Les Houches, Aug. 4th and 5th 2011

High-energy collisions of particles, strings and branes: II

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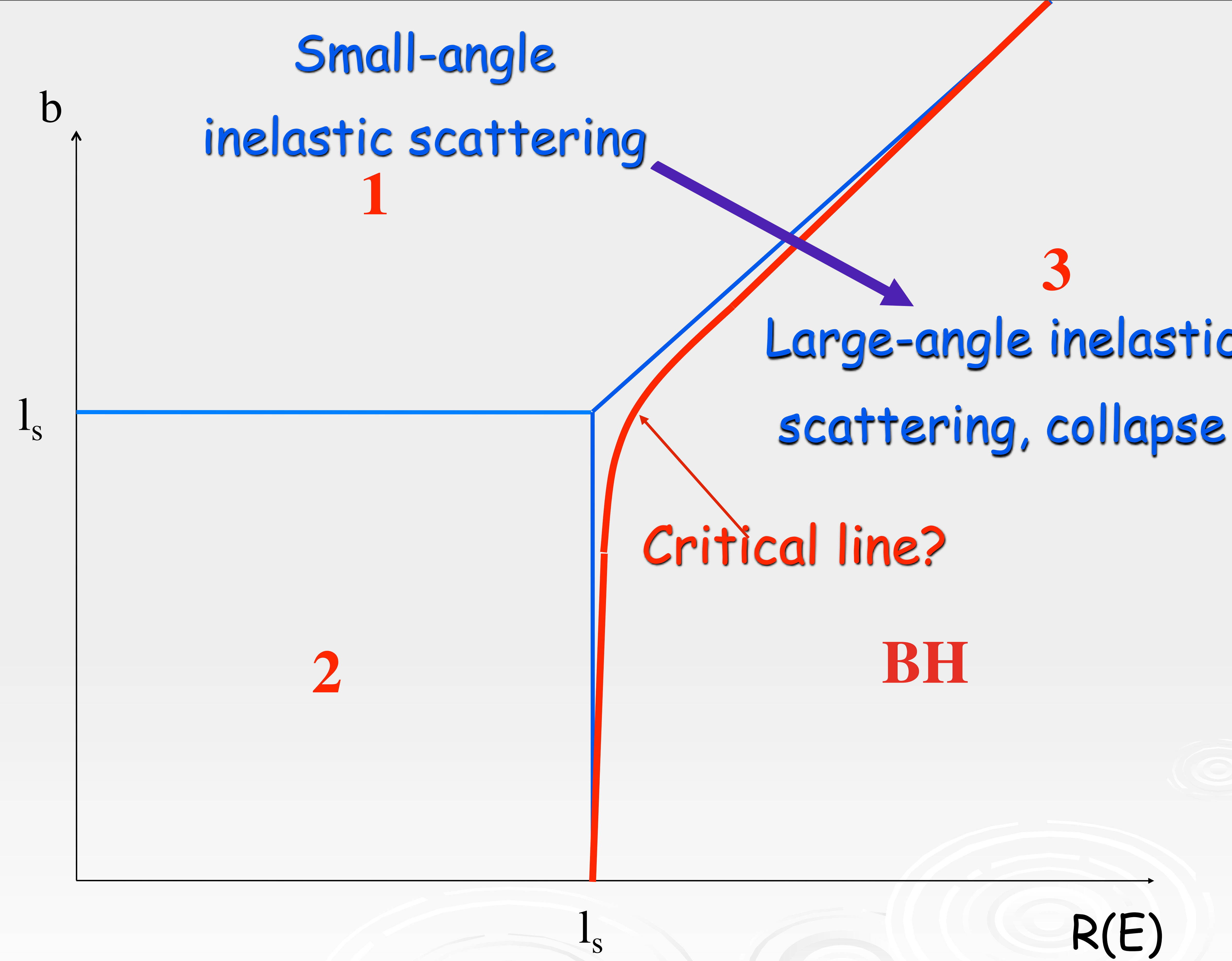
Outline

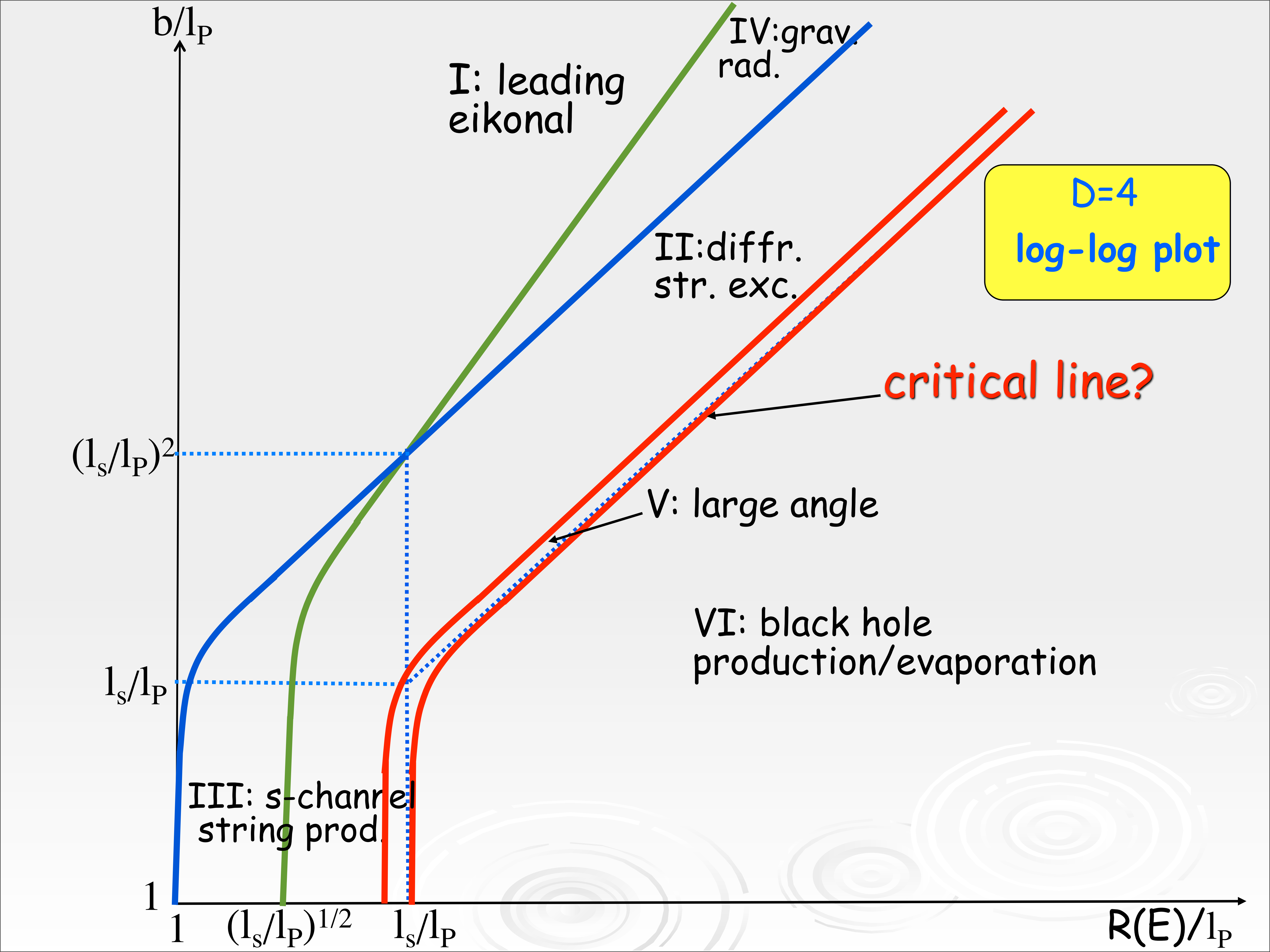
Lecture I

- GR collapse criteria: a brief review.
- Transplanckian energy collisions of particles and strings:
  - The small-angle regime: deflection & tidal forces
  - The stringy regime & precocious BH behaviour

Lecture II

- Transplanckian energy collisions of particles and strings:
  - The large-angle/collapse regime
- High-energy string-brane collisions: an easier problem?
- Outlook, conclusions.


Small-angle
inelastic scattering
1
b
l_s
3
Large-angle inelastic
scattering, collapse
Critical line?
2
BH
l_s
R(E)



b/l_P
I: leading eikonal
IV:grav. rad.
II:diffr. str. exc.
D=4
log-log plot
critical line?
(l_s/l_P)^2
V: large angle
VI: black hole production/evaporation
l_s/l_P
III: s-channel string prod.
1
1
(l_s/l_P)^1/2
l_s/l_P
R(E)/l_P

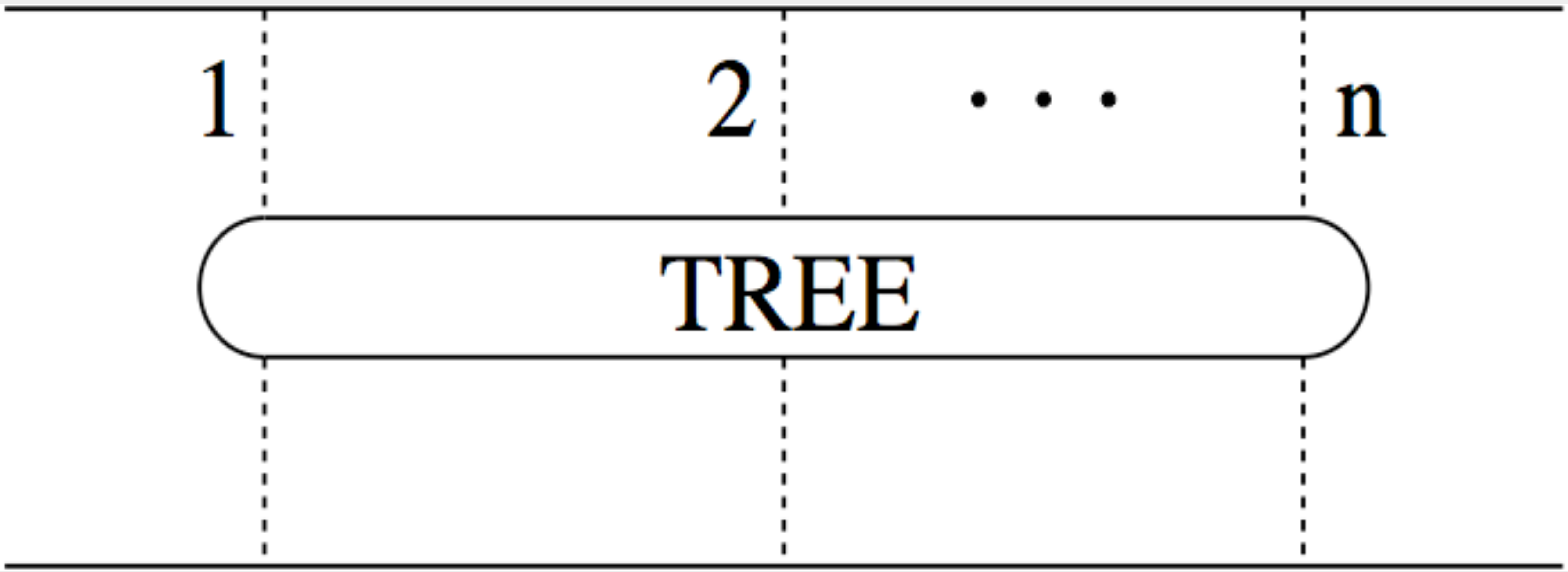
Particle-particle scattering as b->R

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \sim exp\left(-i\frac{Gs}{\hbar}(logb^2 + O(R^2/b^2) + O(l_s^2/b^2) + O(l_P^2/b^2) + \dots)\right)$$

From small to large-angle inelastic scattering... all the way to grav.[al] collapse?

(ACV, hep/th-0712.1209, MO, VW, CC...'08)

Classical corrections are related to "tree diagrams"



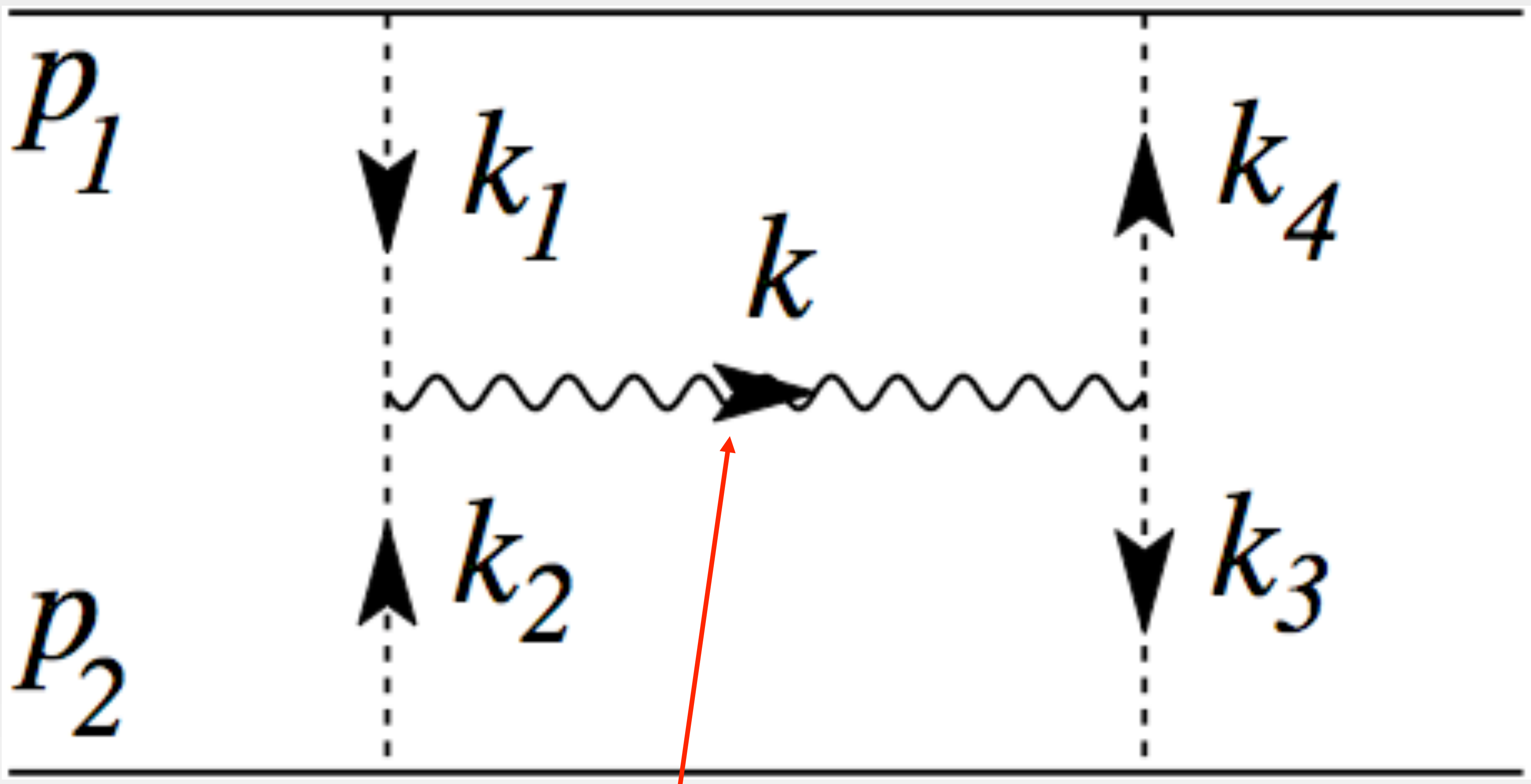
Power counting for connected trees:

$$A_{cl}(E,b) \sim G^{2n-1} s^n \sim Gs\ R^{2(n-1)} \to Gs\ (R/b)^{2(n-1)}$$

Summing tree diagrams => solving a classical field theory.

Q: Which is the effective field theory for TP-scattering?

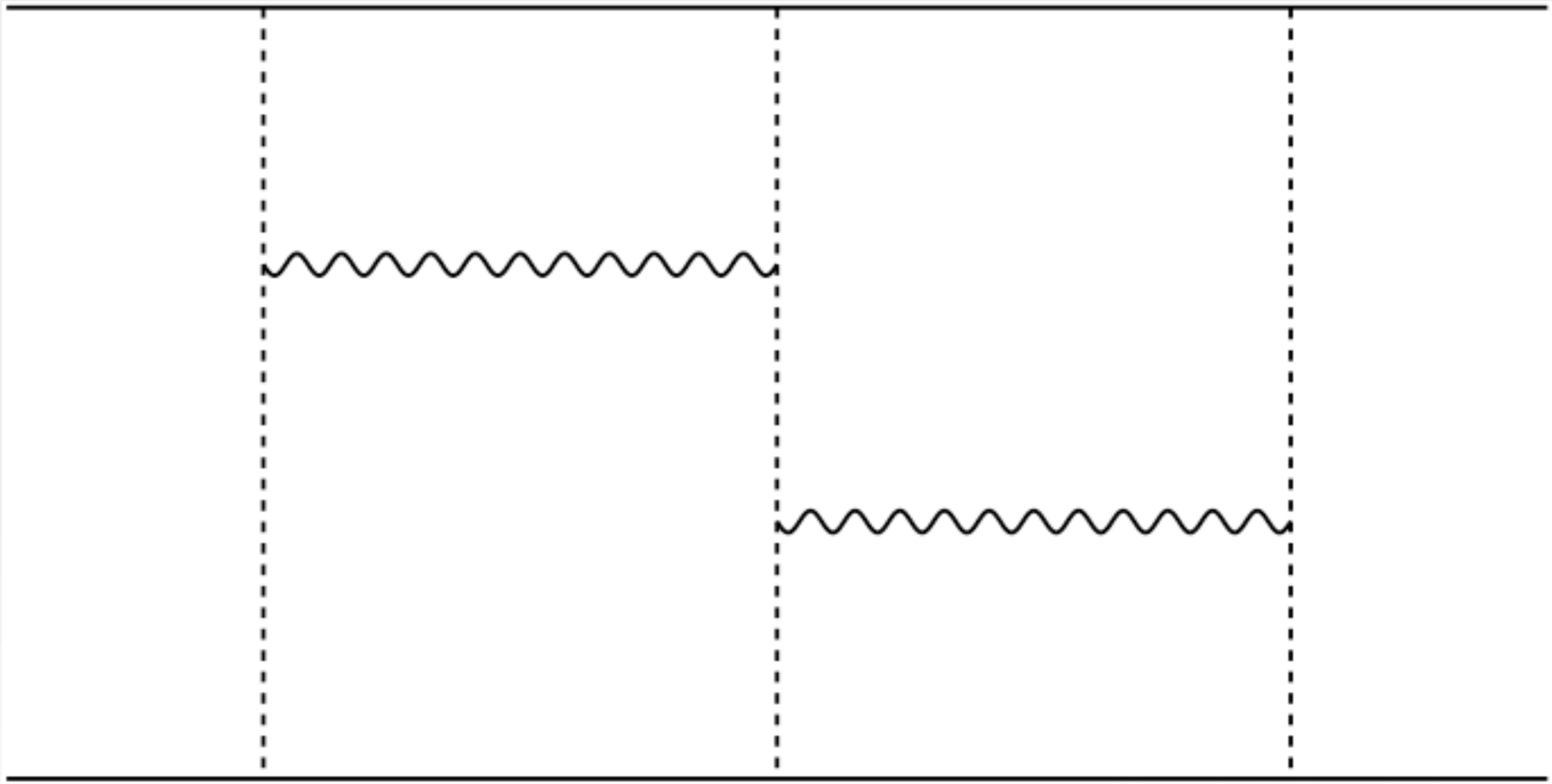
Next to leading order: the H diagram



$$\sim G^3 s^2 = Gs\ G^2 s = GsR^2 \to Gs\ (R/b)^2$$

One of the produced graviton's polarizations ("TT") is IR-safe the other ("LT") is not

NNL-order



$$\sim G^5 s^3 = Gs\ G^4 s^2 = GsR^4 \to Gs\ (R/b)^4$$

Reduced effective action & field equations

There is a D=4 effective action generating the leading diagrams (Lipatov, ACV '93). Too hard to solve!

After (approximately) factoring out the longitudinal dynamics: a D=2 effective action containing 4 fields:

the ++ and -- components of the metric, sourced by the EMT of the two fast particles; a complex field φ representing physical gravitons. One polarization is affected by IR problems and is neglected...

The semiclassical approximation amounts to solving the eom and to computing the classical action on the solution. Still too hard for analytic study...

Numerical solutions

(G. Marchesini & E. Onofri, 0803.0250)

Solved directly PDEs by FFT methods w/ Matlab
Result: real, regular solutions only exist for

$$b > b_c \sim 2.28R$$

Compare with EG's CTS lower bound on b_c

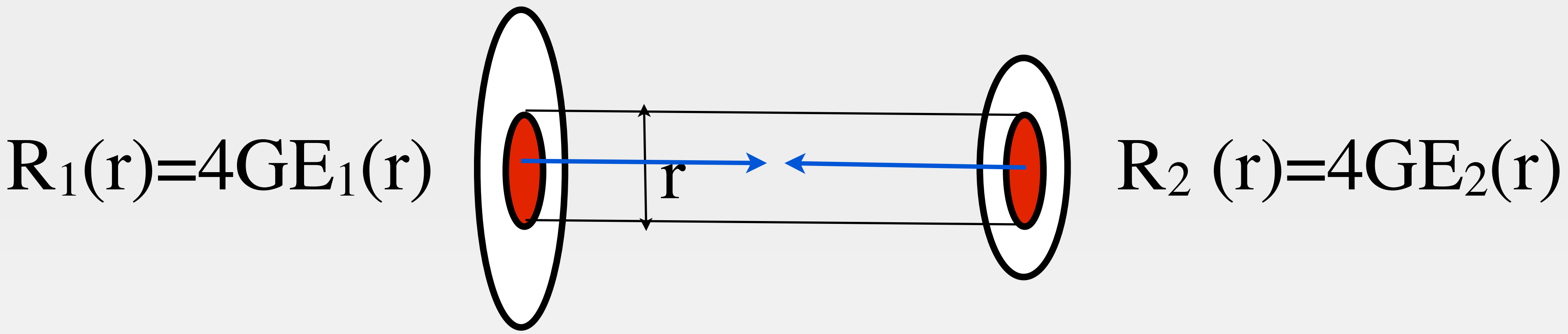
$$b_c > 0.80R$$

b_c is a factor ~ 2.85 above CTS's lower bound

For analytic study we turn to a simpler problem

Axisymmetric beam-beam collisions

(ACV '07, J.Wosiek & GV '08)



A simpler, yet rich, problem:

1. The sources contain several parameters & we can look for critical surfaces in their multi-dim.^{al} space

2. The CTS criterion is simple (see below)

3. Numerical results are coming in (see CP, 2009)

4. "Bad" polarization not produced

5. Last but not least: PDEs become ODEs

Main Results

ACV vs. CTS

Criterion for existence of CTS (KV): if there exists an r_c s.t.

$$R_1(r_c)R_2(r_c) = r_c^2$$

we can construct a CTS and therefore a BH must form.

Theorem (VW08): whenever the KV criterion holds the ACV field equations do not admit regular real solutions.

Thus:

KV criterion ==> ACV criterion

but not necessarily the other way around!

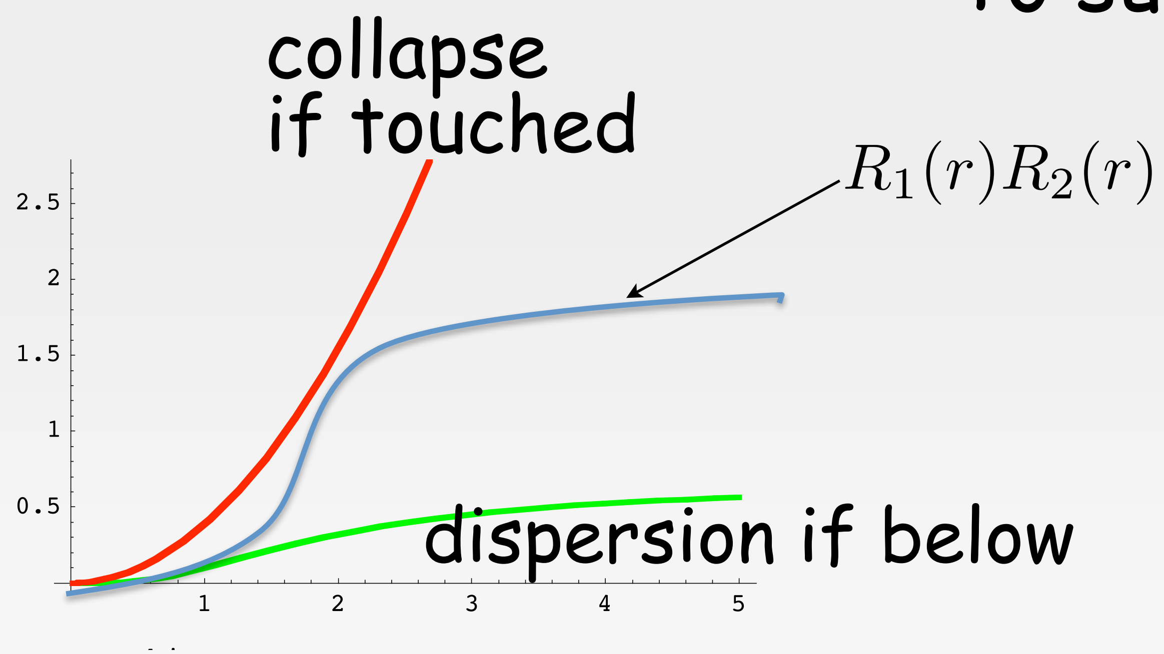
A sufficient criterion for ACV slns.

(P.-L. Lions, private comm.)

If, for all r, $\frac{R_1(r)R_2(r)}{R^2} \leq \frac{4r^4}{(3R^2+2r^2)^2}\left[1-\frac{R^2}{2r^2}\log(1+\frac{2r^2}{3R^2})\right]^2$

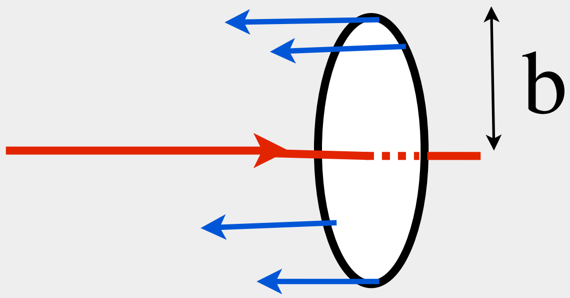
the ACV eqns do admit regular, real solutions.

To summarize



clearly, there is room for improvement...

Examples: 1. Particle-scattering off a ring



Can be dealt with analytically:

=> cubic equation. Has real solutions iff

$$b^2 > \frac{3\sqrt{3}}{2}R^2 \equiv b_c^2$$

$(b/R)_c \sim 1.61$

CTS: $(b/R)_c > 1$

2. Two homogeneous beams of radius L

$$\left(\frac{R}{L}\right)_{cr} \sim 0.47$$ vs. CTS $$\left(\frac{R}{L}\right)_{cr} < 1.0$$

3. Two Gaussian-shaped beams of width L

L_c is a factor ~ 2.70 above CTS's lower bound

An amusing coincidence?

In 0908.1780 Choptuik & Pretorius analyzed a “similar” situation numerically (relativistic central collision of two solitons of fixed mass and transverse size).

BH formation occurs at a critical γ_c (i.e. R_c) which is a factor 2-3 below the naive CTS value (but still in the relativistic regime)

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Conclusions on string-string collisions

The above results are encouraging but real control over the different approximations is lacking, in particular on the freezing of longitudinal dynamics.

This is probably at the origin of some puzzles we find in connection with gravitational radiation at b >> R.

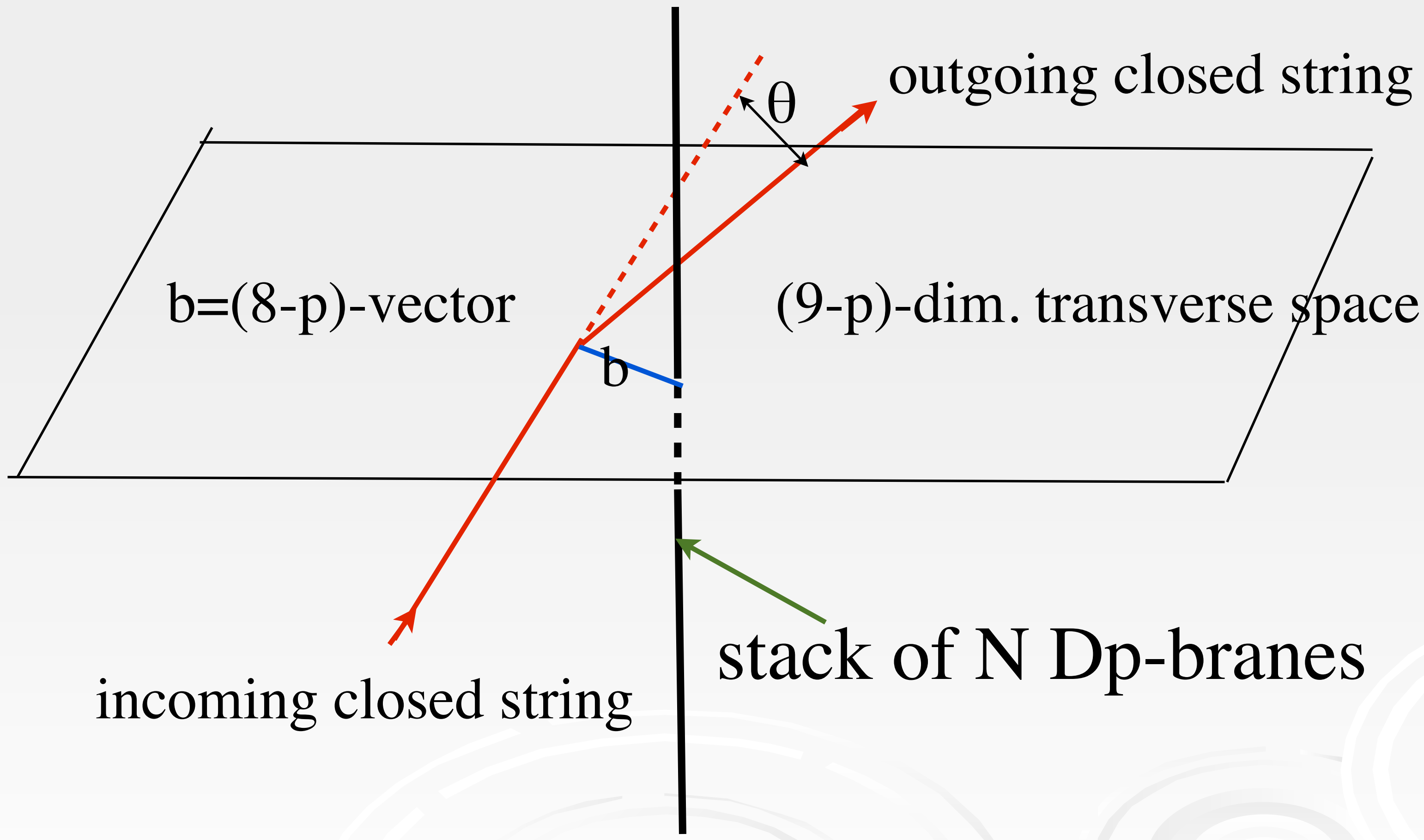
Another big question is the apparent violation of unitarity below b_c. A new elastic-unitarity deficit appears which, unlike the previous ones (related to the opening of inelastic channels), has no simple physical interpretation (BH formation? Too good to be true!).

Recent work by Ciafaloni, Colferai & Falcioni (1106.5628) suggests abandoning the constraint of regularity of the solution at r=0. But then the action blows up below b_c ...

HE string-brane collisions: an even simpler problem?

String scattering off a stack of N Dp-branes

G. D'Apollonio, P. Di Vecchia, R. Russo & G.V.
(1008.4773 and in progress)
W. Black and C. Monni, 1107.4321
(M. Bianchi et al, to appear)

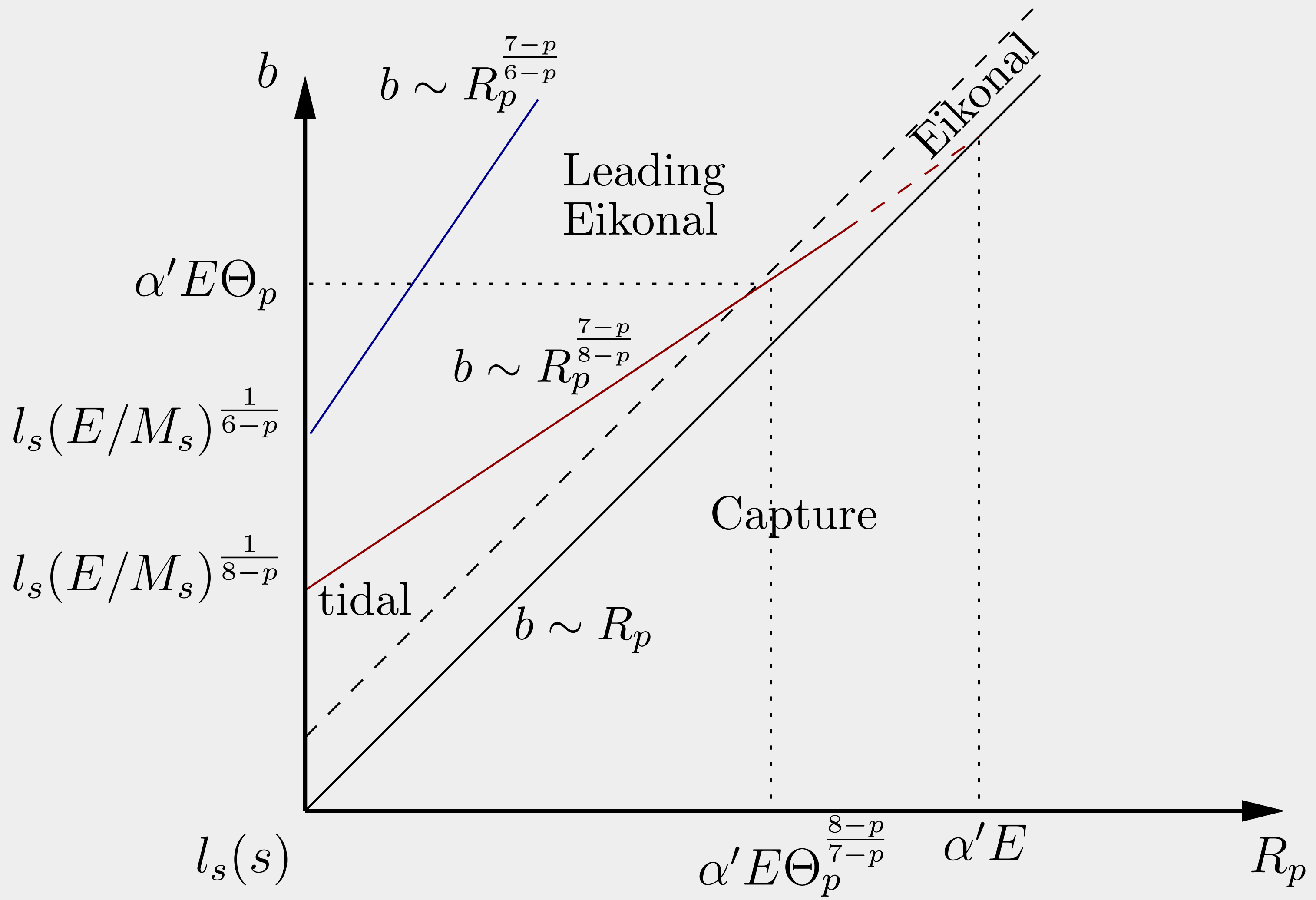


Comments

- We are not assuming any metric: calculations are done in flat spacetime (& in the presence of N-Dp-branes introduced via the boundary state formalism)

- The relevant scales are now:
  - The (orbital) angular momentum, $J = b\,E$, of the incoming string with $J \gg h$ (justifying a semiclassical treatment);
  - The scale R_p of the (expected) emerging geometry;
  - The string length l_s. Ratio R_p/l_s can be tuned by varying g_sN (with $g_s \ll 1$, $N \gg 1$).

- These 3 length scales lead to a phase diagram resembling that of ACV (w/ collapse --> capture)



$$ds^2 = \frac{1}{\sqrt{H(r)}}\left(\eta_{\alpha\beta}dx^\alpha dx^\beta\right) + \sqrt{H(r)}(\delta_{ij}dx^i dx^j) \ ,$$

$$e^{\phi(x)} = g\,[H(r)]^{\frac{3-p}{4}} \ , \qquad \mathcal{C}_{01\ldots p}(x) = \frac{1}{H(r)} - 1 \ ,$$

$$H(r) = 1 + \left(\frac{R_p}{r}\right)^{7-p} \ , \qquad R_p^{7-p} = \frac{gN(2\pi\sqrt{\alpha'})^{7-p}}{(7-p)\Omega_{8-p}} \ , \qquad \Omega_n = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma(\frac{n+1}{2})}$$

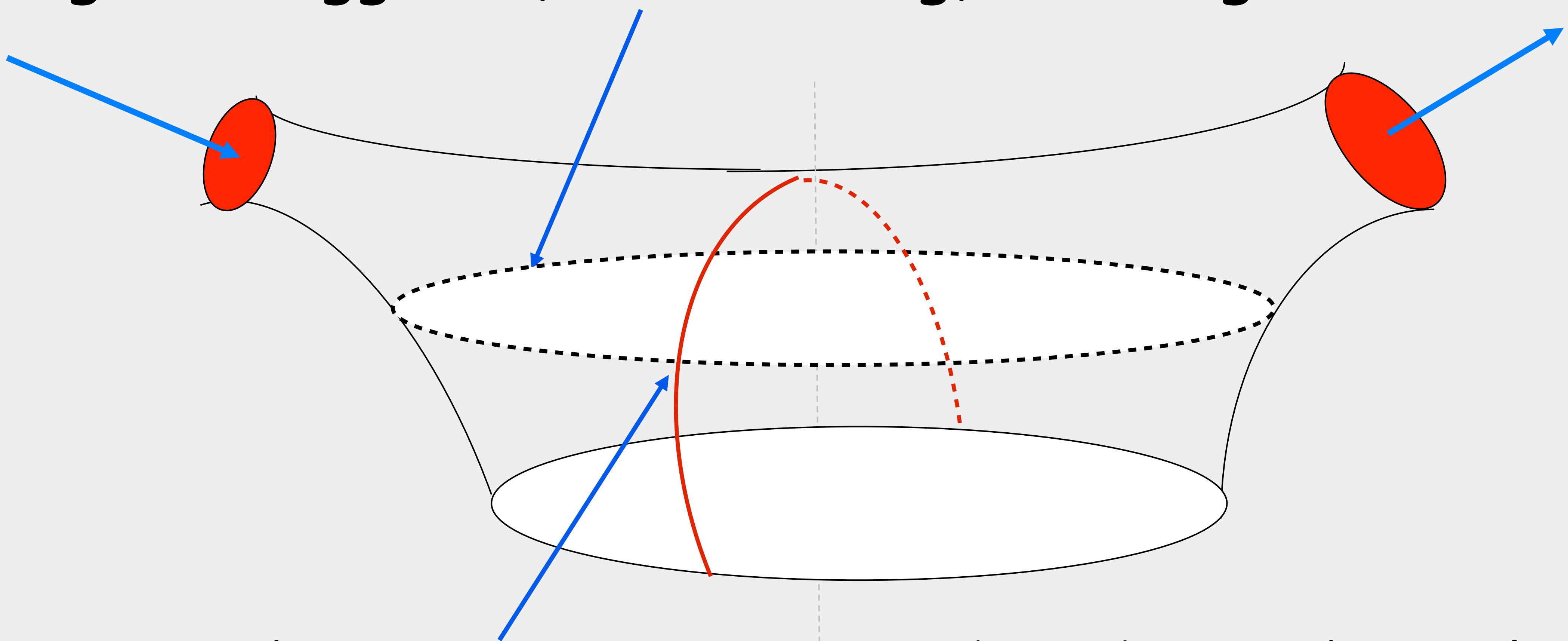
At very high E, gravity dominates. Yet we can neglect closed-string loops below an E_{max} that goes to infinity with N.

- Easier than 2-particle collisions: closed string acts as a probe of the brane-induced geometry (no back-reaction).

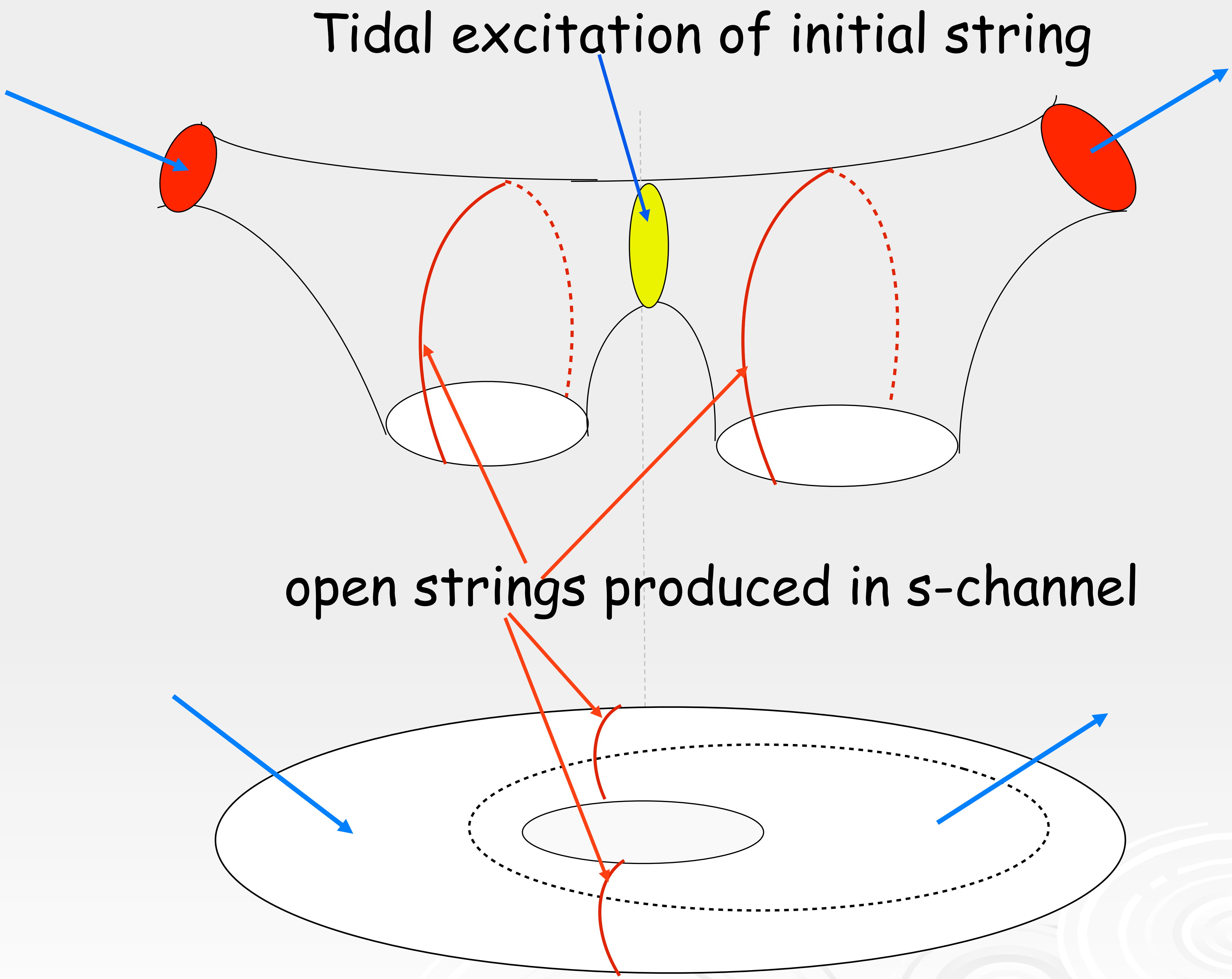
- At the disc and annulus level an effective classical brane geometry emerges through the deflection formulae satisfied at the saddle point of b-integral.

- Unlike in ACV this can be done reliably to next-to-leading order in the deflection angle (extension to all orders in progress)

Disc(tree)-level scattering



Annulus (1-loop) level scattering



another representation of the annulus diagram

A non trivial calculation of a subleading term in the annulus diagram gives:

$$\Theta_p = \sqrt{\pi}\left[\frac{\Gamma\left(\frac{8-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)}\left(\frac{R_p}{b}\right)^{7-p} + \frac{1}{2}\frac{\Gamma\left(\frac{15-2p}{2}\right)}{\Gamma(6-p)}\left(\frac{R_p}{b}\right)^{2(7-p)} + O\left(\left(\frac{R_p}{b}\right)^{3(7-p)}\right)\right]$$

Agree to that order with exact classical formula:

$$\Theta_p = 2\int_0^{\rho_*} d\rho \frac{\hat{b}}{\sqrt{1+\rho^{7-p}-\hat{b}^2\rho^2}} - \pi \qquad \hat{b} \equiv \frac{b}{R_p}$$

Example of p =3

$$\begin{aligned}\theta_3 &= 2\sqrt{1+k^2}\int_0^1 dt \frac{1}{\sqrt{(1-k^2t^2)(1-t^2)}} - \pi = 2\sqrt{1+k^2}K(k) - \pi\ , \\ k^2 &= -1 + \frac{1-\sqrt{1-4\beta^4}}{2\beta^4}\ , \quad \beta \equiv R/b\end{aligned}$$

• Tidal effects can also be computed and come out in complete agreement with what one would obtain (to leading order in R_p/b and l_s/b) by quantizing the string in the D-brane metric (see next slide).
• Indeed one can justify, at least at leading order and at high energy, a "Penrose pp-wave limit" for the metric
• These effects become relevant below a critical $b=b_D$:

$$b_D^{8-p} = \frac{\pi}{2}\alpha' \sqrt{\pi s}(7-p)\frac{\Gamma\left(\frac{8-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)} R_p^{7-p}$$

$$ds^2 = 2dudv - \alpha dv^2 + 2b\alpha dv dz + r^2\alpha C^2 dz^2 + \alpha dx^a dx^a + \beta r^2 \sin^2(z-\bar{\theta})d\Omega_{7-p}^2$$

• Tidal excitation spectrum has been double checked even for an initial massive string by W. Black and C. Monni.

Original metric in adapted (Fermi)coordinates (near a null geodesic):

$$ds^2 = \alpha(r)\left(-dt^2 + \sum_{a=1}^{p}(dx^a)^2\right) + \beta(r)\left(dr^2 + r^2(d\theta^2 + \sin^2\theta d\Omega_{7-p}^2)\right)$$

$$ds^2 = 2dudv - \alpha dv^2 + 2b\alpha dvdz + r^2\alpha C^2 dz^2 + \alpha dx^a dx^a + \beta r^2 \sin^2(z-\bar{\theta})d\Omega_{7-p}^2$$

pp wave form of the D-brane metric justified at high energy

$$C(r) = \sqrt{\frac{\beta(r)}{\alpha(r)} - \frac{b^2}{r^2}}$$

$$du = \pm\frac{\beta dr}{C}$$

$$ds^2 = 2dud\hat{v} + \sum_{a=1}^{p} d\hat{x}_a^2 + \sum_{i=1}^{7-p} d\hat{y}_i^2 + d\hat{y}_0^2 + \mathcal{G}(u, \hat{x}^a, \hat{y}^j, y^0)du^2 \quad ,$$

$$\mathcal{G} = \frac{\partial_u^2\sqrt{\alpha}}{\sqrt{\alpha}}\sum_{a=1}^{p}\hat{x}_a^2 + \frac{\partial_u^2\sqrt{\beta r^2 - b^2\alpha}}{\sqrt{\beta r^2 - b^2\alpha}}\hat{y}_0^2 + \frac{\partial_u^2(\sqrt{\beta}r\sin\bar{\theta})}{\sqrt{\beta}r\sin\bar{\theta}}\sum_{j=1}^{7-p}\hat{y}_j^2$$

$$\equiv \mathcal{G}_x\ \hat{x}_i^2 + \mathcal{G}_0\ \hat{y}_0^2 + \mathcal{G}_y\ \hat{y}_j^2 \ .$$

$$\beta(r) = 1/\alpha(r) = \sqrt{H(r)}$$

and σ-model for string fluctuations in suitable gauge:

$$S - S_0 = \frac{E}{2}\int_0^{2\pi}\frac{d\sigma}{2\pi}\int_{-\infty}^{+\infty} du\ \mathcal{G}(u, X^a(\sigma, u/2\alpha' E), Y^j(\sigma, u/2\alpha' E), Y^0(\sigma, u/2\alpha' E))$$

$$\to \frac{E}{2}\int_0^{2\pi}\frac{d\sigma}{2\pi}\int_{-\infty}^{+\infty} du\left(\mathcal{G}_x(u)X_a^2(\sigma,0) + \mathcal{G}_y(u)Y_j^2(\sigma,0) + \mathcal{G}_0(u)Y_0^2(\sigma,0)\right)$$

$$\equiv \frac{E}{2}\int_0^{2\pi}\frac{d\sigma}{2\pi}\left(c_x X_a^2(\sigma,0) + c_y Y_j^2(\sigma,0) + c_0 Y_0^2(\sigma,0)\right) \quad ,$$

We have definite hopes to be able to resum classical corrections and to study the S-matrix in the strong gravity regime $b \sim b_c \sim R$ (diverging tidal effects?), or even in the classical capture regime $b \ll R$ for which a precise unitary description of the system's evolution is quite non-trivial.

- Absorption of the string by the brane can be studied in some regimes (typically $l_s > b, R_p$) where it becomes important (in analogy with ACV). A crucial difference: the incoming energy now goes into open-string excitations of the D-brane system (described by a gauge theory?).

For $p=3$ we would like to have a new handle on the celebrated AdS/CFT correspondence which is usually limited to situations in which the system lives near the horizon. In our case the initial state is prepared in an asymptotically flat spacetime and the hope is to establish a connection between a CFT on the boundary of AdS and a bona-fide S-matrix.

Conclusions, Outlook

- TPE string-string collisions in flat spacetime are an ideal theoretical lab. for studying several conceptual issues (Cf. inf. paradox) arising from interplay of QM and gravity within a fully consistent framework

- We have been able to reproduce classical expectations (grav. deflection, tidal effects) and extend them within a unitarity-preserving semiclassical description

- When string-size effects dominate we found no evidence for BH formation but, instead, a softening of the final state resembling Hawking radiation

- In the regime of strong gravitational fields our successes are still limited. Amusingly, a drastic approximation of the dynamics appears to reproduce at the semiquantitative level expectations based on CTS collapse criteria.
- No solid conclusion can be drawn without more work. Some features of the present approach may not survive a more complete treatment (e.g. on long.[al] dynamics)
- A general pattern seems to emerge where, at the quantum level, the transition between the dispersive and the collapse phase is smoothed out by QM
- As some critical value of the impact parameter is approached the nature of the final state smoothly changes from that characteristic of a dispersive state to one reminiscent of Hawking's radiation (very high multiplicity and energies O(h/R))
- Many issues remain unsettled (in particular the saturation of unitarity) possibly due to our drastic approximations.

- TPE string collisions off D-branes seem to offer a new tool to study all these issues within an easier set up.
- We have already seen how classical expectations from an effective metric are reproduced both through deflection formulae and from tidal excitations at leading and next to leading order
- Generalization to higher (all) orders within reach
- Extension to classical-capture regime should be possible and will allow to understand how quantum coherence is preserved through the production of a coherent multi-open-string state.
- In the case of 3-branes we hope that this gedanken experiment will shed some new light on the AdS/CFT correspondence within an S-matrix framework.

THANK YOU!

Additional slides

Reduced effective action & field equations

$$\frac{\mathcal{A}}{2\pi G s} = \int d^2x \left[a(x)\bar{s}(x) + \bar{a}(x)s(x) - \frac{1}{2}\nabla_i\bar{a}\nabla_i a\right]$$
$$+ \frac{(\pi R)^2}{2}\int d^2x \left(-(\nabla^2\phi)^2 + 2\phi\nabla^2\mathcal{H}\right) ,$$
$$-\nabla^2\mathcal{H} \equiv \nabla^2 a\ \nabla^2\bar{a} - \nabla_i\nabla_j a\ \nabla_i\nabla_j\bar{a} ,$$

and the corresponding eom

$$\nabla^2 a + 2\delta(x) = 2(\pi R)^2(\nabla^2 a\ \nabla^2\phi - \nabla_i\nabla_j a\ \nabla_i\nabla_j\phi), \quad \bar{a}(x) = a(b-x)$$
$$\nabla^4\phi = -(\nabla^2 a\ \nabla^2\bar{a} - \nabla_i\nabla_j a\ \nabla_i\nabla_j\bar{a})$$

The semiclassical approximation corresponds to solving the eom and computing the classical action on the solution. This is why we took Gs/h >> 1!

Still too hard for analytic study!

Axisymmetric action and eqns (t=r^2)

$$\frac{\mathcal{A}}{2\pi^2 G s} = \int dt\,[a(t)\bar{s}(t) + \bar{a}(t)s(t) - 2\rho\dot{\bar{a}}\dot{a}] - \frac{2}{(2\pi R)^2}\int dt(1-\dot{\rho})^2$$

$$\rho = t\left(1 - (2\pi R)^2\dot{\phi}\right) \qquad \pi\int^t dt' s_i(t') = R_i(t)/R$$

$$\dot{a}_i = -\frac{1}{2\pi\rho}\frac{R_i(r)}{R}$$

$$\ddot{\rho} = \frac{1}{2}(2\pi R)^2\dot{a}_1\dot{a}_2 = \frac{1}{2}\frac{R_1(r)R_2(r)}{\rho^2}$$

2nd order ODE w/ Sturm-Liouville-like b. conditions

Particle Spectra: an "energy crisis"?

(ACV07, VW08/2, M. Ciafaloni & GV in progress)

Within our approximations the spectrum of the produced gravitons gives the following result for GW emission:

$$\frac{dE_{gr}}{d^2k\ d\omega} = Gs\ R^2\ exp\left(-|k||b| - \omega\frac{R^3}{b^2}\right)\ ;\ \frac{Gs}{\hbar}\frac{R^2}{b^2} >> 1$$

Accordingly, the fraction of energy emitted in GWs is O(1) already for b=b*>>R (Gs/h (R/b*)^2 =O(1)). Is this puzzling from a GR perspective? Answer related to:

Q: What is the frequency cutoff on the GWs emitted in an ultra-relativistic small angle (b>>R) 2-body scattering?

The CGR answer to this problem seems to be unknown..

Possible answers: 1/b, 1/R (my present), b/R^2, b^2/R^3 (ACV), γ/b (singular m=0 limit?), E/h (singular classical limit?)

My guess (1/R) would rather give:

$$\frac{dE_{gr}}{d^2k\ d\omega} = Gs\ R^2\ exp(-|k||b| - \omega R) \Rightarrow \frac{E_{gr}}{\sqrt{s}} \sim \frac{R^2}{b^2}$$

In both cases, while for b >> R gravitons are produced at small angles, as b -> b_c ~ R their distribution becomes more and more spherical w/ <n> ~ Gs/h and (again!) characteristic energy O(h/R ~T_H)

Recent work by B. Kol (1103.57410 hep-th) on "weak ultra-relativistic" gravitational scattering could be relevant for this issue.

THE GENERATION OF GRAVITATIONAL WAVES. IV. BREMSSTRAHLUNG*†‡

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ABSTRACT

This paper attempts a definitive treatment of "classical gravitational bremsstrahlung"—i.e., of the gravitational waves produced when two stars of arbitrary relative mass fly past each other with arbitrary relative velocity v, but with large enough impact parameter that

$$\text{(angle of gravitational deflection of stars' orbits)} \ll (1 - v^2/c^2)^{1/2} .$$

For for $\theta < 1/\gamma (b > \gamma R)$ it agrees with GKST.

What's the answer for $\theta > 1/\gamma$?

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High-speed black-hole encounters and gravitational radiation

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(Received 15 March 1977)

Encounters between black holes are considered in the limit that the approach velocity tends to the speed of light. At high speeds, the incoming gravitational fields are concentrated in two plane-fronted shock regions, which become distorted and deflected as they pass through each other. The structure of the resulting curved shocks is analyzed in some detail, using perturbation methods. This leads to calculations of the gravitational radiation emitted near the forward and backward directions. These methods can be applied when the impact parameter is comparable to $Gc^{-2}M\gamma^2$, where M is a typical black-hole mass and γ is a typical Lorentz factor (measured in a center-of-mass frame) of an incoming black hole. Then the radiation carries power/solid angle of the characteristic strong-field magnitude c^5G^{-1} within two beams occupying a solid angle of order γ^{-2}. But the methods are still valid when the black holes undergo a collision or close encounter, where the impact parameter is comparable to $Gc^{-2}M\gamma$. In this case the radiation is apparently not beamed, and the calculations describe detailed structure in the radiation pattern close to the forward and backward directions. The analytic expressions for strong-field gravitational radiation indicate that a significant fraction of the collision energy can be radiated as gravitational waves.