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High-energy collisions of particles, strings and branes: I

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Motivations

- Progress in fundamental physics has often been based on stepping up in energy, either experimentally or through theoretical (gedanken) experiments.
- When dealing with theories of gravity the need for very (unrealistically?) high energies is even more obvious.
- While classical gravity has no intrinsic energy scale, quantum & string gravity do: M_{Pl} & M_{st} respectively.
- Both are presumably too high for real experiments (except for those that occurred naturally in the early Universe?)
- Combining GR and QM poses deep conceptual problems (Cf. Hawking's information paradox).
- String theory claims to be a consistent framework for addressing such questions: what's its answer?

Outline

Lecture I

- GR collapse criteria: a brief review.
- Transplanckian energy collisions of particles and strings:
  - The small-angle regime: deflection & tidal forces
  - The stringy regime & precocious BH behaviour

Lecture II

- Transplanckian energy collisions of particles and strings:
  - The large-angle/collapse regime
- High-energy string-brane collisions: an easier problem?
- Outlook, conclusions.

Brief review of GR collapse criteria

There are many analytic as well as numerical GR results on whether some given initial data should lead to gravit.^{al} collapse or to a completely dispersed final state

The two phases would be typically separated by a critical hypersurface in the parameter space of the initial states

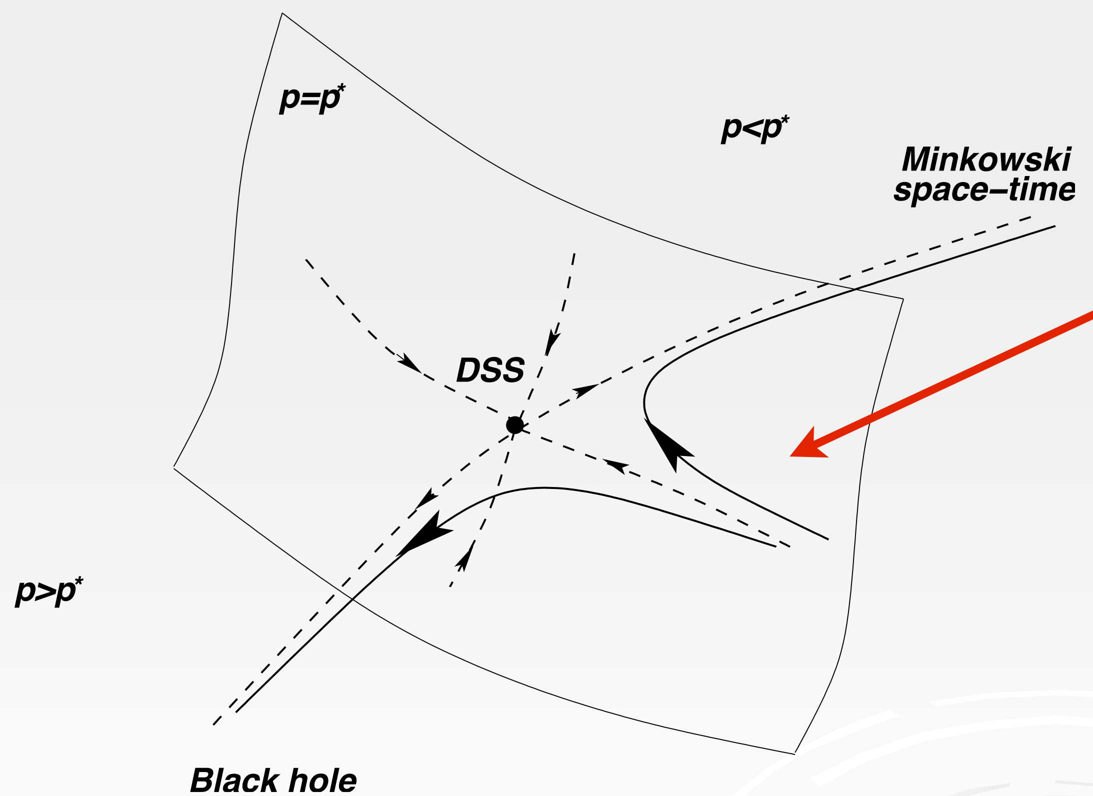


Figure 1: Phase space picture of the critical gravitational collapse.

from C. Gundlach's review ('02)

The approach to criticality resembles that of phase transitions (order, crit. exp. ...)

For pure gravity Christodoulou & Klainerman ('93) have found a region on the dispersion side of the critical surface;

Regions on the collapse side have been found for spherical symmetry by Christodoulou ('91, ...) and, numerically, by Choptuik and collaborators ('93, ...'09);

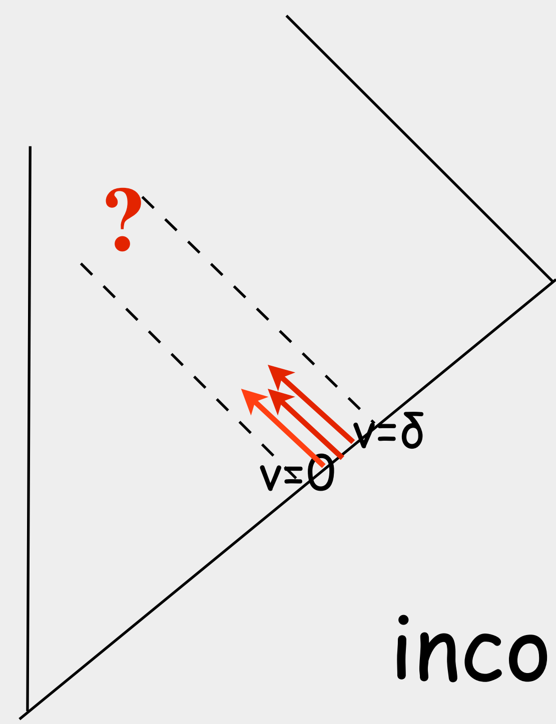
In 0805.3880, Christodoulou identified another such region in which a lower bound on (incoming energy)/(unit adv. time) holds uniformly over the full solid angle; not useful for two-body collisions, the energy being concentrated in two narrow cones (see however Klainerman & Rodnianski, 0912.5097, 1002.2656).

In 0908.1780 Choptuik and Pretorius have obtained new numerical results for a highly-relativistic axisymmetric situation (see below).

A useful (sufficiency) criterion for collapse is the identification of a Closed Trapped Surface (CTS) at a certain point in the system's evolution.

D. Christodoulou, gr-qc 0805.3880

Penrose diagram



incoming energy (G=1) per unit advanced time & solid angle

$$\text{If} \quad M(\theta,\phi,\delta) = \int_0^\delta dv \frac{d\mathcal{M}(v,\phi,\delta)}{dv\ dcos\theta\ d\phi} \geq \frac{k}{8\pi} \quad \text{for all} \quad \theta,\phi$$

then a CTS forms with $R \geq k - O(\delta)$ (provided $R > 0$)

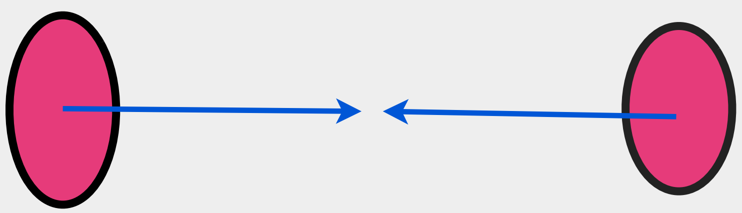
Small sample of results

- Point-particle collisions:

1. b=0: Penrose ('74) : $M_{BH} > E/\sqrt{2} \sim 0.71E$
2. b≠0: Eardley & Giddings ('02), one example:

$$\left(\frac{R}{b}\right)_{cr} \leq 1.25 \qquad (R = 2G\sqrt{s} = 4GE_1 = 4GE_2)$$

- Extended sources:



- Kohlprath & GV ('02), one example: central collision of 2 homogeneous null discs of radius L

$$\left(\frac{R}{L}\right)_{cr} \leq 1$$

(Work in progress with Ph. LeFloch on singularity inside horizon)

What about the quantum problem?

We can prepare pure initial states that correspond, roughly, to the classical data (J ~ bE).

- Does a unitary S-matrix (evolution operator) always describe the evolution of the system?
- If yes, does such an S-matrix develop singularities as one approaches a critical (parameter-space) surface?
- If yes, what happens in its vicinity? Does the nature of the final state change as one goes through it?
- Is there a relation between the classical and quantum critical surfaces?
- What happens to the final state deep inside the collapse region? Does it resemble at all Hawking's thermal spectrum for each initial pure state?
- Qs related to information paradox/puzzle.

A more phenomenological motivation for studying TPE collisions?

Finding signatures of string/quantum gravity @ LHC:

- In KK models with large extra dimensions;
- In brane-world scenarios; in general:
- If the true Quantum Gravity scale is O(few TeV)

NB: In the most optimistic situation the LHC will be quite marginal for producing BH, let alone semiclassical ones

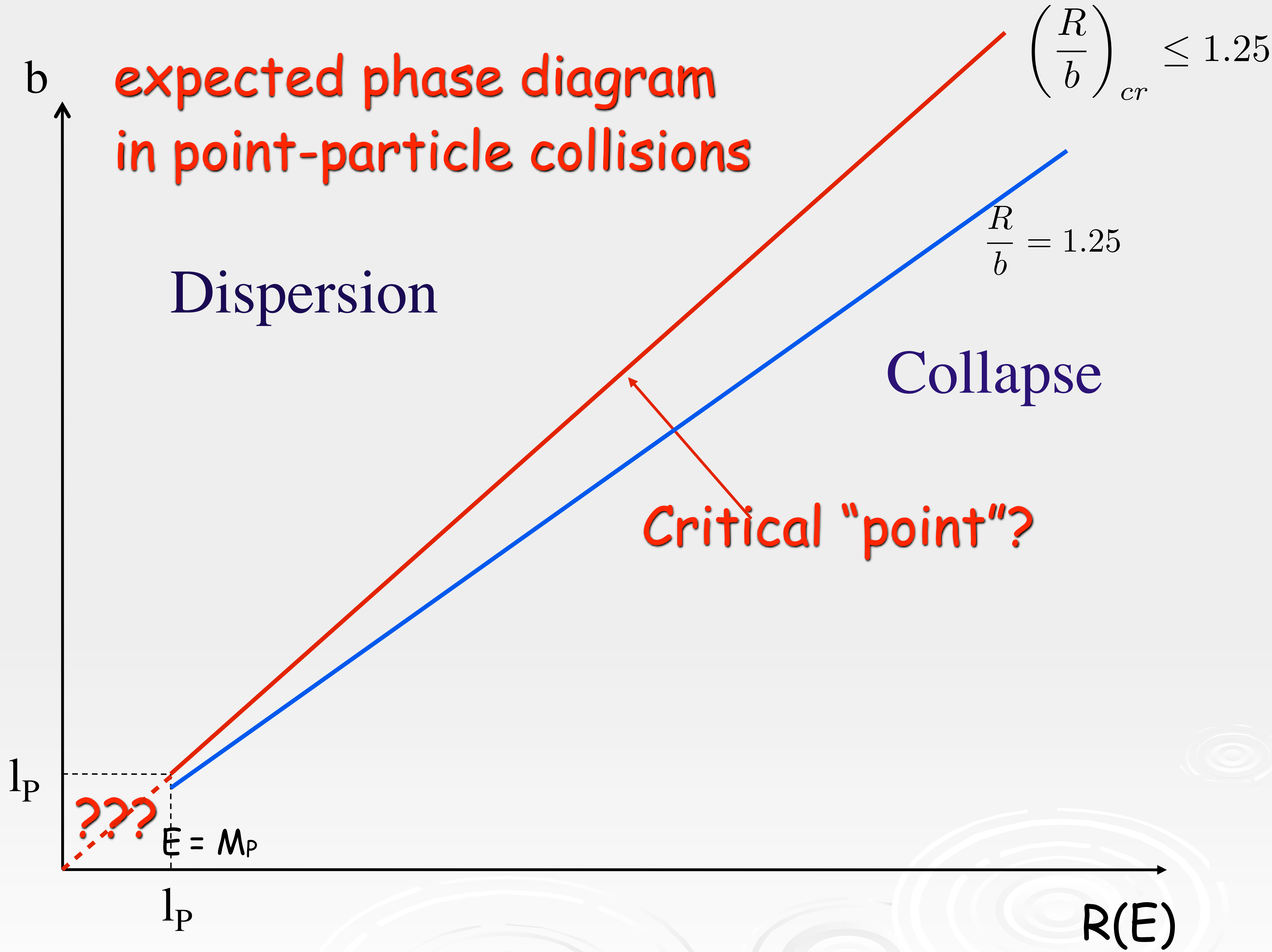
Q: Can there be some precursors of BH behaviour even below the expected BH-production threshold?

Transplanckian-energy collisions of particles and strings
(Amati, Ciafaloni & GV 1987-....)

Trans-Planckian-Energy (TPE => $E \gg M_P c^2$, or $Gs/c^5 h \gg 1$) string collisions represent a perfect theoretical laboratory for studying these questions within a framework that claims to be a fully consistent quantum theory of gravity.

We can hardly imagine a simpler pure initial state that could lead to BH formation and whose unitary evolution we would like to understand/follow.

As it turns out TPE also simplifies the theoretical analysis


expected phase diagram
in point-particle collisions
b
$\left(\frac{R}{b}\right)_{cr} \leq 1.25$
$\frac{R}{b} = 1.25$
Dispersion
Collapse
Critical "point"?
l_P
???
E = M_P
l_P
R(E)

If we collide strings, instead of point particles, there is another length scale, l_s ,the characteristic size of strings

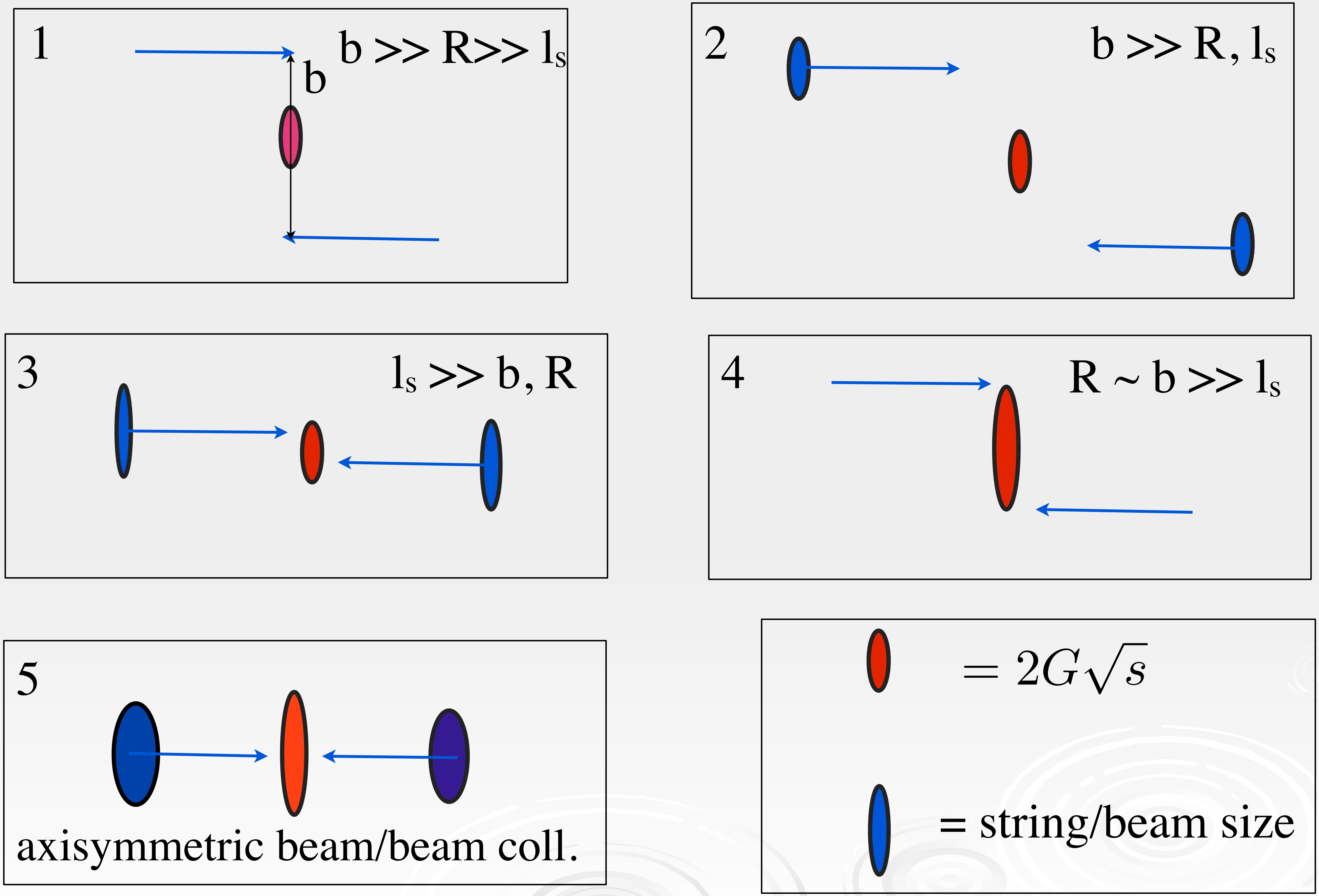
l_s plays the role of the beam size!

3 length scales: b, R and l_s

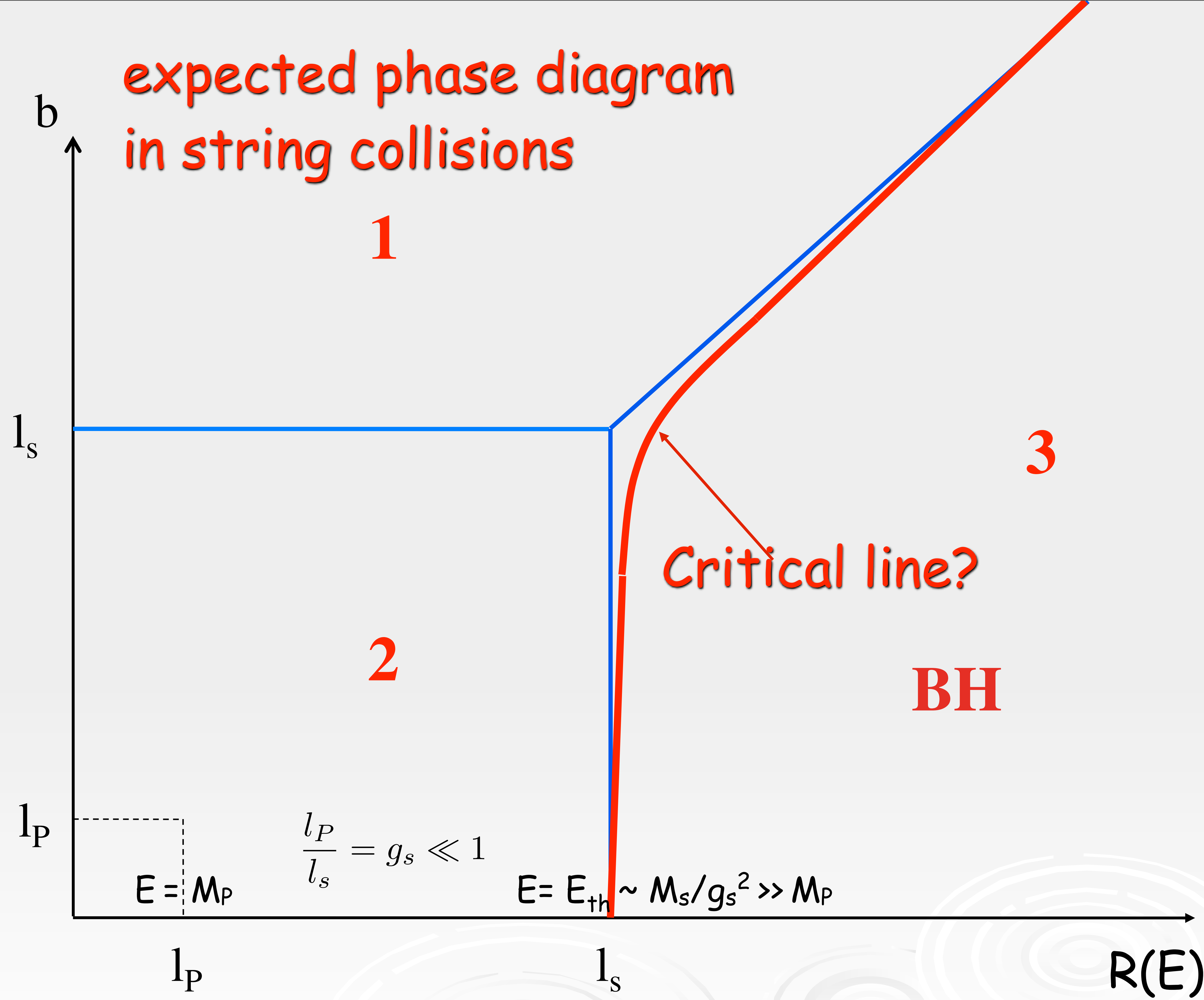
➡ **3 broad-band regimes in transplanckian string collisions**

1) Small angle scattering ($b \gg R, l_s$)
2) Stringy ($l_s > R, b$)
3) Large angle scattering ($b \sim R > l_s$), collapse ($b, l_s < R$)

Various regimes in string-string collisions



expected phase diagram in string collisions

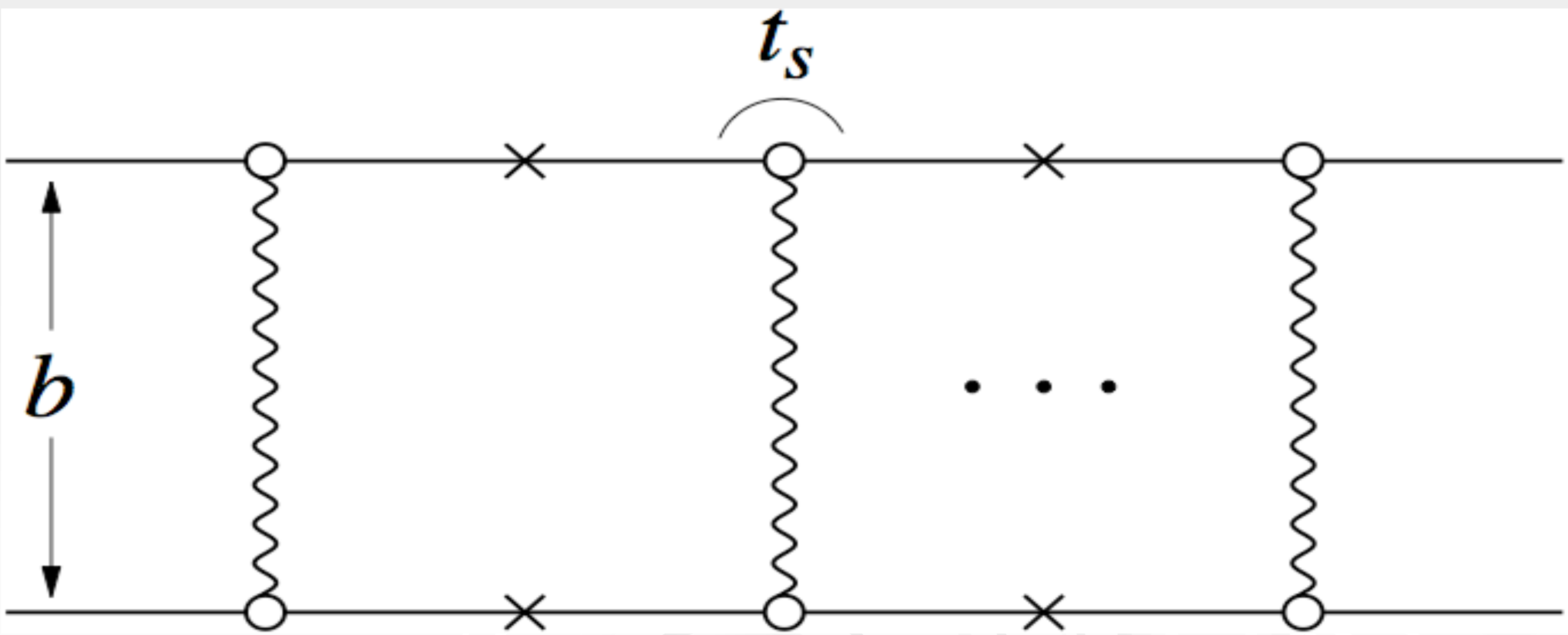


A semiclassical S-matrix @ TPE

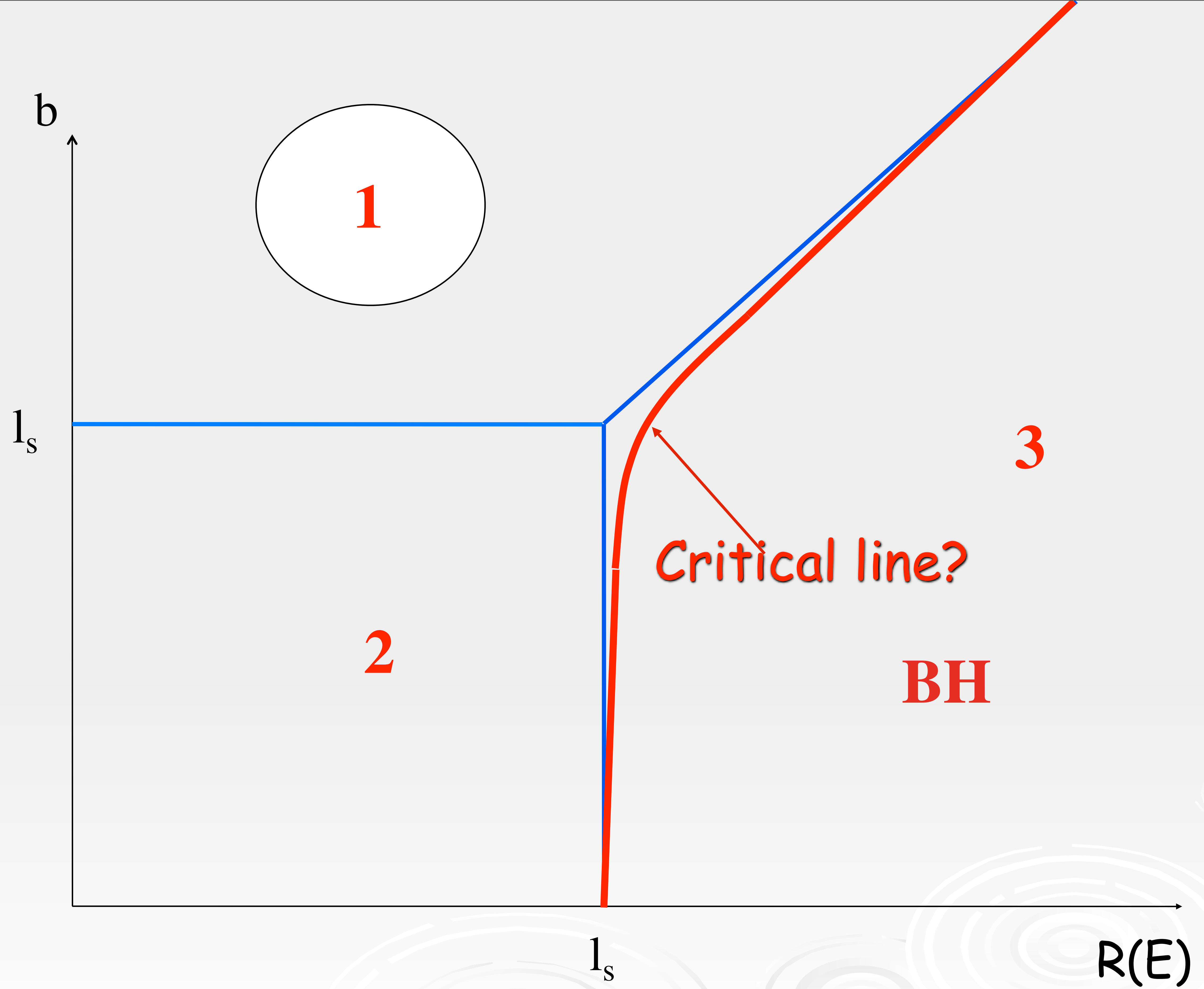
General arguments as well as explicit calculations suggest the following form for the elastic S-matrix:

$$S(E,b) \sim exp\left(i\frac{A}{\hbar}\right) \sim exp\left(-i\frac{Gs}{\hbar}(logb^2 + O(R^2/b^2) + O(l_s^2/b^2) + \cancel{O(l_P^2/b^2) + \ldots})\right)$$

Leading eikonal diagrams (crossed ladders included)



NB: For Im A some terms may be more than just corrections...



b
1
l_s
3
Critical line?
2
BH
l_s
R(E)

Particle-particle scattering @ large b

$$S(E,b) \sim exp\left(-i\frac{Gs}{\hbar}logb^2\right) \;\; ; \;\; S(E,q) = \int d^2b \; e^{-iqb} S(E,b) \;\; ; \quad s = 4E^2 \; , \; q \sim \theta E$$

The integral is dominated by a saddle point at:

$$b_s = \frac{4G\sqrt{s}}{\theta} \; , \; \theta = \frac{4G\sqrt{s}}{b} = 2\frac{R}{b} \; , \;\; R \equiv 2G\sqrt{s}$$

Generalization of Einstein's deflection formula for ultra-relativistic collisions. Also easily extended to arbitrary D. It corresponds precisely to the relation between impact parameter and deflection angle in the (Aichelburg-Sexl) metric generated by a relativistic point-particle of energy E. This effective metric is not put in: it's "emergent"

High-p_T is not necessarily short distance!

$$b_s = \frac{4G\sqrt{s}}{\theta}\,,\ \theta = \frac{4G\sqrt{s}}{b} = 2\frac{R}{b}\ ,\ \ R \equiv 2G\sqrt{s}$$

- At fixed θ, larger E probe larger b
- The reason is simple: because of eikonal exponentiation, Gs/h also gives the average loop-number. The total momentum transfer q = θ E is thus shared among O(s~E^2) exchanged gravitons to give:

$$q_{ind} \sim \frac{\hbar q}{Gs} \sim \frac{\hbar\theta}{R} \sim \frac{\hbar}{b_s}$$

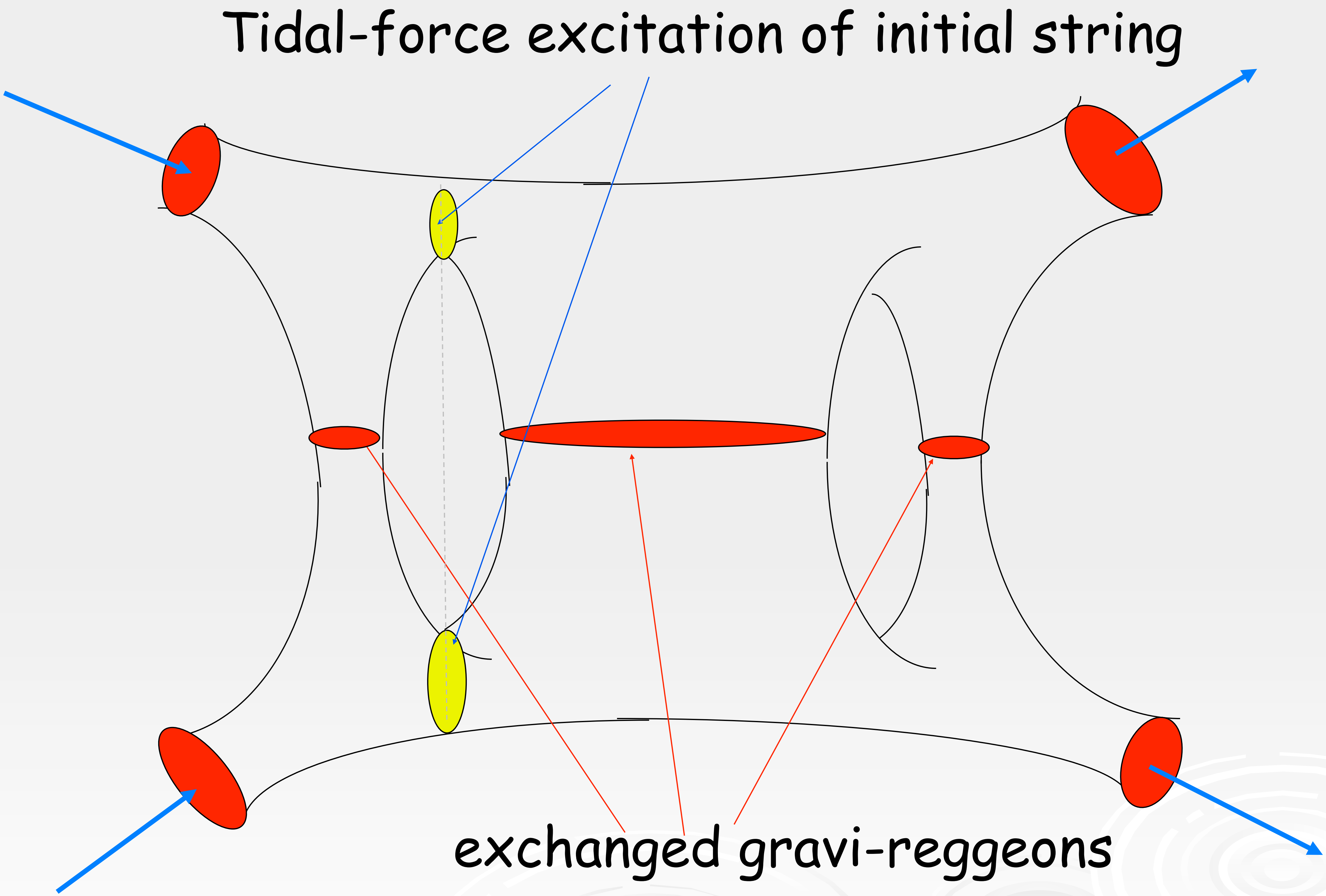
meaning that the process is soft at large s...

String-string scattering @ large b

(new effects because of imaginary part)

$$S(E,b) \sim exp\left(i\frac{A}{\hbar}\right) \sim exp\left(-i\frac{Gs}{\hbar}(logb^2 + \cancel{O(R^2/b^2)} + O(l_s^2/b^2) + \cancel{O(l_P^2/b^2)} + \ldots)\right)$$

Graviton exchanges can excite one or both strings. Reason (Giddings '06): a string moving in a non-trivial metric feels tidal forces as a result of its finite size. A simple argument gives the critical impact parameter b_D below which the phenomenon kicks-in (as found by direct calculation by ACV). It is parametrically larger than l_s.

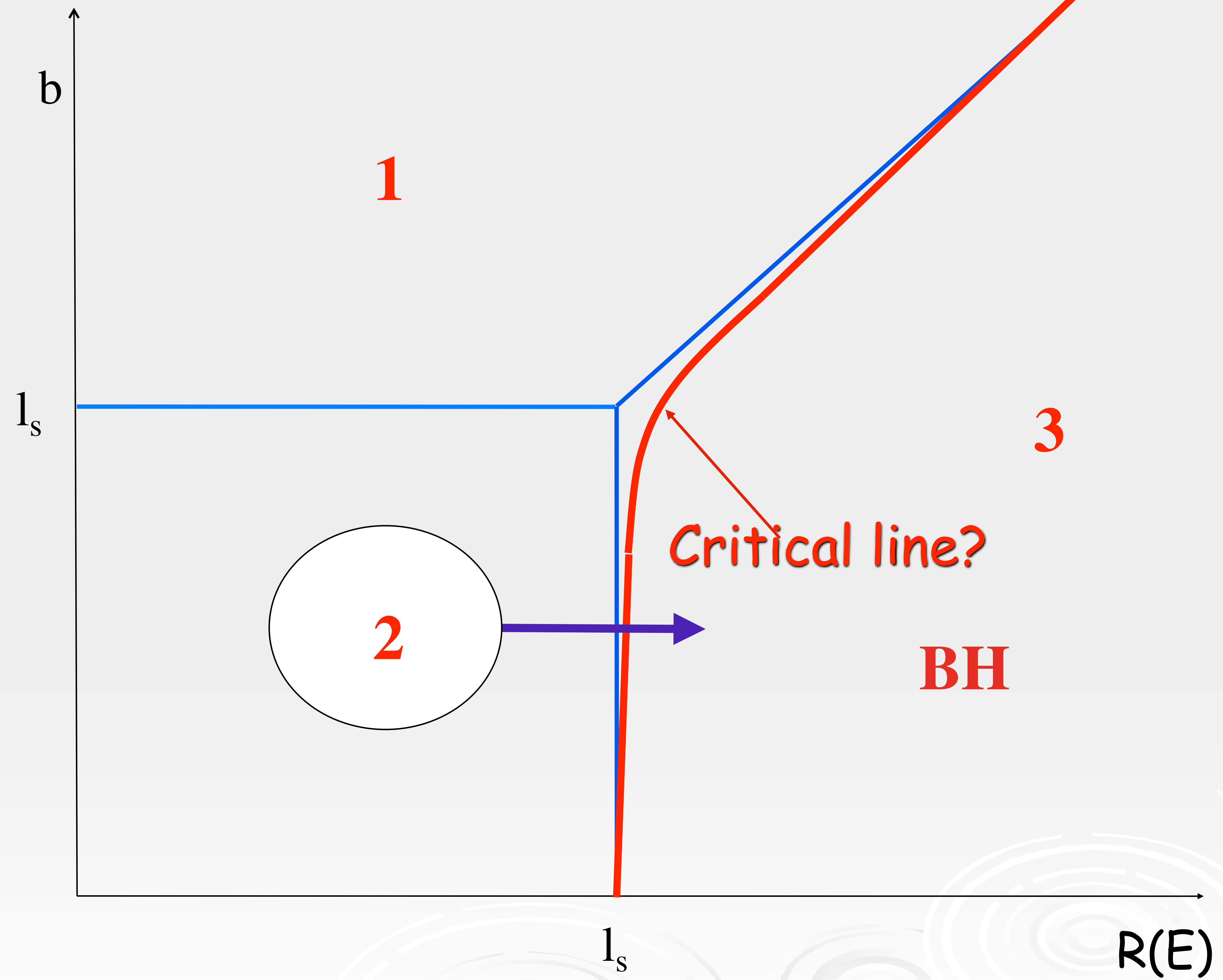

Tidal-force excitation of initial string
exchanged gravi-reggeons

String-string scattering @ b,R < l_s

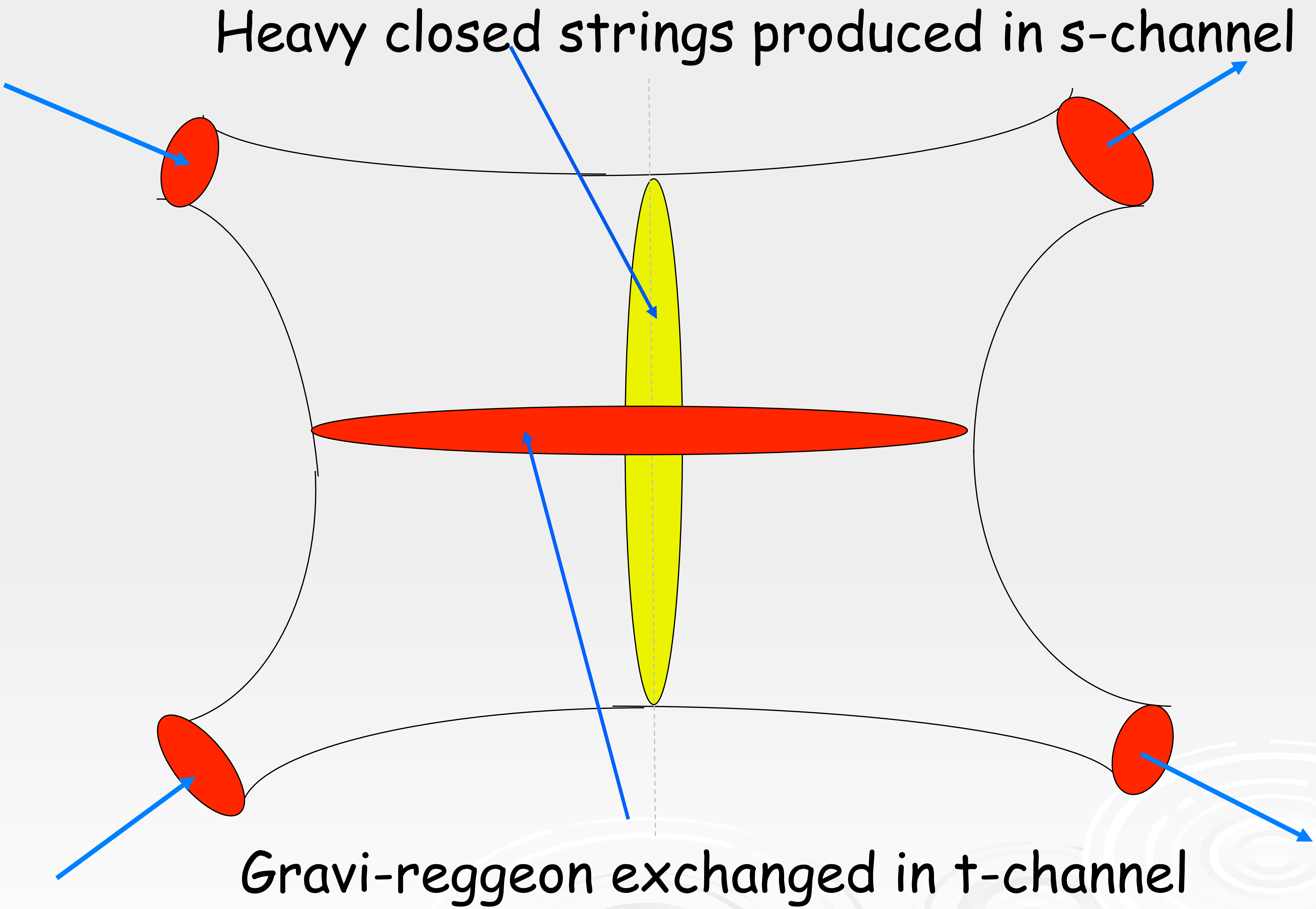
$$S(E,b) \sim exp\left(i\frac{A}{\hbar}\right) \sim exp\left(-i\frac{Gs}{\hbar}(logb^2 + \cancel{O(R^2/b^2)} + O(l_s^2/b^2) + \cancel{O(l_P^2/b^2)} + \ldots)\right)$$

Because of (good old DHS) duality even single graviton exchange does not give a real scattering amplitude. The imaginary part is due to formation of closed-strings in the s-channel.

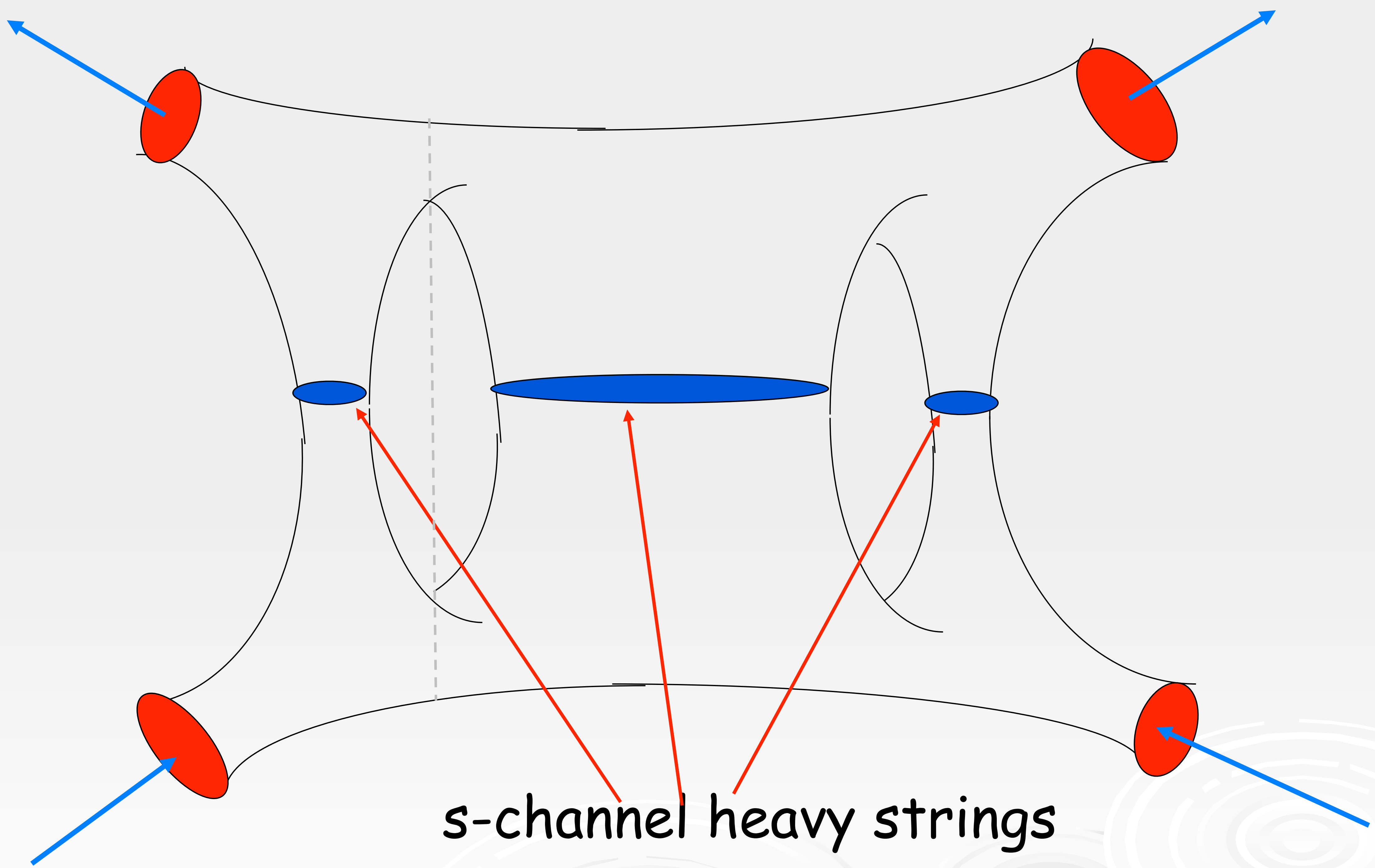
It is exponentially damped at large impact parameter (=> irrelevant in region 1, important in region 2)


b
l_s
1
2
3
Critical line?
BH
l_s
R(E)

Im A is due to closed strings in s-channel (DHS duality)



Turning the previous diagram by 90°



$$\mathrm{Im} A_{cl}(E,b) \sim \frac{Gs}{\hbar} \exp\left(-\frac{b^2}{l_s^2 \log s}\right)$$

As one goes to impact parameters below the string scale one starts producing more and more strings. The average number of produced strings grows (once more!) like $Gs \sim E^2$ so that, above M_{Pl}, the average energy of each final string starts decreasing as the incoming energy is increased

$$\langle E_{final} \rangle \sim \frac{M_{Pl}^2}{\sqrt{s}} \sim T_{BH}$$

Similar to what we expect in BH physics!

An interesting signature even below the actual threshold of BH production!

THANK YOU!

Additional slides

(Generalized) AS metrics

The Aichelbourg-Sexl (AS) metric is the shock-wave metric generated by a point-like massless source carrying an energy E. Its generalization (GAS) refers to a massless source whose total energy E is spread on the plane orthogonal to the motion and is δ-function-like in the direction of the motion (a pancake). Also the metric of a beam of massless particles moving in the same direction with the same $x^{\pm}$ =(v,u). In a convenient set of coordinates:

$$ds^2 = -du dv + \phi(\mathbf{x})\delta(u) du^2 + d\mathbf{x}^2 \quad \text{with} \quad \Delta\phi(\mathbf{x}) = -16\pi G\rho(\mathbf{x})$$

while in another convenient set it becomes:

$$ds^2 = -dU dV + H_{ik} H_{jk} dX^i dX^j \quad ; \quad H_{ij} = \delta_{ij} + \frac{1}{2}\nabla_i \nabla_j \phi(\mathbf{X}) U \Theta(U)$$

Geodesics in GAS metrics

Modulo some subtlelties in dealing with δ and θ-functions, it is quite straighforward to compute the trajectories of massless test particles in a GAS metric. The main features of these geodesics are:

1. A deflection making initially parallel geodesics converge (lensing!). Deflection angles are related to gradients of φ
2. A shift (jump) in v (with no shift in u) controlled by φ itself.
3. Amusing result: for a homogeneous beam of size L all parallel geodesics with a fixed v and b < L converge at the same space-time point after hitting the SW (FPV, GRG 1988)

CTS criteria for the collision of two GAS shock waves

Consider two such shock waves colliding head on. In a suitable frame is at u=t-z=0 (moving to the right) and the other at v=t+z=0 (moving to the left). At t<0 they have not collided yet. They do at t=z=0. At t<0 the metric is given by a superposition of the two GAS metrics:

$$ds^2 = -dUdV + \left[H^{(1)}_{ik} H^{(1)}_{jk} + H^{(2)}_{ik} H^{(2)}_{jk} - \delta_{ij}\right] dX^i dX^j$$

while at t>0 the problem becomes very difficult (only the infinite, homogeneous wavefront case can be reduced to quadratures). However, we can use the above expression to find out whether a CTS can be constructed at $t = 0^-$.
This is the method followed by Eardley-Giddings (AS) and by Kohlprath and GV (for GAS) with the results given above.