Multiplicity dependence of charm production and parton saturation

K.W. in collaboration with

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D multiplicity vs charged multiplicity in pp

Significant deviation from the diagonal (linear increase)
in particular for large $p_t$

ALICE arXiv:1505.00664v1
PYTHIA 8.157
Already understanding a linear increase is a challenge!

(Only recent Pythia versions can do)

Even much more the deviation from linear (towards higher values)
Trying to understand these data in the EPOS framework

Important issues:

- Multiple scattering, parton saturation
- Collectivity
EPOS: Based on multiple scattering and flow

Several steps (even in pp!):

1) Initial conditions:
   Gribov-Regge multiple scattering approach, 
   elementary object = Pomeron = parton ladder,
   Nonlinear effects via saturation scale $Q_s$

2) Core-corona approach
   to separate fluid and jet hadrons

3) Viscous hydrodynamic expansion, $\eta/s = 0.08$

4) Statistical hadronization, final state hadronic cascade

Initial conditions: **Marriage pQCD+GRT+energy sharing**
(Drescher, Hladik, Ostapchenko, Pierog, and Werner, Phys. Rept. 350, 2001)

For pp, pA, AA:

\[
\sigma^{\text{tot}} = \sum_{\text{cut } P} \int \sum_{\text{uncut } P} \int (\text{squared amplitude})
\]

\[
\text{cut Pom : } G = \frac{1}{2\hat{s}} 2\text{Im } \{\mathcal{F}\mathcal{T}\{T\}\}(\hat{s}, b), \quad T = i\hat{s}\sigma_{\text{hard}}(\hat{s}) \exp(R_{\text{hard}}^2 t)
\]

Nonlinear effects considered via saturation scale \( Q_s \)
\[
\sigma_{\text{tot}} = \int d^2b \int \prod_{i=1}^{A} d^2b_i^A \, dz_i^A \rho_A(\sqrt{(b_i^A)^2 + (z_i^A)^2}) \prod_{j=1}^{B} d^2b_j^B \, dz_j^B \rho_B(\sqrt{(b_j^B)^2 + (z_j^B)^2})
\]

\[
\sum_{m_1 l_1} \ldots \sum_{m_{AB} l_{AB}} (1 - \delta_0 \Sigma m_k) \int \prod_{k=1}^{AB} \left( \prod_{\mu=1}^{m_k} dx_{k,\mu}^+ dx_{k,\mu}^- \prod_{\lambda=1}^{l_k} d\tilde{x}_{k,\lambda}^+ d\tilde{x}_{k,\lambda}^- \right) \left\{ \prod_{k=1}^{AB} \left( \frac{1}{m_k! l_k!} \prod_{\mu=1}^{m_k} G(x_{k,\mu}^+, x_{k,\mu}^-, s, |\vec{b} + \vec{b}_\pi^A| - \vec{b}_\tau^B|) \prod_{\lambda=1}^{l_k} -G(\tilde{x}_{k,\lambda}^+, \tilde{x}_{k,\lambda}^-, s, |\vec{b} + \vec{b}_\pi^A| - \vec{b}_\tau^B|) \right) \prod_{i=1}^{A} \left( 1 - \sum_{\pi(k)=i} x_{k,\mu}^+ - \sum_{\pi(k)=i} \tilde{x}_{k,\lambda}^+ \right)^\alpha \prod_{j=1}^{B} \left( 1 - \sum_{\pi(k)=j} x_{k,\mu}^- - \sum_{\pi(k)=j} \tilde{x}_{k,\lambda}^- \right)^\alpha \right\}
\]
Core-corona procedure (for pp, pA, AA):

Pomeron => parton ladder => flux tube (kinky string)

String segments with high pt escape => **corona**, the others form the **core** = initial condition for hydro depending on the local string density

\[
\begin{align*}
\eta &= -1.00 \\
\text{5.7fm, 5 Pomerons} \\
p_{\text{Pb}} 5\text{TeV, 20-40%} \\
p_{\text{t}} &< 6
\end{align*}
\]
Core => Hydro evolution (Yuri Karpenko)

Israel-Stewart formulation, $\eta - \tau$ coordinates, $\eta/S = 0.08$, $\zeta/S = 0$

\[ \partial_\nu T^{\mu\nu} = \partial_\nu T^{\mu\nu} + \Gamma^\mu_{\nu\lambda} T^{\nu\lambda} + \Gamma^\nu_{\nu\lambda} T^{\mu\lambda} = 0 \]

\[ \gamma (\partial_t + v_i \partial_i) \pi^{\mu\nu} = -\frac{\pi^{\mu\nu} - \pi^{\mu\nu}_{\text{NS}}}{\tau_{\pi}} + I_{\pi}^{\mu\nu} \]

\[ \gamma (\partial_t + v_i \partial_i) \Pi = -\frac{\Pi - \Pi_{\text{NS}}}{\tau_{\Pi}} + I_{\Pi} \]

- $T^{\mu\nu} = \epsilon u^\mu u^\nu - (p + \Pi) \delta^{\mu\nu} + \pi^{\mu\nu}$
- $\partial_\nu$ denotes a covariant derivative,
- $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ is the projector orthogonal to $u^\mu$,
- $\pi^{\mu\nu}$, $\Pi$ shear stress tensor, bulk pressure

\[ \pi^{\mu\nu}_{\text{NS}} = \eta (\Delta^{\mu\lambda} \partial_{;\lambda} u^\nu + \Delta^{\nu\lambda} \partial_{;\lambda} u^\mu) - \frac{2}{3} \eta \Delta^{\mu\nu} \partial_{;\lambda} u^\lambda \]

\[ \Pi_{\text{NS}} = -\zeta \partial_{;\lambda} u^\lambda \]

\[ I_{\pi}^{\mu\nu} = -\frac{4}{3} \pi^{\mu\nu} \partial_{;\gamma} u^\gamma - [u^\nu \pi^{\mu\beta} + u^\mu \pi^{\nu\beta}] u^\lambda \partial_{;\lambda} u_\beta \]

\[ I_{\Pi} = -\frac{4}{3} \Pi \partial_{;\gamma} u^\gamma \]

**Freeze out:** at 168 MeV, Cooper-Frye $E \frac{d^3 n}{d^3 p} = \int d\Sigma_{\mu} p^\mu f(up)$, equilibrium distr

**Hadronic afterburner: UrQMD**

Marcus Bleicher, Jan Steinheimer
A crucial ingredient: The saturation scale $Q_s^2$

Single Pomeron contribution $G$ (to the amplitude), computed via pQCD, can be (very well) fitted as

$$G \approx G_{\text{fit}} = \alpha (x^+)^{\beta} (x^-)^{\beta'}$$

($x^\pm$ are light cone momentum fractions)

Extremely useful! Allows analytical calculations of cross sections.

*) (Drescher, Hladik, Ostapchenko, Pierog, and Werner, Phys. Rept. 350, 2001)
Consistency requires adding more diagrams (ladder splitting/fusion, triple Pomeron vertices, gluon fusion in CGC ...)

\[ \text{(Drescher, Hladik, Ostapchenko, Pierog, and Werner, Phys. Rept. 350, 2001)} \]
Motivated by model calculations, we treat ladder fusion via adding an exponent $^1$:

\[ G_{\text{fit}} \rightarrow G_{\text{eff}} = \alpha (x^+)^{\beta + \varepsilon_{\text{proj}}} (x^-)^{\beta' + \varepsilon_{\text{targ}}} \]

(“epsilon method”) with

\[ \varepsilon = \varepsilon(Z), \]

depending on “the number of participating partons”:

\[ Z^{\text{proj}} = \sum_{\text{proj nucleons } i'} f_{\text{part}} \left( |\vec{b} + \vec{b}_{i'} - \vec{b}_j| \right) \]

(j is the target nucleon the Pomeron is connected to)

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Advantages

- Cross section calculations perfectly doable
- Energy dependence of $\sigma_{\text{tot}}$, $\sigma_{\text{el}}$ (and more) correct

Big problems

- Adding $\varepsilon$ does not change the internal Pomeron structure
- No binary scaling in pA at high $p_t$ (tails much too low)
Solution

- Introducing a saturation scale
  
  (K. Werner, B. Guiot, Iu. Karpenko, T. Pierog, 

Before: Compute $G$ with fixed soft cutoff $Q_0$

$\rightarrow$ fit $\rightarrow$ add $\varepsilon$ exponents

New: Compute $G$ with saturation scale $Q_s \propto Z \hat{s}^\lambda$

$\rightarrow$ fit  ($\hat{s} =$ Pomeron invariant mass)

varying $Q_s$ changes internal structure!
Still something missing ... 

- The saturation scale depends on the number of participating nucleons,

- but NOT on the number of Pomerons $N_{\text{Pom}}$ (participating parton pairs)

The number of Pomerons represents the event activity in pp, as the number of participating nucleons does in pA.
The final solution

☐ Combining “epsilon method” and saturation scale in a smart way (T. Pierog and K. Werner, procs. EDS 2015, Borgo, France)

Step 1 Compute \( G = G(Q_0) \) with fixed soft cutoff \( Q_0 \) → fit → add \( \varepsilon \) exponents (→ \( G_{\text{eff}} \)) in order to fit cross sections

Step 2 Introduce saturation scale via

\[
G_{\text{eff}} = k \, G(Q_s)
\]

affecting the internal structure
(We will see what to take to \( k \))
The saturation scale $Q_s^2$

pp at 7 TeV

using $G_{\text{eff}} = k G(Q_s)$

with constant $k$

$(x_{+PE}$ is the LC momentum fraction on the projectile side)
A crucial test:
Multiplicity dependence of spectra at high $p_t$

**preliminary ALICE data**
(digitalized from B.A.Hess, talk at MPI@LHC 2015 Trieste November 27, 2015)

multiplicity bins
(top to bottom):
0-1%, 1-5%, 10-15%, 20-30%, 40-50%, 70-100%

lines to guide the eye
Same data - ratio to 70-100%

non-trivial:
spectra get harder
with multiplicity
Comparing ALICE data with EPOS calculations

(preliminary ALICE data digitalized from B.A.Hess, talk at MPI@LHC 2015 Trieste November 27, 2015)

multiplicity bins
(top to bottom):
0-1%, 1-5%, 10-15%, 20-30%, 40-50%, 70-100%

Not too bad for a first shot ... but tails are not correct
Comparing ALICE data with EPOS calculations

Ratio calculation / data

Multiplicities bins: 0-1% (red), 1-5%, 10-15%, 20-30%, 40-50%, 70-100% (grey)

Tails wrong by factors of two (low pt will be modified by hydro)
Make saturation scale $Q_s^2$ depending on $N_{Pom}$

pp at 7 TeV

using $G_{\text{eff}} = k \, G(Q_s)$

with

$$k = \left( \frac{N_{Pom}}{\langle N_{Pom} \rangle} \right)^{0.75}$$

higher $Q_s^2$ with increasing Pomeron number (like $N_{\text{part}}$ dependence in pA)
Comparing ALICE data with EPOS calculations

Using

\[ k = \left( \frac{N_{\text{Pom}}}{\langle N_{\text{Pom}} \rangle} \right)^{0.75} \]

=> much better
Comparing ALICE data with EPOS calculations

Ratio calculation / data

Using

\[ k = \left( \frac{N_{\text{Pom}}}{\langle N_{\text{Pom}} \rangle} \right)^{0.75} \]

Multiplicity bins:
0-1% (red), 1-5%, 10-15%, 20-30%, 40-50%, 70-100% (grey)

Tails reasonable (low pt will be modified by hydro)
Still finetuning and tests needed, but we use

\[ G_{\text{eff}} = k \, G(Q_s) \]

with

\[ k = \left( \frac{N_{\text{Pom}}}{\langle N_{\text{Pom}} \rangle} \right)^{A_{\text{sat}}} , \quad A_{\text{sat}} = 0.75 \]

to analyse the multiplicity dependence of D-meson production (results depend somewhat on \( A_{\text{sat}} \))

Remark : This new procedure => **EPOS 3.2xx**
Charm – multiplicity correlations

**Notations** (always at midrapidity)  (D-meson = average $D^+, D^0, D^{*+}$)

- $N_{ch}$: Charged particle multiplicity
- $N_{D1}$: D-meson multiplicity for $1 < p_t < 2 \text{ GeV}/c$
- $N_{D2}$: D-meson multiplicity for $2 < p_t < 4 \text{ GeV}/c$
- $N_{D4}$: D-meson multiplicity for $4 < p_t < 8 \text{ GeV}/c$
- $N_{D8}$: D-meson multiplicity for $8 < p_t < 12 \text{ GeV}/c$
Heavy quark (Q) production
in EPOS multiple scattering framework

as light quark production
(but non-zero masses: $m_c = 1.3$, $m_b = 4.2$)

In any of the ladders

- during SLC (space-like cascade)
- during TLC (time-like cascade)
- in Born
Multiple scattering (EPOS3, basic):

\[ N_{Di} \propto N_{ch} \propto N_{Pom} \]

"Natural" linear behavior
(first approximation)

We use \( n = N/\langle N \rangle \) for \( N_{ch} \) and \( N_{Di} \)
The actual calculations

$n_{Di}$ vs $n_{ch}$

... even more than linear increase!

(in particular for large $p_t$)

(less for $A_{sat} = 0$)

(much less in EPOS 3.1xx)

Why this $p_t$ dependence?
Crucial: Fluctuations

$N_{ch}$ and $N_{Pom}$ are correlated, but not one-to-one

(=> two-dimensional probability distribution)

In the following, we consider fixed values $N_{ch}^*$

$\rightarrow$ fixed $n_{ch}^*$
To understand the implications of “fixed $n_{\text{ch}}$”
Strings in multiple scattering event (Schematic view):

basic EPOS

Strings of different lengths, different rapidity coverage, different hardness length
$\sim$ mass $(\sqrt{x^+x^-s})$
To understand the implications of "fixed

Strings in multiple scattering event (Schematic view):

full EPOS (with hydro) string segments => fluid

but
string properties
- number,
- masses,
- hardnesses
determine initial energy density and final multiplicity
Consider $n_{D1}$ for some given $n_{ch^*}$

$$n_{D1} = \sum_{N_{Pom}} \text{prob}(N_{Pom}, n_{ch^*}) \times n_{D1}(N_{Pom}, n_{ch^*})$$

$(60\%) > n_{ch^*}$ having used

$$n_{D1}(N_{Pom}, n_{ch^*})$$

$(60\%) > n_{ch^*}$
The precise calculation (red point)

\[ n_{D1}(n_{ch}) \]

- diagonal

\[ n_{D1}(n_{ch}^*) \]
\[ n_{D8} \text{ for given } n_{ch}^* \]

\[ n_{D8} = \sum_{N_{Pom}} \text{prob}(N_{Pom}, n_{ch}^*) \times n_{D8}(N_{Pom}, n_{ch}^*) \]

\[ > > n_{ch}^* \]

because

\[ n_{D8} > n_{ch}^* \text{ at high } N_{Pom} \]

and

increases strongly towards small \( N_{Pom} \)
The precise calculation \textit{(red point)}

\[ n_{D8} \geq n_{ch} \]

- **significantly above the diagonal!**
- **strongly non-linear!**

How to understand \( n_{D8} \gg n_{ch} \) and why increasing towards small \( N_{Pom} \)?
We compute in addition

- The average invariant Pomeron mass for given $N_{\text{Pom}}$ and $n_{\text{ch}}^*$

- The average Pomeron hardness

$$\left( \langle p_t^2 \rangle / \langle p_t^2 \rangle_{\text{ref}} - 1 \right) \times 100$$

for given $N_{\text{Pom}}$ and $n_{\text{ch}}^*$

(based on string segments)
Pomeron mass and hardness

both increase significantly with decreasing $N_{\text{Pom}}$

red line: $n_{D8}$
blue dashed-dotted: $N_{\text{Pom distr}}$

correspondence hardness - $n_{D8}$ !!
Strong non-linear increase (of $n_{D8}(n_{ch})$)
since

- Pomerons harder with increasing multiplicity (more screening, higher $Q_s^2$)

- The number of Pomerons fluctuates for given multiplicity and smaller Pomeron numbers imply harder Pomerons

- note: $n_{D8}$ is nothing but a “Pomeron hardness” measure (even a very sensitive one)
EPOS 3.204 compared to data
Hydo helps somewhat
(for basic EPOS the increase is somewhat less)

No change for $n_{Di}$

But some reduction of $n_{ch}$

$\Rightarrow n_{D8}(n_{ch})$ with hydro is somewhat steeper compared to basic EPOS
Why multiplicity reduction?

**Basic EPOS:**

**Pomerons > Strings > String fragmentation**

(independent of event activity)

**Full model:**

**Pomerons > Strings > Fluid, collectivity**

(collective energy increases with event activity)
Taking charged-particle multiplicity at forward/backward rapidity

$2.8 < \eta < 5.1 \quad \text{and} \quad -3.7 < \eta < -1.7$

($V_{\text{zero}}$ multiplicity, $N_{vz}$, $n_{vz}$)
Vzero multiplicity: Smaller increase

as in the data
$n_{D8}$ for given $n_{vz}^*$

whereas

$n_{D8}(N_{Pom}, n_{ch}^*)$ increases strongly towards small $N_{Pom}$

$n_{D8}(N_{Pom}, n_{vz}^*)$ decreases slightly

=> Pomerons do not get harder
Why do Pomerons get harder at small $N_{\text{Pom}}$ for fixed $n_{\text{ch}}$ but not for fixed $n_{\text{vz}}$?
In case of $n_{\text{ch}}$, almost all Pomerons cover the corresponding central rapidity range
In case of $n_{\text{ch}}$, almost all Pomerons cover the corresponding central rapidity range, so to keep $n_{\text{ch}}$ fixed for smaller $N_{\text{Pom}}$ requires harder Pomerons (no other way)
In case of $n_{vz}$, only some Pomerons cover the corresponding forward rapidity range,
In case of $n_{vz}$, only some Pomerons cover the corresponding forward rapidity range, so to keep $n_{vz}$ fixed for smaller $N_{Pom}$ can be accommodated with more Pomerons covering that rapidity range.
Summary

- New (and final?) major improvement of the multiple scattering scheme in EPOS: Pomeron number dependence of the saturation scale (and the corresponding technical improvements which make it possible)

- Provides increasing Pomeron hardness with increasing multiplicity (ALICE multipl dependence of spectra)

- Explains strong increase of high pt charm production vs multiplicity, and the modest increase in case of forward multiplicity.