Gauge/String Dualities and Their Applications

Igor Klebanov

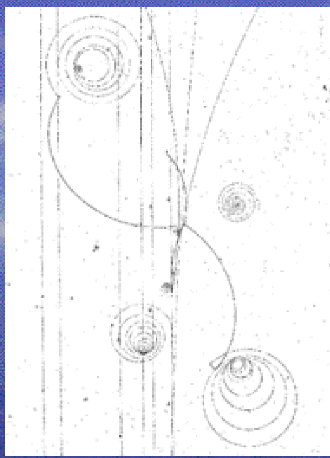
Department of Physics
Princeton University

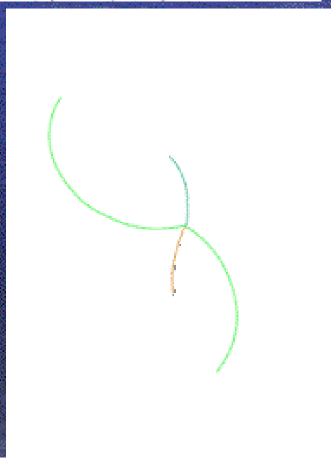
Lectures at Les Houches 2007
String Theory and Real World

Introduction

- One often hears of string theory as the leading hope for unifying all known interactions-- strong (nuclear), electromagnetic, weak (β-decay) and gravitational-- into a consistent quantum theory. Some have dubbed it
`The Theory of Everything.'
- Actually, string theory has had a more humble beginning. It was invented in the late 60's to model `just' the strong (nuclear) interactions.
- A reference for the beginning of my lectures: `QCD and String Theory,' arXiv:hep-ph/0509087

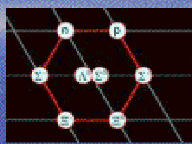
- In the 1950’s and 60’s technological progress led to discovery of many `strongly interacting’ sub-nuclear particles (hadrons).
- They come in two types: baryons (protons, neutrons, Λ, etc.) and mesons (π, ρ, K, etc.).
- Here is a bubble chamber picture of a charged K-meson decaying into three charged π-mesons.

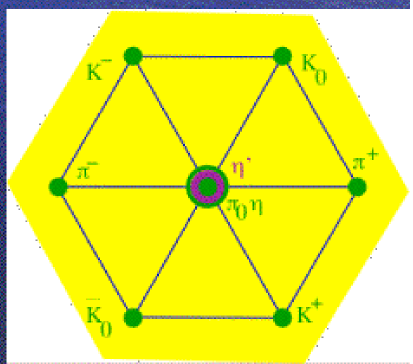




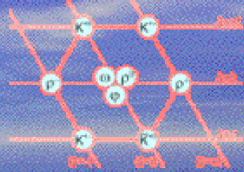
- The strong interactions are short range (~ 1 fm = 10 $^{-15}$ m), but much stronger than the electromagnetic.
- The strong interaction analogue $\alpha_s = g_{YM}^2/(4\pi)$ of the electromagnetic `fine structure constant' (α=1/137) is about 100 times bigger.
- As more hadrons were discovered, they were grouped into multiplets. For example, the lightest spin-0 mesons form an `octet.' This gave impetus to the Quark Model.

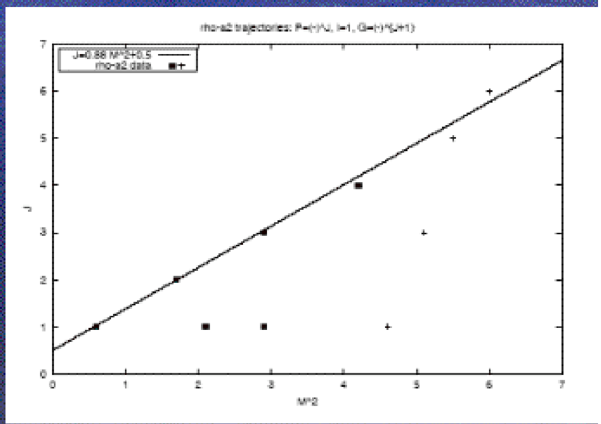
Gell-Mann, Zweig



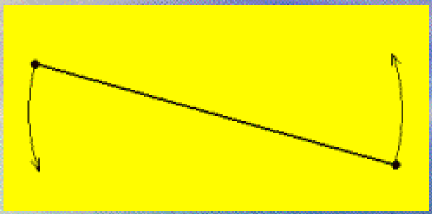


- Lots of heavier mesons, with higher spin, have also been discovered.
- Early empirical evidence for the string-like structure of hadrons comes from arranging mesons and baryons into `Regge trajectories' on plots of angular momentum vs. mass-squared.
- A leading `Regge trajectory' of mesons is shown (ρ, a_2 ...)





Open String Picture of Mesons



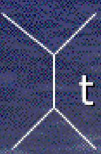
$$J = \alpha' m^2 + \alpha(0)$$

- In the string model, excited mesons are identified with excitations (rotational and vibrational) of a relativistic string of energy density ~ 1 GeV/fm, which is around 1.6 kilo-Joules/cm.
- The rest energy of a ρ meson is 0.78 GeV.
- The linear relation between angular momentum and mass-squared is provided by a spinning relativistic string. Later it was understood that a quark and anti-quark are located at the string endpoints.

Dual models

Dolen-Horn-Schmid duality conjecture: in meson scattering, sum over s-channel exchanges

equals sum over t-channel exchanges

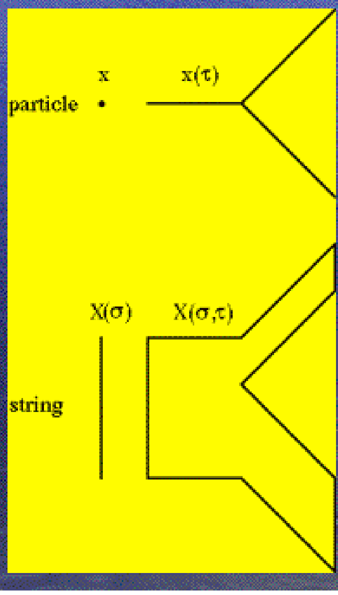


- Veneziano proposed a manifestly dual amplitude for elastic pion scattering: $A(s,t) \sim \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$

  with linear Regge trajectory

  $$\alpha(s) = \alpha(0) + \alpha' s$$

- Nambu, Nielsen and Susskind independently proposed its open string interpretation



- The string world sheet dynamics is governed by the Nambu-Goto area action

$$S_{\mathrm{NG}} = -T \int d\sigma d\tau \sqrt{-\det\ \partial_a X^\mu \partial_b X_\mu}$$

- The string tension is related to the Regge slope through

$$T = \frac{1}{2\pi\alpha'}$$

- The quantum consistency of the Veneziano model requires that the Regge intercept is $\alpha(0) = 1$
 so that the spin 1 state is massless but the spin 0 is a tachyon.

- Calculation of the string zero-point energy gives $\alpha(0) = \frac{d-2}{24}$

- Hence the model has to be defined in 26 space-time dimensions.

Crossroads in the 1970's

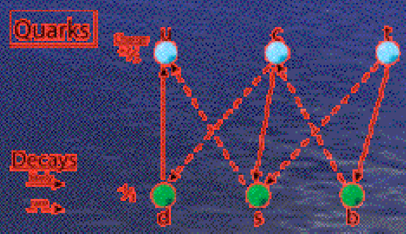
- Attempts to quantize such a string model in 3+1 dimensions lead to tachyons, problems with unitarity.
- Consistent supersymmetric string theories were discovered in 9+1 dimensions, but their relation to strong interaction was initially completely unclear.
- Most importantly, the Asymptotic Freedom of strong interactions was discovered by Gross, Wilczek; Politzer in 1973. This singled out the Quantum Chromodynamics (QCD) as the exact theory of strong interactions.

- Most physicists gave up on strings as a description of strong interactions. Instead, string theory emerged as the leading hope for unifying quantum gravity with other forces (the graviton appears in the closed string spectrum). Scherk, Schwarz; Yoneya

QCD

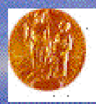
- In 1973 a point particle theory of strong interactions, inspired by the quark model, was proposed: the Quantum Chromodynamics.
- The hadrons are made of spin 1/2 constituents called quarks and spin 1 ones called gluons. Quarks come in 6 known flavors (up, down, strange, charm, bottom, top), and each flavor comes in 3 different color states (red, green, blue).
- The adjoint gluons come in $3^2 - 1 = 8$ different color states:

$$S = -\int d^4x \frac{1}{2g_{YM}^2} \mathrm{Tr} F_{\mu\nu}^2$$



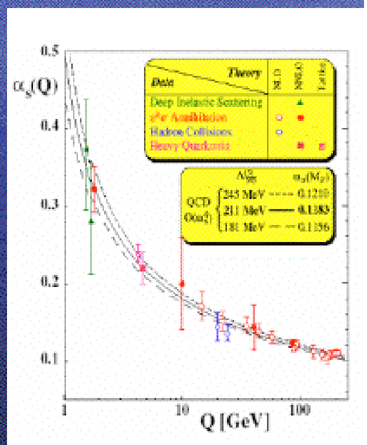
Asymptotic Freedom

- A crucial property of QCD, discovered in 1973 by Gross and Wilczek at Princeton, and Politzer at Harvard.



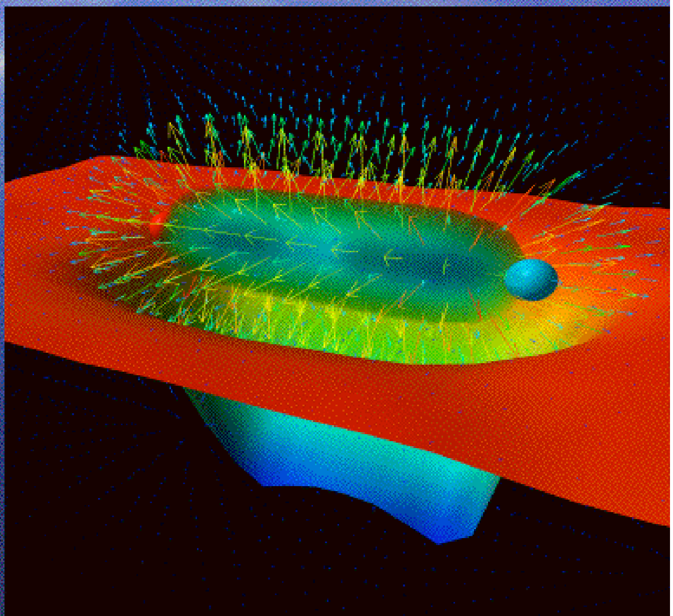
$$\beta(g) = -g^3/16\pi^2\,[11/3\,C_2 - 4/3\,T(R)] + O(g^5)$$

- As momentum transfer increases, the interactions between the quarks and gluons, though still quite strong, gradually weaken. This provided a theoretical explanation of such effects observed in the late 60's at energies around 10 GeV.



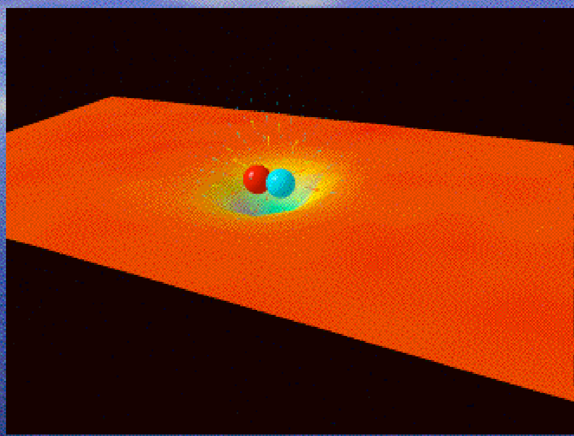
QCD Gives Strings A Chance

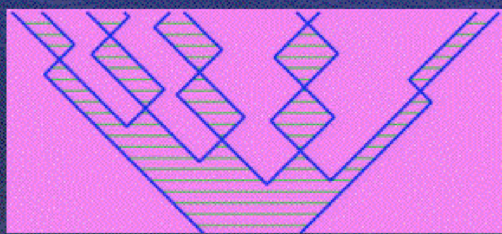
- At short distances, must smaller than 1 fermi, the quark-antiquark potential is approximately Coulombic, due to the Asymptotic Freedom.
- At large distances the potential should be linear (Wilson) due to formation of confining flux tubes.



Flux Tubes in QCD

- These objects may be approximately described by the Nambu strings
  (animation from lattice work by D. Leinweber et al, Univ. of Adelaide)
- The tubes are widely used, for example, in jet hadronization algorithms (the Lund String Model) where they snap through quark-antiquark creation.





Semi-Classical String

- Semi-classical quantization around long straight Nambu string predicts the quark-antiquark potential

$$V(r) = \sigma r + \mu + \gamma/r + \mathrm{O}(1/r^2)$$

- The coefficient of the universal Luescher term depends only on the space-time dimension d and is proportional to the Regge intercept:

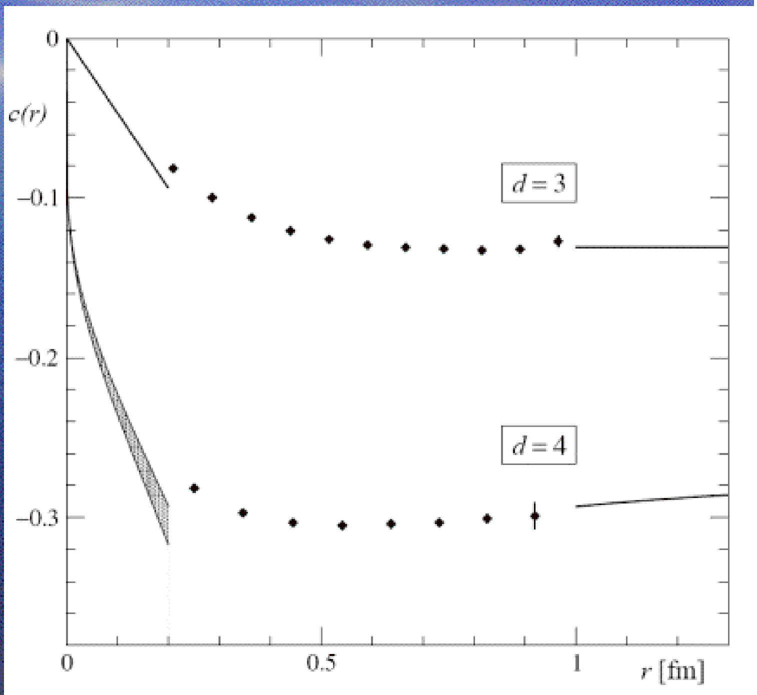
$$\gamma = -\frac{\pi}{24}(d-2)$$

Comparison with Lattice Data

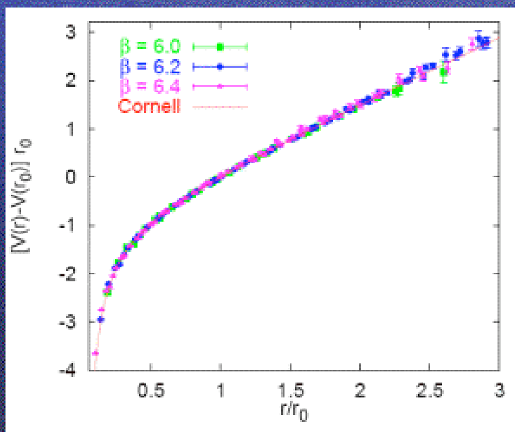
Luescher, Weisz (2002)

- Lattice calculations of the force vs. distance produce good agreement with the semi-classical Nambu string for r>0.7 fm:

$$c(r) = \frac{1}{2} r^3 F'(r)$$

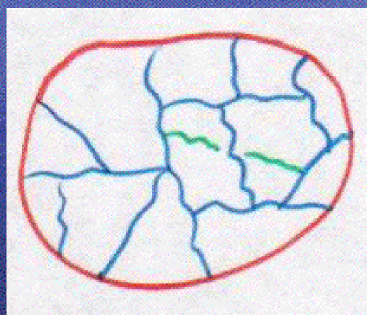


- While the success of this `effective string' description is very nice, one could ask a deeper, more ambitious question: **Are there examples of EXACT duality between gauge theory and string theory?**
- In QCD it would not be limited to a long string approximation, and would incorporate ALL the features of gauge theory including, for example, the nearly Coulombic short distance quark anti-quark potential (lattice results by G. Bali et al with $r_0 \sim 0.5$ fm).



Large N Gauge Theories

- Connection of gauge theory with string theory is strengthened in `t Hooft's generalization from 3 colors (SU(3) gauge group) to N colors (SU(N) gauge group).
- Make N large, while keeping the `t Hooft coupling $\lambda = g_{\rm YM}^2 N$ fixed.
- The probability of snapping a flux tube by quark-antiquark creation (meson decay) is 1/N. The string coupling is 1/N.
- Yet, the planar diagrams needed in the large N limit are very difficult to sum explicitly.

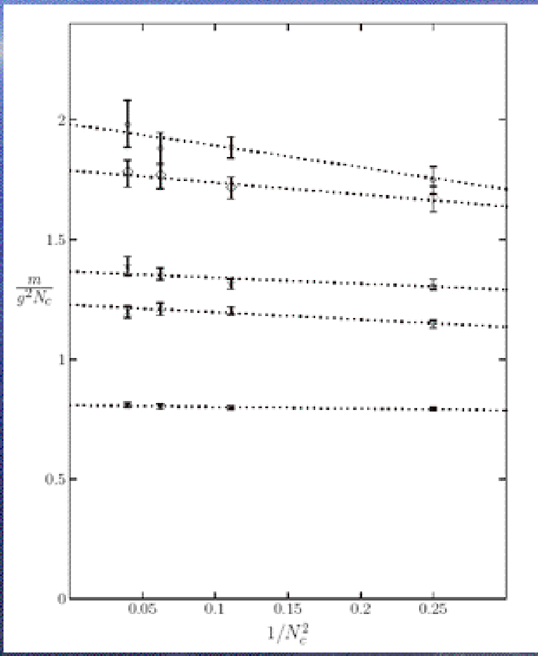


Glueballs in 2+1 SU(N) Theory

- $m/\lambda = a + b/N^2$ provides a good fit even for low N

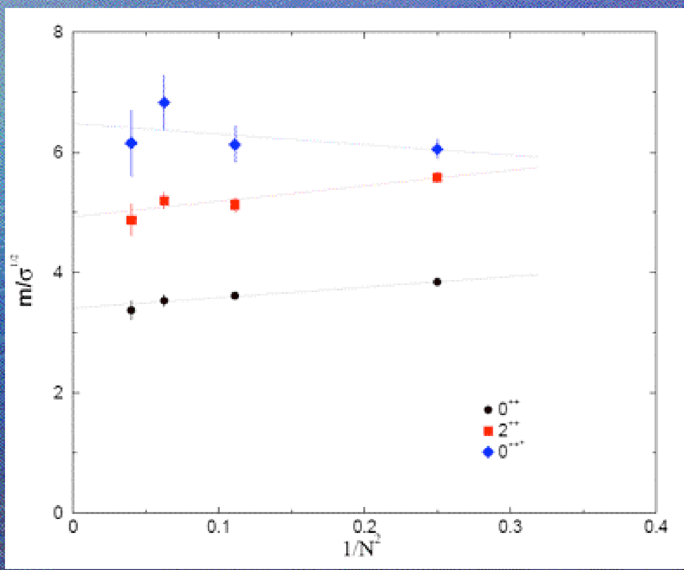
  M. Teper

- This is evidence that the large N limit is approached rapidly, at least in the pure glue theory.

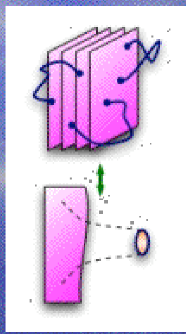


Glueballs in 3+1 SU(N) Theory

Lucini and Teper



D-Branes vs. Geometry



- Dirichlet branes (Polchinski) led string theory back to gauge theory in the mid-90's.
- A stack of N Dirichlet 3-branes realizes $\mathcal{N}$=4 supersymmetric SU(N) gauge theory in 4 dimensions. It also creates a curved background of 10-d theory of closed superstrings (artwork by E.Imeroni)

$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-1/2} \left(-(dx^0)^2 + (dx^i)^2\right) + \left(1 + \frac{L^4}{r^4}\right)^{1/2} \left(dr^2 + r^2 d\Omega_5^2\right)$$

which for small r approaches $AdS_5 \times \mathbf{S}^5$

- The self-dual 5-form background

$$F_5 = \mathcal{F}_5 + \star\mathcal{F}_5 \ , \qquad \mathcal{F}_5 = 16\pi(\alpha')^2 N \mathrm{vol}(\mathbf{S}^5)$$

is adjusted to satisfy the D-brane tension quantization condition

$$\int_{\mathbf{S}^{8-p}} \star F_{p+2} = \frac{2\kappa^2 \tau_p N}{g_s}$$

$$\tau_p = \frac{\sqrt{\pi}}{\kappa}(4\pi^2\alpha')^{(3-p)/2}$$

$$\int_{\mathbf{S}^5} F_5 = (4\pi^2\alpha')^2 N$$

- Comparing with $g_s F_5 = d^4x \wedge dh^{-1} - r^5 \frac{dh}{dr}\mathrm{vol}(\mathbf{S}^5)$

  and using $-r^5 \frac{dh}{dr} = 4L^4$ and $g_{\mathrm{YM}}^2 = 4\pi g_{st}$

  find $L^4 = g_{\mathrm{YM}}^2 N \alpha'^2$

Entropy of Thermal $\mathcal{N}$=4 SUSY SU(N) theory

- Dual to a near-extremal background

$$ds^2 = \frac{r^2}{L^2}\left[-\left(1-\frac{r_0^4}{r^4}\right)dt^2 + d\vec{x}^2\right] + \frac{L^2}{r^2}\left(1-\frac{r_0^4}{r^4}\right)^{-1}dr^2 + L^2 d\Omega_5^2$$

- The CFT temperature is identified with the Hawking T of the horizon located at r_0

$$T = r_0/(\pi L^2)$$

- Any event horizon contains Bekenstein-Hawking entropy $S_{BH} = \frac{2\pi A_h}{\kappa^2}$
- A brief calculation gives the entropy density $s = \frac{\pi^2}{2} N^2 T^3$

Gubser, IK, Peet

- This is interpreted as the strong coupling limit of

$$s = \frac{2\pi^2}{3} f(\lambda) N^2 T^3$$

- The weak `t Hooft coupling behavior of the interpolating function is determined by Feynman graph calculations in the $\mathcal{N}$=4 SYM theory

$$f(\lambda) = 1 - \frac{3}{2\pi^2}\lambda + \frac{3+\sqrt{2}}{\pi^3}\lambda^{3/2} + \dots$$

- We deduce from AdS/CFT duality that

$$\lim_{\lambda\to\infty} f(\lambda) = \frac{3}{4}$$

- The entropy density is multiplied only by _ as the coupling changes from zero to infinity. Gubser, IK, Tseytlin

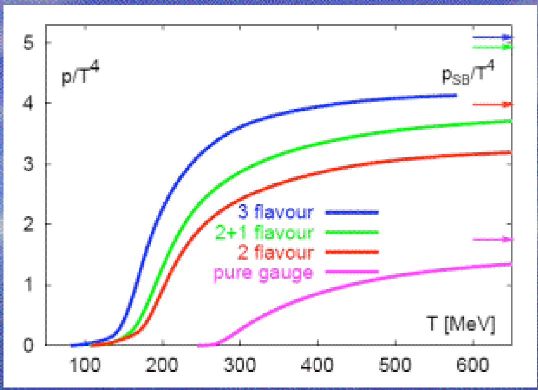
- Corrections to the interpolating function at strong coupling come from the higher-derivative terms in the type IIB effective action:

$$f(g_{\rm YM}^2 N) = \frac{3}{4} + \frac{45}{32}\zeta(3)(g_{\rm YM}^2 N)^{-3/2} + \dots$$

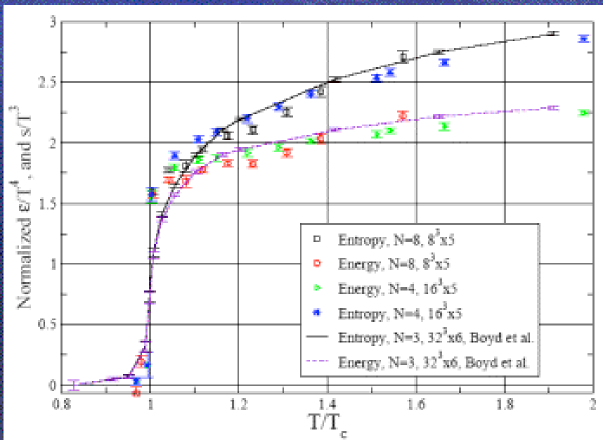
Gubser, IK, Tseytlin

- The interpolating function is usually assumed to have a smooth monotonic form, but so far we do not know its form at the intermediate coupling.

- A similar reduction of entropy by strong-coupling effects is observed in lattice non-supersymmetric gauge theories for N=3: the arrows show free field values.
  Karsch (hep-lat/0106019).



- N-dependence in the pure glue theory enters largely through the overall normalization.
  Bringoltz and Teper (hep-lat/0506034)



Super-Conformal Invariance

- In the $\mathcal{N}$=4 SYM theory there are 6 scalar fields (it is useful to combine them into 3 complex scalars: Z, W, V) and 4 gluinos interacting with the gluons. All the fields are in the adjoint representation of the SU(N) gauge group.
- The Asymptotic Freedom is canceled by the extra fields; the beta function is exactly zero for any complex coupling. The theory is invariant under scale transformations $x^\mu \to a\, x^\mu$. It is also invariant under space-time inversions. The full super-conformal group is SU(2,2|4).

The AdS/CFT Duality

Maldacena; Gubser, IK, Polyakov; Witten

- Relates conformal gauge theory in 4 dimensions to string theory on 5-d Anti-de Sitter space times a 5-d compact space. For the $\mathcal{N}$=4 SYM theory this compact space is a 5-sphere realizing the SU(4) R-symmetry.
- The SO(2,4) geometrical symmetry of the AdS_5 space realizes the conformal symmetry of the gauge theory.
- The d-dimensional AdS space is a hyperboloid

$$(X^0)^2 + (X^d)^2 - \sum_{i=1}^{d-1} (X^i)^2 = L^2 \, .$$

- Its metric is

$$ds^2 = \frac{L^2}{z^2}\left(dz^2 - (dx^0)^2 + \sum_{i=1}^{d-2}(dx^i)^2\right)$$

- When a gauge theory is strongly coupled, the radius of curvature of the dual AdS_5 and of the 5-d compact space becomes large: $\frac{L^2}{\alpha'} \sim \sqrt{g_{\rm YM}^2 N}$

- String theory in such a weakly curved background can be studied in the effective (super)-gravity approximation, which allows for a host of explicit calculations. Corrections to it proceed in powers of $\frac{\alpha'}{L^2} \sim \lambda^{-1/2}$

- Feynman graphs instead develop a weak coupling expansion in powers of λ. At weak coupling the dual string theory becomes difficult.

- Gauge invariant operators in the CFT_4 are in one-to-one correspondence with fields (or extended objects) in AdS_5
- Operator dimension is determined by the mass of the dual field; e.g. for scalar operators GKPW

$$\Delta_{\pm} = 2 \pm \sqrt{4 + m^2 L^2}$$

- The BPS protected operators are dual to SUGRA fields of m~1/L. Their dimensions are independent of λ.
- The unprotected operators (Konishi operator is the simplest) are dual to massive string states. AdS/CFT predicts that at strong coupling their dimensions grow as $\lambda^{1/4}$.

- Correlation functions are calculated from the dependence of string theory path integral on boundary conditions ϕ_0 in AdS_5, imposed near z=0:

$$\langle \exp \int d^4x \phi_0 \mathcal{O} \rangle = Z_{\text{string}}[\phi_0]$$

- In the large N limit the path integral is found from the classical string action:

$$Z_{\text{string}}[\phi_0] \sim \exp(-I[\phi_0])$$

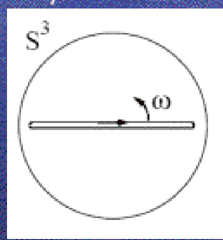
Spinning Strings vs. Highly Charged Operators

- Vibrating closed strings with large angular momentum on the 5-sphere are dual to SYM operators with large R-charge (the number of fields Z) Berenstein, Maldacena, Nastase
- For example a 2-impurity operator $\mathrm{Tr}\, W Z^l W Z^{J-l} e^{ipl}$ with p ~ 1/J is dual to a small vibrating string circling around the equator of S^5 .
- When the number of impurities W becomes comparable to J, the string elongates.

- Generally, semi-classical spinning strings are dual to long operators, e.g. the dual of a high-spin operator

$$\mathrm{Tr}\, F_{+\mu} D_+^{S-2} F_+{}^{\mu}$$

is a folded string spinning around the center of global AdS_5 space Gubser, IK, Polyakov



- Its metric is

$$ds^2 = R^2 \left(-dt^2 \cosh^2 \rho + d\rho^2 + \sinh^2 \rho d\Omega_3^2\right)$$

- The operator dimension is given by the energy in these coordinates.

- The structure of dimensions of high-spin operators is $$\Delta - S = f(g)\ln S \ + O(S^0), \qquad g = \frac{\sqrt{g_{YM}^2 N}}{4\pi}$$
- The function f(g) is independent of the twist; it is universal in the planar limit.
- At strong coupling, the AdS/CFT corresponds predicts via the spinning string energy calculations Gubser, IK, Polyakov; Frolov, Tseytlin

$$f(g) = 4g - \frac{3\ln 2}{\pi} + \ldots$$

- At weak coupling the expansion of the universal function f(g) up to 3 loops is

$$f(g) = 8g^2 - \frac{8}{3}\pi^2 g^4 + \frac{88}{45}\pi^4 g^6 + O(g^8)$$

Exact Integrability

- Perturbative calculations of anomalous dimensions are mapped to integrable spin chains, suggesting exact integrability of the $\mathcal{N}$=4 SYM theory. Minahan, Zarembo; Beisert, Staudacher
- For the `SU(2) sector' operators Tr (ZZZWZW…ZW) the Heisenberg spin chain emerges at 1 loop. Higher loops correct the Hamiltonian but seem to preserve its integrability (infinite number of conserved charges). Such spin chains are solvable via Bethe Ansatz.
- This meshes nicely with earlier findings of integrability in certain subsectors of QCD. Lipatov; Faddeev, Korchemsky; Braun, Derkachov, Manashov
- The string theory approach indicates that in the SYM theory the exact integrability is present at very strong coupling (Bena, Polchinski, Roiban). Hence it is likely to exist for all values of the coupling.

- The coefficients in f(g) are related to the corresponding coefficients in QCD through selecting at each order the term with the highest transcendentality. Kotikov, Lipatov, Onishchenko, Velizhanin
- Recently, great progress has been achieved on finding f(g) at 4 loops and beyond!
- Using the spin chain symmetries, the Bethe ansatz equations were restricted to the form Staudacher, Beisert

$$\left(\frac{x_k^+}{x_k^-}\right)^L = \prod_{\substack{j=1 \\ j\neq k}}^{S} \frac{x_k^- - x_j^+}{x_k^+ - x_j^-}\,\frac{1-g^2/x_k^+x_j^-}{1-g^2/x_k^-x_j^+}\,\exp(2i\theta(u_k,u_j))$$

- The magnon dispersion relation is

$$4C^2 - 16g^2 \sin^2(\tfrac{1}{2}p) = 1$$

- The complex x-variables encode the momentum p and energy C:

$$e^{ip} = \frac{x^+}{x^-}\,, \qquad C = \frac{1}{2} + \frac{ig}{x^+} - \frac{ig}{x^-}$$

$$u = x^+ + \frac{1}{x^+} - \frac{i}{2g} = x^- + \frac{1}{x^-} + \frac{i}{2g}$$

- Of particular importance is the crossing symmetry (Janik)

$$x^\pm \mapsto \frac{1}{x^\pm}, \qquad p \mapsto -p, \qquad C \mapsto -C, \qquad u \mapsto u$$

The Dressing Phase

- The `dressing phase' in the magnon S-matrix appears first at 4-loop order in perturbation theory:

$$\theta(u_k, u_j) = \sum_{r=2}^{\infty} \sum_{\nu=0}^{\infty} \beta_{r,r+1+2\nu}(g) \big(q_r(u_k)\, q_{r+1+2\nu}(u_j) - q_r(u_j)\, q_{r+1+2\nu}(u_k) \big)$$

  where $q_r(u)$ are eigenvalues of the magnon charges. It was determined first in the large g expansion (Beisert, Hernandez, Lopez), and then via appropriate resummation in the small g expansion (Beisert, Eden, Staudacher).
- f(g) is determined through solving an integral equation

$$f(g) = 16 g^2 s(0)$$

$$s(t) = K(2gt, 0) - 4g^2 \int_0^{\infty} dt' K(2gt, 2gt') \frac{t'}{e^{t'} - 1} s(t')$$

- The BES kernel is $K(t,t') = K^{(m)}(t,t') + 2K^{(c)}(t,t')$
- The first term is the ES kernel

$$K^{(m)}(t,t') = \frac{J_1(t)J_0(t') - J_0(t)J_1(t')}{t-t'}$$

while the second one is due to the dressing phase in the magnon S-matrix

$$K^{(c)}(t,t') = 4g^2 \int_0^\infty dt'' K_1(t, 2gt'') \frac{t''}{e^{t''}-1} K_0(2gt'', t')$$

$$K_0(t,t') = \frac{tJ_1(t)J_0(t') - t'J_0(t)J_1(t')}{t^2 - t'^2} = \frac{2}{t\,t'} \sum_{n=1}^{\infty} (2n-1) J_{2n-1}(t) J_{2n-1}(t')$$
$$K_1(t,t') = \frac{t'J_1(t)J_0(t') - tJ_0(t)J_1(t')}{t^2 - t'^2} = \frac{2}{t\,t'} \sum_{n=1}^{\infty} (2n) J_{2n}(t) J_{2n}(t').$$

- Perturbative order-by-order solution of the BES equation gives the 4-loop term in f(g)

$$-16\left(\frac{73}{630}\pi^6 + 4\zeta(3)^2\right)g^8$$

  (it differs by relative sign from the ES prediction which did not include the \`dressing phase')
- Remarkably, an independent 4-loop calculation by Bern, Dixon, Czakon, Kosower and Smirnov yielded a numerical value that prompted them to conjecture exactly the same analytical result.
- This has led the two groups to the same conjecture for the complete structure of the perturbative expansion of f(g): it is the one yielded by the BES integral equation.

- This approach yields analytic predictions for all planar perturbative coefficients

$$\begin{aligned} f(g) = {} & 8g^2 - \frac{8}{3}\pi^2 g^4 + \frac{88}{45}\pi^4 g^6 - 16\left(\frac{73}{630}\pi^6 + 4\,\zeta(3)^2\right) g^8 \\ & + 32\left(\frac{887}{14175}\pi^8 + \frac{4}{3}\pi^2\zeta(3)^2 + 40\,\zeta(3)\,\zeta(5)\right) g^{10} \\ & - 64\left(\frac{136883}{3742200}\pi^{10} + \frac{8}{15}\pi^4\zeta(3)^2 + \frac{40}{3}\pi^2\zeta(3)\,\zeta(5) \right. \\ & \qquad \left. + 210\,\zeta(3)\,\zeta(7) + 102\,\zeta(5)^2\right) g^{12} \\ & + 128\left(\frac{7680089}{340540200}\pi^{12} + \frac{47}{189}\pi^6\zeta(3)^2 + \frac{82}{15}\pi^4\zeta(3)\,\zeta(5) + 70\pi^2\zeta(3)\,\zeta(7) \right. \\ & \qquad \left. + 34\pi^2\zeta(5)^2 + 1176\,\zeta(3)\,\zeta(9) + 1092\,\zeta(5)\,\zeta(7) + 4\,\zeta(3)^4\right) g^{14} \end{aligned}$$

- So far the 4-loop answer is only known numerically. Recently, a new method yielded improved numerical precision and agrees with the analytical prediction to around 0.001%.

Cachazo, Spradlin, Volovich

- The alternation of the series and the geometric behavior of the coefficients remove all singularities from the real axis, allowing smooth extrapolation to infinite coupling.
- The radius of convergence is _. The closest singularities are square-root branch points at

$$g = \pm i/4$$

- The analytic structure of f(g) is not completely clear, but there appear to be an infinite number of branch cuts along the imaginary axis. There is an essential singularity at infinity.

- To solve the equation at finite coupling, we use a basis of linearly independent functions $s(t) = \sum_{n\geq 1} s_n \frac{J_n(2gt)}{2gt}$
- Determination of $f(g) = 8g^2 s_1$ is tantamount to inverting an infinite matrix.

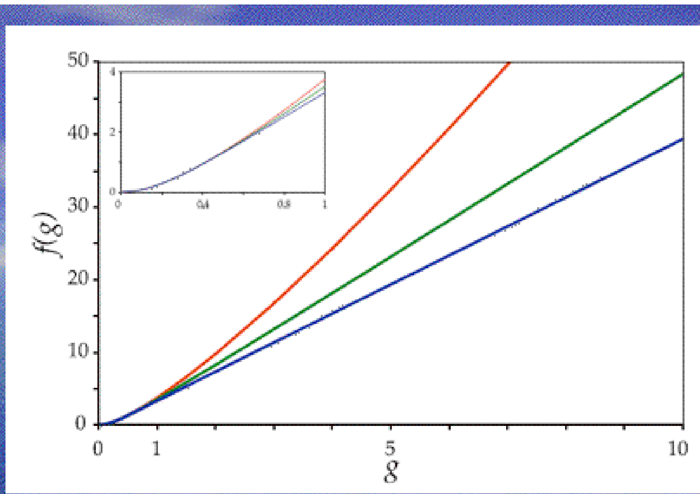
$$s_n \frac{J_n(2gt)}{2gt} = \frac{J_1(2gt)}{2gt} + 8n Z_{2n,1} \frac{J_{2n}(2gt)}{2gt} - 2n Z_{nm} s_m \frac{J_n(2gt)}{2gt}$$
$$-16n(2m-1) Z_{2n,2m-1} Z_{2m-1,r} s_r \frac{J_{2n}(2gt)}{2gt} \ ,$$

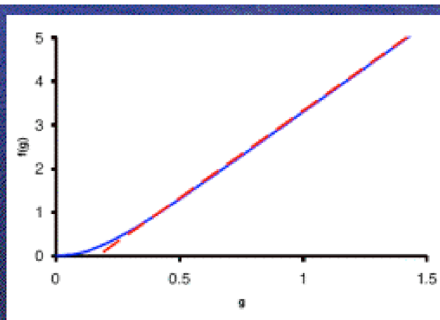
$$Z_{mn} = \int_0^\infty \frac{J_m(2gt) J_n(2gt)}{t(e^t - 1)}$$

- Truncation to finite matrices converges very rapidly. Benna, Benvenuti, IK, Scardicchio

- The blue line refers to the BES equation, red line to the ES, green line to the equation where the dressing kernel is divided by 2.

- The first two terms of the numerical large g asymptotics are in very precise agreement with the AdS/CFT spinning string predictions. The third is an approximate prediction.





$$f(g) = (4.000000 \pm 0.000001)g - (0.661907 \pm 0.000002) - \frac{0.0232 \pm 0.0001}{g} + \dots$$

- Recently, the exact analytic solution was obtained for the leading order BES equation. Alday, Arutyunov, Benna, Eden, IK; Kostov, Serban, Volin; Beccaria, De Angelis, Forini
- Expanding at strong coupling, $s_k = \frac{1}{g} s_k^{\ell} + \frac{1}{g^2} s_k^{s\ell} + \ldots$

  The solution is $s^{\ell}_{2n-1} = s^{\ell}_{2n} = \frac{(-1)^{n-1}(2n-1)!!}{2^n (n-1)!}$
- Since s_1=1/2, this proves that

  f(g)= 4 g + O(1)

Shear Viscosity η and RHIC

- In a comoving frame, $$T_{ij} = \delta_{ij} p - \eta \left(\partial_i u_j + \partial_j u_i - \frac{2}{3} \delta_{ij} \partial_k u_k \right)$$
- Can be also determined through the Kubo formula $$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d\mathbf{x}\, e^{i\omega t} \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0,0)] \rangle$$
- For the $\mathcal{N}$=4 supersymmetric YM theory this 2-point function may be computed from graviton absorption by the 3-brane metric.
- At very strong coupling, Policastro, Son and Starinets found $$\eta = \frac{\pi}{8} N^2 T^3$$

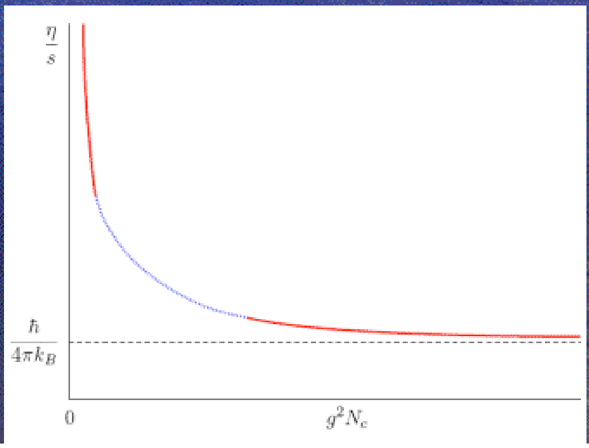
Viscosity/entropy lower bound?

Kovtun, Son, Starinets

- In the SYM theory at very strong coupling $\frac{\eta}{s} = \frac{\hbar}{4\pi}$
- This is reasonable on general grounds. The shear viscosity η ~ energy density times quasiparticle mean free time τ. So, η/s ~ quasiparticle energy $\times$ τ, which is bounded from below by the uncertainty principle.

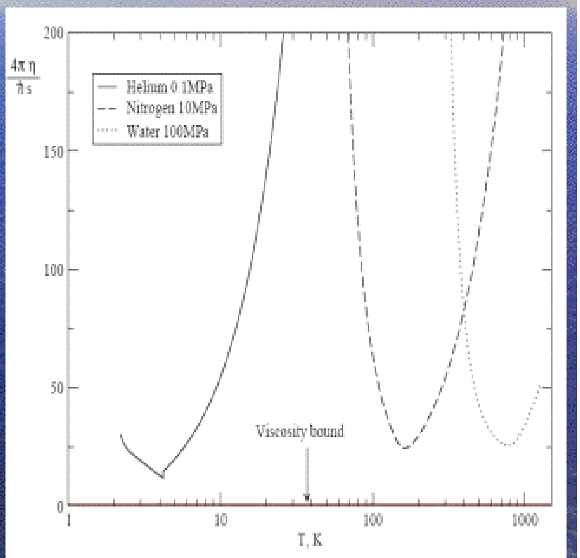
- At weak coupling η/s is large. There is evidence it decreases monotonically.

Buchel, Liu, Starinets



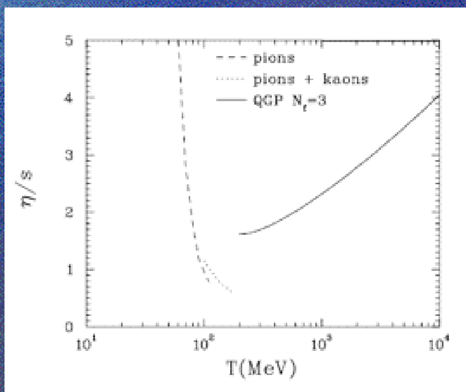
Is very strongly coupled SYM the most perfect fluid?

- For known fluids (e.g. helium, nitrogen, water) η/s is considerably higher.
- The quark-gluon plasma produced at RHIC is believed to be strongly coupled and to have viscosity not far from the KSS bound. Shuryak, Teaney, Gyulassy, McLerran, Hirano, …
- A new term has been coined, sQGP, to describe the state observed at RHIC. A lot of recent string theory research is devoted to developing intuition about this state.



Is QCD close to saturating the bound?

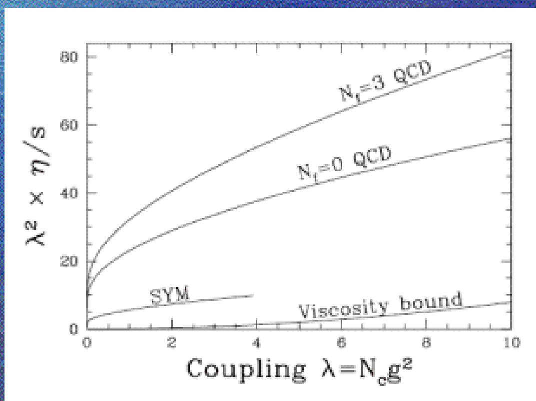
- At low T use chiral perturbation theory; at high T thermal gauge theory. Graph from
  Csernai, Kapusta, McLerran

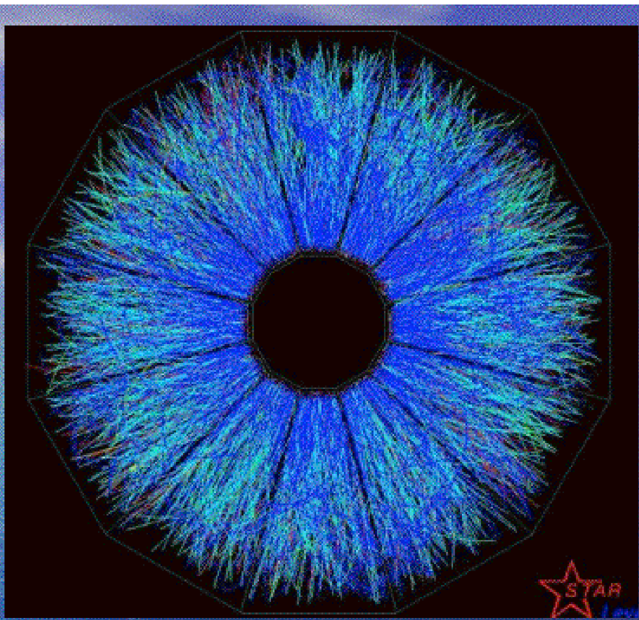


Comparing QCD with $\mathcal{N}$=4 SYM

- At least at weak coupling, the SYM theory gives a much lower result than QCD
  Huot, Jeon, Moore

$$\frac{\eta_{\mathrm{SYM}}}{s_{\mathrm{SYM}}} \simeq \frac{6.174}{\lambda^2 \ln(2.36/\sqrt{\lambda})}$$



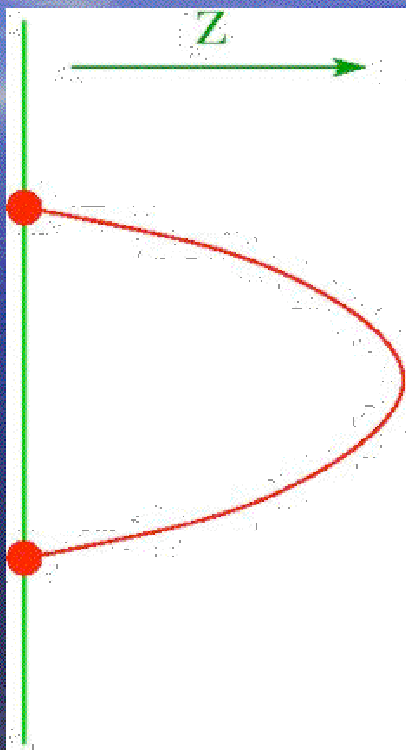


A high-energy collision of gold ions at BNL's Relativistic Heavy Ion Collider (RHIC) produces a fireworks display of particle tracks, as recorded by the STAR detector. In central collisions, about 7500 hadrons are produced.

The quark anti-quark potential

- The z-direction of AdS is dual to the energy scale of the gauge theory: small z is the UV; large z is the IR.
- Because of the 5-th dimension z, the string picture applies even to theories that are conformal. The quark and anti-quark are placed at the boundary of Anti-de Sitter space (z=0), but the string connecting them bends into the interior (z>0). Due to the scaling symmetry of the AdS space, this gives Coulomb potential (Maldacena; Rey, Yee)

$$V(r) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma\left(\frac{1}{4}\right)^4 r}$$



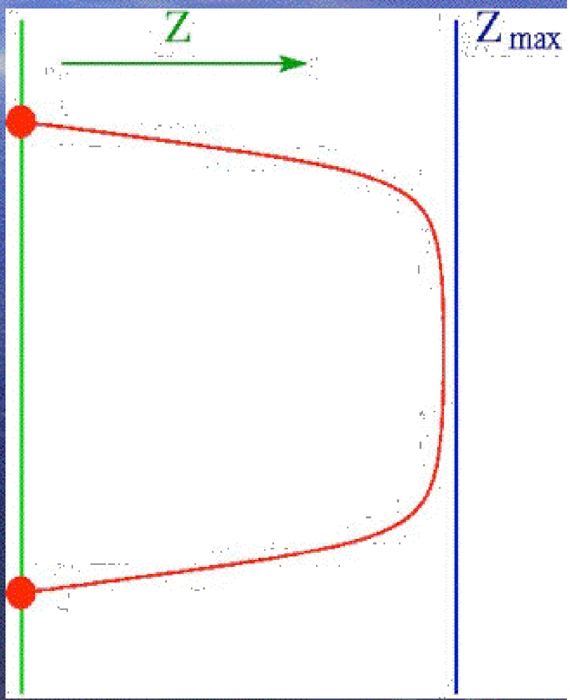
String Theoretic Approaches to Confinement

- It is possible to generalize the AdS/CFT correspondence in such a way that the quark-antiquark potential is linear at large distance but nearly Coulombic at small distance.
- The 5-d metric should have a warped form (Polyakov):

$$ds^2 = \frac{dz^2}{z^2} + a^2(z)\big(-(dx^0)^2 + (dx^i)^2\big)$$

- The space ends at a maximum value of z where the warp factor is finite. Then the confining string tension is

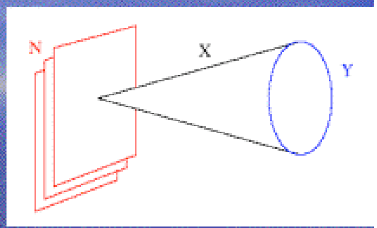
$$\frac{a^2(z_{\max})}{2\pi\alpha'}$$



Conebrane Dualities

- To reduce the number of supersymmetries in AdS/CFT, we may place the stack of N D3-branes at the tip of a 6-d Ricci-flat cone X whose base is a 5-d Einstein space Y:

$$ds_X^2 = dr^2 + r^2 ds_Y^2$$



- Taking the near-horizon limit of the background created by the N D3-branes, we find the space AdS_5 x Y, with N units of RR 5-form flux, whose radius is given by

$$L^4 = \frac{\sqrt{\pi}\kappa N}{2\,\mathrm{Vol}(Y)} = 4\pi g_s N \alpha'^2 \frac{\pi^3}{\mathrm{Vol}(Y)}$$

- This type IIB background is conjectured to be dual to the IR limit of the gauge theory on N D3-branes at the tip of the cone X.

Trace Anomaly

- In a 4-d CFT there are two trace anomaly coefficients, a and c: $\langle T^\alpha_\alpha \rangle = -aE_4 - cI_4$

$$E_4 = \frac{1}{16\pi^2}\left(R^2_{ijkl} - 4R^2_{ij} + R^2\right)$$

$$I_4 = -\frac{1}{16\pi^2}\left(R^2_{ijkl} - 2R^2_{ij} + \frac{1}{3}R^2\right)$$

- Calculations on AdS_5 x Y give their leading large N values Henningson, Skenderis; Gubser

$$a = c = \frac{\pi^3 N^2}{4\,\mathrm{Vol}(Y)}$$

- In super-conformal theories the anomalies are related to the spectrum of R-charges of the chiral fermions: Anselmi, Freedman, Grisaru, Johansen

$$a = \frac{3}{32}(3\mathrm{Tr}R^3 - \mathrm{Tr}R)\ ; \quad c = \frac{1}{32}(9\mathrm{Tr}R^3 - 5\mathrm{Tr}R)$$

- This provides basic checks of the dualities.
- For the $\mathcal{N}$=4 SYM theory the gauginos have R=1, while the fermion fields from the Z^i chiral multiplets have R=-1/3.
- Since Tr R=0, we find a=c, and

$$a = c = \frac{9}{32}\mathrm{Tr}R^3 = (N^2-1)\frac{9}{32}\left(1+3(-1/3)^3\right) = \frac{N^2-1}{4}$$

- On the gravity side, the volume of S^5 is π^3 (the radius of Y is fixed so that $R_{ij} = 4g_{ij}$)
- For large N the two calculations of anomaly coefficients agree.

Orbifold Cones

Kachru, Silverstein; Lawrence, Nekrasov, Vafa

- The simplest set of examples is provided by cones that are orbifolds R^6/Γ, where Γ is a subgroup of the rotation group SU(4).
- For abelian orbifolds, all group elements can be brought to the form

$$\begin{pmatrix} e^{2\pi i x_1} & 0 & 0 & 0 \\ 0 & e^{2\pi i x_2} & 0 & 0 \\ 0 & 0 & e^{2\pi i x_3} & 0 \\ 0 & 0 & 0 & e^{-2\pi i(x_1+x_2+x_3)} \end{pmatrix}$$

- For Z_k orbifolds, the n-th group element is specified by three integers m_i defined mod k: $x_i=nm_i/k$.
- If none of the eigenvalues of the generator = 1, then all SUSY is broken; if one of the eigenvalues = 1, then $\mathcal{N}=1$ SUSY is preserved; if two of the eigenvalues = 1, then $\mathcal{N}=2$ SUSY is preserved.

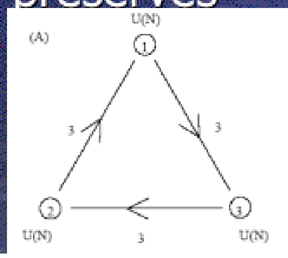
- The action in the n-th twisted sector on 3 complex coordinates of C^3, $Z^1 = X^1 + iX^2\,, \quad Z^2 = X^3 + iX^4\,, \quad Z^3 = X^5 + iX^6$

  and their complex conjugates, is

$$R(g_n) = \mathrm{diag}(\omega_k^{n(m_1+m_2)}, \omega_k^{n(m_1+m_3)}, \omega_k^{n(m_2+m_3)}, \omega_k^{-n(m_1+m_2)}, \omega_k^{-n(m_1+m_3)}, \omega_k^{-n(m_2+m_3)})$$

  where $\omega_k = e^{2\pi i/k}$

- If none of these phases = 1, then the orbifold acts freely on S^5/Γ. (The tip of the cone is a fixed point that is removed in the basic near-horizon limit.)
- A well-known example of a freely-acting orbifold is Z_3 with m_i=1. Since one of the eigenvalues of the generator = 1, i.e. $\Gamma \subset SU(3)$, this orbifold preserves $\mathcal{N}$=1 SUSY.



Construction of the quiver gauge theories Douglas, Moore

- Gauge theory on N D3-branes at the tip of R^6/Γ is found by applying projections to the U(Nk) gauge theory on the covering space. Retain only the fields invariant under the orbifold action combined with conjugation by a U(Nk) matrix γ acting on the gauge indices: $\gamma = \mathrm{diag}(I_N, e^{2\pi i/k} I_N, e^{4\pi i/k} I_N, \ldots, e^{-2\pi i/k} I_N)$

$$\psi^1 \to e^{2\pi i m_1/k} \gamma \psi^1 \gamma^{-1}\,, \qquad \psi^2 \to e^{2\pi i m_2/k} \gamma \psi^2 \gamma^{-1}\,, \ \ldots$$

$$Z^1 \to e^{2\pi i(m_1+m_2)/k} \gamma Z^1 \gamma^{-1}\,, \qquad Z^2 \to e^{2\pi i(m_1+m_3)/k} \gamma Z^2 \gamma^{-1}\,, \ \ldots$$

- In the supersymmetric examples, such as C^3/Z_3 or the conifold, the conebrane dualities have been tested almost as thoroughly as in the maximally supersymmetric case.
- But when all SUSY is broken, problems may arise.
- When the orbifold Γ breaks all SUSY and is not freely acting, then the weakly curved background $AdS_5 \times S^5/\Gamma$ is unstable due to the presence of tachyons that have $(mL)^2<-4$, and therefore violate the BF bound.
- But for freely acting orbifolds, the negative zero-point energy is compensated by the large stretching of the twisted sector closed strings in the compact space. Hence, at large radius, there are no `bad tachyons.' This makes freely acting orbifolds particularly interesting from AdS/CFT point of view.
- Before the formal decoupling limit, the non-SUSY freely acting orbifolds have closed-string tachyons localized at the tip of the cone. These problems seem related to flow of double-trace couplings.

Problem 1

a) Derive the Z_2 quiver gauge theory obtained by the projection on the $U(2N)$ $\mathcal{N}=4$ SYM where the Z_2 is generated by $-I$ in $SU(4)$ accompanied by conjugation with $\gamma = \mathrm{diag}(I_N, -I_N)$. What is the gauge group and field content? Is this a freely acting orbifold?

b) In this gauge theory, calculate the one-loop Coleman-Weinberg potential as a function of the eigenvalues of the adjoint scalars. What are the operators that pick up one-loop beta functions?

D3-branes on the Conifold

- The conifold is a Calabi-Yau 3-fold cone X described by the constraint $\sum_{a=1}^{4} z_a^2 = 0$ on 4 complex variables.
- Its base Y is a coset $T^{1,1}$ which has symmetry $SU(2)_A \times SU(2)_B$ that rotates the z's, and also $U(1)_R$: $z_a \to e^{i\theta} z_a$
- The Sasaki-Einstein metric on $T^{1,1}$ is

$$\mathrm{d}s^2_{T^{1,1}} = \frac{1}{9}\Big(\mathrm{d}\psi + \sum_{i=1}^{2} \cos\theta_i \mathrm{d}\phi_i\Big)^2 + \frac{1}{6}\sum_{i=1}^{2}\Big(\mathrm{d}\theta_i^2 + \sin^2\theta_i \mathrm{d}\phi_i^2\Big)$$

  where $\theta_i \in [0, \pi], \phi_i \in [0, 2\pi], \psi \in [0, 4\pi]$
- The topology of $T^{1,1}$ is $S^2 \times S^3$.

- To `solve' the conifold constraint

  det Z = 0 we introduce another set of convenient coordinates:

$$Z = \begin{pmatrix} z^3 + iz^4 & z^1 - iz^2 \\ z^1 + iz^2 & -z^3 + iz^4 \end{pmatrix} = \begin{pmatrix} w_1 & w_3 \\ w_4 & w_2 \end{pmatrix} = \begin{pmatrix} a_1 b_1 & a_1 b_2 \\ a_2 b_1 & a_2 b_2 \end{pmatrix}$$

- The action of global symmetries is

$$SU(2) \times SU(2) \text{ symmetry} \quad : \quad \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \to L \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \qquad \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \to R \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\text{R-symmetry} \quad : \quad (a_i, b_j) \to e^{i\frac{\alpha}{2}} (a_i, b_j) \ ,$$

- There is a redundancy under

$$a_i \to \lambda \, a_i \quad , \quad b_j \to \frac{1}{\lambda} \, b_j \quad (\lambda \in \mathbf{C})$$

  which is partly fixed by imposing

$$|a_1|^2 + |a_2|^2 - |b_1|^2 - |b_2|^2 = 0$$

- It remains to quotient the space by the phase rotation $a \sim e^{i\alpha}a, b \sim e^{-i\alpha}b$ (in the gauge theory, this will have the meaning of the U(1) baryon number symmetry).
- In the IR gauge theory on D3-branes at the apex of the conifold, the coordinates a_1, a_2, b_1, b_2 are replaced by chiral superfields. For a single D3-brane it is necessary to introduce gauge group U(1) x U(1).

- The $\mathcal{N}$=1 SCFT on N D3-branes at the apex of the conifold has gauge group SU(N)xSU(N) coupled to bifundamental chiral superfields A_1, A_2, in $(\overline{\mathbf{N}}, \mathbf{N})$, and B_1, B_2 in $(\mathbf{N}, \overline{\mathbf{N}})$. IK, Witten
- The R-charge of each field is _. This insures $U(1)_R$ anomaly cancellation.
- The unique $SU(2)_A$x$SU(2)_B$ invariant, exactly marginal quartic superpotential is added:

$$W = \epsilon^{ij}\epsilon^{kl} \,\mathrm{tr}\, A_i B_k A_j B_l$$

- This theory also has a baryonic U(1) symmetry under which $A_k \to e^{ia} A_k$; $B_l \to e^{-ia} B_l$, and a Z_2 symmetry which interchanges the A's with the B's and implements charge conjugation.

Comparison with a Z_2 Orbifold Quiver

- The simplest $\mathcal{N}$=2 SUSY quiver has k=2; $m_1=m_2=1$, $m_3=0$. The gauge group is again SU(N)xSU(N), but in addition to the bifundamentals A_i, B_j, there is one adjoint chiral superfield for each gauge group, with superpotential $g\mathrm{Tr}\Phi(A_1B_1 - A_2B_2) + g\mathrm{Tr}\tilde{\Phi}(B_1A_1 - B_2A_2)$
- Adding a Z_2 odd mass term $\frac{m}{2}(\mathrm{Tr}\Phi^2 - \mathrm{Tr}\tilde{\Phi}^2)$ and integrating out the adjoints, we obtain the superpotential of the conifold theory,

$$-\frac{g^2}{m}\left[\mathrm{Tr}(A_1B_1A_2B_2) - \mathrm{Tr}(B_1A_1B_2A_2)\right]$$

Problem 2

In a supersymmetric field theory, the trace anomaly coefficients a and c are given by the formulae

$$a = \frac{3}{32}\left(3\mathrm{Tr}R^3 - 3\mathrm{Tr}R\right) \ , \qquad c = \frac{1}{32}\left(9\mathrm{Tr}R^3 - 5\mathrm{Tr}R\right) \ ,$$

where R refers to the $U(1)_R$ charges, and the trace is over all the chiral fermion fields.

a) Calculate a and c in the following two gauge theories: the $\mathcal{N}=2$ supersymmetric Z_2 orbifold quiver, and in the $\mathcal{N}=1$ SCFT on N D3-branes at the conifold.

b) For $AdS_5 \times Y$ with N units of RR 5-form flux, it was found at leading order in N that

$$a = c = \frac{N^2\pi^3}{4\mathrm{vol}(Y)} \ ,$$

where the radius of Y is normalized so that $R_{ij} = 4g_{ij}$ on Y. Compare this formula with the gauge theory results of part a).

Wrapped D3-branes

- They are dual to baryon operators such as det B_2.
- A mininal 3-cycle is located at constant θ_2, ϕ_2 and has volume $V_3 = 8\pi^2 L^3/9$
- The mass of such a `heavy particle' in AdS_5 is $m = V_3 \frac{\sqrt{\pi}}{\kappa} = \frac{8\pi^{5/2} L^3}{9\kappa}$
- The corresponding operator dimension agrees with the gauge theory

$$mL = \frac{8\pi^{5/2} L^4}{9\kappa} = \frac{3}{4} N$$

- Need to quantize the `collective coordinates' θ_2, ϕ_2 of the wrapped D3.
- Due to the N units of R-R flux, find spherical harmonics on S^2 in presence of a charge N magnetic monopole.
- They transform in $(\mathbf{1}, \mathbf{N+1})$ of $SU(2) \times SU(2)$
- The corresponding multiplet of baryonic operators is $\mathcal{B}_{2l} = \epsilon^{\alpha_1 \ldots \alpha_N} \epsilon_{\beta_1 \ldots \beta_N} D_l^{k_1 \ldots k_N} \prod_{i=1}^{N} B_{k_i \alpha_i}^{\beta_i}$
- $D_l^{k_1 \ldots k_N}$ is the completely symmetric SU(2) Clebsch-Gordon coefficient.

Resolution and Deformation

- There are two well-known Calabi-Yau blow-ups of the conifold singularity.
- The `deformation' replaces the constraint on the z-coordinates by

$$z_1^2+z_2^2+z_3^2+z_4^2=\epsilon^2$$

- This replaces the singularity by a finite 3-sphere.
- In the `small resolution' the singularity is replaced by a finite 2-sphere. This is implemented by modifying the constraint on the a and b variables

$$|b_1|^2+|b_2|^2-|a_1|^2-|a_2|^2=u^2$$

- This suggests that in the gauge theory the resolution is achieved by giving VEV's to the chiral superfields. IK, Witten
- For example, we may give a VEV to only one of the four superfields: $B_2 = u 1_{N\times N}$
- The dual of such a gauge theory is a resolved conifold, which is warped by a stack of N D3-branes placed at the north pole of the blown up 2-sphere.

$$ds_{10}^2 = \sqrt{H^{-1}(y)}dx^\mu dx_\mu + \sqrt{H(y)}ds_6^2$$

- The explicit CY metric on the resolved conifold is

Pando Zayas, Tseytlin

$$\kappa(r) = \frac{r^2 + 9u^2}{r^2 + 6u^2}$$

$$\begin{aligned} ds_6^2 &= \kappa^{-1}(r)dr^2 + \frac{1}{9}\kappa(r)r^2\left(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2\right)^2 \\ &\quad + \frac{1}{6}r^2(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \frac{1}{6}(r^2 + 6u^2)(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2) \end{aligned}$$

- The warp factor is the Green's function on this space with a source located at the D3-brane stack

IK, Murugan

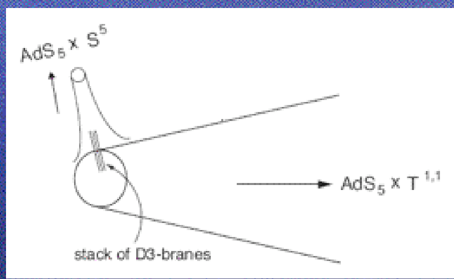
$$H(r,\theta_2) = L^4 \sum_{l=0}^{\infty} (2l+1) H_l^A(r) P_l(\cos\theta_2)$$

- The radial functions are hyper-geometric:

$$H_l^A(r) = \frac{2}{9u^2}\frac{C_\beta}{r^{2+2\beta}}\ {}_2F_1\left(\beta, 1+\beta; 1+2\beta; -\frac{9u^2}{r^2}\right)$$

$$C_\beta = \frac{(3u)^{2\beta}\Gamma(1+\beta)^2}{\Gamma(1+2\beta)}\ , \qquad \beta = \sqrt{1 + (3/2)l(l+1)}$$

- We get an explicit `localized' solution which describes SU(2)xU(1)xU(1) symmetric holographic RG flow from the $\mathcal{N}$=1 SU(N)xSU(N) SCFT in the UV to the $\mathcal{N}$=4 SU(N) SYM in the IR.



- A previously known `smeared' solution corresponds to taking just the l=0 harmonic. This solution is singular Pando Zayas, Tseytlin

$$\frac{2}{9u^2r^2}+\frac{4\beta^2}{81u^4}\ln r+\mathcal{O}(1) \quad \xleftarrow{0\leftarrow r} \quad H_l^A(r) \quad \xrightarrow{r\to\infty} \quad \frac{2C_\beta}{9u^2r^{2+2\beta}}$$

Baryonic Branch

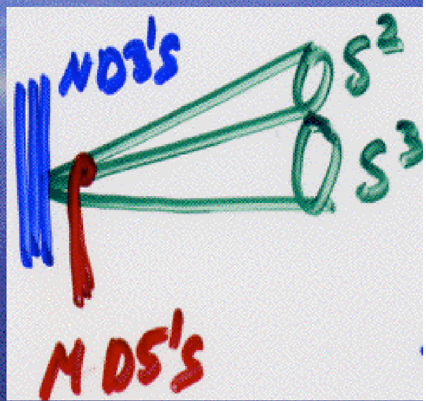
- A baryonic operator det B_2 acquires a VEV. It can be calculated on the string side of the duality using a Euclidean D3-brane wrapping a 4-cycle inside the resolved conifold, located at fixed θ_2, ϕ_2 with a UV cut-off r_c:

$$e^{-S_{BI}} = \left(\frac{3e^{5/12}u}{r_c}\right)^{3N/4} \sin^N(\theta_2/2)$$

- No mesonic operators, e.g. Tr (A_i B_j), have VEV's. This is a `baryonic branch' of the gauge theory.

Confinement in the IR

- A useful tool is to add to the N D3-branes M D5-branes wrapped over the S^2 at the tip of the conifold.



- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold (IK, Strassler)

$$ds_{10}^2 = h^{-1/2}(t)\big(-(dx^0)^2 + (dx^i)^2\big) + h^{1/2}(t)ds_6^2$$

- ds_6^2 is the metric of the deformed conifold, a simple Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

- The warp factor is finite at the `end of space' t=0, as required for the confinement: $h(t) = 2^{-8/3}\,\gamma\, I(t)$

$$I(t) = \int_t^\infty dx \frac{x\coth x - 1}{\sinh^2 x}(\sinh 2x - 2x)^{1/3} \ , \qquad \gamma = 2^{10/3}(g_s M \alpha')^2 \varepsilon^{-8/3}$$

- The standard warp factor a^2 , which measures the string tension, is identified with $h(t)^{-1/2}$ and is minimized at t=0. It blows up at large t (near the boundary).
- The dilaton is exactly constant due to the self-duality of the 3-form background

$$\star_6\, G_3 = i G_3 \ , \qquad G_3 = F_3 - \frac{i}{g_s} H_3$$

- The radius-squared of the S^3 at t=0 is $g_s M$ in string units.
- When $g_s M$ is large, the curvatures are small everywhere, and the SUGRA solution is reliable in `solving’ this confining gauge theory.
- The details of the solution are reviewed in C. Herzog, IK, P. Ouyang, hep-th/0205100.

Log running of couplings in UV

- The large radius asymptotic solution is characterized by logarithmic deviations from $AdS_5 \times T^{1,1}$ IK, Tseytlin
- The near-AdS radial coordinate is $r \sim \varepsilon^{2/3} e^{t/3}$
- The NS-NS and R-R 2-form potentials:

$$F_3 = \frac{M\alpha'}{2}\omega_3 \ , \qquad B_2 = \frac{3g_s M\alpha'}{2}\omega_2 \ln(r/r_0)$$

$$\omega_2 = \frac{1}{2}(g^1 \wedge g^2 + g^3 \wedge g^4) = \frac{1}{2}(\sin\theta_1 d\theta_1 \wedge d\phi_1 - \sin\theta_2 d\theta_2 \wedge d\phi_2)$$

$$\omega_3 = \frac{1}{2} g^5 \wedge (g^1 \wedge g^2 + g^3 \wedge g^4)$$

- This translates into log running of the gauge couplings through

$$\frac{4\pi^2}{g_1^2}+\frac{4\pi^2}{g_2^2}=\frac{\pi}{g_s e^{\Phi}} ,$$

$$\left[\frac{4\pi^2}{g_1^2}-\frac{4\pi^2}{g_2^2}\right] g_s e^{\Phi}=\frac{1}{2\pi\alpha'}\left(\int_{\mathbf{S}^2} B_2\right)-\pi$$

$$\frac{8\pi^2}{g_1^2}-\frac{8\pi^2}{g_2^2}=6M\ln(r/r_s)$$

- This agrees with the β-functions in the gauge theory

$$\frac{d}{d\log(\Lambda/\mu)}\frac{8\pi^2}{g_1^2} = 3(N+M)-2N(1-\gamma)$$
$$\frac{d}{d\log(\Lambda/\mu)}\frac{8\pi^2}{g_2^2} = 3N-2(N+M)(1-\gamma)$$

$$\frac{8\pi^2}{g_1^2}-\frac{8\pi^2}{g_2^2} = M\ln(\Lambda/\mu)[3+2(1-\gamma)]$$

- In the UV the anomalous dimension of operators $\mathrm{Tr} A_i B_j$ is γ ~ -1/2

- The warp factor deviates from the M=0 solution logarithmically.

$$h(r) = \frac{27\pi(\alpha')^2[g_s N + a(g_s M)^2 \ln(r/r_0) + a(g_s M)^2/4]}{4r^4}$$

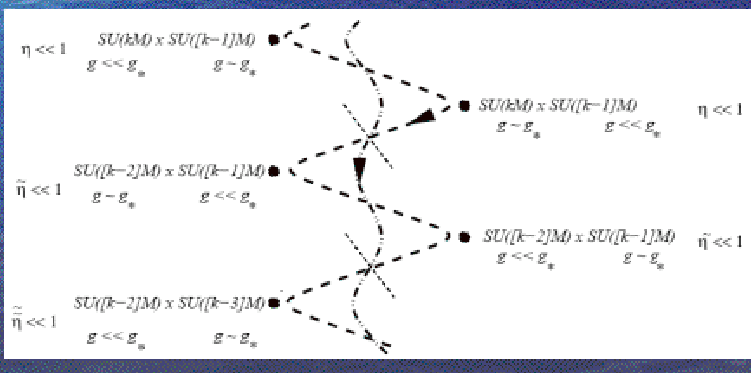
- Remarkably, the 5-form flux, dual to the number of colors, also changes logarithmically with the RG scale.

$$\tilde{F}_5 = \mathcal{F}_5 + \star\mathcal{F}_5 \ , \qquad \mathcal{F}_5 = 27\pi\alpha'^2 N_{eff}(r)\mathrm{vol}(\mathrm{T}^{1,1})$$

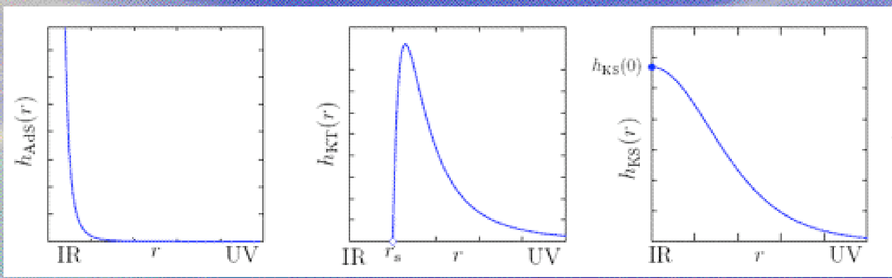
$$N_{eff}(r) = N + \frac{3}{2\pi} g_s M^2 \ln(r/r_0)$$

- What is the explanation in the dual SU(kM)xSU((k-1)M) SYM theory coupled to bifundamental chiral superfields A_1, A_2, B_1, B_2 ? A novel phenomenon, called a **duality cascade**, takes place: k repeatedly changes by 1 as a result of the Seiberg duality IK, Strassler

(diagram of RG flows from a review by M. Strassler)



- There is a scale where the SU(kM) coupling becomes very strong. This gauge group has N_f=2(k-1)M flavors.
- To understand further RG flow, perform Seiberg duality to SU(N_f-N_c)=SU((k-2)M).
- The resulting SU((k-1)M)xSU((k-2)M) theory has the same structure as the original one, but k is reduced by 1.
- The flow proceeds in a quasi-periodic fashion until k becomes O(1) in the IR.



- Comparison of warp factors in the AdS, warped conifold, and warped deformed conifold cases. The warped conifold (KT) solution, which has a naked singularity, should be interpreted as asymptotic (UV) approximation to the correct solution.

- The graph of quark anti-quark potential is qualitatively similar to that found in numerical simulations of QCD. The upper graph, from the recent Senior Thesis of V. Cvicek shows the string theory result for the warped deformed conifold.
- The lower graph shows lattice QCD results by G. Bali et al with $r_0 \sim 0.5$ fm.

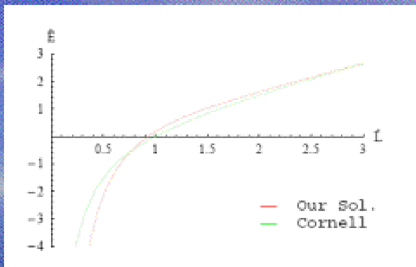
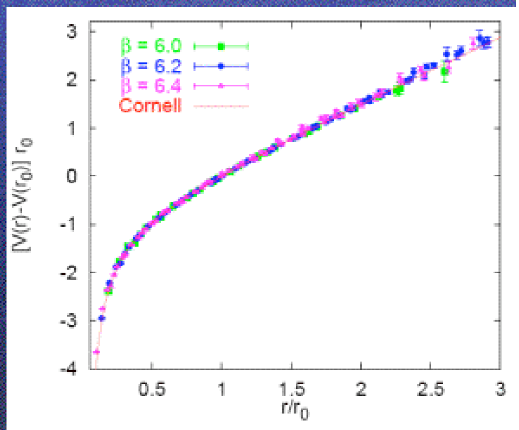


Figure 11: Comparison to the Cornell model



Light glueballs

- The confining string tension is $$T_s = \frac{1}{2^{4/3} a_0^{1/2} \pi} \frac{\varepsilon^{4/3}}{(\alpha')^2 g_s M}$$
- The glueballs are the normalizable modes localized near at small t. In the supergravity limit (at large g_s M) their mass scales are

$$m_{glueball} \sim m_{KK} \sim \frac{\varepsilon^{2/3}}{g_s M \alpha'} \qquad T_s \sim g_s M (m_{glueball})^2$$

- In order to eliminate the anomalously light bound states, we need a small g_s M, which requires a departure from the SUGRA limit.
- As g_s M -> 0 should recover pure $\mathcal{N}$=1 SU(M) theory.
- Even for small g_s M, SUGRA becomes reliable in the UV description of the duality cascade (at large t).

IR Behavior of the Conifold Cascade

- Here the dynamical deformation of the conifold renders the solution smooth, and explains the IR dynamics of the gauge theory.
- Dimensional transmutation in the IR. The dynamically generated confinement scale is

  $\sim \varepsilon^{2/3}$
- The pattern of R-symmetry breaking is the same as in the SU(M) SYM theory: Z_{2M} -> Z_2
- Yet, for large $g_s M$ the IR gauge theory is somewhat more complicated.

- In the IR the gauge theory cascades down to SU(2M) x SU(M). The SU(2M) gauge group effectively has $N_f = N_c$.
- The baryon and anti-baryon operators   Seiberg

$$\mathcal{A} = \epsilon^{i_1 \ldots i_{N_c}} A^{a_1}_{\alpha_1 i_1} \ldots A^{a_{N_c}}_{\alpha_{N_c} i_{N_c}}$$
$$\mathcal{B} = \epsilon_{i_1 \ldots i_{N_c}} B^{i_1}_{\dot\alpha_1 a_1} \ldots B^{i_{N_c}}_{\dot\alpha_{N_c} a_{N_c}}$$

acquire expectation values and break the U(1) symmetry under which $A_k \rightarrow e^{ia} A_k$; $B_l \rightarrow e^{-ia} B_l$. Hence, we observe confinement without a mass gap: due to $U(1)_{baryon}$ chiral symmetry breaking there exist a Goldstone boson and its massless scalar superpartner. There exists a baryonic branch of the moduli space

$$\mathcal{A} = i\Lambda_1^{2M}\zeta \ , \qquad \mathcal{B} = i\Lambda_1^{2M}/\zeta$$

- The KS solution is part of a moduli space of confining SUGRA backgrounds, **resolved warped deformed conifolds**. Gubser, Herzog, IK; Butti, Grana, Minasian, Petrini, Zaffaroni
- To look for them we need to use the PT ansatz:

$$ds_{10}^2 = H^{-1/2} dx_m dx_m + e^x ds_6^2,$$

$$ds_6^2 = (e^g + a^2 e^{-g})(e_1^2 + e_2^2) + e^{-g} \sum_{i=1}^{2} (\epsilon_i^2 - 2a e_i \epsilon_i) + v^{-1}(\tilde{\epsilon}_3^2 + dt^2)$$

- H, x, g, a, v, and the dilaton are functions of the radial variable t.
- Additional radial functions enter into the ansatz for the 3-form field strengths. The PT ansatz preserves the SO(4) but breaks a Z_2 charge conjugation symetry, except at the KS point.

- BGMPZ used the method of SU(3) structures to derive the complete set of coupled first-order equations.
- A result of their integration is that the warp factor and the dilaton are related:

$$H(t) = \tilde{H}\left(e^{-2\phi(t)} - 1\right)$$

Dymarsky, IK, Seiberg

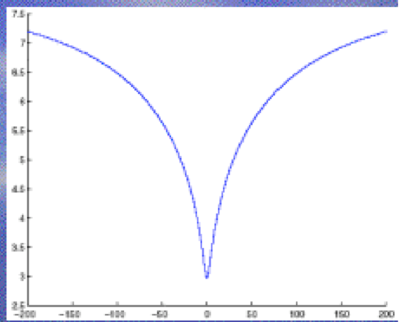
- The integration constant determines the `modulus' U: $\tilde{H} = \gamma U^{-2}$ where $\gamma = 2^{10/3}(g_s M\alpha')^2 \varepsilon^{-8/3}$
- At large t the solution approaches the KT `cascade asymptotics': $a(t) = -2e^{-t} + Ue^{-5t/3}(-t+1) + \ldots$

$$\gamma^{-1}H(t) = \frac{3}{32}e^{-4t/3}(4t-1) - \frac{3}{32\cdot 512}U^2(256t^3 - 864t^2 + 1752t - 847)e^{-8t/3} + O\left(e^{-10t/3}\right)$$

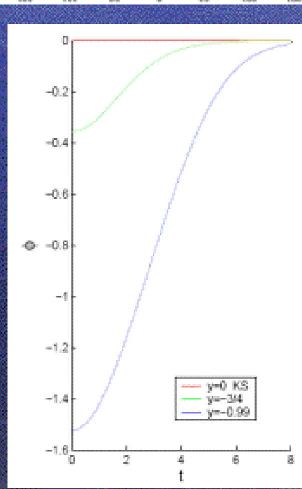
- In a recent paper, Benna, Dymarsky and I confirmed the presence of the baryonic condensates on the string theory side of the duality. These 1-point functions are calculated using the Euclidean D5-brane wrapped over the 6 deformed conifold directions, with certain world volume gauge fields turned on (they are determined by the supersymmetry conditions). The behavior of the D5-brane action as a function of the large radius cut-off is in complete agreement with the expectations in the cascading gauge theory.

- The resolution parameter U is proportional to the VEV of the operator

$$\mathcal{U} = \mathrm{Tr}\left(\sum_{\alpha} A_{\alpha} A^{\dagger}_{\alpha} - \sum_{\dot\alpha} B^{\dagger}_{\dot\alpha} B_{\dot\alpha}\right)$$



- This family of resolved warped deformed conifolds is dual to the `baryonic branch' in the gauge theory (the quantum deformed moduli space).
- At large U the IR part of the solution approaches that of the CVMN solution. But we always have the `cascade' asymptotics at large t.

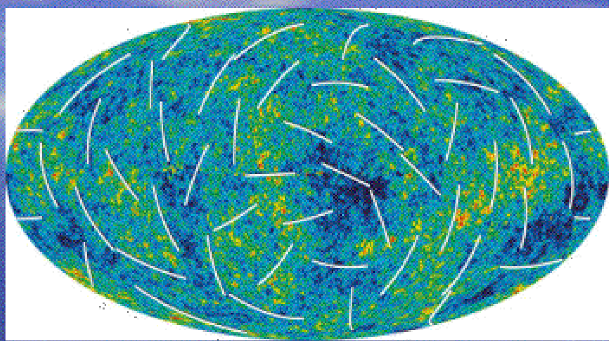


- Here are plots of the string tension (a fundamental string at the bottom of the throat is dual to an `emergent' chromo-electric flux tube) and of the dilaton profiles as a function of the modulus U=ln |ζ|. Dymarsky, IK, Seiberg

- All of this provides us with an exact solution of a class of 4-d large N confining supersymmetric gauge theories.
- This should be a good playground for testing various ideas about strongly coupled gauge theory.
- Some results on glueball spectra are already available, and further calculations are ongoing. Krasnitz; Caceres, Hernandez; Dymarsky, Melnikov; Berg, Haack, Muck
- Could there be applications of these models to new physics?

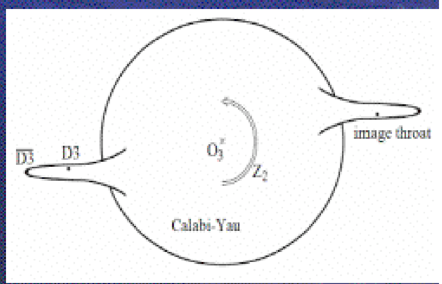
Applications to D-brane Inflation

- The Slow-Roll Inflationary Universe (Linde; Albrecht, Steinhardt) is a very promising idea for generating the CMB anisotropy spectrum observed by the WMAP.
- Finding models with very flat potentials has proven to be difficult. Recent string theory constructions use moving D-branes. Dvali, Tye, …



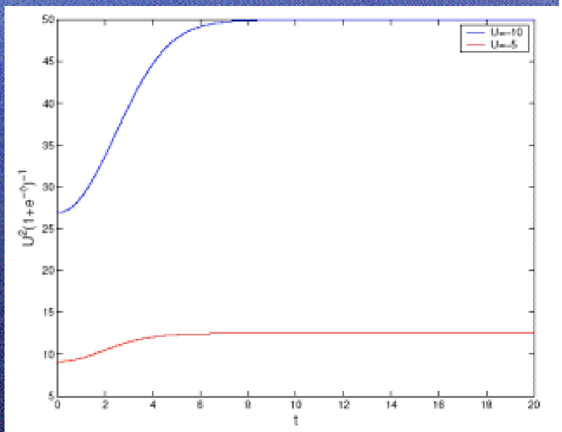
- In the KKLT/KKLMMT model, the warped deformed conifold is embedded into a string compactification. An anti-D3-brane is added at the bottom to break SUSY and generate a potential. A D3-brane rolls in the throat. Its radial coordinate plays the role of an inflaton.

Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi

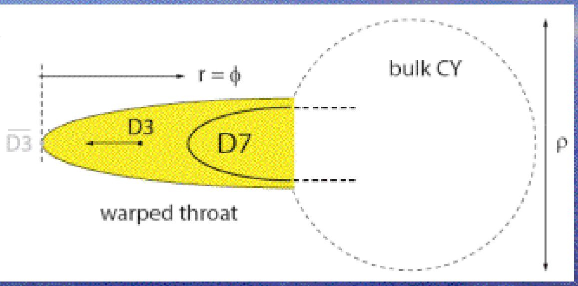


A related suggestion for D-brane inflation (A. Dymarsky, IK, N. Seiberg)

- In a flux compactification, the $U(1)_{baryon}$ is gauged. Turn on a Fayet-Iliopoulos parameter ξ .
- This makes the throat a **resolved** warped deformed conifold.
- The probe D3-brane potential on this space is asymptotically flat, if we ignore effects of compactification and D7-branes. The plots are for two different values of $U \sim \xi$.
- No anti-D3 needed: in presence of the D3-brane, SUSY is broken by the D-term ξ. Related to the `D-term Inflation' Binetruy, Dvali; Halyo



Slow roll D-brane inflation?



- Effects of D7-branes and of compactification generically spoil the flatness of the potential. Non-perturbative effects introduce the KKLT-type superpotential $W = W_0 + A(X)e^{-a\rho}$ where X denotes the D3-brane position. In any warped throat D-brane inflation model, it is important to calculate A(X).

- The gauge theory on D7-branes wrapping a 4-cycle Σ_4 has coupling $\frac{1}{g^2} = \frac{V^w_{\Sigma_4}}{g_7^2} = \frac{T_3 V^w_{\Sigma_4}}{8\pi^2}$
- The non-perturbative superpotential $\propto \exp\left(-\frac{T_3 V^w_{\Sigma_4}}{N_{D7}}\right)$

  depends on the D3-brane location through the warped volume $V^w_{\Sigma_4} \equiv \int_{\Sigma_4} \mathrm{d}^4\xi \sqrt{g^{ind}}\, h(X)$
- In the throat approximation, the warp factor can be calculated and integrated over a 4-cycle explicitly. Baumann, Dymarsky, IK, Maldacena, McAllister, Murugan.
- If the D7-brane emedding is specified by $f(z_\alpha) = 0$

$$A(z_\alpha) = A_0 \left(\frac{f(z_\alpha)}{f(0)}\right)^{1/n}$$

- Using the DeWolfe-Giddings Kaehler potential for the volume modulus ρ and the three D3-brane coordinates z_α on the conifold

$$\mathcal{K}(\rho, \bar{\rho}, z_\alpha, \bar{z}_\alpha) = -3 \log[\rho + \bar{\rho} - \gamma k] \equiv -3 \log U$$

$$k = \frac{3}{2}\left(\sum_{i=1}^{4} |z_i|^2\right)^{2/3} = \frac{3}{2} r^2$$

The F-term potential is found to be

Burgess, Cline, Dasgupta, Firouzjahi; Baumann, Dymarsky, IK, McAllister, Steinhardt

$$\begin{aligned} V_F &= \frac{1}{3U^2}\Big[(\rho + \bar{\rho})|W_{,\rho}|^2 - 3(\overline{W} W_{,\rho} + c.c.) \\ &\quad + \frac{3}{2}(\overline{W_{,\rho}} z^\alpha W_{,\alpha} + c.c.) + \frac{1}{\gamma} k^{\alpha\bar{\beta}} W_{,\alpha} \overline{W_{,\beta}}\Big] \end{aligned}$$

- This generally gives Hubble-scale corrections to the inflaton potential, so fine-tuning is needed.
- The `uplifting' is accomplished by the D-term potential $V_D = D(r)U^{-2}\,, \qquad D(r) \equiv D\left(1 - \frac{3D}{16\pi^2}\frac{1}{(T_3 r^2)^2}\right)$
- For the KKLMMT model with anti-D3 brane, where h $D = 2T_3/h_0$ warp factor at the bottom of the throat.
- We have studied a simple and symmetric Kuperstein embedding $z_1 = \mu$
- The stable trajectory for positive μ is

$$z_1 = -\frac{1}{\sqrt{2}} r^{3/2}$$

$$\phi \equiv r\sqrt{\tfrac{3}{2}T_3}$$

- The effective potential for the inflaton generically has a local maximum and minium. It can be fine-tuned to have an inflection point.
- Motion near the inflection point can produce enough e-folds of inflation.
- But cosmological predictions are very sensitive, e.g. $n_s - 1 = (2\eta - 6\epsilon)|_{\phi_{\rm CMB}} \approx 2\eta(\phi_{\rm CMB})$
- The sign of $\eta(\phi_{\rm CMB})$ depends on its position relative to inflection point. This is a `Delicate Universe.' Baumann, Dymarsky, IK, McAllister, Steinhardt