corresponds to **injecting angular momentum**

$$\hat{H}_{\text{eff}} \sim \hat{L}_z^2 \Longrightarrow \delta E \sim 2L_z \delta L_z$$

$\longrightarrow$ activating the mode leads to **chiral current on the plaquette**

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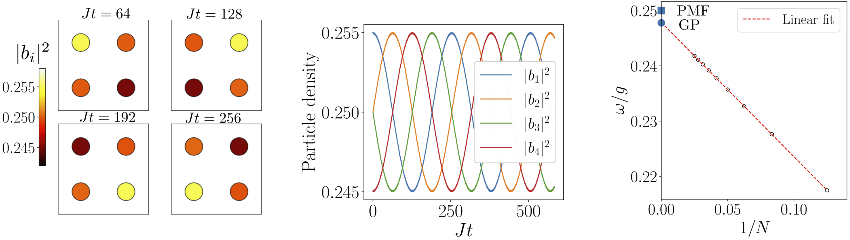
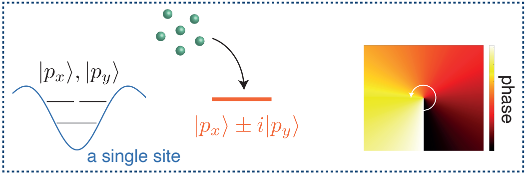
- **Illustration** : We quench a small impurity potential $\hat{H}_{\text{imp}} = -\Delta \hat{b}_1^\dagger b_1 \to \delta\theta \neq 0$



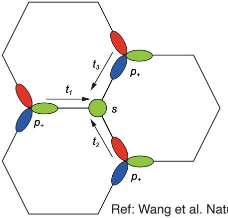
$\longrightarrow$ **chiral motion on the plaquette** of frequency $\omega \approx \omega_0 = g/4$

IV. From the building block to an extended lattice

- **P-band physics** : Assembling vortices on a lattice



2D lattice



Ref: Wang et al. Nature 596, 227 (2021)

TRS-broken phases (local/global) :

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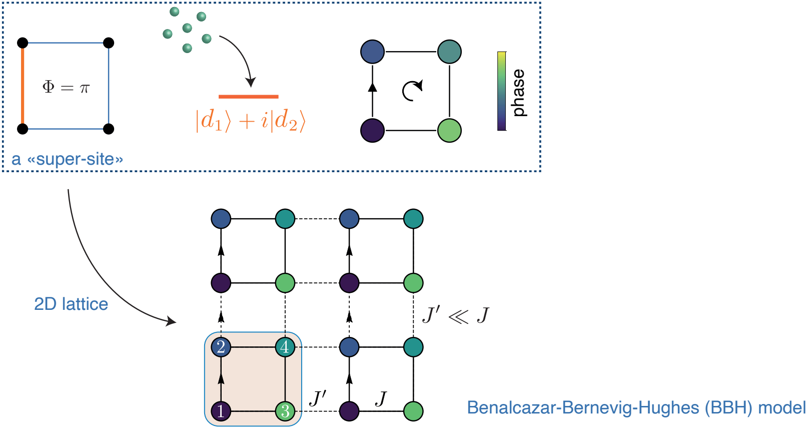
$\Longrightarrow$ Let us build a lattice using our **building block** (pi-flux plaquette) . . .

- **Building the extended lattice :** the BBH model with interacting bosons



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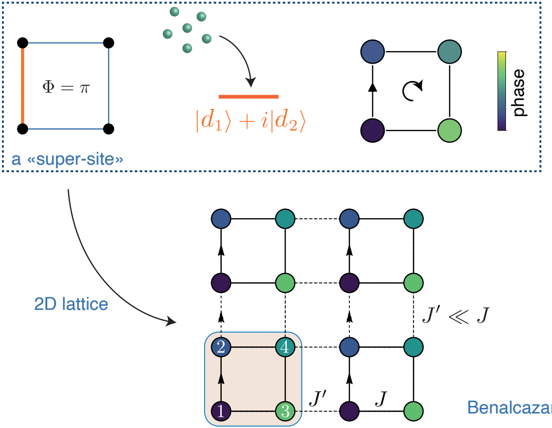
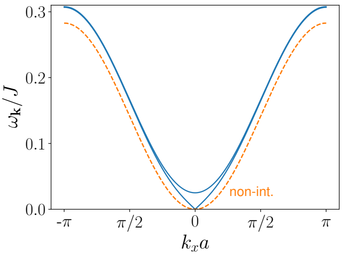


Benalcazar-Bernevig-Hughes (BBH) model

The ground state is a uniform condensate (Γ point) forming a **superfluid vortex lattice**

$\Longrightarrow$ chiral superfluid

- **The excitation spectrum** is obtained within Bogoliubov theory



- **Goldstone mode** : $\omega_{1,\mathbf{k}} \approx c_s |\mathbf{k}|$, with sound velocity $c_s \sim \sqrt{J' g}$

- **Massive mode** : $\omega_{2,\mathbf{k}} = g/4 + \xi_{\mathbf{k}} \longrightarrow$ gap $= \omega_0$ (chiral mode on plaquette)

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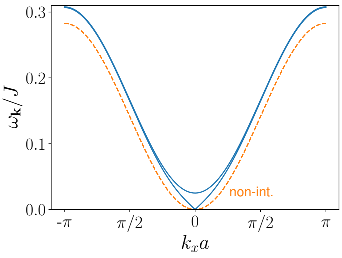

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- When $g \ll J'$: Decay channel $|\beta_{2,\mathbf{k}=\mathbf{0}}\rangle \longrightarrow |\beta_{1,\mathbf{k}}, \beta_{1,-\mathbf{k}}\rangle$

- The decay rate Γ is estimated beyond the Bogoliubov approximation

Long-lived gapped mode : $\Gamma < \omega_0 = g/4 \Longrightarrow U \lesssim 18\, J'$

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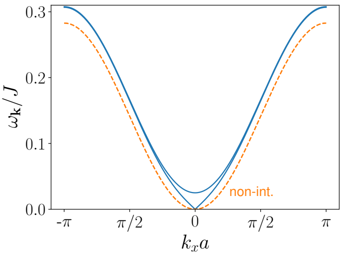

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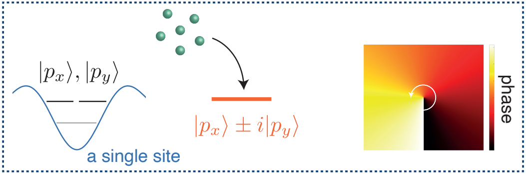
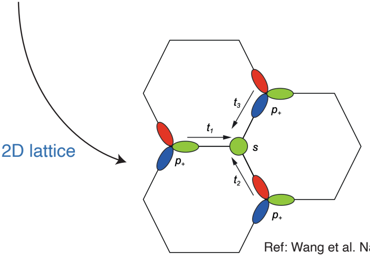
- **Open question** : Transfer of **topology** from the BBH band to Bogoliubov excit. ?

V. Another extended lattice : towards topological matter (in progress)

- **P-band physics and topological superfluids** :





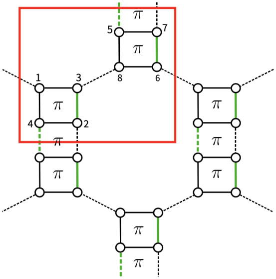
TRS-broken phases (**global**)

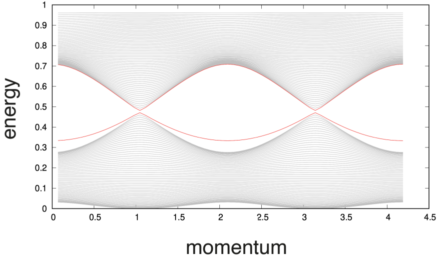
topological superfluids (chiral edge modes)

Ref: Wang et al. Nature 596, 227 (2021)

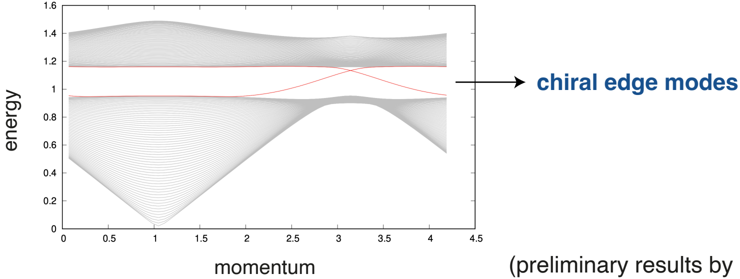
$\Longrightarrow$ "Similar" lattices using our **building block** ?

- Lattice and single-particle spectrum :





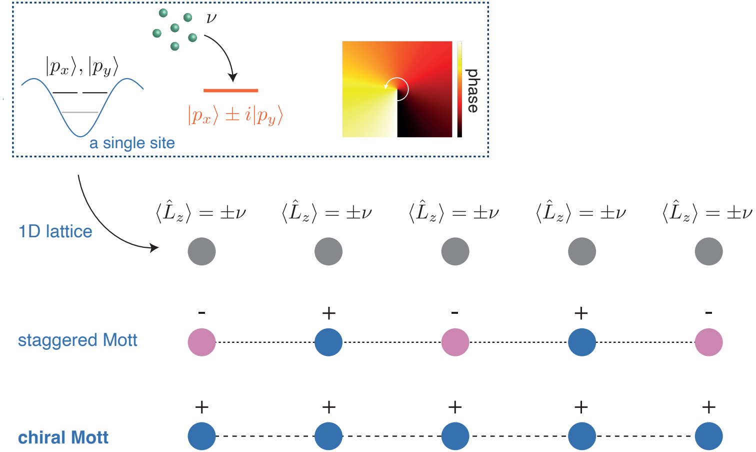
- **Bogoliubov spectrum**



(preliminary results by BoYe Sun)

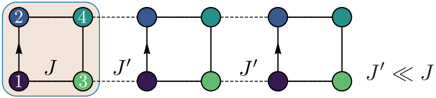
VI. Strong interactions and chiral Mott phases (in progress)

- **Mott phases with orbital order in p bands** (for integer filling $\nu > 1$)



Review : X. Li and W. V. Liu, Rep. Prog. Phys. '16

- Consider a ladder of pi-flux “super-sites” connected by **weak links**



- GS in mean-field limit : superfluid vortex lattice (**global TRS-breaking**)

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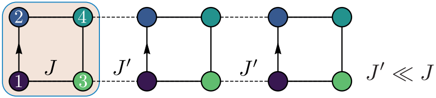

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- We now consider a filling factor $\nu = 1/2$ (i.e. 2 bosons per plaquette)

Note : equivalent to $\nu = 2$ in p-bands (within the effective two-orbital model !)

$\Longrightarrow$ we explore a “chiral superfluid to chiral Mott” transition by varying Hubbard U

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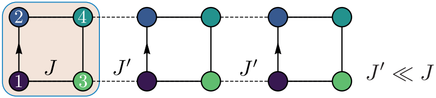

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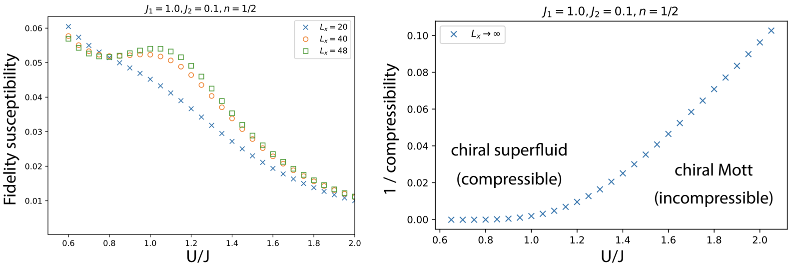
$\Longrightarrow$ we explore a "chiral superfluid to chiral Mott" transition by varying Hubbard U

- What do we expect ?

  - $U/J \ll 1$: chiral superfluid (vortex lattice)
  - $U/J \gg 1$: normal superfluid (breakdown of effective two-orbital model) ?
  - $U/J \sim 1$: chiral Mott ... ?

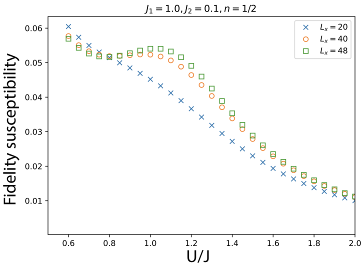
- We explore a "chiral superfluid to chiral Mott" transition by varying Hubbard U

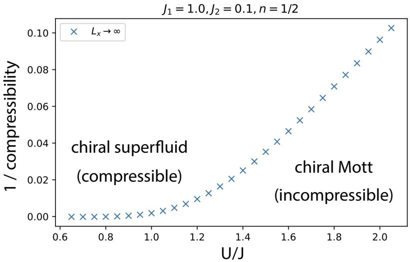


- Transition takes place around $U \approx J$

two-orbital effective model still valid $\Longrightarrow$ **chiral** Mott phase (to be confirmed)

- We explore a "chiral superfluid to chiral Mott" transition by varying Hubbard U





- Transition takes place around $U \approx J$

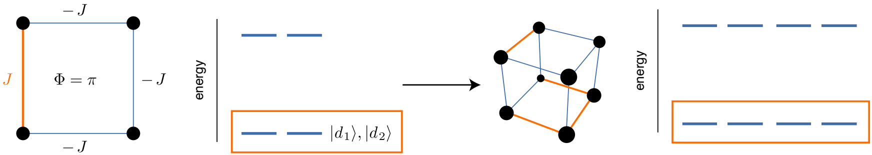
two-orbital effective model still valid $\Longrightarrow$ **chiral** Mott phase (to be confirmed)

- Transition to the normal superfluid at larger U ? (to be confirmed)

Marco Di Liberto and NG, arXiv :2111.13572, to be updated soon ...

VII. Going beyond 2D (a few thoughts to trigger curiosity ...)

- **Outlook** : Higher-dimensional building blocks with $d > 2$ degenerate orbitals

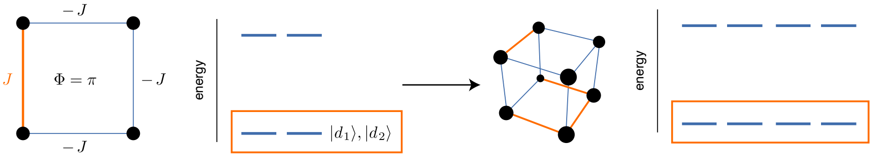


- Degenerate $|\psi_{\text{GS}}\rangle$'s for N interacting bosons

$\Longrightarrow$ Generalized angular momentum operator ? Symmetry breaking ?

$\Longrightarrow$ Interaction-induced chirality / TRS-breaking ? Nature of collective modes ?

- **Outlook** : Higher-dimensional building blocks with $d > 2$ degenerate orbitals



- Degenerate $|\psi_{\text{GS}}\rangle$'s for N interacting bosons

$\Longrightarrow$ Generalized angular momentum operator ? Symmetry breaking ?

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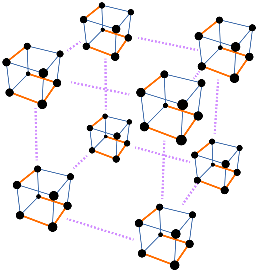
- **Outlook** : Build an extended lattice from 3D building blocks :



Ex: 3D Benalcazar-Bernevig-Hughes (BBH) model

Bogoliubov modes ? Topology ?

Exotic Mott phases and superfluids ?

Other extended models of interest ?

Today on arXiv !

Realization of a fractional quantum Hall state with ultracold atoms

Julian Léonard,[1,*] Sooshin Kim,[1] Joyce Kwan,[1] Perrin Segura,[1]
Fabian Grusdt,[2,3] Cécile Repellin,[4] Nathan Goldman,[5] and Markus Greiner[1]

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[2] *Department of Physics and ASC, LMU München, Theresienstr. 37, München D-80333, Germany*
[3] *Munich Center for Quantum Science and Technology (MCQST), Schellingstr. 4, D-80799 München, Germany*
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[5] *CENOLI, Université Libre de Bruxelles, CP 231, Campus Plaine, B-1050 Brussels, Belgium*

Strongly interacting topological matter [1] exhibits fundamentally new phenomena with potential applications in quantum information technology [2, 3]. Emblematic instances are fractional quantum Hall states [4], where the interplay of magnetic fields and strong interactions gives rise to fractionally charged quasi-particles, long-ranged entanglement, and anyonic exchange statistics. Progress in engineering synthetic magnetic fields [5–21] has raised the hope to create these exotic states in controlled quantum systems. However, except for a recent Laughlin state of light [22], preparing fractional quantum Hall states in engineered systems remains elusive. Here, we realize a fractional quantum Hall (FQH) state with ultracold atoms in an optical lattice. The state is a lattice version of a bosonic $\nu = 1/2$ Laughlin state [4, 23] with two particles on sixteen sites. This minimal system already captures many hallmark features of Laughlin-type FQH states [24–28]: we observe a suppression of two-body interactions, we find a distinctive vortex structure in the density correlations, and we measure a fractional Hall conductivity of $\sigma_H/\sigma_0 = 0.6(2)$ via the bulk response to a magnetic perturbation. Furthermore, by tuning the system parameters, we observe a transition from the Laughlin state to a normal state, where the particles are well localized. Exploring such states in the bulk of large systems reveals the fractionally charged quasi-particles, long-ranged entanglement, and anyonic exchange statistics [4].

The desire to study these phenomena in a controlled environment has triggered effort to realize FQH states in quantum-engineered systems. Since the constituents of those platforms are typically charge neutral, synthetic magnetic fields are introduced through the Coriolis force in rotating systems [5–8, 20, 35], or by engineering ge-

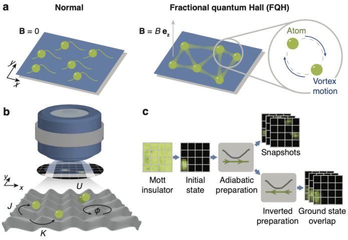

FIG. 1. **Realizing a fractional quantum Hall state in an optical lattice.** **a,** Without magnetic field, a two-