

String Theory and its challenges

I

Historical overview

- String theory has reached the respectable age of 43
- Hadronic birth
- Desert crossing, the 70's with some treasures along the way
- The 1984 revolution, 1st quantised decade, phenomenology, CFT mirror symmetry...
- The dual decade, S-T-U duality, branes
- Phenomenology explosion
- AdS/CFT correspondence
- Application in Heavy Ion Collisions, fluid/gravity correspondence
- String Cosmology, the String Landscape
- Integrability, MHV technology, N=8 finiteness....



Some general remarks

There are so many possible phases of the theory that we still do not know which one (if any) corresponds to the world we live in, with its low-energy properties

There are however properties that are quite insensitive to this and which shed important light into our understanding of Black Holes, QFT with and without supersymmetry

There are instances where we can get important information on strongly coupled systems that are not accessible without the use of string theory tools

String appear also in other contexts: QCD and superconductivity. Thanks to AdS/CFT we may study some of their properties in well-defined environments

On a more philosophical side, the large number of options has been used to propose a tentative solution to various fine-tuning problems. The cosmological constant being one of them. To some appealing to the anthropic principle is rather unpalatable, but it may end up being the only solution. There may be many basic properties of our universe that are purely "environmental" without fundamental explanation.



The fate of theorems in Physics

There is a famous theorem in QFT: The Coleman-Mandula Theorem. What is remarkable is that the assumptions that looked reasonable at the time, when weakened have yielded two remarkable new symmetries in QFT:

Supersymmetry

Yangians although this one may be more attached to the large-N limit

In the 1st case because the conserved charges cannot be used to label external states

In the second because bosonic conserved charges do not act in terms of tensor products in multi-particle states, as seen schematically below suppressing all indices

Then one can have a non-trivial S-matrix with the required analyticity properties required by the C-M theorem, and yet a group of symmetries bigger than Poincare $\{Q_a^I, \bar{Q}_{\dot{b}}^J\} = 2 \sigma^{\mu}_{a\dot{b}} \delta^{IJ} + Z^{IJ}$

 $\Delta(J) = J \otimes 1 + 1 \otimes J \Longrightarrow \Delta(J) = J \otimes 1 + 1 \otimes J + \sum J \otimes J$



Hadronic origin of Strings



Think string...



Fundamental I-dimensional objects have very remarkable properties, they come in two varieties, open and closed

The Veneziano amplitude can be reproduced

They admit a field theory limit

There is a single interaction vertex

Consistency puts stringent constraints

String Theory is the origin of supersymmetry

Space-time has dimension= 10 !!

Gravity and gauge symmetry are not unified, they are inseparable

• Only known fundamental origin of general covariance and gauge invariance.

First quantisation, CFT

The first quantisation of the theory led to a very fruitful interplay between string theory, CFT, the understanding on how space-time supersymmetry is embedded in the world-sheet theory. The basic objects in 1st quantisation contain the embedding of a superriemann surface into some target manifold or a more abstract object. Questions relating to finiteness, general behavior of the scattering amplitudes at higher genus were considered in detail, and at the same time there was a large amount of work on perturbative KK compactifications.

It was argued from Lorentz invariance that for open strings only Nbc's were necessary. We had to wait for Polchinski to tell us how wrong we were, and to open up the Brane revolution.

For the closed string we have L,R oscillators. Only one type for the open string due to the BC's. Consistency requirements imply that the world-sheet 2D theory should be a CFT. The spectrum of physical operators fit into well defined representation of the Virasoro algebra. The L's are the components of the EM tensor in 2D. The central extension c is related to the dimension of spacetime





The lightest physical state of the open string corresponds to massless vector bosons. Those of the close string contain a dilaton, a graviton and an anstisymmetric two-form field. Gauge field emitted from the end of the open strings

$$S_g = rac{1}{4\pilpha'} \int_M \sqrt{h} \, h^{lphaeta} g_{\mu
u}(X) \partial_lpha X^\mu \partial_eta X^
u d^2 z.$$

$$S_B = \frac{1}{4\pi\alpha'} \int_M \varepsilon^{\alpha\beta} B_{\mu\nu}(X) \partial_\alpha X^\mu \partial_\beta X^\nu d^2 z$$

$$egin{aligned} S_{\Phi} &= rac{1}{4\pi} \int_M \sqrt{h} \, \Phi(X) R^{(2)}(h) \, d^2 z \ S_A &= q \, \int A_\mu \dot{x}^\mu d au. \end{aligned}$$

The graviton and GR in ST



Requiring conformal invariance yields the string form of the Einstein equations. For the bosonic string the critical dimension is 26, for the fermionic string it is 10, and (we still have to deal with the tachyon field). Note that the expectation value of the dilaton acts as the string coupling constant

$$S_{\text{eff.}} = \frac{1}{2k_0^2} \int d^{26} X \, (-G)^{1/2} \, e^{-2\phi} \left\{ R - \frac{1}{12} \, H_{MNP} \, H^{MNP} + 4 \partial_M \phi \partial^M \phi \right\} + \mathcal{O}(\alpha')$$

Including fermions

Fermions are easy to include in the worldsheet, and they come in two varieties related to periodic (Ramond) or anti-periodic (Neveu-Schwarz) boundary conditions.

The R-fermions have zero modes, hence their canonical anticommutation relations satisfy a Clifford algebra, this is the origin of space-time fermion.

The two-dimensional theory in fact has a full 2d superconformal algebra, and similar physical conditions as those of the bosonic theory.

We have left and right moving theories, and in principle also tachyon. The great discovery of GSO was to show the existence of projection that cancels the tachyons, maintains locality, and generates space-time supersymmetry.

By studying N=2 supersymmetry on the worldsheet BFMS determined under what conditions the world-sheet theory implies space-time supersymmetry.

GS constructed the explicit formulation of space-time superstrings in d=10: Type-I and Type-IIA IIB characterised by the number of chiral space-time supersymmetries

$$S = -\frac{1}{2\pi} \int d^2\sigma \left(\partial_lpha X_\mu \partial^lpha X^\mu + ar{\psi}^\mu
ho^lpha \partial_lpha \psi_\mu
ight),$$

 $\psi_\pm(\sigma) = \pm \psi_\pm(\sigma + \pi),$

$$\psi^{\mu}_+(\sigma,\tau) = \sum_{n \in \mathbb{Z}} \tilde{d}^{\mu}_n e^{-2in(\tau+\sigma)} \qquad \text{or} \qquad \psi^{\mu}_+(\sigma,\tau) = \sum_{r \in \mathbb{Z}+1/2} \tilde{b}^{\mu}_r e^{-2ir(\tau+\sigma)}.$$

$$\psi^{\mu}_{-}(\sigma,\tau) = \sum_{n \in \mathbb{Z}} d^{\mu}_{n} e^{-2in(\tau-\sigma)} \quad \text{ or } \quad \psi^{\mu}_{-}(\sigma,\tau) = \sum_{r \in \mathbb{Z}+1/2} b^{\mu}_{r} e^{-2ir(\tau-\sigma)},$$

$$\begin{split} [L_m,L_n] &= (m-n)L_{m+n} + \frac{D}{8}m(m^2-1)\delta_{m+n,0},\\ [L_m,G_r] &= \left(\frac{m}{2} - r\right)G_{m+r},\\ \{G_r,G_s\} &= 2L_{r+s} + \frac{D}{2}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}. \end{split}$$

$$(-1)^{F_{\rm NS}} = -1,$$



Types I, II A B

The massless spectrum of these theories differ substantially. They both share a dilaton, 2-form and graviton coming from the NS,NS sector.

There are bosonic fields coming from the RR-sector. In the IIA theory they are odd-form potentials (1,3) while IIB contains even potentials (0,2,4+), where the latter means a four-form field whose field strength is a self-dual five form

On the fermionic side IIA contains two Weyl-Majorana gravitinos of opposite chirality and two Weyl-Majorana spin $\frac{1}{2}$ fermions of opposite chirality

The IIB theory contains two gravitinos of the same chirality and two WM spin $\frac{1}{2}$ with the same chirality and opposite to the one of the gravitinos. This was the first theory to be shown to be anomaly free in ten dimensions.

Type-I contains N=I supergravity coupled to N=I super-YM theory with SO(N) or Sp(N) gauge groups.

After the work of GS it was shown that the only groups where the anomalies could cancel were SO(32) and E8 \times E8



Heterotic theories

Soon after the discovery of the anomaly cancellation, GHMR constructed the heterotic string by combining a left-moving Type I superstring and a 26-dimensional right moving string, where the extra 16 dimensions have to be compactified on a 16 dimensional even self-dual lattice. There are only two of those: the root lattices of E8+E8 and Spin(32)/Z2

This began an explosion of work in studying their structural properties as well as their possible phenomenological implications. The explosion has continued since. CHSW in fact showed that the simplest KK compactification involve very special manifolds: complex 3 dimensionalCalabi-Yau manifolds with rather unique properties. At the same time DHVW constructed a large class of very tractable compactifications based on orbifolds, which often can be seen as special limits of CY compactifications.

There there were all kinds of objects constructed: Gepner models, coset models, linear dilaton theories, compatifications with vector-bundles, F-theory formulations.

With the advent of branes and flux compactifications, the taxidermy and variety of possible low energy theories is remarkably large. It can contain the SM of particle physics, but it contains many more possibilities, including most of the BSM scenarios constructed so far, the last word has not yet been spoken...



Remarks on finiteness

The question of finiteness is a rather long and delicate one, truffled with technicalities. In doing string loop integrals the analog of Feynman parameters are the moduli of Riemann surfaces. These integrals may have singularities associated to the boundary of moduli space, where surfaces degenerate.

It is believe that for superstring the GSO projection generates finite and well-defined answers, at least for strings in flat space.

Are there string-loop divergences correcting Einstein's equations? Do they induce additional constraints on the spectrum for more complicated manifolds? A rather large consensus is that they do not













Since we live in 4 dimensions, the extra 6 have to be curled up into closed manifolds. Not any manifold qualifies, the simplest are CY-folds.

The extra dimensions have to be curled up into interesting algebrogeometric structures, Calabi-Yau manifolds and their generalisations once branes and other objects are included.

Exotic objects to say the least, that could populate the extra dimensions to determine the basic quantum numbers of the particles we observe at low energies, as well as their numbers, Yukawa couplings etc

$$Z_{1}^{4} + Z_{2}^{4} + Z_{3}^{4} + Z_{4}^{4} = 0 \quad \text{in} \qquad CP^{3}$$
$$Z_{1}^{5} + Z_{2}^{5} + Z_{3}^{5} + Z_{4}^{5} + Z_{4}^{5} = 0 \quad \text{in} \qquad CP^{4}$$
$$N_{g} - N_{\bar{g}} = \frac{1}{2} |\chi|$$
$$y_{ijk} = \int_{K} \omega_{i} \wedge \omega_{j} \wedge \omega_{k}$$



T-duality

The probing of the embedding geometry by string theory is more subtle than it might seem at first sight. The first example of this, is a perturbative string symmetry that is called T-duality.

On a p-dimensional toroidal compactification the full duality group is rather large, both for ordinary strings and for heterotic strings



$$X^{25}(\sigma,\tau) = x^{25} + 2\alpha' p^{25}\tau + 2RW\sigma + \dots,$$

$$lpha' M^2 = lpha' \left[\left(rac{K}{R}
ight)^2 + \left(rac{WR}{lpha'}
ight)^2
ight] + 2N_{
m L} + 2N_{
m R} - 4.$$
 $R \rightarrow \widetilde{R} = lpha'/R$

Duality Group = SO(p, p, Z) or SO(p, p + 16, Z)



Electric-Magnetic duality

In QFT we are familiar with the Dirac-Schwinger Zwanziger quantisation rule. In a theory with electric and magnetic charges, dyons, the possible states fall into a lattice, whose simplest examples is the



$$\mathcal{L} = Z + \tau Z; \qquad \tau = \frac{\theta}{2\pi} + \frac{4\pi}{g^2} i$$

E&M duality, DSZ quantisation

$$\nabla \cdot (\mathbf{E} + \mathbf{i}\mathbf{B}) = \rho_e + i\rho_m$$
$$i\frac{\partial}{\partial t}(\mathbf{E} + \mathbf{i}\mathbf{B}) - \nabla \wedge (\mathbf{E} + \mathbf{i}\mathbf{B}) = -(\mathbf{j_e} + \mathbf{i}\mathbf{j_m})$$
$$\operatorname{Im}(q_1 + iq_1)(q_1 + iq_1)^* = 2\pi n, \quad n \in \mathbb{Z}$$

In E&M we cannot consider monopoles and electrons at the same time. In non-abelian gauge theories this is a different issue. In fact, in supegravity theories this form of duality is vastly generalised and provides the low-energy basis for S-duality in String theory. The general features are:

> particles \iff solitons $g \longleftrightarrow \frac{1}{g}$ weak \iff strong



S-T-U Dualities

The Cremmer-Julia continuous groups are not symmetries of the compactification of M-theory once non-perturbative semiclassical states are taken into consideration. Discrete subgroups survive, they form the U-duality group.

It contains the T- and S-duality transformations. A generalised version of electric-magnetic duality.

T-duality as shown before is world-sheet E-M duality, while S-duality is space-time E-M duality.

In the context of Type IIB in d=10 we have a SL(2,Z) Sduality group. After toroidal compactification, we have a T-duality group SO(10-6,10-d, Z) which does not commute with SL(2,Z). They generate the U-duality group

D	d	$E_{d(d)}(\mathbb{R})$	$E_{d(d)}(\mathbb{Z})$
10	1	1	1
9	2	$Sl(2,\mathbb{R})$	$Sl(2,\mathbb{Z})$
8	3	$Sl(3,\mathbb{R}) \times Sl(2,\mathbb{R})$	$Sl(3,\mathbb{Z}) \times Sl(2,\mathbb{Z})$
7	4	$Sl(5,\mathbb{R})$	$Sl(5,\mathbb{Z})$
6	5	$SO(5,5,\mathbb{R})$	$SO(5,5,\mathbb{Z})$
5	6	$E_{6(6)}(\mathbb{R})$	$E_{6(6)}(\mathbb{Z})$
4	7	$E_{7(7)}(\mathbb{R})$	$E_{7(7)}(\mathbb{Z})$
3	8	$E_{8(8)}(\mathbb{R})$	$E_{8(8)}(\mathbb{Z})$







Using compactifications, T- S-dualities one can see that the apparently distinct 5 theories I, IIA, IIB, HO, HE and M-theory turn out to be different phases of the same theory whose intrinsic formulation still escapes us. In going from M to HE one has to compactify M-theory on an interval.

The "interval" compactification follows from a remarkable work of Horava-Witten that is at the origin of subsequent work of LED, RS etc. It once again shows the fate of theorems in physics. In the mid 80's Witten used the Atiyah-Hirzebruch to exclude the generation of chirality from pure KK compactification. The key is to do compactification on a manifold with boundary, the Z2 orbifolding S^I to an interval.

If we compactify with more sophisticated tools, including brane-intersections, fluxes etc, we end up in the String Landscape







In 1996 Polchinski found the elephants in the room that were roaming freely without being noticed. The Dboundary conditions are perfectly acceptable, they tell us of existence of extended object in the theory, the Dbranes, like BPS states, they have fascinating properties and deeply transformed the subject in all possible ways, including having a highly non-trivial impact on our understanding of BHs and also opening the way to the next deep revolution, the AdS/CFT or Maldacena conjecture





Brane intersection theories







The proliferation of models is staggering. They come with many scalar fields whose expectation values have to be stabilised. They provide models compatible with the SM but also with many of its extensions. It is not known how to choose among them. There is no selection principle. This makes String Theory very hard to test. So far we have not yet found a theory that contains the SM without undesired features. Low energy supersymmetry is not a necessary prediction of the theory. This may simply be a technical problem, or there may be some yet unknown fundamental

properties of String Theory that we do not know



BHs and Hawking radiation





$$T_{H} = \frac{\hbar c^{3}}{8\pi GMk} \qquad S = \frac{A_{h}}{4A_{P}} \qquad A_{P} = \frac{\hbar G_{N}}{c^{3}}$$
$$T = 10^{-8} K \left(\frac{M_{Sun}}{M}\right) \qquad \tau = \tau_{Univ} \left(\frac{M}{10^{12} \text{Kg}}\right)^{3} \qquad S = \log \Omega(M, J, Q)$$

This is a huge entropy. This is larger than you might think No-hair theorems tell that the final BH state depends on 3 parameters ρ_{out} Where are the missing states? One bit per Planck area

$$Tr\rho^2 = 1 \rightarrow Tr\rho^2 < 1$$

$$\rho_{out} = \$ \rho_{in} \qquad \$ \neq S \otimes S^{\dagger}$$
$$\Delta x \sim \frac{\hbar}{E} + l_s^2 E$$

String count of BH states



One of the major theoretical achievements of String Theory has been to provide a way of reproducing the BH entropy formula in terms of a genuine state count, a la Boltzmann. They in fact provide Wald's generalisation.

Strominger and Vafa considered a type IIB theory compactified in K3 \times S1, the corresponding BPS states they counted carried axion and electric charges. By using Cardy's formula counting the number of states at a given level in a CFT, they found the following remarkable result:

$$S_{stat} = 2\pi \sqrt{Q_H(\frac{1}{2}Q_F^2 + 1)}, \qquad S_{BH} = 2\pi \sqrt{\frac{Q_H Q_F^2}{2}}$$

Very active area of research: to include the stringy and quantum corrections, understanding why BH evaporation does not violate QM in detail. Some approaches involve fuzzballs, LQG...



Holographic principle



'tHooft's entropy bound

In QFT suppose we have a system with energy E, volume V and area A. How many states would a QFT allow under these conditions, rather what is the largest number of states? The most probable state would be a gas at some temperature T, with energy and entropy:

To avoid falling inside its Schwarzschild horizon, we need to require:

The maximal entropy however is the one given by the BH area law S=A/4!

$$E \sim n_{d.o.f.} T^4 \qquad S \sim n_{d.o.f.} T^3$$

 $2E < (V/\frac{4}{3}\pi)^{1/3}$

$$S < n^{1/4} A^{3/4}$$



AdS/CFT Maldacena conjecture

In gravity there are no local observables, hence it makes sense to parametrize the quantum state of gravity in a given volume in terms of degrees of freedom on the boundary of the bulk region.

Non-locality at the horizon appears always one way or another in solving the puzzle

$$-X_0^2 + X_1^2 + \dots + X_d = R^2$$

$$-X_{-1}^2 - X_0^2 + X_1^2 + \dots + X_{d-1} = -R^2$$

$$SU(N) \qquad g_{YM} \qquad \left(\frac{R}{l_s}\right)^4 = g_{YM}^2 N$$

Gravitational processes in the bulk can be represented in terms of gauge states in the boundary. Since gauge theories represent well defined unitary gauge theories, the process of BH formation and evaporation should be described. So far however, there are too many details missing...

GAUGE----GRAVITY DUALITY





Maldacena's abstract

Abstract

We show that the large N limit of certain conformal field theories in various dimensions include in their Hilbert space a sector describing supergravity on the product of Anti-deSitter spacetimes, spheres and other compact manifolds. This is shown by taking some branes in the full M/string theory and then taking a low energy limit where the field theory on the brane decouples from the bulk. We observe that, in this limit, we can still trust the near horizon geometry for large N. The enhanced supersymmetries of the near horizon geometry correspond to the extra supersymmetry generators present in the superconformal group (as opposed to just the super-Poincare group). The 't Hooft limit of 3+1 $\mathcal{N} = 4$ super-Yang-Mills at the conformal point is shown to contain strings: they are IIB strings. We conjecture that compactifications of M/string theory on various Anti-deSitter spacetimes is dual to various conformal field theories. This leads to a new proposal for a definition of M-theory which could be extended to include five non-compact dimensions.

$$\lambda = g_{\rm YM}^2 N, \qquad g_{\rm YM}^2 = 4\pi g_s, \qquad R = \lambda^{1/4} l_s$$

$$ds^{2} = \frac{dz^{2} + \eta_{\mu\nu}dx^{\mu}\,dx^{\nu}}{z^{2}} + ds_{K}^{2}$$



Some applications

The number of applications and the ramifications of the conjecture are staggering, and will occupy many of the theoretical talks at this school

Hawking-Page transition in AdSBH and Confinement-Deconfinement transition, Unitary BH evolution...

Integrability of N=4 SYM in d=4 in the planar strong coupling limit. The amount of results collected so far is very impressive, and at least in this limit we may be close to a proof of the conjecture.

Anomalous dimensions related to integrable spin chains: Bethe ansatz, thermodynamic Bethe ansatz, contains integrable chain in the BFKL limit discovered by Faddeev and Korchemsky. Type IIB in the weak coupling limit on AdS5 \times S5 satisfies integrability as well

Inteplay with MHV amplitudes, light-like Wilson loops, minimal surfaces... This is a booming area where results come out almost weekly...

Lower-dimensional instances of conjecture, ABJM, AdS3/CFT2,...

Gravity-Fluid duality

"Real" applications: Strongly coupled non-abelian plasma, deconfined QCD, shear viscosity prediction and other transport properties...

AdS/QCD, AdS/CMT, etc., etc... You will hear plenty during the conference



More topics

There has been a lot of activity in cosmology. GV, B-V started the use of String Theory in cosmology, prebig-bang scenarios...

There are also large numbers of brane-like scenarios, KKLT-like theories, many candidates for inflatons, several ways of solving the \eta problem in supergravity cosmology... Still no distinctive natural model has emerged

We expect that BH formation and evaporation should be describable by a unitary process in the context of AdS/CFT but many details are still missing, like for instance if there is a holographic description of the BH interior, if it ever forms... The problem is rather complex, hence it is not clear the level of progress to be achieved in the future



Farewell

The amount of progress in String Theory since its discovery has been remarkable

Many new ideas with profound implications in our understanding of Gravity and Gauge Theory have been generated: Holography, Supersymmetry, Branes,... We have learned a great deal of new things in QFT, although for the time being mostly with a lot of supersymmetry, and also deep results on the theory of BHs and strong gravity.

Integrability in the large-N limit SYM theory in d=4 may be around the corner

We have been less lucky in low-E phenomenology. The amount of models is large, but still do not have a sound candidate theory for the SM with some distinct predictions. One cannot make LHC-accessible predictions. We should concentrate on results that depend less on the details of the vacuum we live in

Interesting "experimental" applications in deconfined QCD, HIC, perhaps even Condensed Matter Theory

This decade could be a golden decade with plenty of experimental discoveries, one might hope that some of them will provide hints on the UV completion of gravity and perhaps on String Theory





Thank you