Resummation of large transverse logarithms in the high energy evolution of color dipoles

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E. Iancu, J.D. Madrigal, A.H. Mueller, G. Soyez and D.T., Phys. Lett. **B744** (2015) 293, **B750** (2015) 643 and w.i.p.

Outline

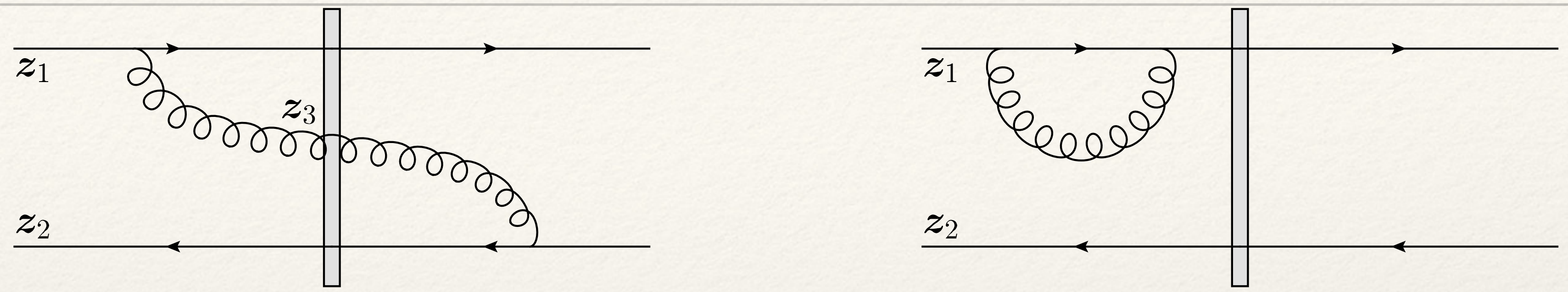
- NLO BK equation and large transverse logarithms
- Unphysical solutions
- Resummation of logarithms to all orders
- Restoration of stability, numerical solutions
- Fits to HERA data

CGC and BK in Heavy Ion Collisions

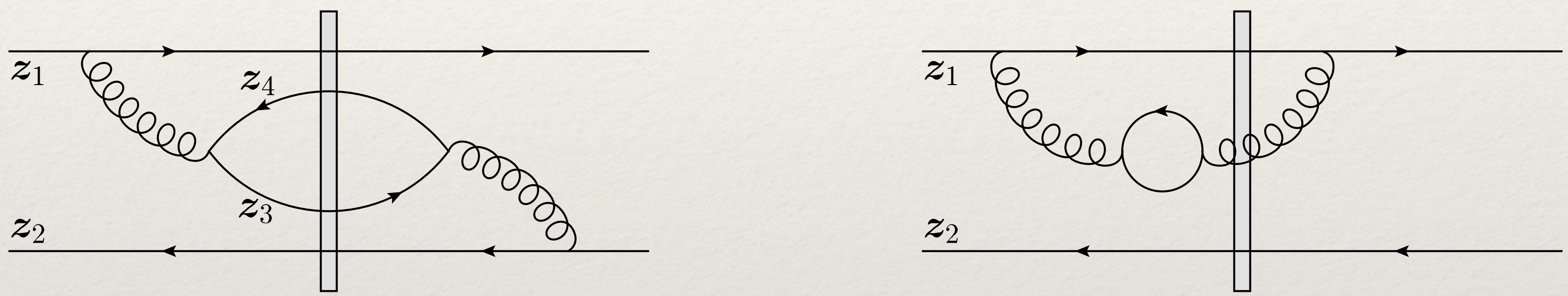
- CGC: high energy evolution of hadronic wave function
- Best d.o.f. : Wilson lines for projectile partons scattering
- Cross section in DIS, single/double particle production at forward rapidity in pA collisions, energy density just after an AA collision, … :
  Local in rapidity observables: correlators of Wilson lines.
- To excellent accuracy, all such correlators expressed in terms of dipole scattering. BK equation.

Diagrams for dipole evolution

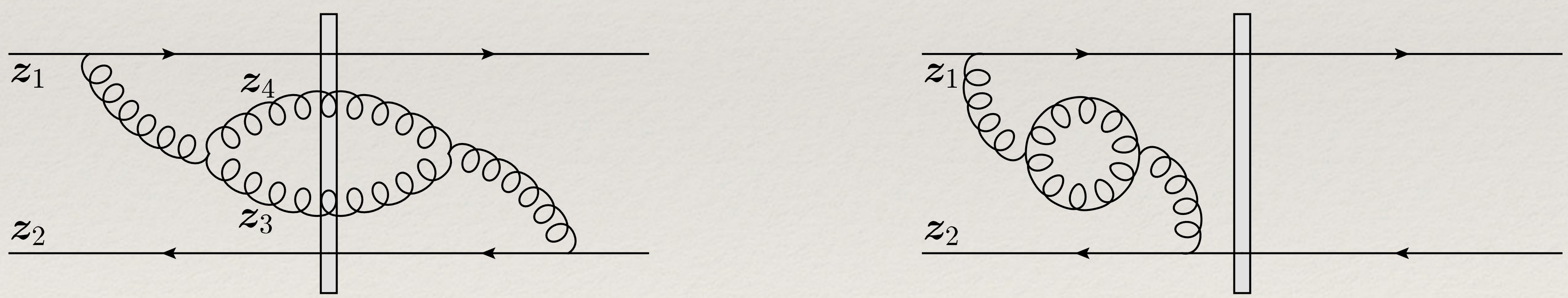
- LO



- NLO N_f



- NLO N_c



- First emission soft, second non-soft in general

The BK equation at NLO

$$\frac{\mathrm{d}S_{12}}{\mathrm{d}Y} = \frac{\bar{\alpha}_s}{2\pi}\int \mathrm{d}^2 z_3\, \frac{z_{12}^2}{z_{13}^2 z_{23}^2}\left[1 + \bar{\alpha}_s\left(\bar{b}\,\ln z_{12}^2\mu^2 - \bar{b}\,\frac{z_{13}^2 - z_{23}^2}{z_{12}^2}\ln\frac{z_{13}^2}{z_{23}^2} + \frac{67}{36} - \frac{\pi^2}{12} - \frac{5}{18}\frac{N_\mathrm{f}}{N_\mathrm{c}} - \frac{1}{2}\ln\frac{z_{13}^2}{z_{12}^2}\ln\frac{z_{23}^2}{z_{12}^2}\right)\right](S_{13}S_{32} - S_{12})$$

$$+\frac{\bar{\alpha}_s^2}{8\pi^2}\int \frac{\mathrm{d}^2 z_3\,\mathrm{d}^2 z_4}{z_{34}^4}\left[-2 + \frac{z_{13}^2 z_{24}^2 + z_{14}^2 z_{23}^2 - 4 z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2 - z_{14}^2 z_{23}^2}\ln\frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2} + \frac{z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2}\left(1 + \frac{z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2 - z_{14}^2 z_{23}^2}\right)\ln\frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2}\right]$$

$$\left[S_{13}S_{34}S_{42} - \frac{1}{2N_\mathrm{c}^3}\mathrm{tr}\big(V_1 V_3^\dagger V_4 V_2^\dagger V_3 V_4^\dagger\big) - \frac{1}{2N_\mathrm{c}^3}\mathrm{tr}\big(V_1 V_4^\dagger V_3 V_2^\dagger V_4 V_3^\dagger\big) - S_{13}S_{32} + \frac{1}{N_\mathrm{c}^2}S_{12}\right]$$

$$+\frac{\bar{\alpha}_s^2}{8\pi^2}\frac{N_\mathrm{f}}{N_\mathrm{c}}\int \frac{\mathrm{d}^2 z_3\,\mathrm{d}^2 z_4}{z_{34}^4}\left[2 - \frac{z_{13}^2 z_{24}^2 + z_{14}^2 z_{23}^2 - z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2 - z_{14}^2 z_{23}^2}\ln\frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2}\right]$$

$$\left[S_{14}S_{32} - \frac{1}{N_\mathrm{c}^3}\mathrm{tr}\big(V_1 V_2^\dagger V_3 V_4^\dagger\big) - \frac{1}{N_\mathrm{c}^3}\mathrm{tr}\big(V_1 V_4^\dagger V_3 V_2^\dagger\big) + \frac{1}{N_\mathrm{c}^2}S_{12}S_{34} - S_{13}S_{32} + \frac{1}{N_\mathrm{c}^2}S_{12}\right]$$

$$z_{ij} = z_i - z_j \qquad S_{ij} = \frac{1}{N_\mathrm{c}}\mathrm{tr}\big(V_i^\dagger V_j\big) \qquad V_i^\dagger = \mathrm{P}\exp\left[ig\int \mathrm{d}z^+ A_a^-(z^+, \boldsymbol{z}_i)t^a\right] \qquad \bar{b} = \frac{11}{12} - \frac{1}{6}\frac{N_\mathrm{f}}{N_\mathrm{c}}$$

Large transverse logs

- Strongly ordered large ``perturbative'' dipoles (DLA)

$$1/Q_s \gg z_{14} \simeq z_{24} \simeq z_{34} \gg z_{13} \simeq z_{23} \gtrsim z_{12} \exp\left(\bar{\alpha}_s^{-1/2,-1}\right) \gg z_{12}$$

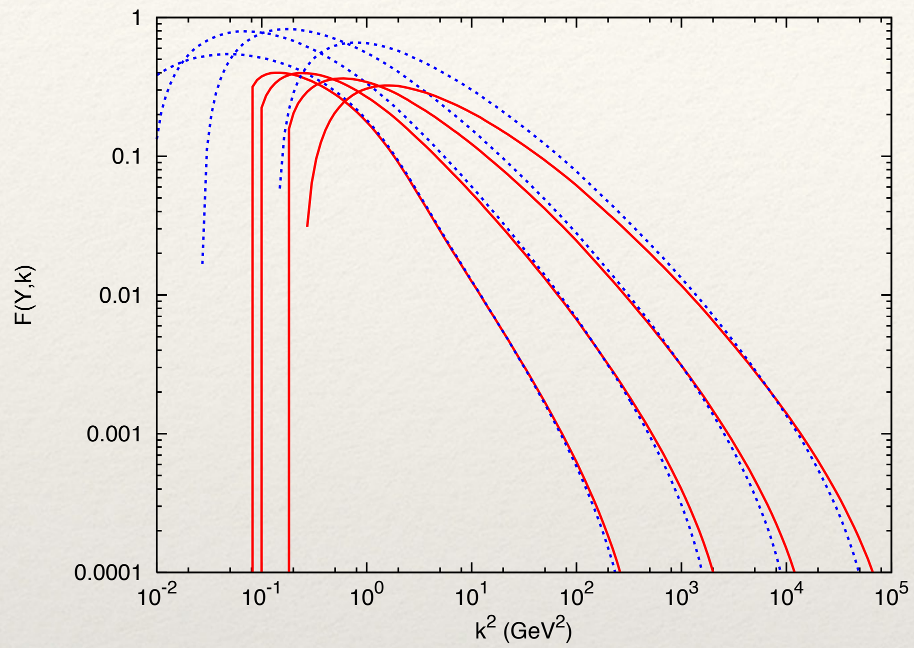
$$\frac{\mathrm{d}T_{12}}{\mathrm{d}Y} = \bar{\alpha}_s \int_{z_{12}^2}^{1/Q_s^2} \mathrm{d}z_{13}^2 \, \frac{z_{12}^2}{z_{13}^4} \left(1 - \bar{\alpha}_s \frac{1}{2} \ln^2 \frac{z_{13}^2}{z_{12}^2} - \bar{\alpha}_s \frac{11}{12} \ln \frac{z_{13}^2}{z_{12}^2}\right) T_{13}$$

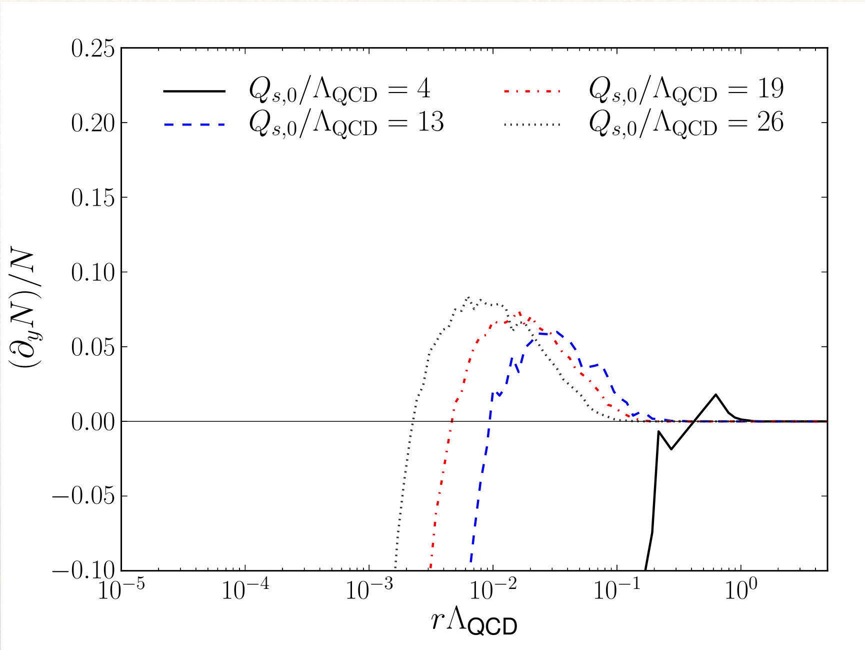
- Large dipoles interact stronger, real terms dominate
- NLO correction > LO term, unstable expansion in α_s
- Simplest thing: one step for GBW-type initial condition

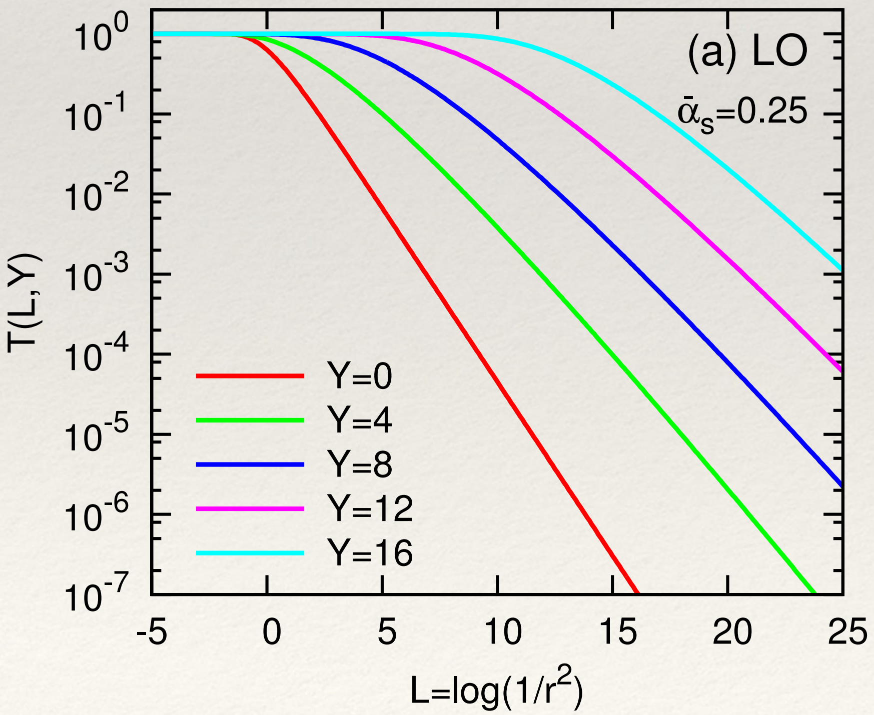
$$T_{12} = \begin{cases} z_{12}^2 Q_{s0}^2 & \text{for } \; z_{12} Q_{s0} \ll 1 \\ 1 & \text{for } \; z_{12} Q_{s0} \gg 1 \end{cases} \Rightarrow \Delta T_{12} \simeq \Delta Y \bar{\alpha}_s z_{12}^2 Q_{s0}^2 \left(\ln \frac{1}{z_{12}^2 Q_{s0}^2} - \bar{\alpha}_s \frac{1}{6} \ln^3 \frac{1}{z_{12}^2 Q_{s0}^2} - \bar{\alpha}_s \frac{11}{24} \ln^2 \frac{1}{z_{12}^2 Q_{s0}^2}\right)$$

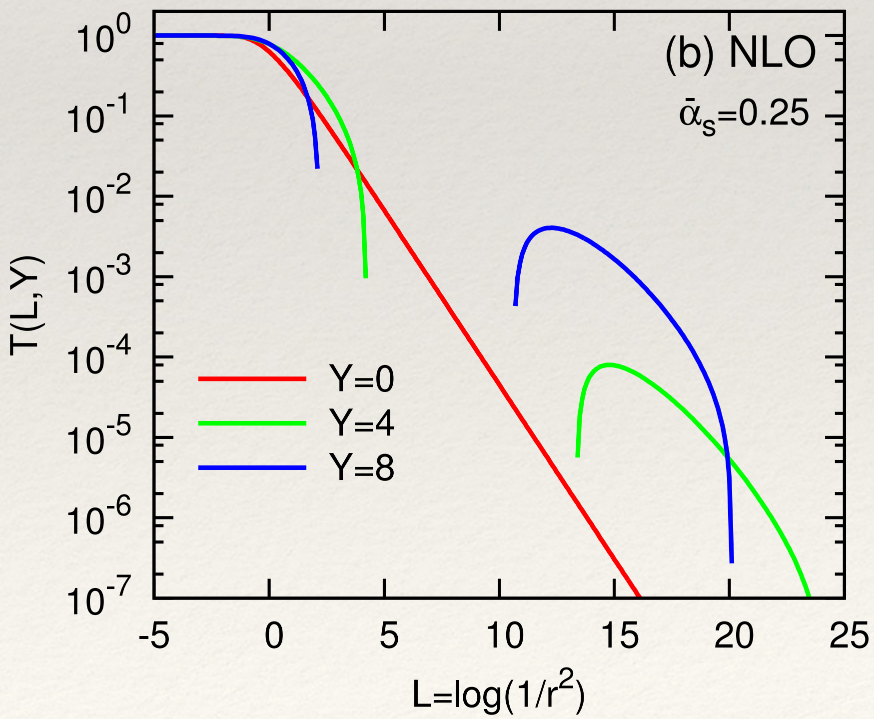
- Expect unstable numerical solution, as in BFKL

Unstable numerical solutions



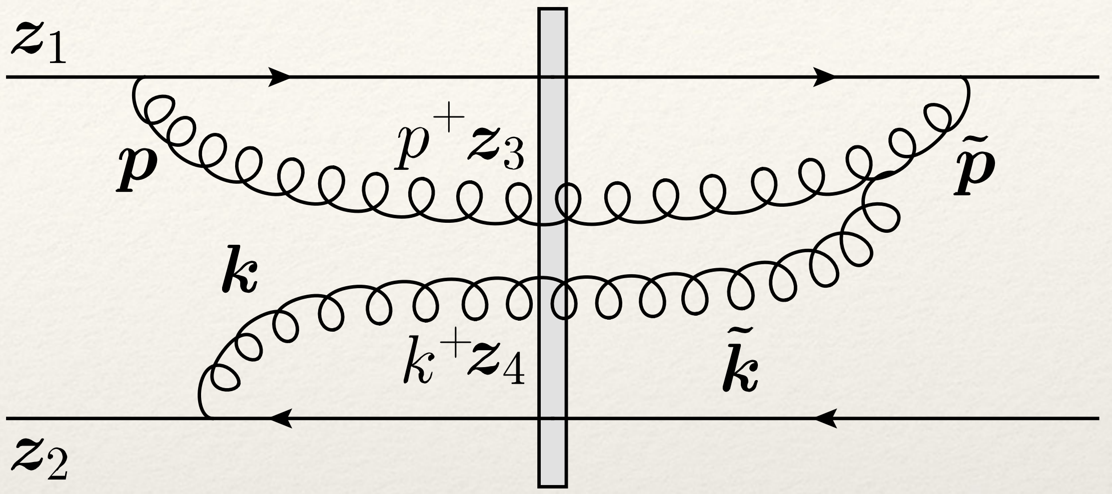






Two gluons and time ordering (kinematics)

- Take $\boldsymbol{k} \ll \boldsymbol{p}$ and $k^+ \ll p^+$



$$\Delta T_{12} = -\frac{g^4 N_c^2}{(2\pi^2)} \int \frac{\mathrm{d}p^+}{p^+} \int \frac{\mathrm{d}k^+}{k^+} \int \mathrm{d}^2 z_3 \mathrm{d}^2 z_4 \int_{\boldsymbol{p}\boldsymbol{k}\tilde{\boldsymbol{p}}\tilde{\boldsymbol{k}}} \frac{\boldsymbol{p}\cdot\tilde{\boldsymbol{p}}\ \boldsymbol{k}\cdot\tilde{\boldsymbol{k}}}{\boldsymbol{p}^2\tilde{\boldsymbol{p}}^2\boldsymbol{k}^2\tilde{\boldsymbol{k}}^2} \frac{\mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{z}_{31}+\mathrm{i}\tilde{\boldsymbol{p}}\cdot\boldsymbol{z}_{13}+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{z}_{42}+\mathrm{i}\tilde{\boldsymbol{k}}\cdot\boldsymbol{z}_{34}}}{\left[1+\frac{k^+}{p^+}\frac{\boldsymbol{p}^2}{\boldsymbol{k}^2}\right]\left[1+\frac{k^+}{p^+}\frac{(\tilde{\boldsymbol{p}}-\tilde{\boldsymbol{k}})^2}{\tilde{\boldsymbol{k}}^2}\right]} 2T(z_4)$$

- Largest logs when time ordering $\tau_k = k^+/\boldsymbol{k}^2 \ll \tau_p = p^+/\boldsymbol{p}^2$
- Add all diagrams: WW kernels, phases and trans. mom. int. $\rightarrow$ two dipole kernels. For $z_4 \gg z_3 \gg z_{12}$

$$\Delta T_{12} = \bar{\alpha}_s^2 \int \frac{\mathrm{d}p^+}{p^+}\frac{\mathrm{d}k^+}{k^+}\Theta\Big(p^+\frac{z_3^2}{z_4^2} - k^+\Big)\mathrm{d}z_3^2\mathrm{d}z_4^2\ \frac{z_{12}^2}{z_3^2 z_4^4}\, T(z_4) \rightarrow -\frac{\bar{\alpha}_s^2 \Delta Y}{2}\int_{z_{12}^2} \frac{\mathrm{d}z_4^2}{z_4^4} \ln^2\frac{z_4^2}{z_{12}^2} T(z_4)$$

Resummation of double logs (DLA)

- Resum double logs to all orders in non-local equation

$$T(Y,\rho) = T(0,\rho) + \bar{\alpha}_s \int_0^{\rho} \mathrm{d}\rho_1 \int_0^{Y-\rho+\rho_1} \mathrm{d}Y_1\, \mathrm{e}^{-(\rho-\rho_1)} T(0,\rho_1), \qquad \rho = \ln 1/z_{12}^2 Q_0^2$$

- Mathematically equivalent to local equation

$$\tilde{T}(Y,\rho) = \tilde{T}(0,\rho) + \bar{\alpha}_s \int_0^{Y} \mathrm{d}Y_1 \int_0^{\rho} \mathrm{d}\rho_1\, \mathrm{e}^{-(\rho-\rho_1)} \frac{\mathrm{J}_1\big(2\sqrt{\bar{\alpha}_s(\rho-\rho_1)^2}\big)}{\sqrt{\bar{\alpha}_s(\rho-\rho_1)^2}} \tilde{T}(0,\rho_1)$$

- I.C. $T(0,\rho) = \mathrm{e}^{-\rho} \;\; \rightarrow \qquad \tilde{T}(0,\rho) = \mathrm{e}^{-\rho} \mathrm{J}_1\big(2\sqrt{\bar{\alpha}_s \rho^2}\big)$

- Leads to physical amplitude T for $Y > \rho$

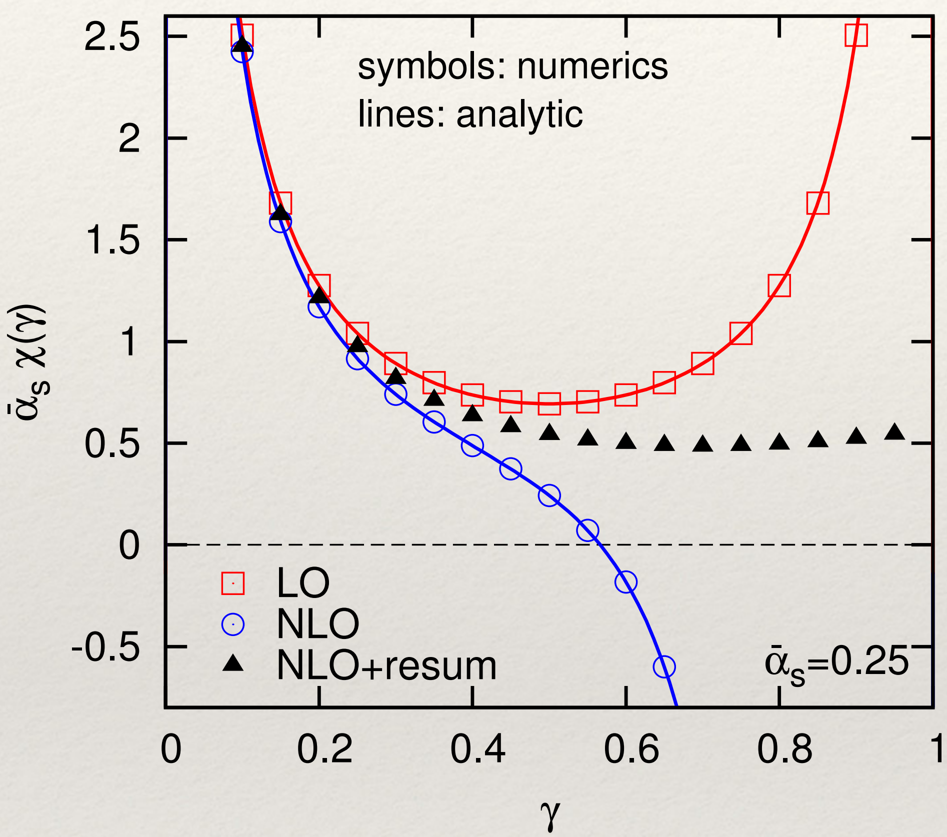
Double logs resummed in BK

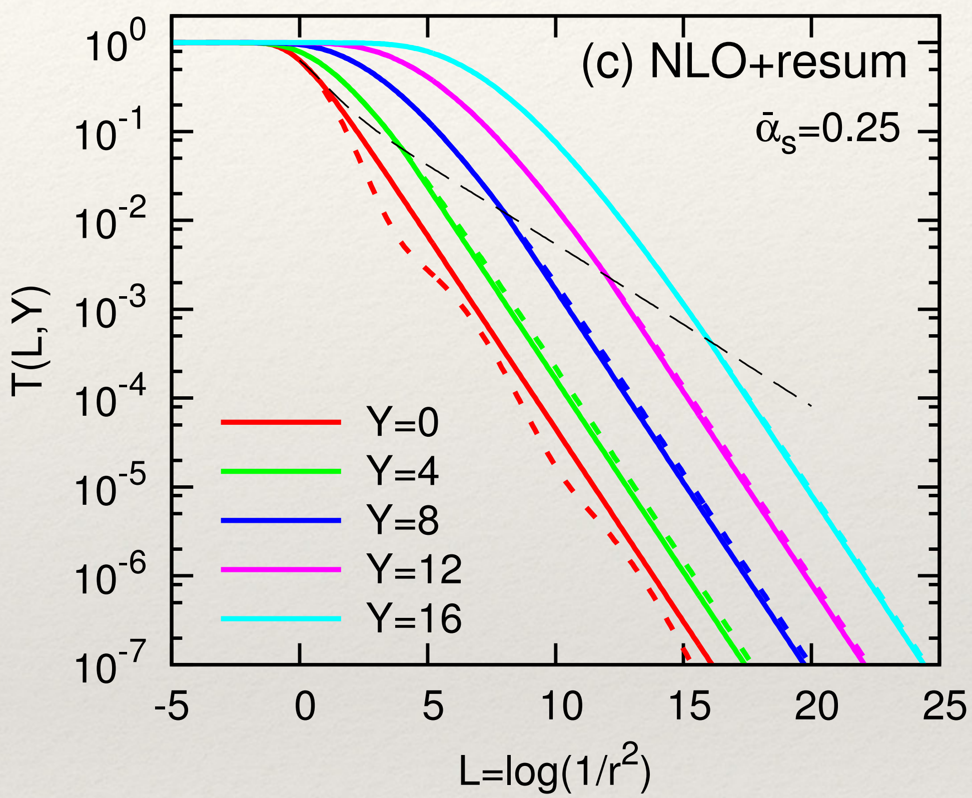
$$\frac{\mathrm{d}\tilde{T}_{12}}{\mathrm{d}Y} = \frac{\bar{\alpha}_s}{2\pi} \int \mathrm{d}^2 z_3 \, \frac{z_{12}^2}{z_{13}^2 z_{23}^2} \, \frac{\mathrm{J}_1\big(2\sqrt{\bar{\alpha}_s L_{13} L_{23}}\big)}{\sqrt{\bar{\alpha}_s L_{13} L_{23}}} \, (\tilde{T}_{13} + \tilde{T}_{23} - \tilde{T}_{12} - \tilde{T}_{13}\tilde{T}_{23})$$

- Agrees with NLO BK (double log term) when truncated to order $\bar{\alpha}_s^2$
- Resums double log terms to all orders
- Solution should behave nicely
- Initial condition: $\tilde{T}(0,\rho) = 1 - \mathrm{e}^{-\tilde{T}_{2\mathrm{g}}(0,\rho)}$

Numerical solution

BFKL on $z_{12}^{2\gamma} \equiv r^{2\gamma}$

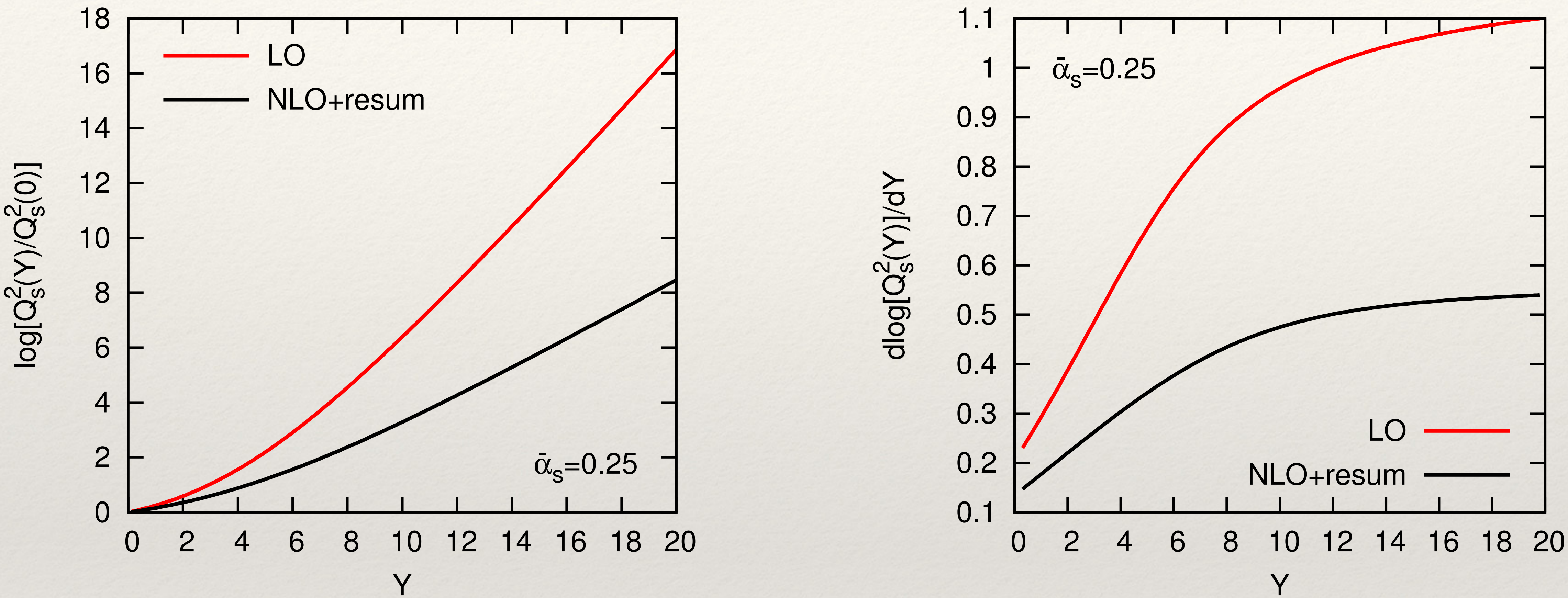




$$\omega_{\rm LO} = \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s}{1-\gamma} + \text{finite} \qquad \omega_{\rm NLO} = \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s}{1-\gamma} - \frac{\bar{\alpha}_s^2}{(1-\gamma)^3} + \text{finite}$$

$$\omega_{\rm NLO}^{\rm res} = \omega_{\rm NLO} - \frac{\bar{\alpha}_s}{1-\gamma} + \frac{\bar{\alpha}_s^2}{(1-\gamma)^3} + \frac{1}{2}\left[-(1-\gamma) + \sqrt{(1-\gamma)^2 + 4\bar{\alpha}_s}\right] + \text{finite}$$

Numerical solution

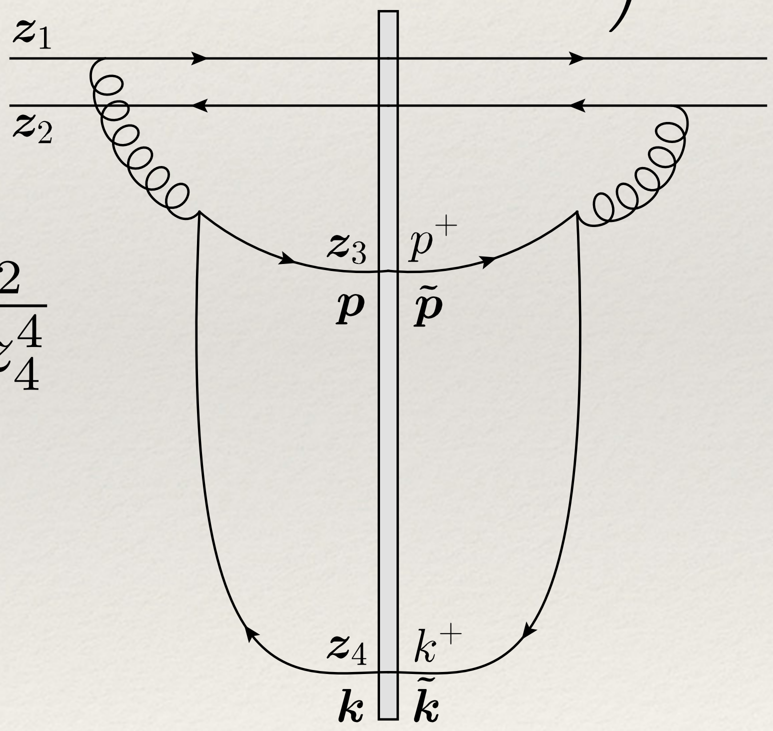


- Considerable speed reduction, roughly factor of 1/2

Single log in quark contribution (dynamics)

- Take $\boldsymbol{k} \ll \boldsymbol{p}$ and $\zeta = k^+/p^+ \ll 1$

$$\Sigma\mathcal{A}_{ij} = 8\alpha_s^2 N_f \int \frac{\mathrm{d}p^+}{p^+} \int_0^1 \mathrm{d}\zeta \int_{\boldsymbol{p}\boldsymbol{k}\tilde{\boldsymbol{p}}\tilde{\boldsymbol{k}}} \frac{\boldsymbol{p}\cdot\boldsymbol{k}\,\tilde{\boldsymbol{p}}\cdot\tilde{\boldsymbol{k}} + \boldsymbol{p}\cdot\tilde{\boldsymbol{p}}\,\boldsymbol{k}\cdot\tilde{\boldsymbol{k}} - \boldsymbol{p}\cdot\tilde{\boldsymbol{k}}\,\boldsymbol{k}\cdot\tilde{\boldsymbol{p}}}{\boldsymbol{p}^2\tilde{\boldsymbol{p}}^2(\zeta\boldsymbol{p}^2+\boldsymbol{k}^2)(\zeta\tilde{\boldsymbol{p}}^2+\tilde{\boldsymbol{k}}^2)}$$
$$\left(\mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{z}_{31}+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{z}_{41}} - \mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{z}_{32}+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{z}_{42}}\right)\left(\mathrm{e}^{\mathrm{i}\tilde{\boldsymbol{p}}\cdot\boldsymbol{z}_{13}+\mathrm{i}\tilde{\boldsymbol{k}}\cdot\boldsymbol{z}_{14}} - \mathrm{e}^{\mathrm{i}\tilde{\boldsymbol{p}}\cdot\boldsymbol{z}_{23}+\mathrm{i}\tilde{\boldsymbol{k}}\cdot\boldsymbol{z}_{24}}\right)$$



- Integrate transverse momenta

$$\Sigma\mathcal{A}_{ij} = \frac{\alpha_s^2 N_\mathrm{f}}{2\pi^4}\,\Delta Y \int_0^1 \mathrm{d}\zeta\, \frac{z_{12}^2}{z_4^2}\,\frac{z_3^4+\zeta^2 z_4^4}{(z_3^2+\zeta z_4^2)^4} \simeq \frac{\alpha_s^2 N_\mathrm{f}}{3\pi^4}\,\Delta Y\,\frac{z_{12}^2}{z_3^2 z_4^4}$$

z_3^2 scales like ζz_4^2, no ordering in time

- Insert color structure and Wilson lines

$$\frac{\Delta T_{12}}{\Delta Y} = \frac{\bar{\alpha}_s^2}{6}\,\frac{N_\mathrm{f}}{N_\mathrm{c}}\left(1-\frac{1}{N_\mathrm{c}^2}\right) z_{12}^2 \int_{z_{12}^2}^{\infty} \frac{\mathrm{d}z_4^2}{z_4^4}\,\ln\frac{z_4^2}{z_{12}^2}\,T(z_4)$$

Relationship to splitting functions

- Pole residues related to coefficients in splitting functions

$$\int_0^1 \mathrm{d}z\, z^\omega P_{\mathrm{gg}}(z) = \frac{1}{\omega} - \frac{11}{12} - \frac{N_f}{6N_c} + \mathcal{O}(\omega)$$

$$\frac{C_\mathrm{F}}{N_\mathrm{c}} \int_0^1 \mathrm{d}z\, z^\omega P_{\mathrm{qg}}(z) = \frac{N_\mathrm{f}}{6N_\mathrm{c}} - \frac{N_\mathrm{f}}{6N_\mathrm{c}^3} + \mathcal{O}(\omega)$$

- Starting from DGLAP, write in Mellin space

$$T_{12} \approx \int \frac{\mathrm{d}\gamma}{2\pi\mathrm{i}} \frac{\mathrm{d}\omega}{2\pi\mathrm{i}} \frac{\mathrm{e}^{\omega Y - \gamma\rho}}{1 - \gamma + \omega - \bar{\alpha}_s \underbrace{\tilde{P}(\omega)}_{\simeq \frac{1}{\omega} - A_1}}$$

- Integrate $\omega \to$ BFKL

Resum single logs

- Keep $1/\omega$-$A_1 \rightarrow$ *local* in Y equation

$$\omega_{\rm coll}(\gamma) = \frac{1}{2}\left[-(1-\gamma+\bar{\alpha}_s A_1) + \sqrt{(1-\gamma+\bar{\alpha}_s A_1)^2 + 4\bar{\alpha}_s}\right]$$

$\mathrm{e}^{-A_1 \bar{\alpha}_s (\rho - \rho_1)}$ extra factor in kernel: anomalous dimension

- Extend to BK as in double logs case

$$\frac{\mathrm{d}\tilde{T}_{12}}{\mathrm{d}Y} = \frac{\bar{\alpha}_s}{2\pi}\int \mathrm{d}^2 z_3\, \frac{z_{12}^2}{z_{13}^2 z_{23}^2}\left(\frac{z_{12}^2}{z_<^2}\right)^{\pm A_1 \bar{\alpha}_s} \frac{\mathrm{J}_1\big(2\sqrt{\bar{\alpha}_s L_{13} L_{23}}\big)}{\sqrt{\bar{\alpha}_s L_{13} L_{23}}}\,(\tilde{T}_{13} + \tilde{T}_{23} - \tilde{T}_{12} - \tilde{T}_{13}\tilde{T}_{23})$$

- $z_< = \min\{z_{13}, z_{23}\}$. - sign when $z_< < z_{12}$, resums logs

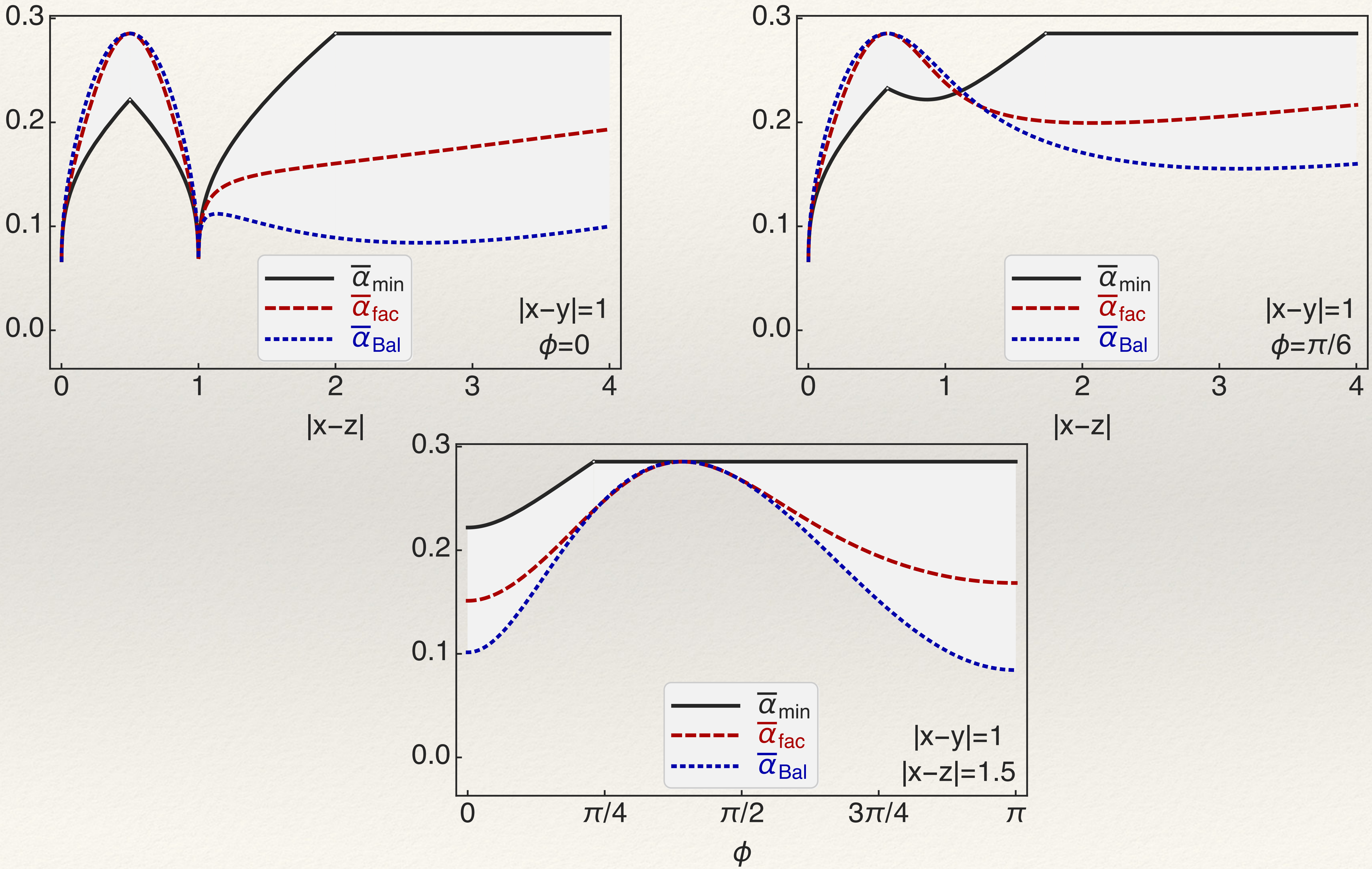
Running coupling

$$\frac{\mathrm{d}T_{12}}{\mathrm{d}Y} = \frac{\bar{\alpha}_s(\mu)}{2\pi} \int \mathrm{d}^2 z_3 \, \frac{z_{12}^2}{z_{13}^2 z_{23}^2} \left[1 + \bar{\alpha}_s(\mu)\left(\bar{b}\, \ln z_{12}^2\mu^2 - \bar{b}\, \frac{z_{13}^2 - z_{23}^2}{z_{12}^2} \ln \frac{z_{13}^2}{z_{23}^2}\right)\right] (T_{13} + T_{23} - T_{12} - T_{13}T_{23})$$

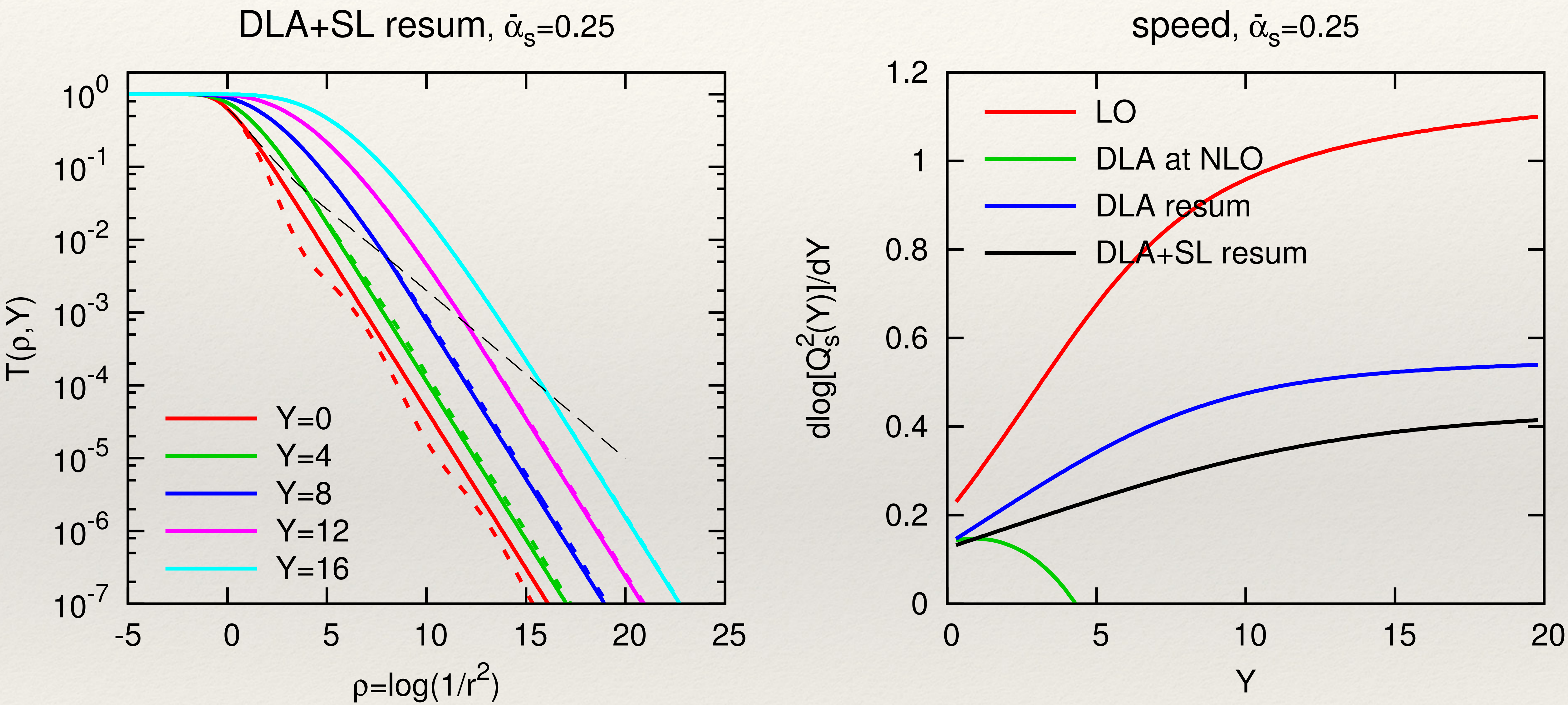
- Choose μ to cancel potentially large log in all regions
  Large daughter dipoles : $\mu \approx 1/z_{12}$
  Small daughter dipoles : $\mu \approx 1/\min\{z_{13}, z_{23}\}$
  In general : $\mu \approx 1/\min\{z_{ij}\}$ ✓ Hardest scale
- Balitsky-prescription: ✓, albeit unphysical slow
- Choose coefficient of $\bar{b}$ to vanish: ✓, better choice

$$\alpha_s = \left[\frac{1}{\alpha_s(z_{12})} + \frac{z_{13}^2 - z_{23}^2}{z_{12}^2} \frac{\alpha_s(z_{13}) - \alpha_s(z_{23})}{\alpha_s(z_{13})\alpha_s(z_{23})}\right]^{-1}$$

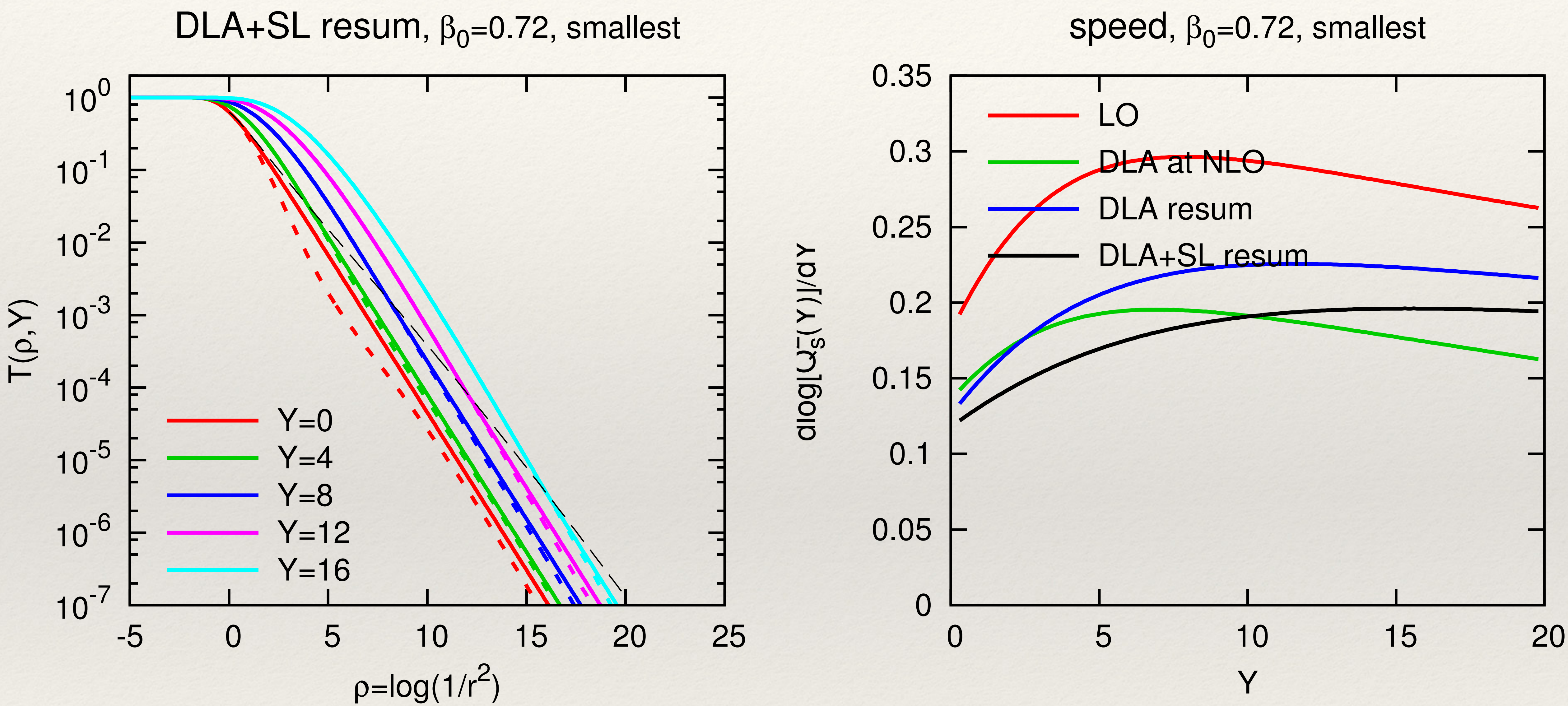
Couplings comparison



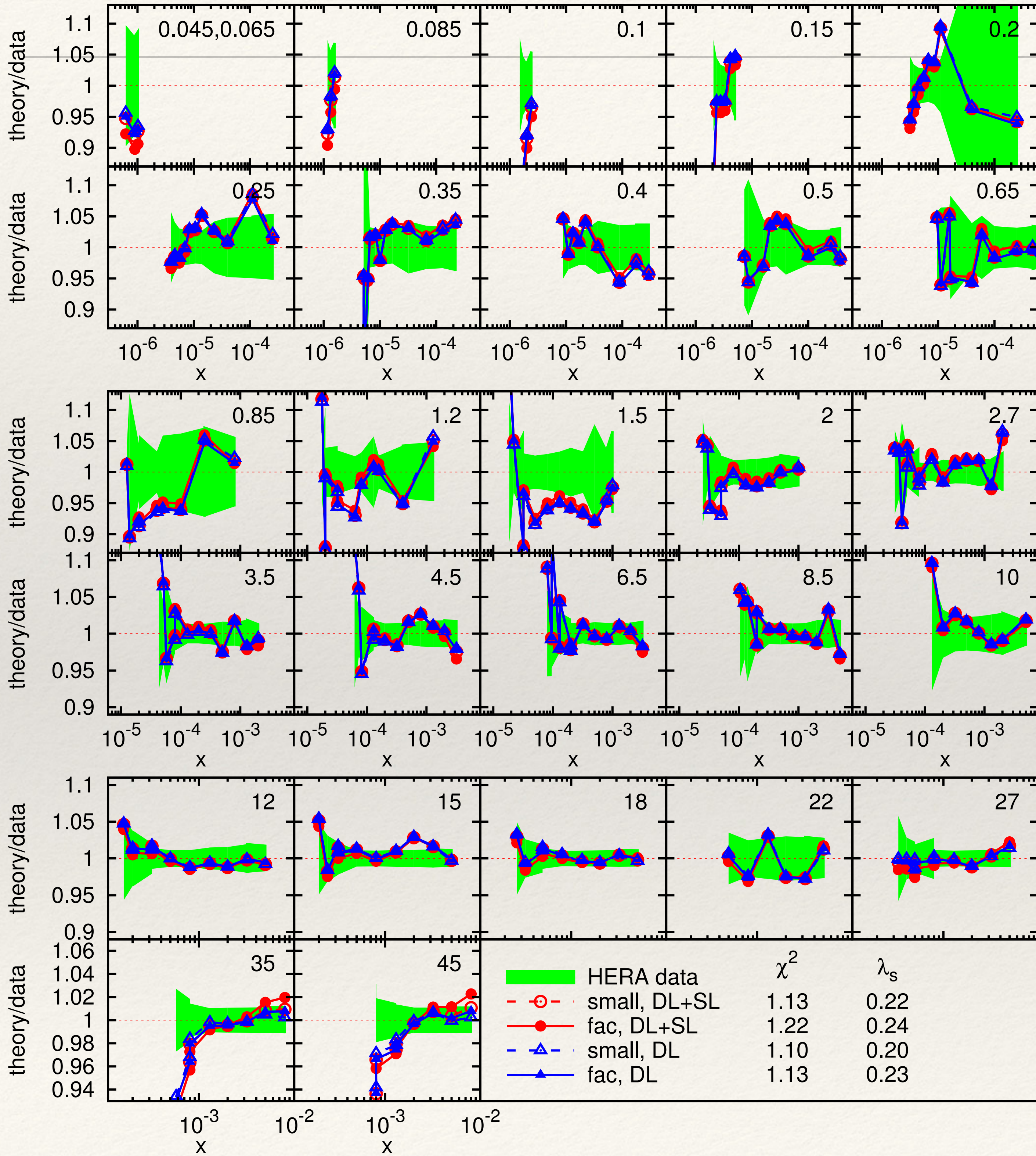
Numerical solution (fixed)



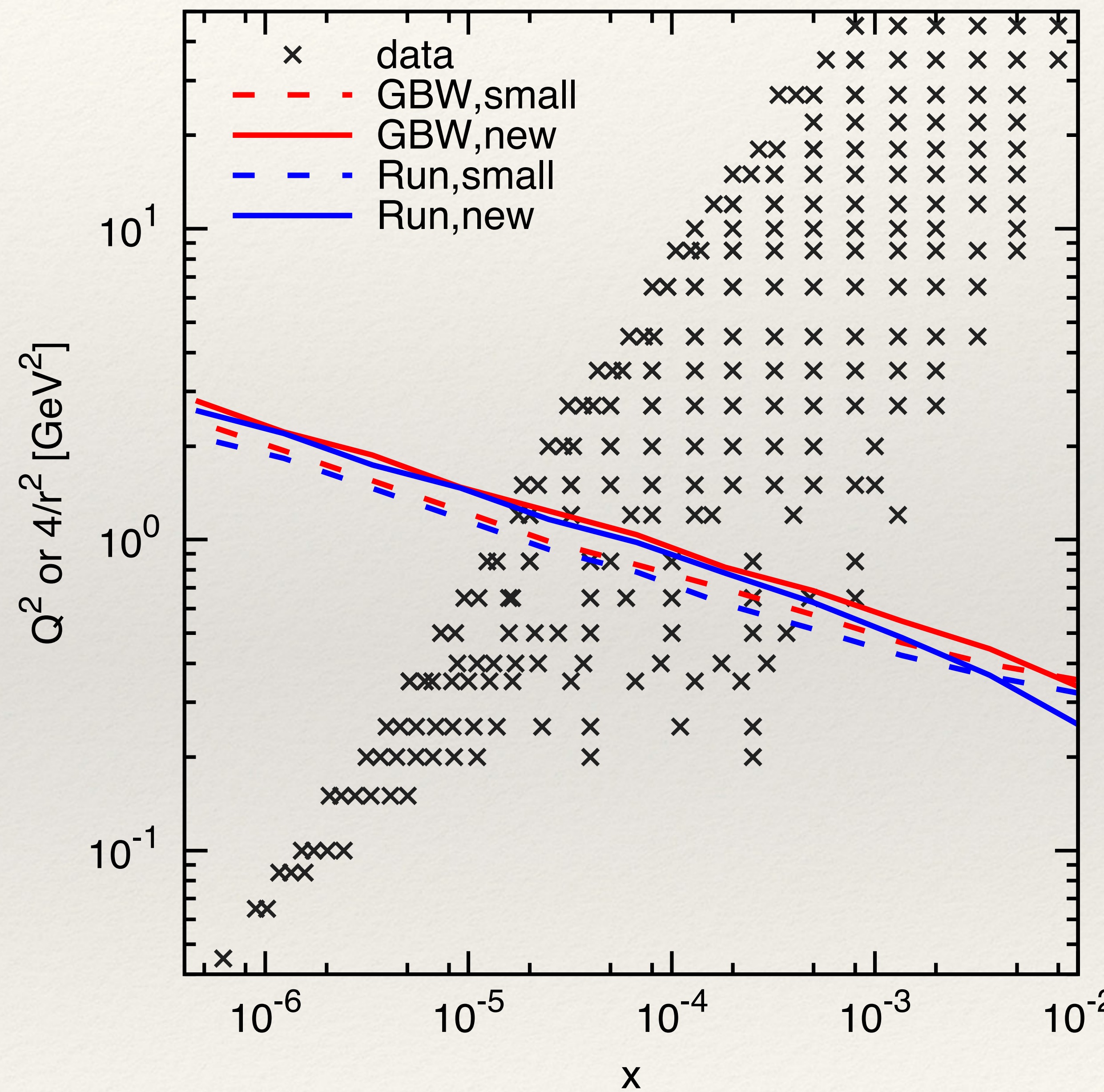
Numerical solution (prescirption:small)



Fit



0.045,0.065
0.085
0.1
0.15
0.2
0.25
0.35
0.4
0.5
0.65
0.85
1.2
1.5
2
2.7
3.5
4.5
6.5
8.5
10
12
15
18
22
27
35
45
theory/data
x
HERA data
small, DL+SL
fac, DL+SL
small, DL
fac, DL
χ2 λs
1.13 0.22
1.22 0.24
1.10 0.20
1.13 0.23



data
GBW,small
GBW,new
Run,small
Run,new
Q2 or 4/r2 [GeV2]
x

Fit

init cdt.	RC schm	sing. logs	χ^2 per data point			parameters				
			$\sigma_{\rm red}$	$\sigma_{\rm red}^{c\bar{c}}$	F_L	R_p[fm]	Q_0[GeV]	C_α	p	$C_{\rm MV}$
GBW	small	yes	1.135	0.552	0.596	0.699	0.428	2.358	2.802	-
GBW	fac	yes	1.262	0.626	0.602	0.671	0.460	0.479	1.148	-
rcMV	small	yes	1.126	0.578	0.592	0.711	0.530	2.714	0.456	0.896
rcMV	fac	yes	1.222	0.658	0.595	0.681	0.566	0.517	0.535	1.550
GBW	small	no	1.121	0.597	0.597	0.716	0.414	6.428	4.000	-
GBW	fac	no	1.164	0.609	0.594	0.697	0.429	1.195	4.000	-
rcMV	small	no	1.097	0.557	0.593	0.723	0.497	7.393	0.477	0.816
rcMV	fac	no	1.128	0.573	0.591	0.703	0.526	1.386	0.502	1.015

init cdt.	RC schm	sing. logs	χ^2/npts for $Q^2_{\rm max}$			
			50	100	200	400
GBW	small	yes	1.135	1.172	1.355	1.537
GBW	fac	yes	1.262	1.360	1.654	1.899
rcMV	small	yes	1.126	1.172	1.167	1.158
rcMV	fac	yes	1.222	1.299	1.321	1.317
GBW	small	no	1.121	1.131	1.317	1.487
GBW	fac	no	1.164	1.203	1.421	1.622
rcMV	small	no	1.097	1.128	1.095	1.078
rcMV	fac	no	1.128	1.177	1.150	1.131

- No anomalous dimension in initial condition
- Including single logs: more physical parameters
- rcMV model: can be extrapolated to higher Q^2
- Small and ``new'' prescription: good fit
  B-prescription: ``doesn't make it''

Backup slides

Resummation of double logs in BK

Extend DLA to match BK

- Let $2\tilde{T}(Y,\rho_1) \to \tilde{T}_{13} + \tilde{T}_{23}$
- Restore dipole kernel $\mathrm{d}\rho_1 \mathrm{e}^{-(\rho-\rho_1)} = \dfrac{\mathrm{d}z_{13}^2\, z_{12}^2}{z_{13}^4} \to \dfrac{1}{\pi}\dfrac{\mathrm{d}z_3^2\, z_{12}^2}{z_{13}^2 z_{23}^2}$
- Insert virtual term $-\tilde{T}_{12}$, non-linear term, remove cutoffs
- Replace argument in argument of DLA kernel to switch off resummation for small daughter dipoles

$$\rho - \rho_1 = \ln\frac{z_{13}^2}{z_{12}^2} \to \sqrt{\ln\frac{z_{13}^2}{z_{12}^2}\ln\frac{z_{23}^2}{z_{12}^2}} \equiv \sqrt{L_{13}L_{23}}$$

ω-shift

- In DLA need x^+ ordering → p^- ordering
- Proper variable $Y^- = \ln \frac{p^-_{\rm max}}{p^-} = \ln \frac{p^+}{p^+_{\rm min}} + \ln z_{12}^2 Q_0^2 = Y - \rho$
- Double Mellin representation of BFKL solution

$$T_{12} = \int \frac{{\rm d}\xi}{2\pi {\rm i}} \frac{{\rm d}\omega}{2\pi {\rm i}} \frac{\bar{T}^0(\xi)}{\omega - \bar{\alpha}_s \chi_0(\xi)} {\rm e}^{\omega Y^-} (z_{12}^2 Q_0^2)^{\xi} \quad \Rightarrow$$

$$T_{12} = \int \frac{{\rm d}\gamma}{2\pi {\rm i}} \frac{{\rm d}\omega}{2\pi {\rm i}} \frac{\bar{T}^0(\gamma - \omega)}{\omega - \bar{\alpha}_s \chi_0(\gamma - \omega)} {\rm e}^{\omega Y} (z_{12}^2 Q_0^2)^{\gamma} \quad \Rightarrow$$

$$\omega = \frac{\bar{\alpha}_s}{1 - \gamma + \omega}$$

Leads to √ seen earlier

Large single logs from large dipoles

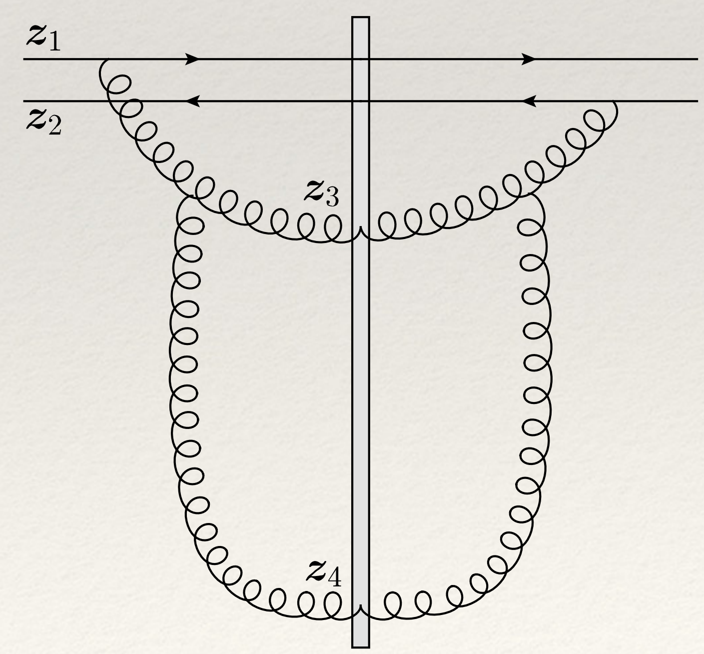
- Easily seen in BFKL, "some effort" to uncover in BK

$$\frac{\mathrm{d}T_{12}}{\mathrm{d}Y} = \frac{\bar{\alpha}_s^2}{8\pi^2} \int \frac{\mathrm{d}^2 z_3\, \mathrm{d}^2 z_4}{z_{34}^4} \left[-2 + \frac{z_{13}^2 z_{24}^2 + z_{14}^2 z_{23}^2 - 4 z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2 - z_{14}^2 z_{23}^2} \ln \frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2} \right] T_{34}$$

- Ordered dipoles $z_{14} \simeq z_{24} \simeq z_{34} \gg z_{13} \simeq z_{23} \gg z_{12}$

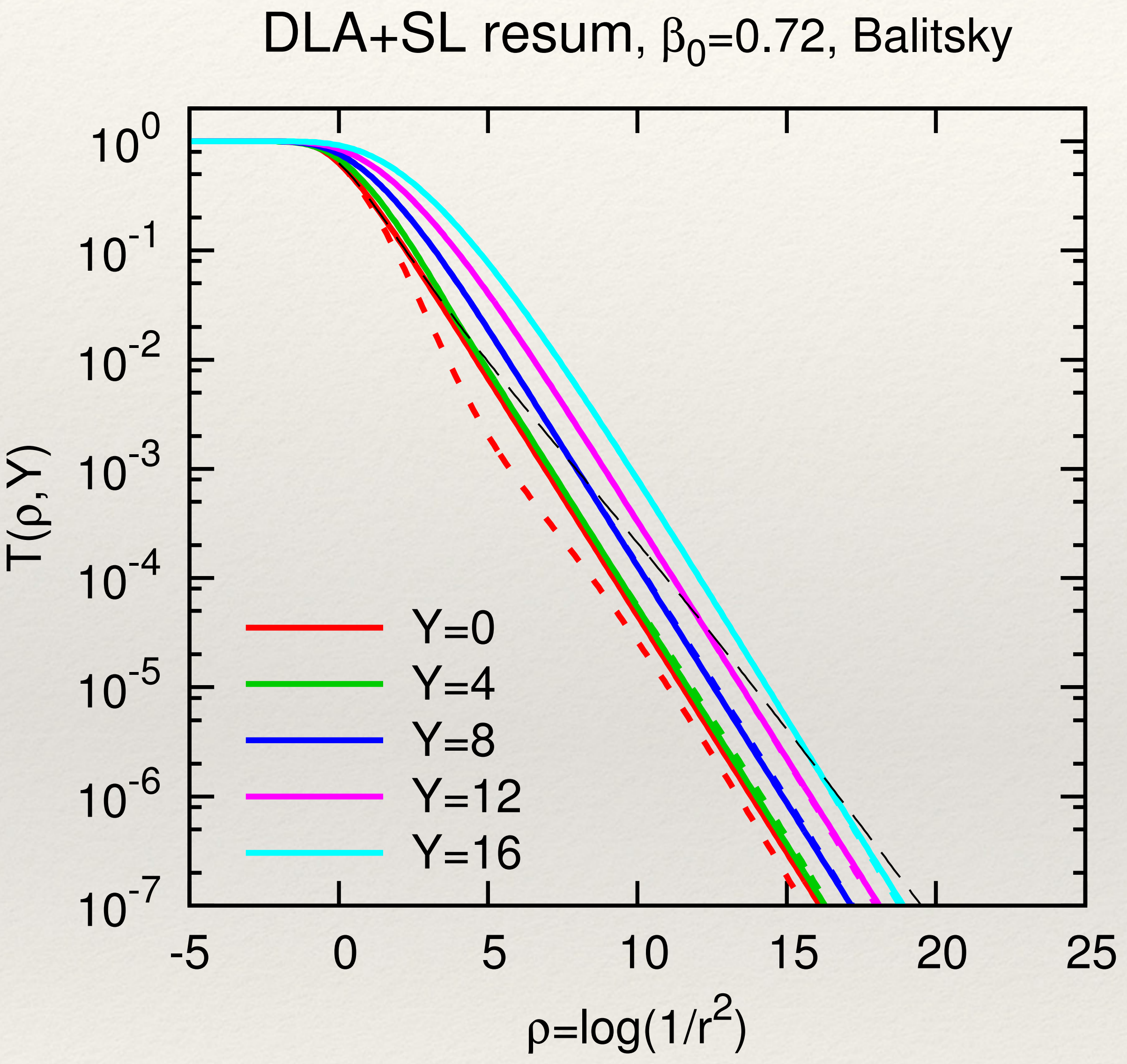
$$\mathcal{K}_{\mathrm{SL}}^{\mathrm{g}} \simeq -\frac{6 - \cos^2\phi}{12} \frac{\bar{\alpha}_s^2}{\pi^2} \frac{z_1^2}{z_3^2 z_4^4} \rightarrow -\frac{11}{24} \frac{\bar{\alpha}_s^2}{\pi^2} \frac{z_1^2}{z_3^2 z_4^4}.$$

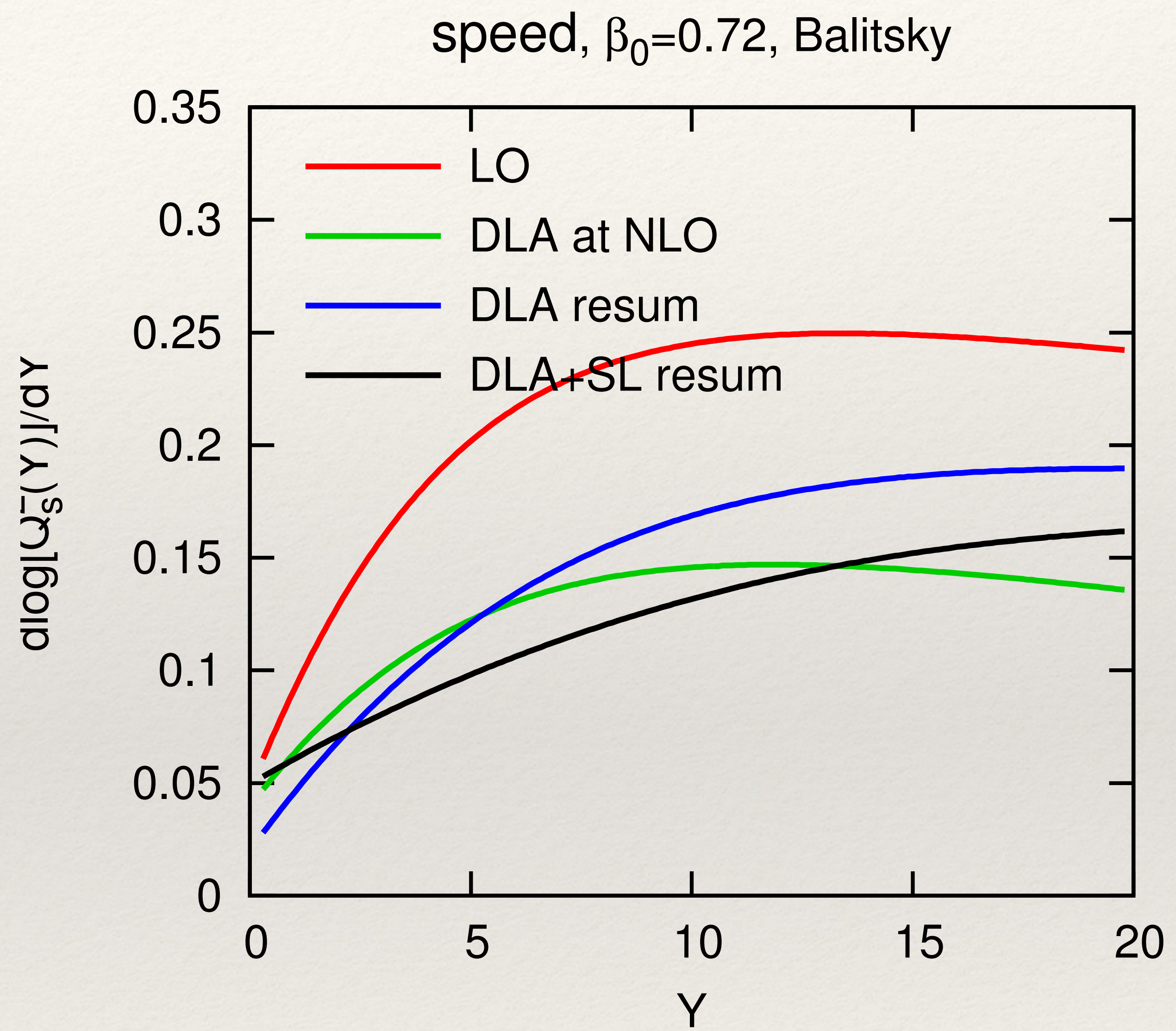
$$\left.\frac{\Delta T(z_{12})}{\Delta Y}\right|_{\mathrm{SL}}^{\mathrm{g}} \simeq -\frac{11}{12} \bar{\alpha}_s^2 z_{12}^2 \int_{z_{12}^2}^{\infty} \frac{\mathrm{d}z_4^2}{z_4^4} \ln \frac{z_4^2}{z_{12}^2}\, T(z_4)$$



- Dominates the LO term

Numerical solution (Balitsky prescirption)





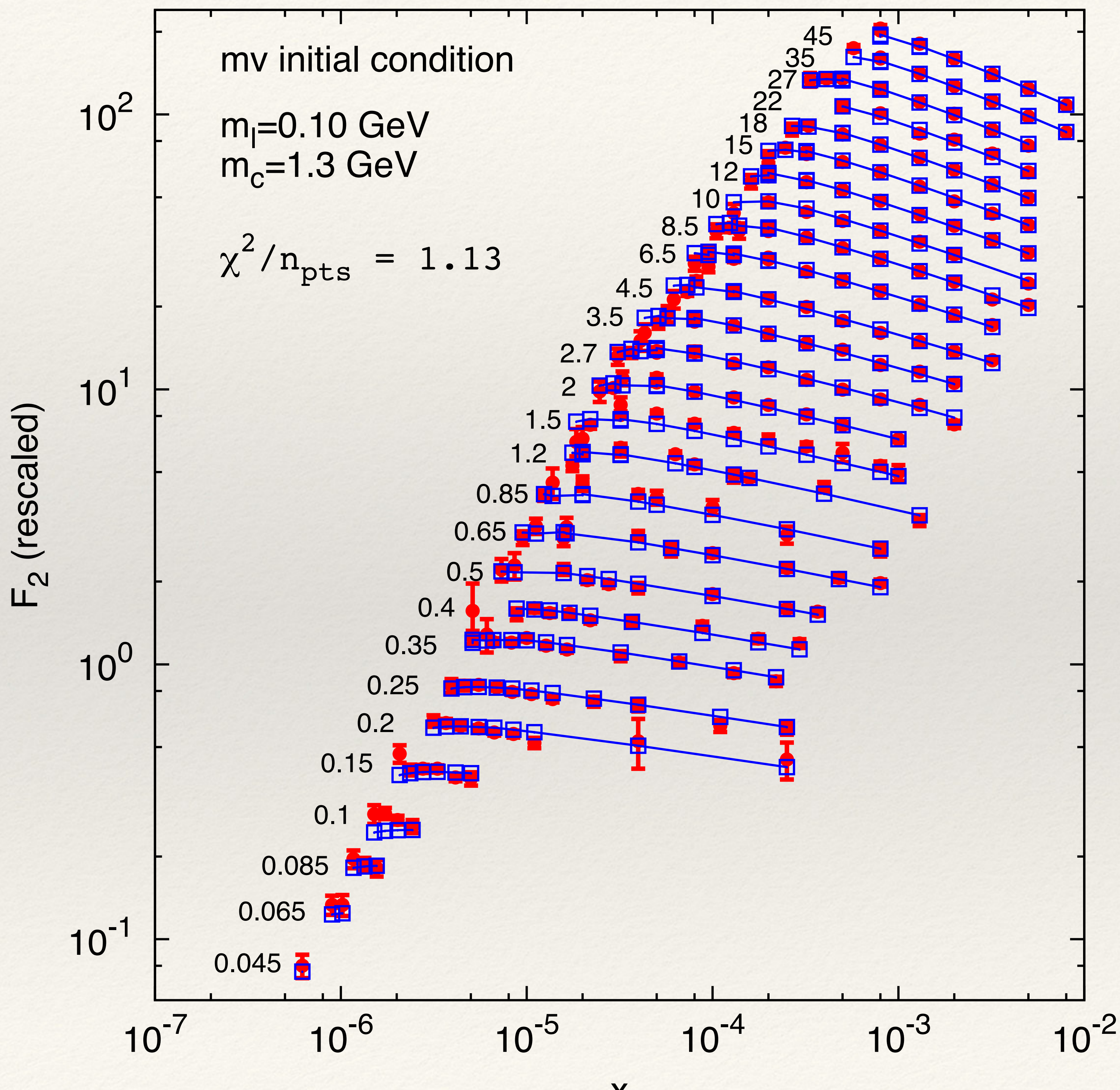
Some fitting expressions

GBW $$T(Y_0, r) = \left\{1 - \exp\left[-\left(\frac{r^2 Q_{s0}^2}{4}\right)^p\right]\right\}^{1/p}$$

rcMV $$T(Y_0, r) = \left\{1 - \exp\left[-\left(\frac{r^2 Q_{s0}^2}{4}\,\bar{\alpha}_s(C_{\rm MV} r)\left[1 + \log\left(\frac{\bar{\alpha}_{\rm sat}}{\bar{\alpha}_s(C_{\rm MV} r)}\right)\right]\right)^p\right]\right\}$$

RC $$\alpha_s(r) = \frac{1}{b_{N_{\rm f}} \ln\left[4C_\alpha^2/(r^2 \Lambda_{N_{\rm f}}^2)\right]}$$ (plus freezing)

Fit



mv initial condition
m_l=0.10 GeV
m_c=1.3 GeV
χ^2/n_{pts} = 1.13
45
35
27
22
18
15
12
10
8.5
6.5
4.5
3.5
2.7
2
1.5
1.2
0.85
0.65
0.5
0.4
0.35
0.25
0.2
0.15
0.1
0.085
0.065
0.045
10^2
10^1
10^0
10^{-1}
F_2 (rescaled)
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
x