Séminaire Poincaré 1 (2002) 67 – 78

Séminaire Poincaré

Measurement of the Atom-Wall Interaction: from London to Casimir-Polder

Alain ASPECT[1] and Jean DALIBARD[2]
[1] Laboratoire Charles Fabry de l'Institut d'Optique*,
B.P. 147
91403 Orsay Cedex, France
[2] Laboratoire Kastler Brossel†
24, rue Lhomond
75005 Paris, France

Abstract. We first present the Casimir–Polder result, giving the interaction potential between a ground state atom and a mirror. This result, obtained within the framework of quantum electrodynamics, is valid for any separation z between the atom and the mirror, provided the electronic cloud does not overlap with the mirror. For large z, this interaction potential varies as $U_{\rm CP}(z) \propto z^{-4}$. This results from the modification of vacuum fluctuations by the mirror and this is quite different from the simple electrostatic result obtained by neglecting any retardation effect, $U(z) \propto z^{-3}$. We also indicate how the Casimir–Polder potential is modified when the mirror is replaced by a dielectric (Lifshitz theory). We then describe three recent experiments which give a clear evidence for the existence of retardation terms in the atom-wall problem, and which are in good agreement with the Casimir-Polder prediction.

1 Introduction

The fact that the electromagnetic vacuum can interact with atomic particles and produce a measurable effect is certainly one of the most striking features of Quantum Mechanics. The name of the Dutch physicist H.B.G. Casimir is attached to some very spectacular manifestations of this interaction. In 1948 he predicted his famous result concerning the attractive force between two perfectly conducting plates [1]. The review of the current experimental state-of-the art for this problem will be done in the next presentation by Reynaud. The same year, Casimir made another essential contribution, together with his colleague Polder [2]. They addressed the following problem: what is the asymptotic behavior of the long range interaction between two atoms, or between an atom and its mirror image?

The existence of long range forces, acting when the constituents are separated by more than a typical atomic size, was predicted by van der Waals in 1881. The first quantitative estimate of these forces was performed by London [3], using an analysis based on classical electrodynamics. The question raised by Casimir and Polder, and that we would like to address here, is the existence of sizeable effects, originating from the quantization of the electromagnetic field, in the long range interaction between an atom prepared in its ground electronic state and a mirror.

*Unité mixte de recherche du CNRS.
†Unité de recherche de l'Ecole normale supérieure et de l'Université Pierre et Marie Curie, associée au CNRS.

2 The Casimir-Polder problem

2.1 The (relatively) short range result

When a static electric dipole $\boldsymbol{d}$ is placed in front of an ideally conducting wall, it interacts with its mirror image and the corresponding energy is

$$U(z) = -\frac{d_x^2 + d_y^2 + 2d_z^2}{64\pi\epsilon_0 z^3} \tag{1}$$

The $-$ sign means that the corresponding interaction is attractive. Here Oz denotes the axis normal to the plane and z is the distance between the atom and the plane. Consider now an atom in its internal ground state $|0\rangle$, placed in front of such a wall. A similar effect may occur, as long as the distance z is notably larger than the atom size, to avoid the overlap between the electron cloud of the atom and the wall itself. The reason for this attraction is clear: although the atom possesses no electric dipole moment in its ground state ($\langle 0|d_i|0\rangle = 0$, for $i = x, y, z$), the average values $\langle d_i^2\rangle$ are strictly positive. A simple picture then emerges in which a fluctuating dipole is associated to the atom, which polarizes the conducting charges of the wall; the induced charge distribution then interacts with the initial atomic dipole. This effect, predicted by Lennard-Jones in 1932 [4], leads to the following interaction potential:

$$U_{\rm LJ}(z) = -\frac{\langle 0|d_x^2 + d_y^2 + 2d_z^2|0\rangle}{64\pi\epsilon_0 z^3} \tag{2}$$

The passage from a truly static to a time-dependent atomic dipole introduces a time scale τ in the problem and hence, due to the finite speed of light, a length scale $c\tau$. The z^{-3} dependence of the electric field created by the dipole is valid only at distances smaller than $c\tau$. For larger distances, a new approach is needed to account for *retardation effects*, as pointed out in 1941 by J. A. Wheeler [5]. For instance, if one deals with a classical oscillating dipole $\boldsymbol{d}e^{-i\omega t}$, it is well known that the electromagnetic field which is radiated at long distances varies like the inverse of the distance to the dipole. The leading term in the interaction energy between the oscillating dipole and the conducting wall is then [6]:

$$\text{Large } z: \qquad U(z) \sim \frac{(d_x^2 + d_y^2)k^2}{32\pi\epsilon_0 z}\ \cos 2kz\ , \tag{3}$$

where we set $k = \omega/c$.

For an atom or a molecule, several questions now emerge. What is the relevant time scale τ? Is the picture of an oscillating dipole valid? Does the physics depend on the internal level (ground or excited) of the atom?

2.2 Retardation effects in the atom-wall problem

2.2.1 Atom in its ground electronic state

The problem for a ground state atom was solved in 1948 in a brilliant manner by Casimir and Polder, using the formalism of quantum electrodynamics [2, 7]. Their results show that the retardation effect anticipated above is indeed essential and that it leads to the replacement of the Lennard-Jones z^{-3} variation of the interaction energy by a z^{-4} variation. The length scale on which the transition between the z^{-3} and the z^{-4} regimes occurs is c/ω, where ω is a typical Bohr frequency of the atom. We shall not give the exact result of Casimir and Polder (denoted hereafter as $U_{\rm CP}(z)$). We simply recall that it is valid for any distance z, and that it coincides with $U_{\rm LJ}(z)$

for short distances. We now comment on its asymptotic form for large z. In this case, considering an atom with a single valence electron prepared in its ground state $|0\rangle$, one obtains:

$$\text{Large } z: \qquad U_{\rm CP}(z) \sim -\frac{3}{32\pi^2\epsilon_0}\frac{\hbar c\alpha}{z^4} \ . \tag{4}$$

Here α denotes the static polarisability of the atom in the state $|0\rangle$, of energy E_0:

$$\alpha = \frac{2q^2}{3}\sum_{n\neq 0}\frac{|\langle n|\hat{\boldsymbol{r}}_{\rm e}|0\rangle|^2}{E_n - E_0} \tag{5}$$

where the sum runs over all the atomic excited states n of energy E_n, and where $\hat{\boldsymbol{r}}_{\rm e}$ is the position operator of the electron with respect to the atom center-of-mass. Note that for hydrogen and for alkali atoms, the largest contribution to the sum (5) comes from the resonance line (1s$\leftrightarrow$2p and $ns\leftrightarrow$ np respectively).

The question now arises to interpret this result in terms either of vacuum fluctuations (modification of the atomic electron dynamics by the quantized electromagnetic field) or radiation reaction (action of the field radiated by the atom upon itself). Such a separation is possible in an unambiguous manner when one expresses the measurable physical quantities in terms of the correlation functions and linear susceptibilities of the two interacting systems, the atom and the electromagnetic field [8]. Using this formalism, the authors of [9] have shown that the result (4) is entirely due to vacuum fluctuations.

Actually one can recover (4) within a numerical factor by the following simple reasoning [10] (see also [11]). The physical origin of (4) is similar to that of the Casimir d^{-4} force between two perfectly conducting walls separated by a distance d (for a review, see the contributions of Balian and Duplantier, and of Reynaud in the same issue). At a distance z from the mirror, the modes of the electromagnetic field which are strongly modified by the presence of the conducting wall are those with a frequency ω such that $\omega \leq c/z$. The electric field associated with each mode is $\mathcal{E}_\omega = \left(\hbar\omega/2\epsilon_0 L^3\right)^{1/2}$, where L^3 is an arbitrarily large quantization volume. The contribution of each mode to the Lamb shift of the ground state of the atom is $-\alpha\mathcal{E}_\omega^2/2$. Here we use the static polarizability α; indeed we assume that the atom is far enough from the wall so that all the considered modes have a frequency ω much lower than any atomic Bohr frequency. This crude estimation of the modification of the Lamb shift of the atomic ground state, due to the presence of the wall, then gives

$$U(z) \simeq -\sum_{\omega<c/z} 2\times\alpha\mathcal{E}_\omega^2/2 = -\frac{\alpha\hbar}{4\pi^2\epsilon_0 c^3}\int_0^{c/z}\omega^3\, d\omega = -\frac{1}{16\pi^2\epsilon_0}\frac{\hbar c\alpha}{z^4} \ ,$$

where the multiplicative factor 2 accounts for the two polarizations basis states for a given wave vector. This is a remarkably good approximation of the exact asymptotic result (4).

2.2.2 Atom in an excited state

The result that we just obtained for a ground state atom is very different from the one obtained for an atom prepared in an excited electronic state $|n\rangle$. In this case, one can show indeed that the leading term is [9, 10]:

$$\text{Large } z: \qquad U(z) \sim \frac{q^2}{8\pi\epsilon_0 z}\sum_{n'<n} k_{nn'}^2 \ \left(|\langle n|\hat{x}_{\rm e}|n'\rangle|^2 + |\langle n|\hat{y}_{\rm e}|n'\rangle|^2\right) \ \cos(2k_{nn'}z) \tag{6}$$

where we have set $k_{nn'} = (E_n - E_{n'})/(\hbar c)$ and where the sum over n' runs only on levels with an energy $E_{n'}$ lower than E_n. Here we recover the $\cos(2kz)/z$ behavior characteristic of a classical oscillating dipole (3). As shown in [9], vacuum fluctuations and radiation reaction contribute equally to this result.

2.3 The Lifshitz approach

A few years after the work of Casimir and Polder, Lifshitz also addressed the problem of long range interactions between atomic particles and a macroscopic body [12]. He did not consider a metallic surface, but a bulk dielectric material characterized by a linear susceptibility $\epsilon(\omega)$. We shall not review Lifshitz theory in detail, and we simply give the long range potential for an atom in its ground electronic state, assuming that one electronic transition at frequency ω_A is dominating (for a review, see e.g. [13, 14, 15]):

$$\text{Large } z: \qquad U(z) \sim -\frac{3}{32\pi^2\epsilon_0}\frac{\hbar c\alpha}{z^4}\,\frac{\epsilon(\omega_A)-1}{\epsilon(\omega_A)+1}\,\Phi(\epsilon)\ . \tag{7}$$

The function $\Phi(\epsilon)$ is nearly constant and equal to 0.77 when the index of refraction $n = \sqrt{\epsilon}$ varies between 1 and 2, which accounts for most glasses. Note that one recovers the case of a perfectly conducting plate by taking the limit $\epsilon \to \infty$, in which case $\Phi(\epsilon) \to 1$.

We conclude this brief section on Lifshitz theory by noting that the use of a dielectric opens new perspectives with respect to a perfectly conducting wall. One can arrange the response function $\epsilon(\omega)$ of the dielectric to be resonant with some particular Bohr frequencies of the atoms. It is then possible to enhance or decrease the contribution of some atomic transitions to the interaction potential. For example, one can modify the coefficient appearing in front of the short range z^{-3} variation (2). It is even possible to change the sign of the interaction energy if the atom is prepared in an excited state, so that the Lennard-Jones attractive potential is turned into a repulsive one [16, 17].

3 Experimental results

3.1 A brief review of the experimental status

The main motivation of this presentation is to discuss the experimental tests of the Casimir-Polder-Lifshitz prediction, i.e. the long range z^{-4} interaction energy for an atom in front of a conducting wall or a dielectric material. We shall not address here the results obtained recently in cavity quantum electrodynamics, where the atom is surrounded by a cavity with a large quality factor, so that it couples resonantly to only one (or a few) of the cavity modes. We refer the reader to [6, 10], where these experiments are discussed in detail. We shall not discuss either the possible manifestations of long range forces inside an atom. These can occur for instance within a Rydberg helium atom, for which the outer electron sees a field which can be significantly different from the Coulomb field from the nucleus+inner electron. We refer the reader to [18], which present several interesting contributions on this topic.

Before addressing the Casimir-Polder z^{-4} prediction, we shall say a few words on experimental studies in the short range regime, where the Lennard-Jones z^{-3} variation dominates. This regime has first been studied in [19], for an atom or a molecule in front of a conducting material. The idea is to send an atomic or molecular beam very close to a metal cylinder and to look for the deflection of the beam. A deviation is actually detected, but it is difficult to extract quantitative conclusions from these experiments, the main reason being that the impact parameters are uniformly distributed over all possible values. The effect of the atom-wall attraction on the deflected beam is then strongly dominated by the atoms having the smallest impact parameter, where retardation effects

play no role, and only the Lennard-Jones potential can be tested. Similar experiments are reported in [20, 21]. Note that the results of these experiments were only in qualitative agreement with the theoretical prediction (2). One can also prepare the atoms in a highly excited Rydberg state, so that the corresponding dipole is much larger, which allows for a more precise study of the Lennard-Jones prediction [22].

The Lifshitz prediction has been tested using liquid-helium films on cleaved surfaces of alkaline-earth fluoride crystals [23]. By varying the thickness of the film between 1 and 25 nm, the authors could obtain a test of Lifschitz theory over 5 orders of magnitude for the potential strength. These experiments showed a first evidence for the deviation from the z^{-3} law at long distances (i.e. thick films).

High resolution spectroscopy experiments can also reveal a position-dependent frequency shift of the atomic energy levels in the relatively short range (z^{-3}) regime. These methods have been used to test the Lennard-Jones prediction for excited atoms [24, 25, 26], and the atom-wall repulsion resulting from a well chosen dielectric response of the wall has been observed [27].

We now turn to three experiments where the Casimir-Polder retardation effect for an atom in its ground state has been observed and studied. We note that this observation cannot be performed using a spectroscopic measurement. Indeed one measures in this case an energy difference between the ground state and an excited state. Since the shift for any excited state (6) is much larger than the shift of the ground state (4), one would only access in this way to the excited level physics, and not to the ground state one. Clearly, one has to rely on a measurement dealing only with the ground state to test this prediction. This leaves several possibilities opened, as pointed out in [6]. One could use atomic interferometry to measure the shift [28, 29, 30]. One could also measure a differential shift between various sublevels of the ground state, in case the non-scalar part of the static susceptibility is significant. Finally, as done in the three experiments described below, one can look for a mechanical effect of the Casimir-Polder potential [31, 33, 41].

3.2 Atom metal force : the Yale experiment [31]

This remarkable experiment constitutes to our knowledge the first quantitative study of retardation effects in the interaction between an isolated ground state atom and a conducting wall. This experiment is precise enough to clearly discriminate between the Casimir-Polder value of the interaction energy and the Lennard-Jones result, in which the interaction is modelled by the instantaneous electrostatic interaction between the atomic dipole and its image in the metal. Figure 1 is a sketch of the experiment. A beam of sodium atoms travels inside a cavity formed by two almost parallel, gold coated, plates. The distance L between the plates can be varied between 0.7 μm and 8 μm. The length of the cavity is $D = 8$ mm.

The experiment consists in measuring the transmission T (or rather the opacity $1/T$) as a function of the separation L. For $L > 3$ μm, the transmission is found equal (within error bars) to the "geometrical" expectation. This geometrical expectation is determined using a Monte-Carlo simulation, in which one neglects any interaction between the atoms and the walls of the cavity. The straight (classical) atomic trajectories are determined by the initial Maxwell-Boltzmann distribution, and only atoms that do not hit the walls are transmitted.

For smaller separations L, the contribution of the atom-wall interaction to the opacity becomes appreciable, and the measured transmission is smaller than the geometrical one (see figure 2). This reduction can be easily understood if one remembers that the atom-wall interaction is attractive, both for short and long distances. When an atom comes close enough to one of the walls, its trajectory is bent towards this wall. Therefore the number of atoms hitting the walls is larger than what is given by the geometrical analysis, and the effective aperture of the cavity is thus smaller than the geometrical aperture.

To obtain an order of magnitude of the critical wall spacing L_c for which the losses due to the

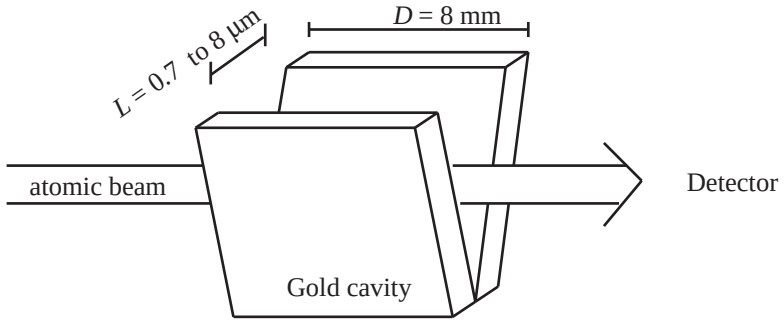

Figure 1: *Yale experiment: an atomic beam is sent through a cavity made of two gold coated plates making a small wedge. The number of transmitted atoms is measured as a function of the distance L between the plates.*

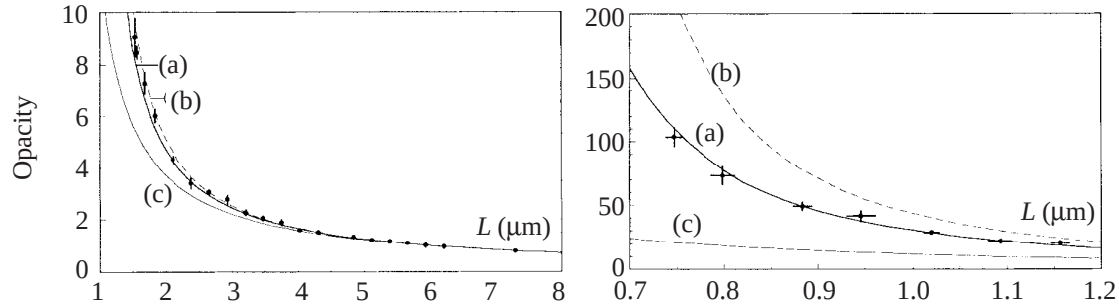

Figure 2: *Inverse of the measured transmission (opacity) as a function of the plate separation L. Curves (a), (b), and (c) result from a Monte-Carlo calculation, assuming various atom-cavity interaction potentials : (a) Casimir Polder interaction (exact); (b) Lennard Jones interaction (no retardation); (c) no interaction (geometrical model).*

atom–wall interaction become significant, we can compare the maximal transverse kinetic energy $E_{K\perp} \sim k_B T\,(L/2D)^2$ of an atom transmitted by the cavity (within the geometrical analysis), and the atom-wall interaction energy $U_{\rm cav}(L/2)$ for an atom located at the center of the cavity. For simplicity, we evaluate $U_{\rm cav}(L/2)$ using the short distance approximation (2). We notice that the value of $\langle d_i^2 \rangle$ essentially results from the contribution of the sodium resonance line 3s↔3p at $\lambda_{\rm res} = 589$ nm, so that (2) can be written $U_{LJ}(z) = (3/16)\,\hbar\Gamma/(kz)^3$, where Γ is the radiative lifetime of the 3p level and $k = 2\pi/\lambda_{\rm res}$ [10]. A back-of-the-envelope calculation then yields $L_{\rm c} \sim 1$ μm, in good agreement with the observed value of the separation below which the measured transmission becomes significantly smaller than the geometrical one.

These experimental results constitute more than a mere evidence of the dramatic role of the atom-wall interaction at a distance $z \sim \lambda_{\rm res}$. They allow a precise comparison with the exact Casimir-Polder result [32] and they clearly rule out a model which would simply extend the Lennard-Jones prediction (2) to any distance. Note that for the relevant atom-wall distances in this experiment ($z \sim \lambda_{\rm res}$), the Casimir-Polder result significantly differs from the simple asymptotic form (4). One must use the exact Casimir-Polder potential $U_{\rm CP}(z)$, which connects the short and long distance asymptotic forms. Now, if one fits the experimental data using the potential $\xi\, U_{\rm CP}(z)$, where ξ is an adjustable parameter, one finds $\xi = 1$, within an uncertainty factor of 10% (at 1 standard deviation). To our knowledge, this is the most precise measurement of the interaction

energy of a ground state atom and a metal wall at a distance sensitive to the retardation effects.

A close look at the data of fig. 2 reveals an a priori paradoxical fact. The discrimination between the theoretical expressions with and without retardation effects is more dramatic for the smallest values of the cavity width. In fact, only the atoms travelling close to the center of the cavity are transmitted, and it is only for these atoms that the precise form of the interaction energy is important. Now, even for the smallest value $L = 0.7$ μm used in this experiment, the atom wall distance (0.35 μm) is not small compared to the wavelength $\lambda_{\rm res}$ of the dominant transition. It is therefore not surprising that the retardation effects play a significant role in this case. The fact that the relevant atoms are travelling at the center of the cavity is important in another respect. For these atoms, the wavelengths of the modes of the electromagnetic field which are affected by the walls are larger than L, that is 0.7 μm. At these wavelengths, gold behaves as an almost perfect conductor. It would not be so for shorter wavelengths, i.e. for smaller atom-wall distances.

3.3 Atom dielectric force : the Orsay experiment [33]

A key ingredient for the success of the experiment above is the fact that, for a small plate separation, the detected atoms are at a well defined distance from the attracting plates, since they travel close to the center of the cavity (atoms departing from this symmetry plane are attracted and stick to the plates). With a well defined impact parameter, it is possible to test the interaction energy law with a good accuracy.

With the advent of methods for laser cooling and manipulating atoms [34, 35, 36], it has become possible to accurately control atomic trajectories, and this offers new possibilities to define precisely the impact parameter. As suggested in [37], atomic mirrors allow to control the distance of minimum approach to a dielectric wall, and to measure the interaction energy. Figure 3 sketches an experiment recently performed in Orsay in this purpose. Laser cooled and trapped rubidium (^{87}Rb) atoms, at a temperature of about 10 μK (i.e. a *r.m.s.* velocity of 4 cm/s), are released on an atomic mirror located 15 mm below. The incident atoms, with a kinetic energy dispersion less than 1%, are reflected from the quasi resonant evanescent wave resulting from the total internal reflection of a laser beam in the prism. The reflecting potential is due to the interaction between the evanescent wave electric field and the atomic electric dipole induced by this field. This *dipole potential* is proportional to the square of the electric field (intensity) in the evanescent wave, and therefore decays exponentially as a function of the distance to the surface [38]:

$$U_{\rm dip}(z) = \frac{\hbar\Gamma}{8}\,\frac{I}{I_{\rm sat}}\,\frac{\Gamma}{\delta}\,e^{-2\kappa z} = \hbar\Lambda\,e^{-2\kappa z}\ , \tag{8}$$

where I is the light intensity at the surface of the prism, and $\delta = \omega_L - \omega_A$ is the detuning between the laser frequency ω_L and the atomic resonance frequency ω_A ($\lambda_{\rm res} = 2\pi c/\omega_A = 780$ nm). The quantity $I_{\rm sat} = 16$ W/m^2 is the saturation intensity of the atomic transition and $\Gamma = 3.7 \times 10^7$ s^{-1} is the radiative width of the relevant excited state. The decay length κ^{-1} is of the order of $\lambda_{\rm res}/2\pi$, the exact value depending on the laser direction (in this experiment, $\kappa^{-1} = 114$ nm).

The reflecting potential is repulsive, in contrast to the Casimir-Polder potential which is attractive at all distances. For the choice of parameters of the experiment, the Casimir-Polder potential, which varies with z as a sum of power laws, dominates at short and very large distances, but there is an intermediate range of position z for which the dipole potential dominates. In this case, a clear maximum of the total potential exists (Figure 4a). The height of this potential barrier depends univocally on the ratio I/δ, and the experiment consists in decreasing this parameter to find the threshold value $(I/\delta)_{\rm T}$ below which the atoms are no longer reflected. This measured value can then be compared to the value predicted with different expressions of the atom-dielectric potential, by stating that the potential barrier height is exactly equal to the kinetic energy of the incident atoms.

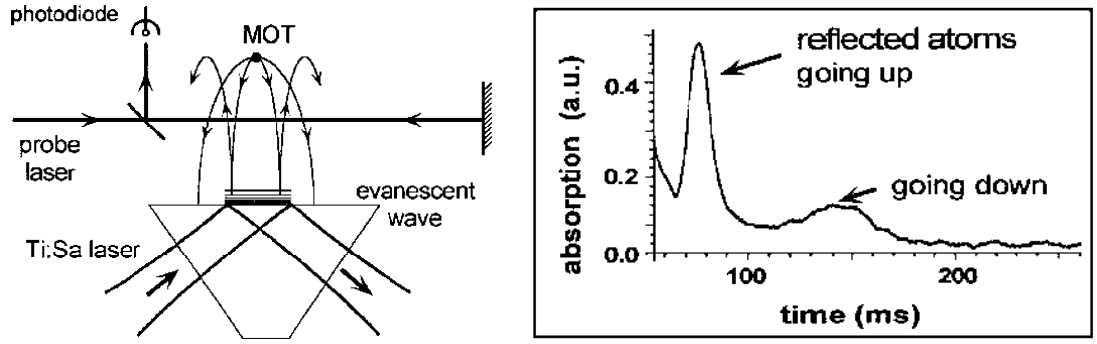

Figure 3: *Orsay experiment: trapped cold atoms at 10 μK are released on an evanescent wave atomic mirror located 15 mm below. The number of reflected atoms is measured by monitoring the absorption of a resonant probe laser above the atomic mirror.*

One may, at this point, raise the experimental problem of having a perfectly uniform evanescent wave intensity, in order to have an abrupt threshold. This is so difficult that the authors of [37] renounced to make a precise measurement. The Orsay group has circumvented the difficulty by keeping the standard gaussian transverse profile of the laser beam, and by noticing the following fact. When one changes the parameter I_0/δ (where I_0 is now the intensity *at the center* of the laser beam), the number $N_{\rm R}$ of reflected atoms varies as $\ln\left((I_0/\delta)/(I_0/\delta)_{\rm T}\right)$. Indeed the effective mirror – *i.e.* the location where the potential barrier height is larger than the kinetic energy of the incident atoms – is an ellipse of area proportional to that quantity.

We have plotted in figure 4b the number of reflected atoms as a function of $\ln\left(I_0/\delta\right)$. One clearly sees that the experimental points are aligned. A fit to a straight line then yields the measured threshold value $(I_0/\delta)_{\rm T}$. We have indicated on the x axis the various threshold values corresponding to the various potentials shown in Figure 4a. The threshold $\Lambda_{\rm T}^{\rm dip}$ is calculated for the dipole potential alone, without any atom-dielectric interaction. It differs from the observed value by a factor of 3, clearly showing the dramatic effect of this atom-dielectric interaction.

The threshold $\Lambda_{\rm T}^{\rm LJ}$ is calculated with the non retarded Lennard-Jones potential (2). Here we take into account that we deal with a dielectric prism and not an ideal mirror; we assume a dielectric constant ϵ independent of the frequency [39] so that:

$$U_{\rm LJ}(z) = -\frac{\epsilon-1}{\epsilon+1}\frac{\langle d^2\rangle}{48\pi\epsilon_0 z^3} = -A\frac{\hbar\Gamma}{(2kz)^3}\ , \tag{9}$$

where we used the fact that the dipole is isotropic: $\langle d^2\rangle = 3\langle d_i^2\rangle$, $i = x, y, z$. We take the value of ϵ at the wavelength $\lambda_{\rm res}$ of the resonant transition, that completely dominates the dipole fluctuations. Using known atomic data, we calculate the square of the atomic electric dipole in the ground state and find $A = 0.88$ with an accuracy of 1%.

We see on Figure 4b that the threshold $\Lambda_{\rm T}^{\rm LJ}$ slightly exceeds the experimental value. Actually, the difference is of the order of our estimation of the uncertainty, which is dominated by the uncertainty on the absolute value of the laser intensity. Therefore the agreement between our result and a model using the Lennard Jones potential is only marginal. On the other hand, it is clear that the threshold $\Lambda_{\rm T}^{\rm CP}$ agrees better with the experimental result. To calculate this threshold, we have used an expression of the Casimir Polder potential given by [40]. As for the case of the Yale experiment, this measurement is done at an intermediate distance, where one cannot use the asymptotic form of equation (4). More precisely, at the position of the potential barrier – *i.e.* at 48 nm from the wall, see Figure 4a – the correction to (9) due to retardation is 30%.

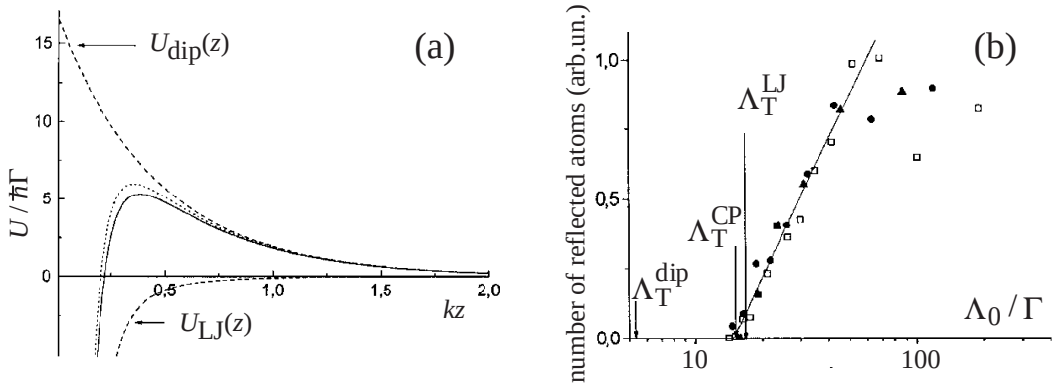

Figure 4: *(a) Atoms incident on the atomic mirror experience a total potential which is the sum of the evanescent wave reflecting potential $U_{\rm dip}(z)$ and of the atom-dielectric interaction $U(z)$. The height of the resulting potential barrier is controlled by changing the parameter I/δ of the evanescent wave and it depends on the mathematical form assumed for $U(z)$. Solid line: total potential neglecting retardation in the atom-dielectric interaction $U(z)$ (Lennard-Jones). Dotted line: total potential with the Casimir-Polder-Lifshitz expression for the atom-dielectric interaction. (b) Number of reflected atoms as a function of $\ln(I_0/\delta)$ (expressed in suitable units, hence the notation Λ_0). The various symbols correspond to different laser intensities. The results can be fitted by a straight line, whose extrapolation to 0 gives the measured value of the threshold, to be compared to the values calculated with the various potentials of figure 4a, and indicated by arrows. $\Lambda_{\rm T}^{\rm dip}$: no atom-dielectric interaction; $\Lambda_{\rm T}^{\rm LJ}$: Lennard-Jones form of the atom-dielectric potential (no retardation); $\Lambda_{\rm T}^{\rm CP}$: Casimir-Polder-Lifshitz potential.*

3.4 Quantum reflection by a Casimir-Polder potential: the Tokyo experiment [41]

We start by explaining briefly the concept of quantum reflection. For $z > 0$, consider a potential $U(z) < 0$ which tends to zero at infinity. We assume that this potential is attractive ($dU/dz > 0$) and we consider incident atoms with an energy E_i at $z = +\infty$. Quantum reflection is predicted to occur for atoms with a low incident kinetic energy E_i, if the potential changes rapidly enough. In this case, the atoms are reflected well before reaching the minimum of the potential $U(z)$ located in $z = 0$, so that the presence probability of the atoms remains vanishingly small around this minimum (fig. 5a).

More precisely, the condition for quantum reflection is

$$\frac{d\lambda_{\rm dB}}{dz} \geq 1 \ , \tag{10}$$

where $\lambda_{\rm dB}(z)$ is the local de Broglie wavelength of the particle at a distance z, calculated in a semi-classical analysis ($\lambda_{\rm dB}(z) = h/\sqrt{2m(E_i - U(z))}$). This condition can be seen as a breakdown of the validity of the semi-classical (WKB) approximation, which would imply that an incident particle always reaches $z = 0$, whatever its initial energy. For a power law potential $U(z) = -C_n/z^n$ with $n > 2$, and for particles with a sufficiently low incident energy E_i, the condition (10) is fulfilled over some range of distances z. Indeed the maximum of $\phi(z) = d\lambda_{\rm dB}/dz$ is found in $z_{\rm max} = ((n-2)C_n/(2(n+1)E_i))^{1/n}$ and this maximum scales as $E_i^{(2-n)/(2n)}$. Both quantities $z_{\rm max}$ and $\phi(z_{\rm max})$ tend to infinity as E_i tends to zero.

Suppose now that the potential $U(z)$ is created by a bulk material located in the domain $z < 0$. In the case of quantum reflection on the surface of the material, the particle is reflected

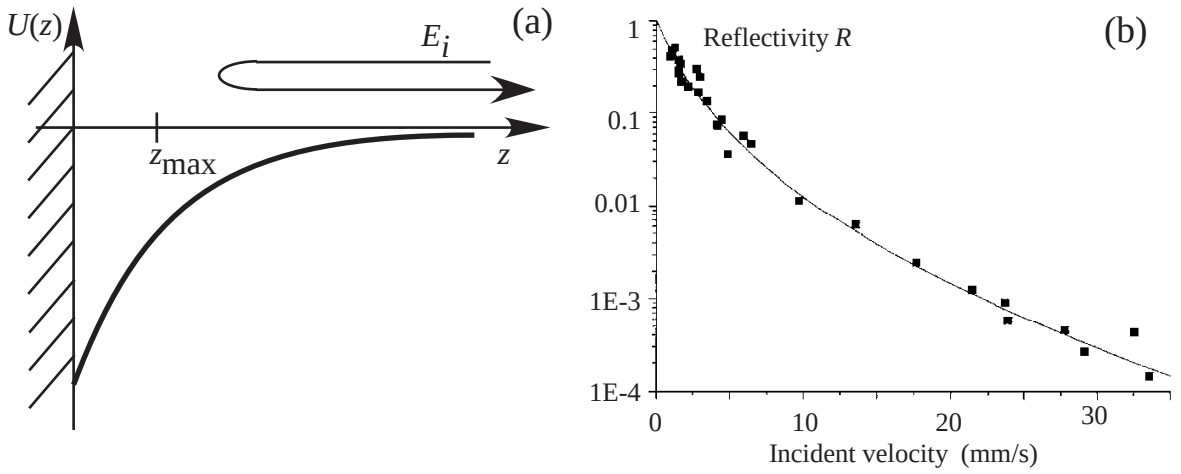

Figure 5: *Tokyo experiment. (a) Quantum reflection of a particle with incident energy E_i on a purely attractive potential $U(z)$. (b) Reflectivity vs. velocity for metastable neon atoms impinging on a silicon surface. The solid curve is the reflectivity calculated using the model potential (11) with $C_4 = 6.8\ 10^{-56}\ J/m^4$ and $z_0 = 64\ nm$.*

before it reaches the immediate vicinity of the material, where it could stick. One therefore expects an elastic reflection coefficient $R(E_i)$ which tends to 1 when the incident energy E_i goes to zero.

This phenomenon has first been observed for the reflection of helium and hydrogen atoms on a liquid helium surface [42, 43, 44, 45]. In the experiment that we wish to describe here [41], quantum reflection has been demonstrated for a solid surface: very slow metastable Neon atoms bounce elastically and specularly on the purely attractive potential created by a piece of silicon (semi conductor) or glass (dielectric). The idea is then to extract information on the potential $U(z)$ from the measurement of $R(E_i)$.

As shown in fig. 5b, obtained with silicon, the author measures the reflectivity for a range of incident velocities between 1 mm/s and 30 mm/s. As expected, he finds that the reflectivity increases when the velocity decreases. The largest reflectivity is $R = 0.5$ at 1 mm/s, and the data are consistent with the extrapolated value $R = 1$ at zero velocity.

The data are fitted by a simple theoretical model, which consists in connecting the asymptotic behaviors of the semi-classical atom wave function for short and long distances z. In this model, one assumes that a particle which can reach the location $z = 0$ sticks to the surface (absorptive boundary conditions). The atom-bulk silicon potential is modelled by

$$U(z) = -\frac{C_4}{(z + z_0)z^3}\ , \tag{11}$$

which gives an account for the behavior seen above for both short (z^{-3}) and long (z^{-4}) distances. The C_4 coefficient deduced from the fit is in agreement with the one expected from the Casimir-Polder theory: $C_4/C_4^{\rm CP} = 0.7 \pm 0.4$. The value of λ is $z_0 = 0.06\ \mu$m, with a range within σ confidence of $0 - 0.7\mu$m. This value for z_0 is much smaller than the distance between the turning point and the surface that one derives from the above considerations (typically 1 μm). This shows that, in this experiment, one is sensitive mostly to the retarded z^{-4} Casimir-Polder potential.

4 Conclusion

Thanks to experimental results obtained during the last ten years, there is now a clear evidence for retardation effects in the interaction between a ground state atom and a wall, either a metal, a semi-conductor, or a dielectric. The experiments clearly rule out a pure Lennard-Jones z^{-3} potential, which would exist in absence of retardation.

For the three experiments that we have described, the typical minimum distance between the atom and the surface varies between 0.05 μm to 1 μm. The most accurate fit to the Casimir-Polder potential is obtained in the Yale experiment, where an agreement between theory and experiment is found at the 10 % level. There is a strong hope that the theoretical predictions can be tested with an improved accuracy when coherent atom sources, emerging from Bose-Einstein condensates, will be easily available. With these *atom lasers* [46], one will be able to have a better control of the parameters of the atomic beam incident on the surface. Together with the possibility of performing an interferometric measurement of the shift induced by the atom-wall potential, this should allow for an extension of the range of distances over which the Casimir-Polder potential is tested.

References

[1] H.B.G. Casimir, Proc. K. Ned. Akad. Wet **60**, 793 (1948).

[2] H.B.G. Casimir and D. Polder, Phys. Rev. **73**, 360 (1948).

[3] F. London, Z. Physik **63**, 245 (1930).

[4] J.E. Lennard-Jones, Trans. Faraday Soc. **28**, 334 (1932).

[5] J.A. Wheeler, Phys. Rev. **59**, 928 (1941).

[6] E.A. Hinds, Adv. At. Mol. Opt. Phys., sup. 2, 1 (1994).

[7] The same paper contains a second calculation, addressing the long-range force between two atoms. Due to the lack of space, we only address the first part of the Casimir-Polder paper.

[8] J. Dalibard, J. Dupont-Roc and C. Cohen-Tannoudji, J. Phys. (Paris) **43**, 1617 (1982).

[9] D. Meschede, W. Yhe and E. Hinds, Phys. Rev. A **41**, 1587 (1990).

[10] S. Haroche, in *Fundamental Systems in Quantum Optics*, Les Houches LIII, J. Dalibard, J.-M. Raimond and J. Zinn-Justin Eds., North-Holland (Amsterdam, 1992).

[11] L. Spruch, in *Long Range Casimir Forces*, F.S. Levin and D.A. Micha eds., Plenum Press (New-York).

[12] E.M. Liftshitz, Sov. Phys. JETP **2**, 73 (1956).

[13] E. Elizalde and A. Romeo, Am. J. Phys. **59**, 711 (1991).

[14] Y. Tikochinsky and L. Spruch, Phys. Rev. A **48**, 4223 (1993).

[15] A. Landragin, Thèse d'université (Orsay, 1997).

[16] J.M. Wylie and J.E. Sipe, Phys. Rev. A **32**, 2030 (1985).

[17] M. Fichet, F. Schuller, D. Bloch, and M. Ducloy, Phys. Rev. A **51**, 1553 (1995).

[18] F.S. Levin and D.A. Micha eds., *Long Range Casimir Forces*, Plenum Press (New-York).

[19] D. Raskin and P. Kusch, Phys. Rev. **179**, 712 (1969).

[20] A. Shih and V.A. Parsigian, Phys. Rev. A **12**, 835 (1975).

[21] M.J. Mehl and W.L. Schaich, Phys. Rev. A **21**, 1177 (1980).

[22] A. Anderson, S. Haroche, E.A. Hinds, W. Jhe and D. Meschede, Phys. Rev. A **37**, 3594 (1988).

[23] E.S. Sabisky and C.H. Anderson, Phys. Rev. A **7**, 790 (1973).

[24] M. Oria, M. Chevrollier, D. Bloch, M. Fichet and M. Ducloy, Europhys. Lett. **14**, 527 (1991).

[25] V. Sandoghdar, C.I. Sukenik, E.A. Hinds, and S. Haroche, Phys. Rev. Lett. **68**, 3432 (1992).

[26] M. Boustimi, B. Viaris de Lesegno, J. Baudon, J. Robert, and M. Ducloy, Phys. Rev. Lett. **86**, 2766 (2001) and refs. in.

[27] H. Failache, S. Saltiel, M. Fichet, D. Bloch, and M. Ducloy, Phys. Rev. Lett. **83**, 5467 (1999).

[28] P. Szriftgiser, D. Guéry-Odelin, M. Arndt, and J. Dalibard, Phys. Rev. Lett. **77**, 4 (1996).

[29] L. Cognet, V. Savalli, G.Z.K. Horvath, D. Holleville, R. Marani, N. Westbrook, C.I. Westbrook, and A. Aspect, Phys. Rev. Lett. **81**, 5044 (1998).

[30] M. Gorlicki, S. Feron, V. Lorent, and M. Ducloy, Phys. Rev. A **61**, 013603 (2001).

[31] C.I. Sukenik, M.G. Boshier, D. Cho, V. Sandoghdar, and E.A. Hinds, Phys. Rev. Lett. **70**, 560 (1993).

[32] For the generalisation of the Casimir-Polder result to the case where an atom interacts with the two walls of a cavity, see G. Barton, Proc. Roy. Soc. London A **410**, 175 (1987).

[33] A. Landragin, J.-Y. Courtois, G. Labeyrie, N. Vansteenkiste, C. I. Westbrook, and A. Aspect, Phys. Rev. Lett. **77**, 1464 (1996).

[34] S. Chu, Rev. Mod. Phys. **70**, 685 (1998).

[35] C. Cohen-Tannoudji, Rev. Mod. Phys. **70**, 707 (1998).

[36] W.D. Phillips, Rev. Mod. Phys. **70**, 721 (1998).

[37] M. Kasevich, K. Moler, E. Riis, E. Sunderman, D. Weis, and S. Chu, in Atomic Physics 12, ed. by J. C. Zorn and R. R. Lewis, AIP Conf. Proc. No 233 (AIP, New York 1991), p.47.

[38] R.J. Cook and R.K. Hill, Opt. Commun.**43**, 258 (1982);Phys. Rev. A**32**, 2030 (1985).

[39] Since the dipole fluctuations are totally dominated by the first resonance transitions at 780 nm and 795 nm, it is legitimate to take ϵ constant.

[40] J. M. Wylie and J.E. Sipe, Phys. Rev. A**30**, 1185 (1984).

[41] F. Shimizu, Phys. Rev. Lett. **86**, 987 (2001).

[42] V. U. Nayak, D. O. Edwards, and N. Masuhara, Phys. Rev. Lett. **50**, 990 (1983).

[43] J. J. Berkhout et al., Phys. Rev. Lett. **63**, 1689 (1989).

[44] J. M. Doyle et al., Phys. Rev. Lett. **67**, 603 (1991).

[45] I. A. Yu et al., Phys. Rev. Lett. **71**, 1589 (1993).

[46] K. Helmerson *et al.*, Physics World, August 1999, p. 31.