Beyond Einstein's Gravity

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Abstract. Despite the important experimental success of General Relativity, there are several theoretical reasons indicating that gravitational phenomena may change radically from the predictions of Einstein's theory at very short distances. A main motivation comes from studies of unifying all fundamental forces in the framework of a consistent quantum theory, called string theory. This theory introduces a new physical constant, the string length, under which a new elementary structure shows up, changing drastically all physical laws of nature. In particular, lowering the string scale in the TeV region provides a theoretical framework for solving the so-called mass hierarchy problem: the apparent weakness of gravity can then be accounted by the existence of large internal dimensions, in the submillimeter region, and transverse to a braneworld where our observed universe is confined. I review the main properties of this scenario and its implications for new gravitational phenomena that can be observed at both particle colliders, and in non-accelerator experiments searching for new short range forces at submillimeter distances. I also discuss the warped metric case and possible localization of gravity in the presence of infinite size extra dimensions that can modify Newton's law at cosmological distance scales.

1 Introduction

During the last few decades, physics beyond the Standard Model (SM) was guided by the problem of mass hierarchy. This can be formulated as the question of why gravity appears to us so weak compared to the other three known fundamental interactions corresponding to the electromagnetic, weak and strong nuclear forces. Indeed, gravitational interactions are suppressed by a very high energy scale, the Planck mass $M_P \sim 10^{19}$ GeV, associated to a length $l_P \sim 10^{-35}$ m, where they are expected to become important. In a quantum theory, the hierarchy implies a severe fine tuning of the fundamental parameters in more than 30 decimal places in order to keep the masses of elementary particles at their observed values. The reason is that quantum radiative corrections to all masses generated by the Higgs vacuum expectation value (VEV) are proportional to the ultraviolet cutoff which in the presence of gravity is fixed by the Planck mass. As a result, all masses are "attracted" to become about 10^{16} times heavier than their observed values.

Besides compositeness, there are two main ideas that have been proposed and studied extensively during the last years, corresponding to different approaches of explaining the mass hierarchy problem. (1) Low energy supersymmetry with all superparticle masses in the TeV region. Indeed, in the limit of exact supersymmetry, quadratically divergent corrections to the Higgs self-energy are exactly cancelled, while in the softly broken case, they are cutoff by the supersymmetry breaking mass splittings. (2) TeV scale strings, in which quadratic divergences are cutoff by the string scale and low energy supersymmetry is not needed. Both ideas are experimentally testable at high-energy particle colliders and in particular at LHC. Below, I discuss their implementation in string theory.

The appropriate and most convenient framework for low energy supersymmetry and grand unification is the perturbative heterotic string. Indeed, in this theory,

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gravity and gauge interactions have the same origin, as massless modes of the closed heterotic string, and they are unified at the string scale M_s. As a result, the Planck mass M_P is predicted to be proportional to M_s:

$$M_P = M_s/g\,, \tag{1}$$

where g is the gauge coupling. In the simplest constructions all gauge couplings are the same at the string scale, given by the four-dimensional (4d) string coupling, and thus no grand unified group is needed for unification. In our conventions $\alpha_{\text{GUT}} = g^2 \simeq 0.04$, leading to a discrepancy between the string and grand unification scale M_{GUT} by almost two orders of magnitude. Explaining this gap introduces in general new parameters or a new scale, and the predictive power is essentially lost. This is the main defect of this framework, which remains though an open and interesting possibility.

The other idea can be naturally realized in the framework of type I string theory with D-branes. Unlike in the heterotic string, gauge and gravitational interactions have now different origin. The latter are described again by closed strings, while the former emerge as excitations of open strings with endpoints confined on D-branes [1]. This leads to a braneworld description of our universe, which should be localized on a hypersurface, i.e. a membrane extended in p spatial dimensions, called p-brane (see Fig. 1). Closed strings propagate in all nine dimensions of string theory: in those extended along the p-brane, called parallel, as well as in the transverse ones. On the contrary, open strings are attached on the p-brane. Obviously, our p-brane world must
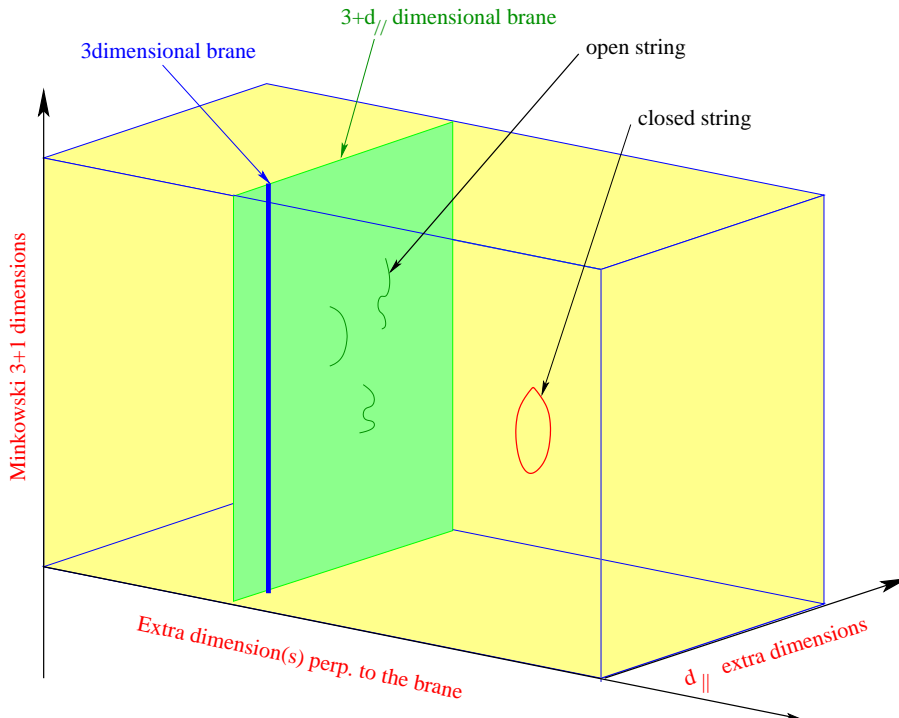

Figure 1: In the type I string framework, our Universe contains, besides the three known spatial dimensions (denoted by a single blue line), some extra dimensions ($d_\parallel = p - 3$) parallel to our world p-brane (green plane) where endpoints of open strings are confined, as well as some transverse dimensions (yellow space) where only gravity described by closed strings can propagate.

have at least the three known dimensions of space. But it may contain more: the extra

$d_\parallel = p - 3$ parallel dimensions must have a finite size, in order to be unobservable at present energies, and can be as large as $\mathrm{TeV}^{-1} \sim 10^{-18}$ m [2]. On the other hand, transverse dimensions interact with us only gravitationally and experimental bounds are much weaker: their size should be less than about 0.1 mm [3]. In the following, I review the main properties and experimental signatures of low string scale models [4, 5].

2   Framework

In type I theory, the different origin of gauge and gravitational interactions implies that the relation between the Planck and string scales is not linear as (1) of the heterotic string. The requirement that string theory should be weakly coupled, constrain the size of all parallel dimensions to be of order of the string length, while transverse dimensions remain unrestricted. Assuming an isotropic transverse space of $n = 9 - p$ compact dimensions of common radius $R_\perp$, one finds:

$$M_P^2 = \frac{1}{g^4} M_s^{2+n} R_\perp^n \;, \qquad g_s \simeq g^2 \;. \tag{2}$$

where g_s is the string coupling. It follows that the type I string scale can be chosen hierarchically smaller than the Planck mass [6, 4] at the expense of introducing extra large transverse dimensions felt only by gravity, while keeping the string coupling small [4]. The weakness of 4d gravity compared to gauge interactions (ratio M_W/M_P) is then attributed to the largeness of the transverse space $R_\perp$ compared to the string length $l_s = M_s^{-1}$.

   An important property of these models is that gravity becomes effectively $(4+n)$-dimensional with a strength comparable to those of gauge interactions at the string scale. The first relation of Eq. (2) can be understood as a consequence of the $(4 + n)$-dimensional Gauss law for gravity, with

$$M_{(4+n)} = M_s/g^{4/2+n} \tag{3}$$

the effective scale of gravity in $4 + n$ dimensions. Taking $M_s \simeq 1$ TeV, one finds a size for the extra dimensions $R_\perp$ varying from 10^8 km, .1 mm, down to a Fermi for $n = 1, 2,$ or 6 large dimensions, respectively. This shows that while $n = 1$ is excluded, $n \geq 2$ is allowed by present experimental bounds on gravitational forces [3, 7]. Thus, in these models, gravity appears to us very weak at macroscopic scales because its intensity is spread in the "hidden" extra dimensions. At distances shorter than $R_\perp$, it should deviate from Newton's law, which may be possible to explore in laboratory experiments (see Fig. 2).

3   Experimental implications in accelerators

The main experimental signal is gravitational radiation in the bulk from any physical process on the world-brane. In fact, the very existence of branes breaks translation invariance in the transverse dimensions and gravitons can be emitted from the brane into the bulk. During a collision of center of mass energy $\sqrt{s}$, there are $\sim (\sqrt{s} R_\perp)^n$ KK excitations of gravitons with tiny masses, that can be emitted. Each of these states looks from the 4d point of view as a massive, quasi-stable, extremely weakly coupled (s/M_P^2 suppressed) particle that escapes from the detector. The total effect is a missing-energy cross-section roughly of order:

$$\frac{(\sqrt{s} R_\perp)^n}{M_P^2} \sim \frac{1}{s} \left(\frac{\sqrt{s}}{M_s} \right)^{n+2} . \tag{4}$$

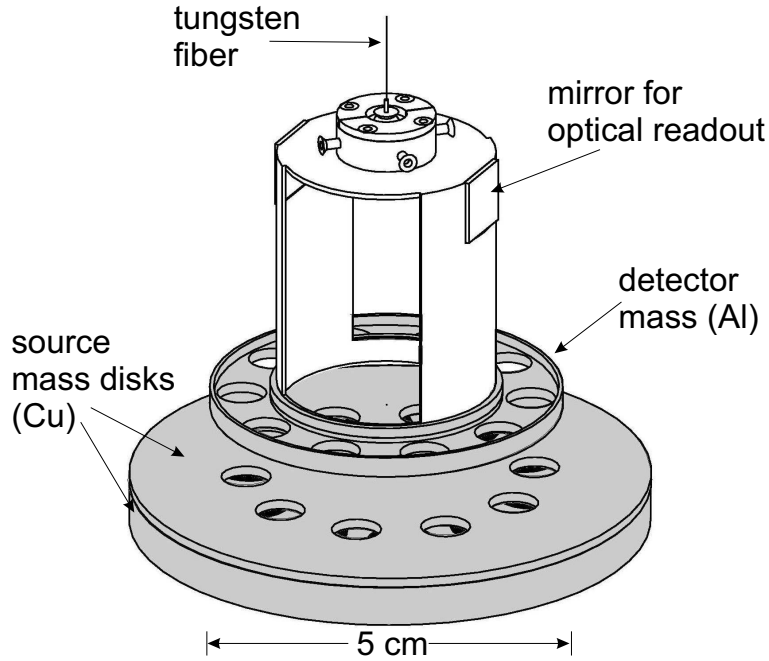Explicit computation of these effects leads to the bounds given in Table 1. However,

Figure 2: Torsion pendulum that tested Newton's law at 130 nm.

Table 1: Limits on $R_\perp$ in mm.

Experiment	$n = 2$	$n = 4$	$n = 6$
Collider bounds			
LEP 2	5×10^{-1}	2×10^{-8}	7×10^{-11}
Tevatron	5×10^{-1}	10^{-8}	4×10^{-11}
LHC	4×10^{-3}	6×10^{-10}	3×10^{-12}
NLC	10^{-2}	10^{-9}	6×10^{-12}
Present non-collider bounds			
SN1987A	3×10^{-4}	10^{-8}	6×10^{-10}
COMPTEL	5×10^{-5}	-	-

larger radii are allowed if one relaxes the assumption of isotropy, by taking for instance two large dimensions with different radii.

Fig. 3 shows the cross-section for graviton emission in the bulk, corresponding to the process $pp \rightarrow jet + graviton$ at LHC, together with the SM background [8]. For a given value of M_s, the cross-section for graviton emission decreases with the number of large transverse dimensions, in contrast to the case of parallel dimensions. The reason is that gravity becomes weaker if there are more dimensions because there is more space for the gravitational field to escape. There is a particular energy and angular distribution of the produced gravitons that arise from the distribution in mass of KK states of spin-2. This can be contrasted to other sources of missing energy and might be a smoking gun for the extra dimensional nature of such a signal.

In Table 1, there are also included astrophysical and cosmological bounds. Astrophysical bounds [9, 10] arise from the requirement that the radiation of gravitons should not carry on too much of the gravitational binding energy released during core collapse of supernovae. In fact, the measurements of Kamiokande and IMB for SN1987A suggest that the main channel is neutrino fluxes. The best cosmological
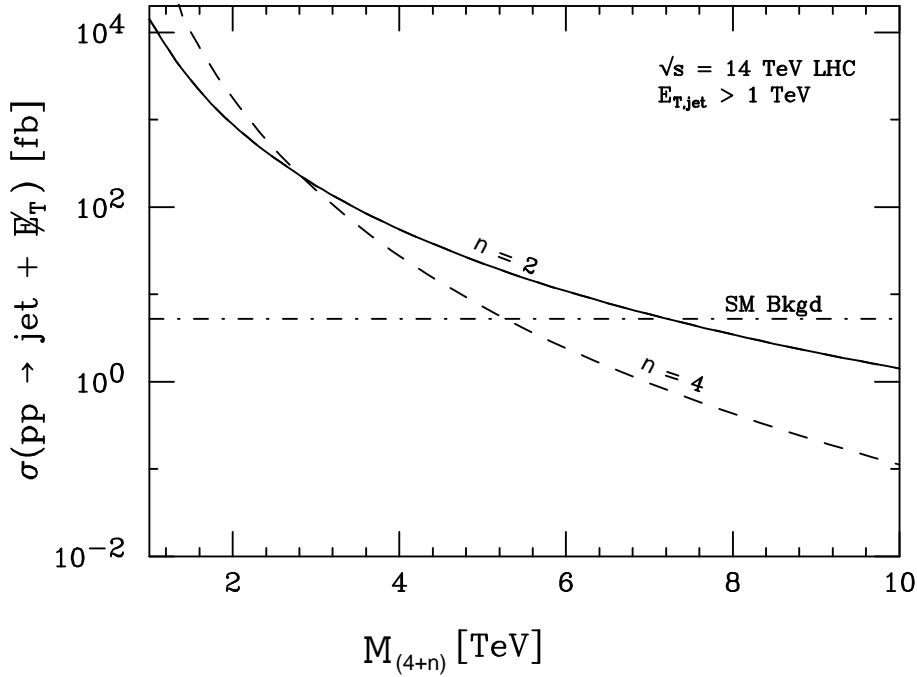
Figure 3: Missing energy cross-section due to graviton emission at LHC, as a function of the higher-dimensional gravity scale $M_{(4+n)}$, for n extra dimensions, produced together with a hadronic jet.

bound [11] is obtained from requiring that decay of bulk gravitons to photons do not generate a spike in the energy spectrum of the photon background measured by the COMPTEL instrument. Bulk gravitons are expected to be produced just before nucleosynthesis due to thermal radiation from the brane. The limits assume that the temperature was at most 1 MeV as nucleosynthesis begins, and become stronger if temperature is increased.

At energies higher than the string scale, new spectacular phenomena are expected to occur, related to string physics and quantum gravity effects, such as possible micro-black hole production [12]. Particle accelerators would then become the best tools for studying quantum gravity and string theory.

4   Supersymmetry in the bulk and short range forces

Besides the spectacular predictions in accelerators, there are also modifications of gravitation in the sub-millimeter range, which can be tested in "table-top" experiments that measure gravity at short distances. There are three categories of such predictions:

(i) Deviations from the Newton's law $1/r^2$ behavior to $1/r^{2+n}$, which can be observable for $n = 2$ large transverse dimensions of sub-millimeter size. This case is particularly attractive on theoretical grounds because of the logarithmic sensitivity of SM couplings on the size of transverse space [13], that allows to determine the hierarchy [14].

(ii) New scalar forces in the sub-millimeter range, related to the mechanism of supersymmetry breaking, and mediated by light scalar fields φ with masses [15, 4]:

$$m_\varphi \simeq \frac{m_{susy}^2}{M_P} \simeq 10^{-4} - 10^{-6} \text{ eV},\qquad(5)$$

for a supersymmetry breaking scale $m_{susy} \simeq 1-10$ TeV. They correspond to Compton wavelengths of 1 mm to 10 μm. m_{susy} can be either $1/R_{\parallel}$ if supersymmetry is broken by compactification [15], or the string scale if it is broken "maximally" on our world-brane [4]. A universal attractive scalar force is mediated by the radion modulus $\varphi \equiv M_P \ln R$, with R the radius of the longitudinal or transverse dimension(s). In the former case, the result (5) follows from the behavior of the vacuum energy density $\Lambda \sim 1/R_{\parallel}^4$ for large $R_{\parallel}$ (up to logarithmic corrections). In the latter, supersymmetry is broken primarily on the brane, and thus its transmission to the bulk is gravitationally suppressed, leading to (5). For $n = 2$, there may be an enhancement factor of the radion mass by $\ln R_{\perp} M_s \simeq 30$ decreasing its wavelength by an order of magnitude [14].

The coupling of the radius modulus to matter relative to gravity can be easily computed and is given by:

$$\sqrt{\alpha_{\varphi}} = \frac{1}{M} \frac{\partial M}{\partial \varphi} \; ; \;\; \alpha_{\varphi} = \begin{cases} \frac{\partial \ln \Lambda_{\text{QCD}}}{\partial \ln R} \simeq \frac{1}{3} & \text{for } R_{\parallel} \\[2ex] \frac{2n}{n+2} = 1 - 1.5 & \text{for } R_{\perp} \end{cases} \tag{6}$$

where M denotes a generic physical mass. In the longitudinal case, the coupling arises dominantly through the radius dependence of the QCD gauge coupling [15], while in the case of transverse dimension, it can be deduced from the rescaling of the metric which changes the string to the Einstein frame and depends slightly on the bulk dimensionality ($\alpha = 1 - 1.5$ for $n = 2 - 6$) [14]. Such a force can be tested in microgravity experiments and should be contrasted with the change of Newton's law due the presence of extra dimensions that is observable only for $n = 2$ [3, 7]. The resulting bounds from an analysis of the radion effects are [3]:

$$M_* \gtrsim 3 - 4.5 \, \text{TeV} \;\; \text{for} \;\; n = 2 - 6 \,. \tag{7}$$

In principle there can be other light moduli which couple with even larger strengths. For example the dilaton, whose VEV determines the string coupling, if it does not acquire large mass from some dynamical supersymmetric mechanism, can lead to a force of strength 2000 times bigger than gravity [16].
(iii) Non universal repulsive forces much stronger than gravity, mediated by possible abelian gauge fields in the bulk [9, 17]. Such fields acquire tiny masses of the order of M_s^2/M_P, as in (5), due to brane localized anomalies [17]. Although their gauge coupling is infinitesimally small, $g_A \sim M_s/M_P \simeq 10^{-16}$, it is still bigger that the gravitational coupling E/M_P for typical energies $E \sim 1$ GeV, and the strength of the new force would be $10^6 - 10^8$ stronger than gravity. This is an interesting region which will be soon explored in micro-gravity experiments (see Fig. 4). Note that in this case supernova constraints impose that there should be at least four large extra dimensions in the bulk [9].

In Fig. 4 we depict the actual information from previous, present and upcoming experiments [7, 14]. The solid lines indicate the present limits from the experiments indicated. The excluded regions lie above these solid lines. Measuring gravitational strength forces at short distances is challenging. The dashed thick lines give the expected sensitivity of the various experiments, which will improve the actual limits by roughly two orders of magnitude, while the horizontal dashed lines correspond to the theoretical predictions for the graviton in the case $n = 2$ and for the radion in the transverse case. These limits are compared to those obtained from particle accelerator experiments in Table 1. Finally, in Figs. 5 and 6, we display recent improved bounds for new forces at very short distances by focusing on the right hand side of Fig. 4, near the origin [7].
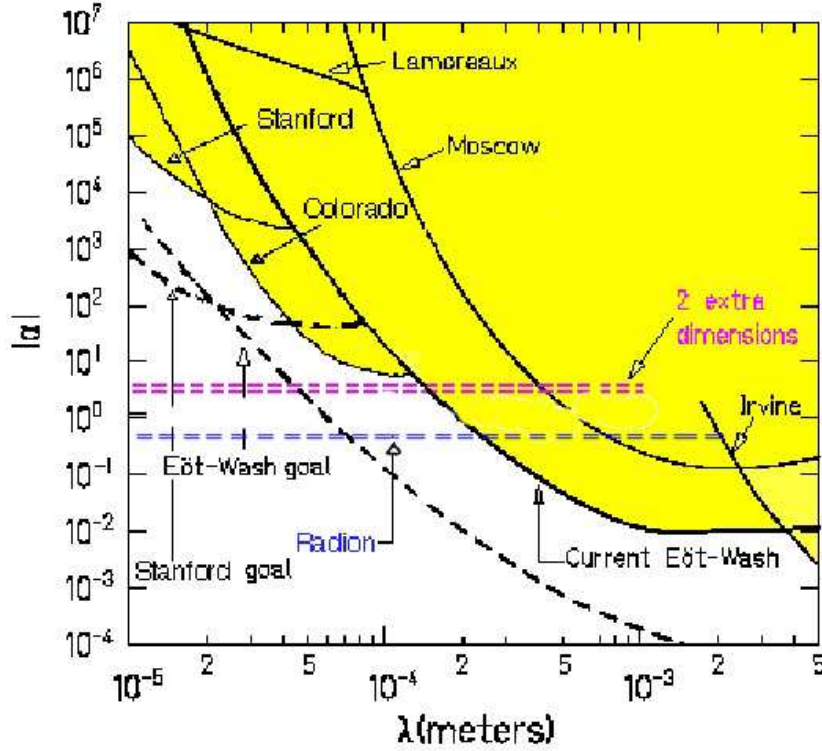
Figure 4: Present limits on non-Newtonian forces at short distances (yellow regions), as a function of their range λ and their strength relative to gravity α. The limits are compared to new forces mediated by the graviton in the case of two large extra dimensions, and by the radion.

5   Non-compact extra dimensions and localized gravity

There are several motivations to study localization of gravity in non-compact extra dimensions: (i) it avoids the problem of fixing the moduli associated to the size of the compactification manifold; (ii) it provides a new approach to the mass hierarchy problem; (iii) there are modifications of gravity at large distances that may have interesting observational consequences. Two types of models have been studied: warped metrics in curved space [18], and infinite size extra dimensions in flat space [19]. The former, although largely inspired by stringy developments and having used many string-theoretic techniques, have not yet a clear and calculable string theory realization [20]. In any case, since curved space is always difficult to handle in string theory, in the following we concentrate mainly on the latter, formulated in flat space with gravity localized on a subspace of the bulk. It turns out that these models of induced gravity have an interesting string theory realization [21] that we describe below, after presenting first a brief overview of the warped case [22].

5.1   Warped spaces

In these models, space-time is a slice of anti de Sitter space (AdS) in $d = 5$ dimensions while our universe forms a four-dimensional (4d) flat boundary [18]. The corresponding line element is:

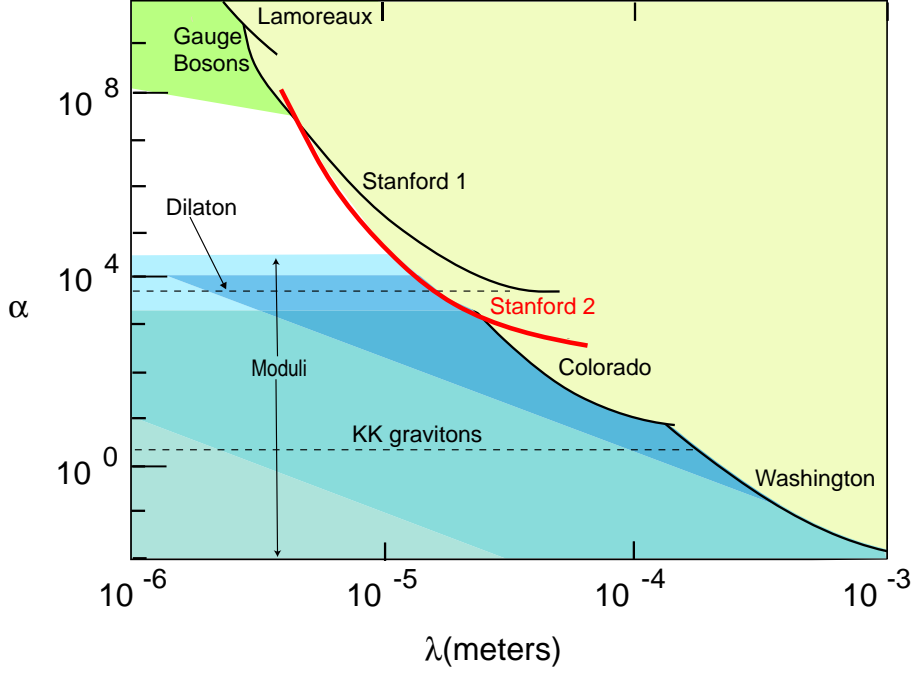$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \quad ; \quad \Lambda = -24M^3k^2 \,, \tag{8}$$

Figure 5: Bounds on non-Newtonian forces in the range 6-20 μm (see S. J. Smullin et al. in Ref. [7]).

where M, Λ are the 5d Planck mass and cosmological constant, respectively, and the parameter k is the curvature of AdS$_5$. The fifth coordinate y is restricted on the interval $[0, \pi r_c]$. Thus, this model requires two 'branes', a UV and an IR, located at the two end-points of the interval, $y = 0$ and $y = \pi r_c$, respectively. The vanishing of the 4d cosmological constant requires to fine tune the two tensions: $T = -T' = 24M^3 k^2$. The 4d Planck mass is given by:

$$M_P^2 = \frac{1}{k}(1 - e^{-2\pi k r_c})M^3 \,. \tag{9}$$

Note that the IR brane can move to infinity by taking the limit $r_c \to \infty$, while M_P is kept finite and thus 4d gravity is always present on the brane. The reason is that the internal volume remains finite in the non-compact limit along the positive y axis. As a result, gravity is kept localized on the UV brane, while the Newtonian potential gets corrections, $1/r + 1/k^2 r^3$, which are identical with those arising in the compact case of two flat extra dimensions. Using the experimental limit $k^{-1} \lesssim 0.1$ mm and the relation (9), one finds a bound for the 5d gravity scale $M \gtrsim 10^8$ GeV, corresponding to a brane tension $T \gtrsim 1$ TeV. Notice that this bound is not valid in the compact case of six extra dimensions, because their size is in the fermi range and thus the $1/r^3$ deviations of Newton's law are cutoff at shorter distances.

5.2   The induced gravity model

The DGP model and its generalizations are specified by a bulk Einstein-Hilbert (EH) term and a four-dimensional EH term [19]:

$$M^{2+n} \int_{\mathcal{M}_{4+n}} d^4 x d^n y \sqrt{G} \, \mathcal{R}_{(4+n)} + M_P^2 \int_{\mathcal{M}_4} d^4 x \sqrt{g} \, \mathcal{R}_{(4)} \,; \quad M_P^2 \equiv r_c^n M^{2+n} \tag{10}$$

with M and M_P the (possibly independent) respective Planck scales. The scale $M \geq 1$ TeV would be related to the short-distance scale below which UV quantum gravity
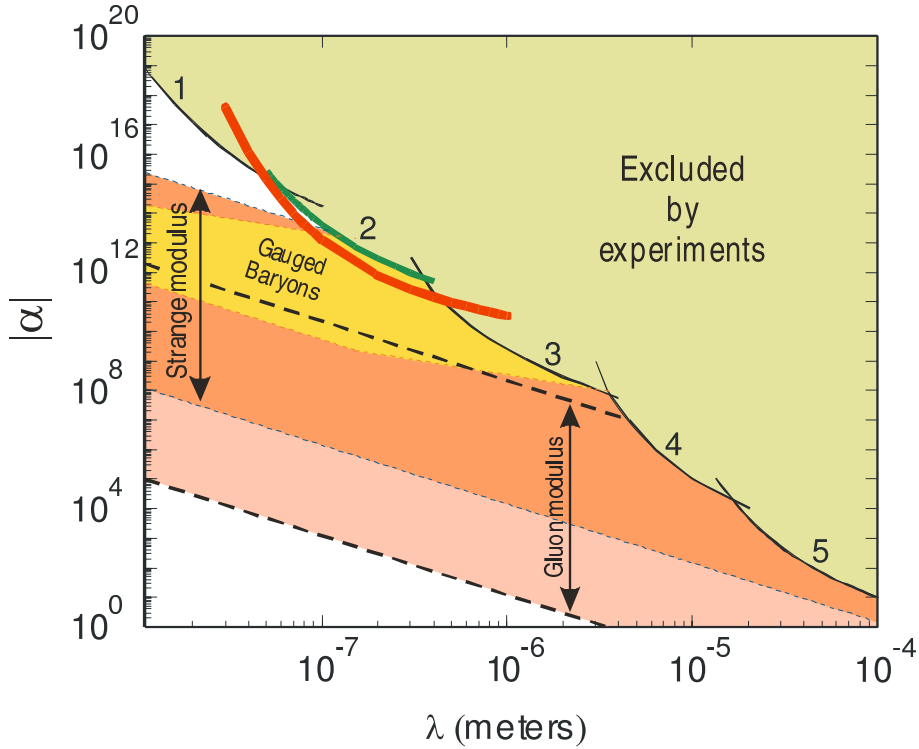
Figure 6: Bounds on non-Newtonian forces in the range around 200 nm (see R. S. Decca et al. in Ref. [7]). Curves 4 and 5 correspond to Stanford and Colorado experiments, respectively, of Fig. 5 (see also J C. Long and J. C. Price of Ref. [7]).

or stringy effects are important. The four-dimensional metric is the restriction of the bulk metric $g_{\mu\nu} = G_{\mu\nu}|$ and we assume the WORLD[1] rigid, allowing the gauge $G_{i\mu}| = 0$ with $i \geq 5$. Finally, only intrinsic curvature terms are omitted but no Gibbons–Hawking term is needed.

5.2.1   Co-dimension one

In the case of co-dimension one bulk $(n = 1)$ and δ-function localization, it is easy to see that r_c is a crossover scale where gravity changes behavior on the WORLD. Indeed, by Fourrier transform the quadratic part of the action (10) with respect to the 4d position x, at the WORLD position $y = 0$, one obtains $M^{2+n}(p^{2-n} + r_c^n p^2)$, where p is the 4d momentum. It follows that for distances smaller than r_c (large momenta), the first term becomes irrelevant and the graviton propagator on the "brane" exhibits four-dimensional behavior $(1/p^2)$ with Planck constant $M_P = M^3 r_c$. On the contrary, at large distances, the first term becomes dominant and the graviton propagator acquires a five-dimensional fall-off $(1/p)$ with Planck constant M. Imposing r_c to be larger than the size of the universe, $r_c \gtrsim 10^{28}$ cm, one finds $M \lesssim 100$ MeV, which seems to be in conflict with experimental bounds. However, there were arguments that these bounds can be evaded, even for values of the fundamental scale $M^{-1} \sim 1$ mm that one may need for suppressing the quantum corrections of the cosmological constant [19].

On the other hand, in the presence of non-zero brane thickness w, a new crossover

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[1]We avoid calling $\mathcal{M}_4$ a brane because, as we will see below, gravity localizes on singularities of the internal manifold, such as orbifold fixed points. Branes with localized matter can be introduced independently of gravity localization.

length-scale seems to appear, $R_c \sim (wr_c)^{1/2}$ [23] or $r_c^{3/5} w^{2/5}$ [24].

		4d	5d

w R_c r_c

↗   ↑

UV cutoff   5d or strong coupling
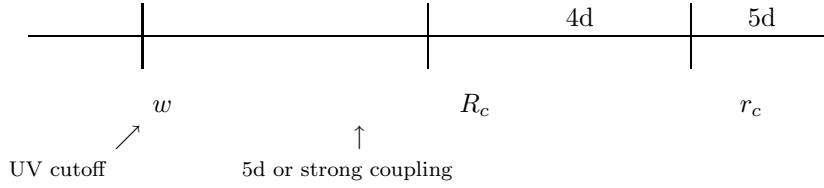
Below this scale, the theory acquires either again a five-dimensional behavior, or a strong coupling regime. For $r_c \sim 10^{28}$ cm, the new crossover scale is of order $R_c \sim 10^{-4} - 10$ m.

5.2.2   Higher co-dimension

The situation changes drastically for more than one non-compact bulk dimensions, $n > 1$, due to the ultraviolet properties of the higher-dimensional theories. Indeed, from the action (10), the effective potential between two test masses in four dimensions

$$\int [d^3x]\, e^{-ip\cdot x}\, V(x) \;=\; \frac{D(p)}{1 + r_c^n\, p^2\, D(p)} \left[\tilde{T}_{\mu\nu} T^{\mu\nu} - \frac{1}{2+n}\, \tilde{T}^\mu_\mu T^\nu_\nu \right] \qquad (11)$$

$$D(p) \;=\; \int [d^n q]\, \frac{f_w(q)}{p^2 + q^2} \qquad (12)$$

is a function of the bulk graviton retarded Green's function $G(x,0;0,0) = \int [d^4p]\, e^{ip\cdot x} D(p)$ evaluated for two points localized on the WORLD ($y = y' = 0$). The integral (12) is UV-divergent for $n > 1$ unless there is a non-trivial brane thickness profile $f_w(q)$ of width w. If the four-dimensional WORLD has zero thickness, $f_w(q) \sim 1$, the bulk graviton does not have a normalizable wave function. It therefore cannot contribute to the induced potential, which always takes the form $V(p) \sim 1/p^2$ and Newton's law remains four-dimensional at all distances.

For a non-zero thickness w, there is only one crossover length scale, R_c:

$$R_c = w \left(\frac{r_c}{w} \right)^{\frac{n}{2}} , \qquad (13)$$

above which one obtains a higher-dimensional behaviour [25]. Therefore the effective potential presents two regimes: (i) at short distances ($w \ll r \ll R_c$) the gravitational interactions are mediated by the localized four-dimensional graviton and Newton's potential on the WORLD is given by $V(r) \sim 1/r$ and, (ii) at large distances ($r \gg R_c$) the modes of the bulk graviton dominate, changing the potential. Note that for $n = 1$ the expressions (11) and (12) are finite and unambiguously give $V(r) \sim 1/r$ for $r \gg r_c$. For a co-dimension bigger than 1, the precise behavior for large-distance interactions depends *crucially* on the UV completion of the theory.

4d	higher d

R_c
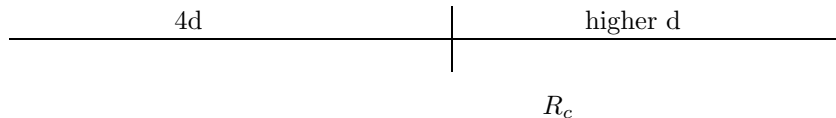
At this point we stress a fundamental difference with the *finite extra dimensions* scenarios. In these cases Newton's law gets higher-dimensional at distances smaller than the characteristic size of the extra dimensions. This is precisely the opposite of the case of infinite volume extra dimensions that we discuss here.

As mentioned above, for higher co-dimension, there is an interplay between UV regularization and IR behavior of the theory. Indeed, several works in the literature raised unitarity [26] and strong coupling problems [27] which depend crucially on the UV completion of the theory. A unitary UV regularization for the higher co-dimension version of the model has been proposed in [28]. It would be interesting to address these questions in a precise string theory context. Actually, using for UV cutoff on the "brane" the 4d Planck length $w \sim l_P$, one gets for the crossover scale (13): $R_c \sim M^{-1}(M_P/M)^{n/2}$. Putting $M \gtrsim 1$ TeV leads to $R_c \lesssim 10^{8(n-2)}$ cm. Imposing $R_c \gtrsim 10^{28}$ cm, one then finds that the number of extra dimensions must be at least six, $n \geq 6$, which is realized nicely in string theory and provides an additional motivation for studying possible string theory realizations.

5.3   String theory realization

In the following, we explain how to realize the gravity induced model (10) with $n \geq 6$ as the low-energy effective action of string theory on a non-compact six-dimensional manifold $\mathcal{M}_6$ [21]. We work in the context of $\mathcal{N} = 2$ supergravities in four dimensions but the mechanism for localizing gravity is independent of the number of supersymmetries. Of course for $\mathcal{N} \geq 3$ supersymmetries, there is no localization. We also start with the compact case and take the decompactification limit. The localized properties are then encoded in the different volume dependences.

In string perturbation, corrections to the four-dimensional Planck mass are in general very restrictive. In the heterotic string, they vanish to all orders in perturbation theory [29]; in type I theory, there are moduli-dependent corrections generated by open strings [30], but they vanish when the manifold $\mathcal{M}_6$ is decompactified; in type II theories, they are constant, independent of the moduli of the manifold $\mathcal{M}_6$, and receive contributions only from tree and one-loop levels that we describe below (at least for supersymmetric backgrounds) [21, 31]. Finally, in the context of M-theory, one obtains a similar localized action of gravity kinetic terms in five dimensions, corresponding to the strong coupling limit of type IIA string [21].

The origin of the two EH terms in (10) can be traced back to the perturbative corrections to the eight-derivative effective action of type II strings in ten dimensions. These corrections include the tree-level and one-loop terms given by:

$$\frac{1}{l_s^8} \int_{M_{10}} \frac{1}{g_s^2} \mathcal{R}_{(10)} - \frac{1}{l_s^2} \int_{M_{10}} \left(\frac{2\zeta(3)}{g_s^2} \mp 4\zeta(2) \right) R \wedge R \wedge R \wedge R \wedge e \wedge e + \cdots \qquad (14)$$

where ϕ is the dilaton field determining the string coupling $g_s = e^{\langle \phi \rangle}$, and the $\pm$ sign corresponds to the type IIA/B theory. On a direct product space-time $\mathcal{M}_6 \times \mathbb{R}^4$, at the level of zero modes, the second term in (14) splits as:

$$\int_{M_6} R \wedge R \wedge R \times \int_{M_4} \mathcal{R}_{(4)} = \chi \int_{M_4} \mathcal{R}_{(4)} \,, \qquad (15)$$

where χ is the Euler number of the M_6 compactification manifold. We thus obtain the expressions for the Planck masses M and M_p:

$$M^2 \sim M_s^2/g_s^{1/2} \quad ; \quad M_P^2 \sim \chi(\frac{c_0}{g_s^2} + c_1)M_s^2 \,, \qquad (16)$$

with $c_0 = -2\zeta(3)$ and $c_1 = \pm 4\zeta(2) = \pm 2\pi^2/3$.

It is interesting that the appearance of the induced 4d localized term preserves $\mathcal{N} = 2$ supersymmetry and is independent of the localization mechanism of matter fields (for instance on D-branes). Localization requires the internal space M_6 to have a non-zero Euler characteristic $\chi \neq 0$. Actually, in type IIA/B compactified on a Calabi-Yau manifold, χ counts the difference between the numbers of $\mathcal{N} = 2$ vector

multiplets and hypermultiplets: $\chi = \pm 4(n_V - n_H)$ (where the graviton multiplet counts as one vector). Moreover, in the non-compact limit, the Euler number can in general split in different singular points of the internal space, $\chi = \sum_I \chi_I$, giving rise to different localized terms at various points y_I of the internal space. A number of conclusions (confirmed by string calculations in [21]) can be reached by looking closely at (14)-(16):

     ▷ $M_p \gg M$ requires a large non-zero Euler characteristic for M_6, and/or a weak string coupling constant $g_s \to 0$.

     ▷ Since χ is a topological invariant the localized $\mathcal{R}_{(4)}$ term coming from the closed string sector is universal, independent of the background geometry and dependent only on the internal topology. It is a matter of simple inspection to see that if one wants to have a localized EH term in less than ten dimensions, namely something linear in curvature, with non-compact internal space in all directions, *the only possible dimension is four* (or five in the strong coupling M-theory limit).

     ▷ In order to find the width w of the localized term, one has to do a separate analysis. On general grounds, using dimensional analysis in the limit $M_P \to \infty$, one expects the effective width to vanish as a power of $l_P \equiv M_P^{-1}$: $w \sim l_P^\nu / l_s^{\nu-1}$ with $\nu > 0$. The computation of ν for a general Calabi-Yau space, besides its technical difficulty, presents an additional important complication: from the expression (16), $l_P \sim g_s l_s$ in the weak coupling limit. Thus, w vanishes in perturbation theory and one has to perform a non-perturbative analysis to extract its behavior. Alternatively, one can examine the case of orbifolds. In this limit, $c_0 = 0$, $l_P \sim l_s$, and the hierarchy $M_P > M$ is achieved only in the limit of large χ. One then finds that the width is given by the four-dimensional induced Planck mass

$$w \simeq l_P = l_s \, \chi^{-1/2} \,, \tag{17}$$

and the power $\nu = 1$.

5.3.1   Summary of the results

Using $w \sim l_P$ and the relations (16) in the weak coupling limit (with $c_0 \neq 0$), the crossover radius of eq. (13) is given by the string parameters ($n = 6$)

$$R_c = \frac{r_c^3}{w^2} \sim g_s \frac{l_s^4}{l_P^3} \simeq g_s \times 10^{32} \text{ cm} \,, \tag{18}$$

for $M_s \simeq 1$ TeV. Because R_c has to be of cosmological size, the string coupling can be relatively small, and the Euler number $|\chi| \simeq g_s^2 \, l_P \sim g_s^2 \times 10^{32}$ must be very large. The hierarchy is obtained mainly thanks to the large value of χ, so that lowering the bound on R_c lowers the value of χ. Our actual knowledge of gravity at very large distances indicates [32] that R_c should be of the order of the Hubble radius $R_c \simeq 10^{28}$ cm, which implies $g_s \geq 10^{-4}$ and $|\chi| \gtrsim 10^{24}$. A large Euler number implies only a large number of closed string massless particles with no a-priori constraint on the observable gauge and matter sectors, which can be introduced for instance on D3-branes placed at the position where gravity localization occurs. All these particles are localized at the orbifold fixed points (or where the Euler number is concentrated in the general case), and should have sufficiently suppressed gravitational-type couplings, so that their presence with such a huge multiplicity does not contradict observations. Note that these results depend crucially on the scaling of the width w in terms of the Planck length: $w \sim l_P^\nu$, implying $R_c \sim 1/l_P^{2\nu+1}$ in string units. If there are models with $\nu > 1$, the required value of χ will be much lower, becoming $\mathcal{O}(1)$ for $\nu \geq 3/2$. In this case, the hierarchy could be determined by tuning the string coupling to infinitesimal values, $g_s \sim 10^{-16}$.

     The explicit string realization of localized induced gravity models offers a consistent framework that allows to address a certain number of interesting physics problems. In particular, the effective UV cutoff and the study of the gravity force among

matter sources localized on D-branes. It would be also interesting to perform explicit model building and study in detail the phenomenological consequences of these models and compare to other realizations of TeV strings with compact dimensions.

Acknowledgments

References

[1] C. Angelantonj and A. Sagnotti, *Phys. Rept.* **371**, 1 (2002) [Erratum-ibid. **376**, 339 (2003)] [arXiv:hep-th/0204089].

[2] I. Antoniadis, *Phys. Lett.* B **246**, 377 (1990).

[3] C. D. Hoyle, D. J. Kapner, B. R. Heckel, E. G. Adelberger, J. H. Gundlach, U. Schmidt and H. E. Swanson, *Phys. Rev.* D **70**, 042004 (2004).

[4] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *Phys. Lett.* B **429**, 263 (1998) [arXiv:hep-ph/9803315]; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *Phys. Lett.* B **436**, 257 (1998) [arXiv:hep-ph/9804398].

[5] For a review see e.g. I. Antoniadis, *Prepared for NATO Advanced Study Institute and EC Summer School on Progress in String, Field and Particle Theory, Cargese, Corsica, France (2002)*; and references therein.

[6] J. D. Lykken, *Phys. Rev.* D **54**, 3693 (1996) [arXiv:hep-th/9603133].

[7] J. C. Long and J. C. Price, *Comptes Rendus Physique* **4**, 337 (2003); R. S. Decca, D. Lopez, H. B. Chan, E. Fischbach, D. E. Krause and C. R. Jamell, *Phys. Rev. Lett.* **94**, 240401 (2005); S. J. Smullin, A. A. Geraci, D. M. Weld, J. Chiaverini, S. Holmes and A. Kapitulnik, arXiv:hep-ph/0508204; H. Abele, S. Haeßler and A. Westphal, in 271th WE-Heraeus-Seminar, Bad Honnef (2002).

[8] G. F. Giudice, R. Rattazzi and J. D. Wells, *Nucl. Phys.* B **544**, 3 (1999); E. A. Mirabelli, M. Perelstein and M. E. Peskin, *Phys. Rev. Lett.* **82**, 2236 (1999); T. Han, J. D. Lykken and R. Zhang, *Phys. Rev.* D **59**, 105006 (1999); K. Cheung and W.-Y. Keung, *Phys. Rev.* D **60**, 112003 (1999); C. Balázs *et al.*, *Phys. Rev. Lett.* **83**, 2112 (1999); L3 Collaboration (M. Acciarri *et al.*), *Phys. Lett.* B **464**, 135 (1999) and **470**, 281 (1999): J. L. Hewett, *Phys. Rev. Lett.* **82**, 4765 (1999).

[9] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Rev.* D **59**, 086004 (1999).

[10] S. Cullen and M. Perelstein, *Phys. Rev. Lett.* **83**, 268 (1999); V. Barger, T. Han, C. Kao and R. J. Zhang, *Phys. Lett.* B **461**, 34 (1999).

[11] K. Benakli and S. Davidson, *Phys. Rev.* D **60**, 025004 (1999); L. J. Hall and D. Smith, *Phys. Rev.* D **60**, 085008 (1999).

[12] S. B. Giddings and S. Thomas, *Phys. Rev.* D **65**, 056010 (2002); S. Dimopoulos and G. Landsberg, *Phys. Rev. Lett.* **87**, 161602 (2001).

[13] I. Antoniadis, C. Bachas, *Phys. Lett.* B **450**, 83 (1999).

[14] I. Antoniadis, K. Benakli, A. Laugier and T. Maillard, *Nucl. Phys.* B **662**, 40 (2003) [arXiv:hep-ph/0211409].

[15] I. Antoniadis, S. Dimopoulos and G. Dvali, *Nucl. Phys.* B **516**, 70 (1998); S. Ferrara, C. Kounnas and F. Zwirner, *Nucl. Phys.* B **429**, 589 (1994).

[16] T. R. Taylor and G. Veneziano, *Phys. Lett.* B **213**, 450 (1988).

[17] I. Antoniadis, E. Kiritsis and J. Rizos, *Nucl. Phys.* B **637**, 92 (2002).

[18] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999) and *Phys. Rev. Lett.* **83**, 3370 (1999).

[19] G. R. Dvali, G. Gabadadze and M. Porrati, *Phys. Lett.* B **485**, 208 (2000).

[20] H. Verlinde, *Nucl. Phys.* B **580**, 264 (2000); S. B. Giddings, S. Kachru and J. Polchinski, *Phys. Rev.* D **66**, 106006 (2002).

[21] I. Antoniadis, R. Minasian and P. Vanhove, *Nucl. Phys.* B **648**, 69 (2003) [arXiv:hep-th/0209030].

[22] For a recent review see e.g. R. Maartens, Living Rev. Rel. **7**, 7 (2004) [arXiv:gr-qc/0312059]; same proceedings and references therein.

[23] E. Kiritsis, N. Tetradis and T. N. Tomaras, *JHEP* **0108**, 012 (2001).

[24] M. A. Luty, M. Porrati and R. Rattazzi, arXiv:hep-th/0303116.

[25] G. R. Dvali and G. Gabadadze, *Phys. Rev.* D **63**, 065007 (2001); G. R. Dvali, G. Gabadadze, M. Kolanovic and F. Nitti, *Phys. Rev.* D **64**, 084004 (2001).

[26] S. L. Dubovsky and V. A. Rubakov, *Phys. Rev.* D **67**, 104014 (2003) [arXiv:hep-th/0212222].

[27] V. A. Rubakov, arXiv:hep-th/0303125.

[28] M. Kolanovic, M. Porrati and J. W. Rombouts, *Phys. Rev.* D **68**, 064018 (2003) [arXiv:hep-th/0304148].

[29] I. Antoniadis, E. Gava and K. S. Narain, *Phys. Lett.* B **283**, 209 (1992).

[30] I. Antoniadis, C. Bachas, C. Fabre, H. Partouche and T. R. Taylor, *Nucl. Phys.* B **489**, 160 (1997); I. Antoniadis, H. Partouche and T. R. Taylor, *Nucl. Phys.* B **499**, 29 (1997).

[31] I. Antoniadis, S. Ferrara, R. Minasian and K. S. Narain, *Nucl. Phys.* B **507**, 571 (1997).

[32] A. Lue and G. Starkman, *Phys. Rev.* D **67**, 064002 (2003).