

QCD at Cosmic energies VII, 2016

Outline



Importance of coherence in high energy scattering example: positronium propagation through the medium



Color fluctuations in hadron - new pattern of high energy hadron - nucleus scattering - going beyond single parton structure of nucleon.



Evidence for x -dependent color fluctuations in nucleons





Color fluctuations in coherent photoproduction of vector mesons



Color fluctuations in incoherent photon - nucleus scattering

Several feature of NN interactions at the LHC relevant for pA and AA

two seem to be most important:

- * Fluctuations of overall strength of NN interaction
- * A factor of two difference of the transverse area scales for soft and hard NN interaction

Other fluctuations - gluon density in nucleon, nuclei, LT shadowing effects -- will mention briefly

Fluctuations of overall strength of high energy NN interaction



High energy projectile stays in a frozen configuration distances $I_{coh} = c\Delta t$

$$\Delta t \sim 1/\Delta E \sim \frac{2p_h}{m_{int}^2-m_h^2} \quad \text{At LHC for} \quad m_{int}^2-m_h^2 \sim 1 \text{GeV}^2 \text{ I}_{\text{coh}} \sim \text{IO}^7 \text{ fm} >> 2 \text{R}_{\text{A}} >> 2 \text{r}_{\text{N}}$$
 coherence up to
$$m_{int}^2 \sim 10^6 \text{GeV}^2$$

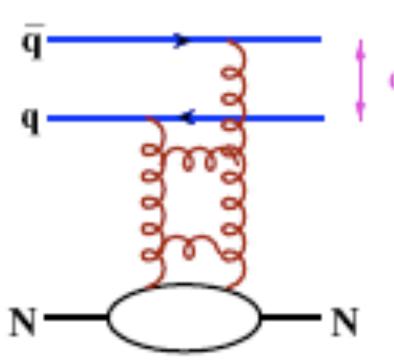
Hence system of quarks and gluons passes through the nucleus interacting essentially with the same strength but changes from one event to another different strength



Strength of interaction of white small system is proportional to the area occupies by color.

QCD factorization theorem for the interaction of small size color singlet wave package of quarks and gluons.

For quark - antiquark dipole:



$$\begin{array}{c} \begin{tabular}{|c|c|c|c|} \hline & \sigma(d,x) = \frac{\pi^2}{3} \alpha_s(Q_{eff}^2) d^2 \left[x G_N(x,Q_{eff}^2) + \frac{2}{3} x S_N(x,Q_{eff}^2) \right] \\ & Q_{eff}^2 = \lambda/d^2, \lambda = 4 \div 10 \end{array}$$

Baym, Blättel, Frankfurt, MS, 93; Frankfurt, Miller, MS 93

compare: $\sigma(d, x) = cd^2$ in QED or two gluon exchange model of Low - Nussinov (1975)

Instructive example: propagation of a very fast positronium (bound state of electron and positron) through a foil

$$\frac{P_{pos}}{2m_e} \cdot \frac{1}{\Delta E (\sim few \ m_e \alpha^2)} \gg L(f$$
 Positronium beam foil

 $rac{P_{pos}}{2m_e} \cdot rac{1}{\Delta E(\sim few \ m_e lpha^2)} \gg L(foil)$ first qualitative discussion - Nemenov, 1981, quantitative treatment Frankfurt and MS 91)

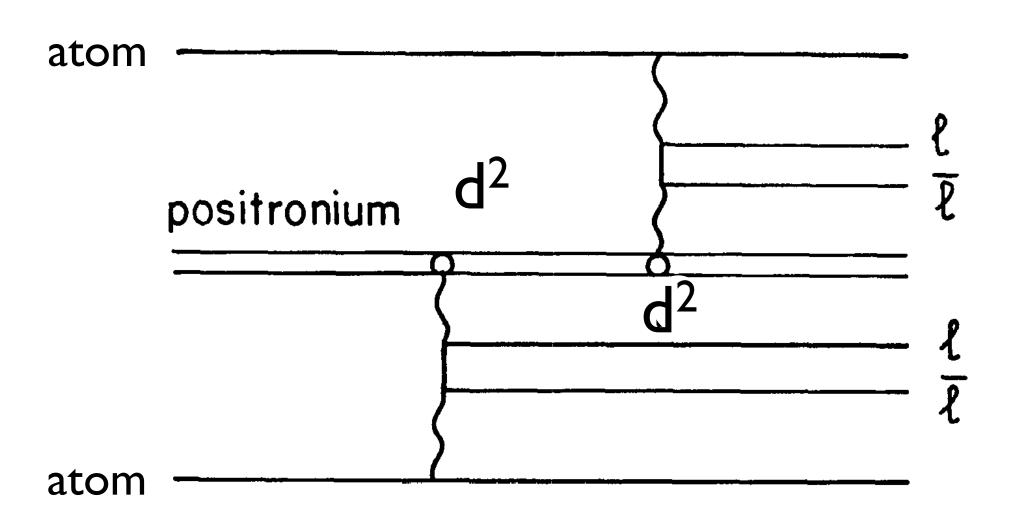
For the positronium at high energies transverse size is frozen during traversing through the foil - so interaction is of dipole-dipole type $\sigma(d) \propto d^2$ where $d=r_t^e-r_t^{e^+}$

Amplitude of i
$$\rightarrow$$
 f transition: $|M_{if}| = \left[\int d^3r \Psi_{pos} \Psi_f^* \exp(-\sigma(d)\rho L/2)\right]^2$

For large L: survival probability $\frac{16}{(<\sigma>\rho L)^2}$ absorption is not exponential !!!

Even larger probability to transform to electron - positron pair of the same momentum as positronium $\frac{2}{<\sigma>\rho L}$

Can we instead trigger on larger than average size configuration in positronium?

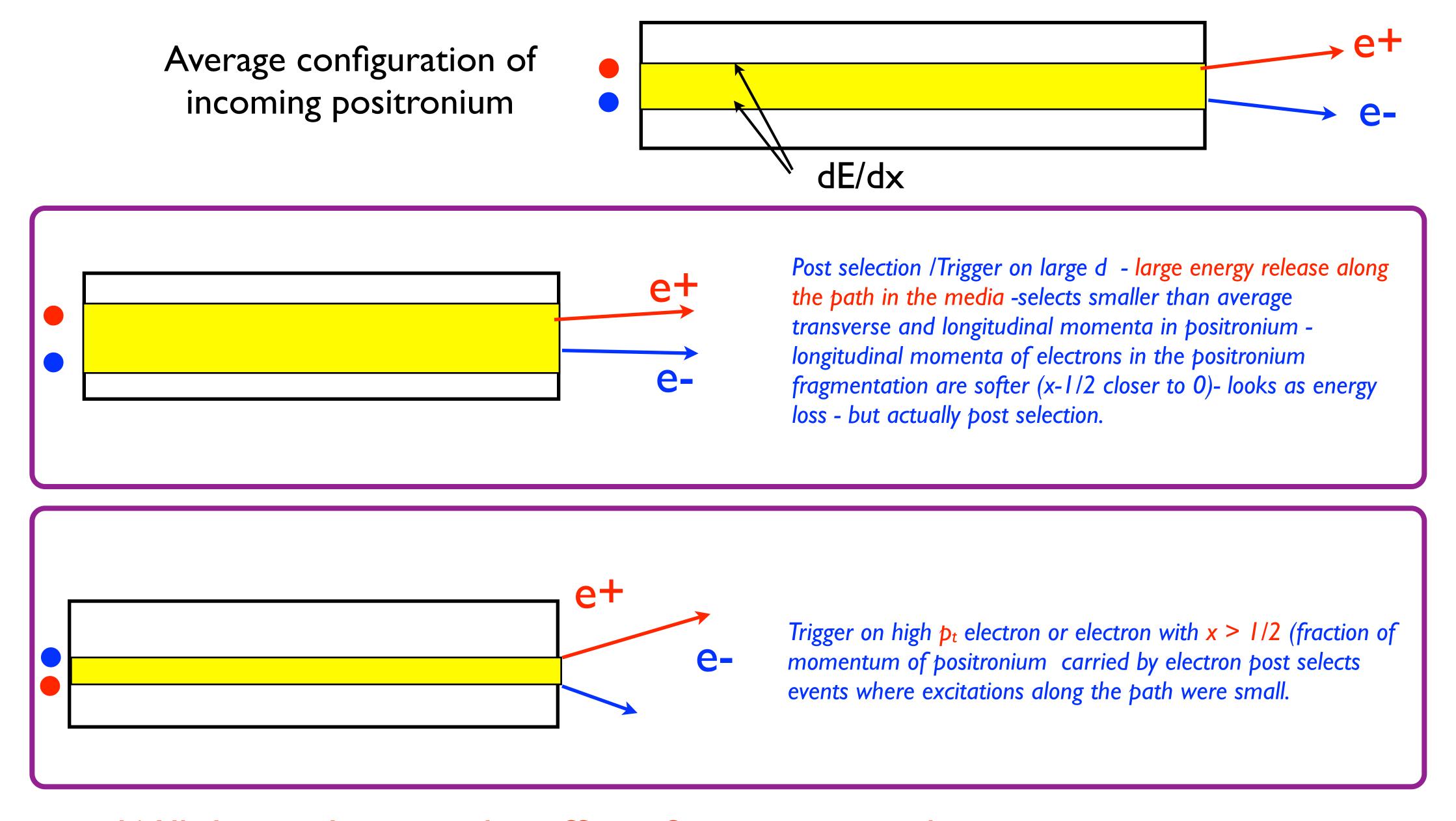


Consider production of one (two) lepton pairs with small momenta in the center of mass:

$$<$$
d $^2>$ for these events is larger than in $\Psi_{pos}^2(d)=\int \Psi_{pos}^2(r)dz$ $\langle d_{2l\bar{l}}^2\rangle > \langle d_{l\bar{l}}^2\rangle > \langle d^2\rangle$

Effects:

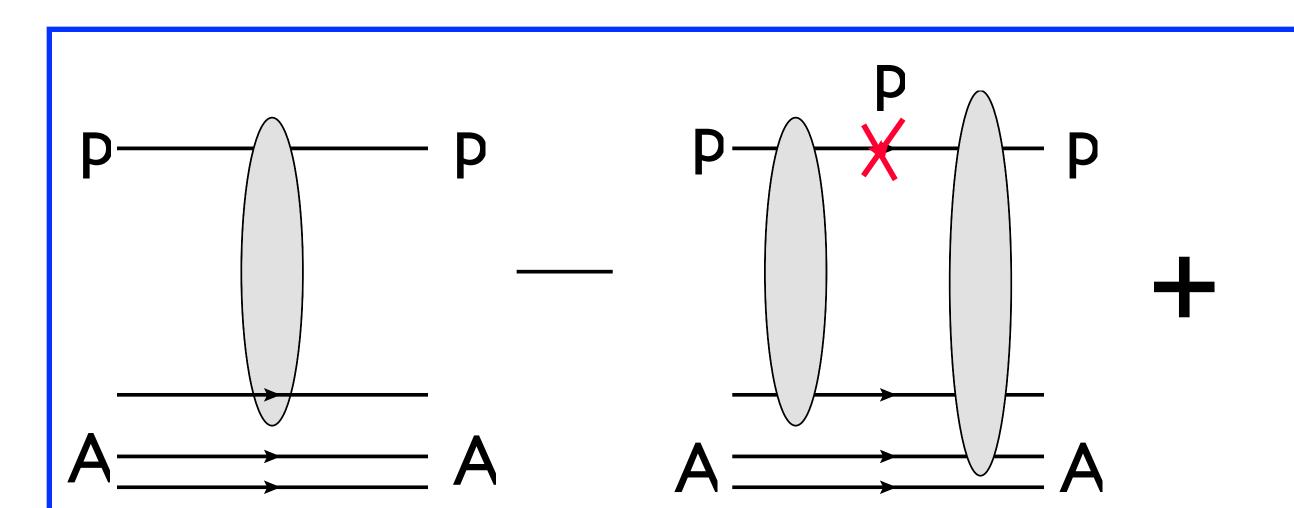
- Positive correlation between production of one and two pairs
- Correlation between energy release along the positronium path and final momenta of e- e+ (next slide)



Will discuss later similar effects for proton - nucleus interactions

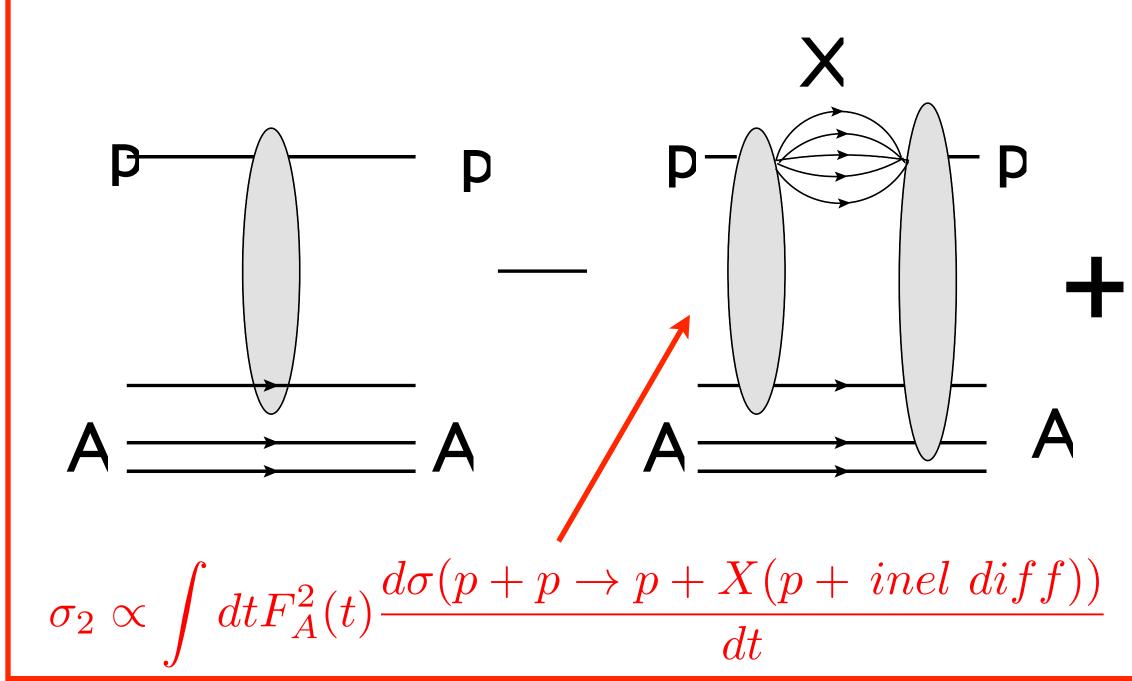
- The non exponential behavior is a manifestation of high energy coherence slow down of space-time evolution
- Various triggers allow to change proportion of small and large configurations in the data sample
- Inelastic processes are sensitive to presence of large & small size configurations in projectile longer the target (nucleus) —higher the sensitivity.

Formal account of large $l_{coh} \rightarrow p$ A scattering is described by different set of diagrams:



Glauber model

in rescattering diagrams proton propagates in intermediate state - zero at high energy - cancelation of planar diagrams (Mandelstam & Gribov)- no time for a proton to come back between interactions.



High energies = Gribov -Glauber model

X= set of frozen intermediate states the same as in pN diffraction

deviations from Glauber are small for E_{inc} < 10 GeV as inelastic diffraction is still small.

Convenient quantity - $P(\sigma)$ -probability that hadron/photon interacts with cross section σ with the

target.
$$\int P(\sigma) d\sigma = I$$
, $\int \sigma P(\sigma) d\sigma = \sigma_{tot}$,

$$\frac{\frac{d\sigma(pp\to X+p)}{dt}}{\frac{d\sigma(pp\to p+p)}{dt}} = \frac{\int (\sigma-\sigma_{tot})^2 P(\sigma) d\sigma}{\sigma_{tot}^2} \equiv \omega_{\sigma} \qquad \begin{array}{c} \textit{variance} \\ \textit{Pumplin & Miettinen} \end{array}$$

$$\int (\sigma - \sigma_{tot})^3 P(\sigma) d \sigma = 0,$$

Baym et al from pD diffraction

$$P(\sigma)_{|\sigma \to 0} \propto \sigma^{n_q-2}$$
 Baym et al 1993 - analog of QCD counting rules probability for all constituents to be in a small transverse area

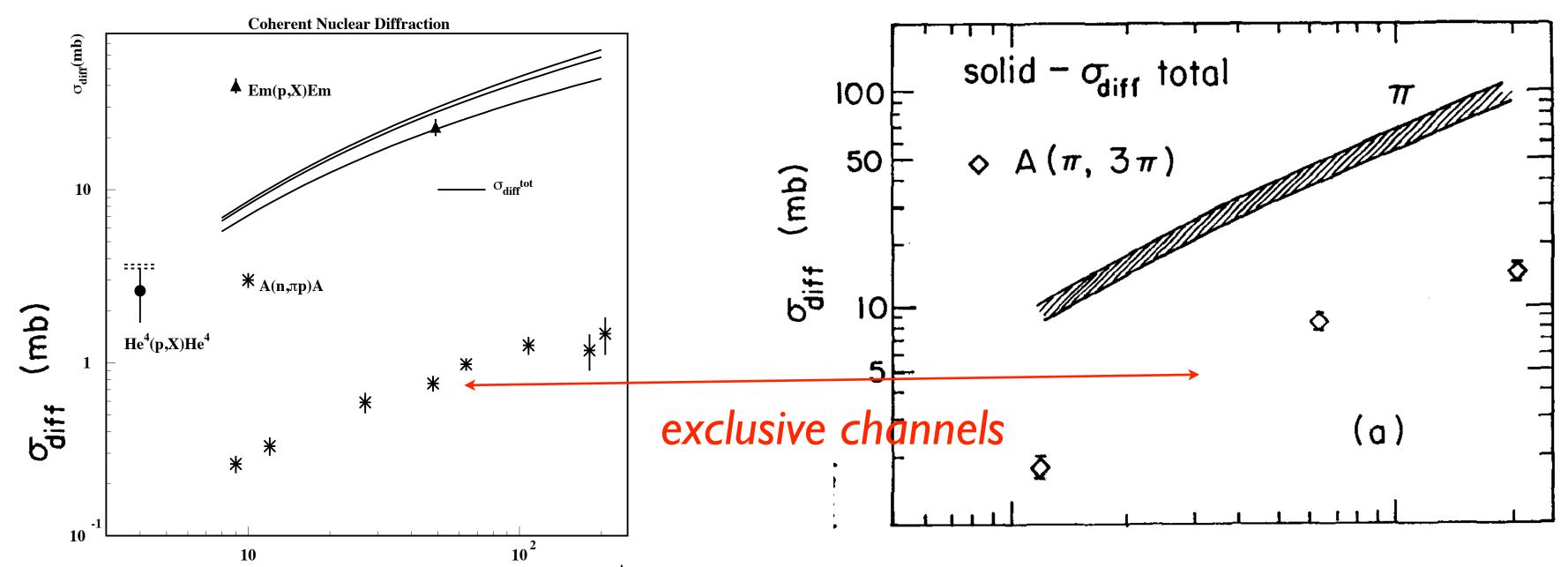
+ additional consideration that for a many body system fluctuations near average value should be Gaussian

$$P_{N}(\sigma_{tot}) = r \frac{\sigma_{tot}}{\sigma_{tot} + \sigma_{0}} exp\{-\frac{(\sigma_{tot}/\sigma_{0} - 1)^{2}}{\Omega^{2}}\}$$

$$P_{\gamma}(\sigma)_{|\sigma
ightarrow 0} \propto \sigma^{-1}$$
 Y =mix of small qq and mesonic configurations

Test: calculation of coherent diffraction off nuclei: $\pi A \rightarrow XA$, $p A \rightarrow XA$ through $P_h(\sigma)$

Test: Calculate inelastic diffraction off nuclei - no free parameters

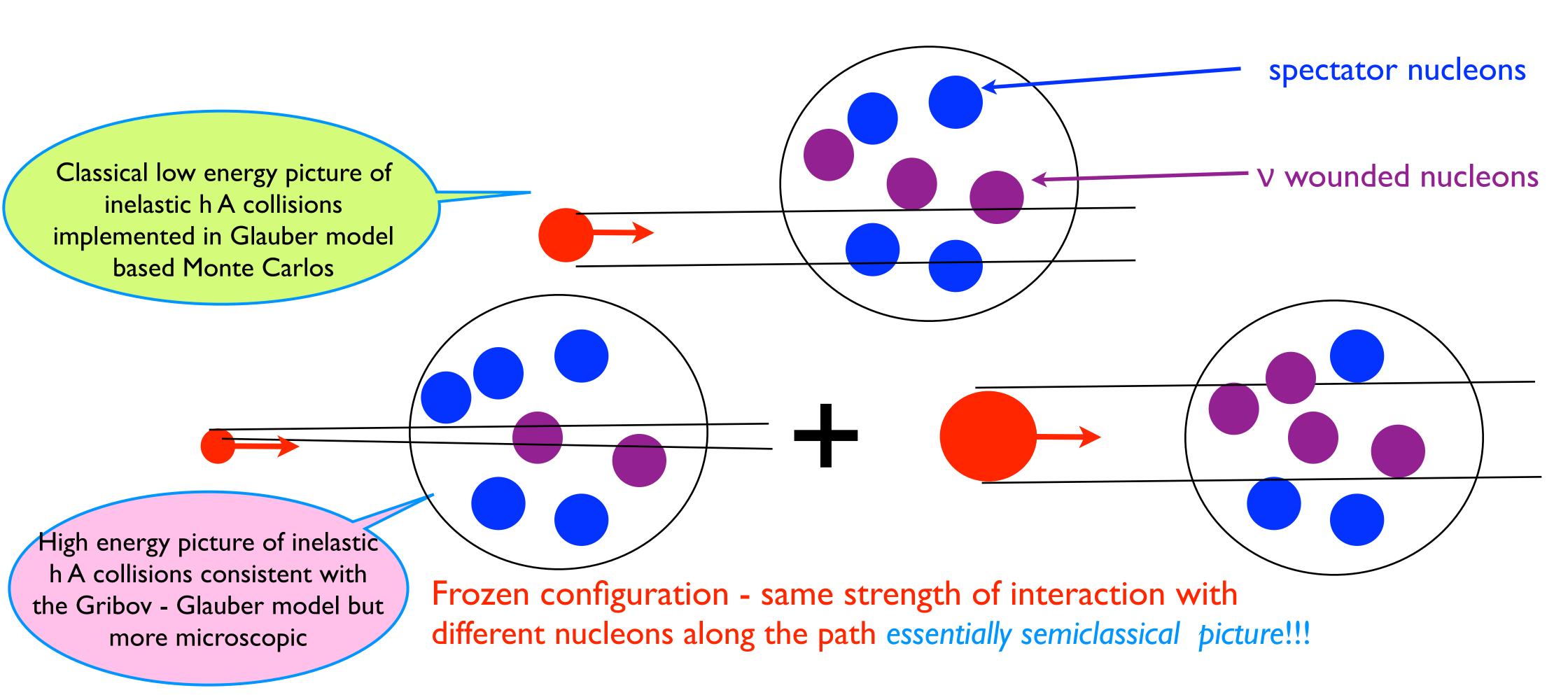


The inelastic small t coherent diffraction off nuclei provides one of the most stringent tests of the presence of the fluctuations of the strength of the interaction in NN interactions. The answer is expressed through $P(\sigma)$ -probability distribution for interaction with the strength σ . (Miller &FS 93)

$$\sigma_{diff}^{hA} = \int d^2b \left(\int d\sigma P_h(\sigma) |\langle h| F^2(\sigma,b) |h\rangle| - \left(\int d\sigma P(\sigma) |\langle h| F(\sigma,b) |h\rangle| \right)^2 \right).$$

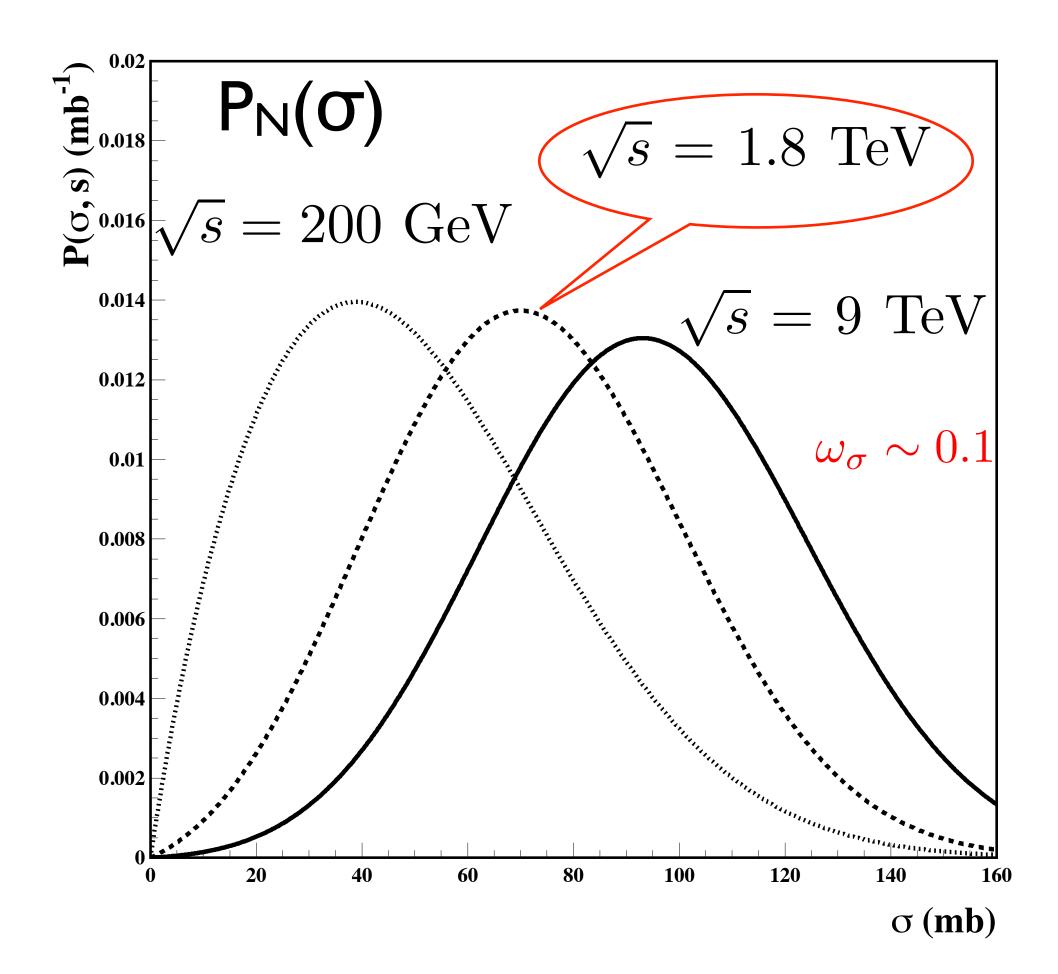
Here $F(\sigma,b)=1-e^{-\sigma T(b)/2}$, $T(b)=\int_{-\infty}^{\infty}\rho_A(b,z)dz$, and $\rho_A(b,z)$ is the nuclear density.

Constructive way to account for coherence of the high-energy dynamics is Fluctuations of interaction cross section formalism.



$$\sigma_{\nu} = \int d\sigma P_h(\sigma) \cdot \frac{A!}{(A-\nu)! \,\nu!} \cdot \int d\boldsymbol{b} \left(\sigma T(b)/A\right)^{\nu} \left[1 - \sigma T(b)/A\right]^{A-\nu}$$

simplified expression (optical limit)

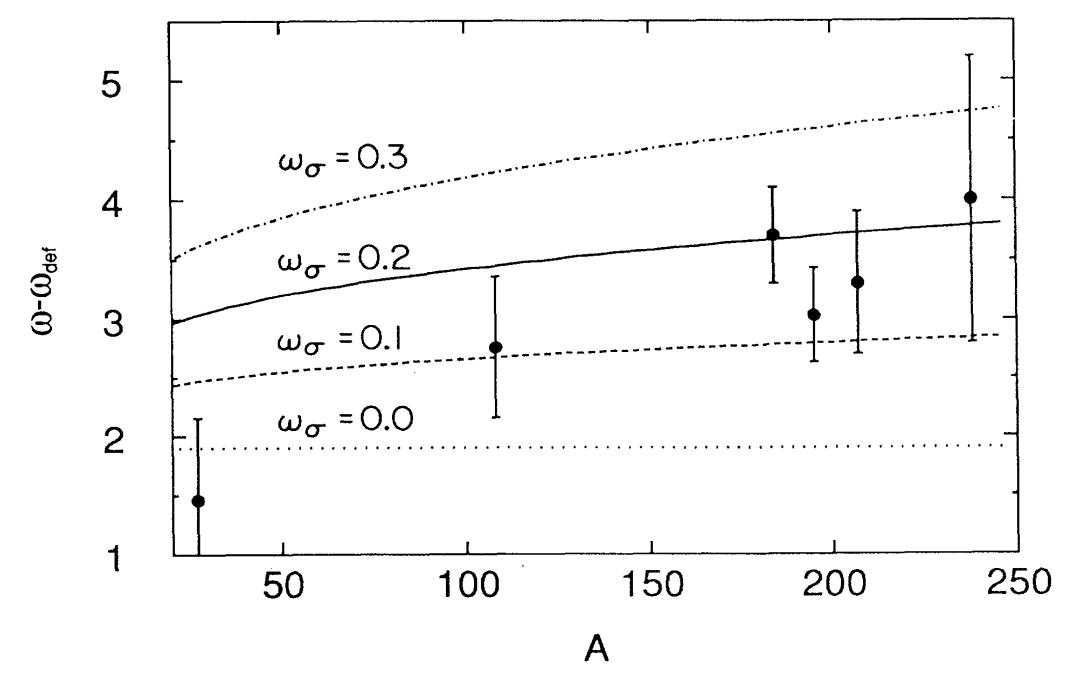


Extrapolation of Guzey & MS before the LHC data consistent with LHC data which are still not too accurate

Qualitative expectation: CF increase fluctuations of a number of observables in pA and AB collisions.

First example: study of dispersion of E_T distribution in AB collisions as superposition of emission from binary collisions with variance ω_0 :

$$\frac{\omega - \omega_{def}}{\text{nucl. deform.}} = \omega_0 + 2 - \alpha - \beta + (N_{pB} + N_{pA} - \alpha - \beta)\omega_{\sigma}$$



H. Heiselberg, G. Baym, B. Blattel, L. L. Frankfurt, "and M. Strikman PRL 1991

Dispersion of E_T distribution in central ^{32}SA collisions at SPS at E/A = 200 GeV

Large fluctuations in the number of wounded nucleons at fixed impact parameter

Simple illustration - two component model

≡ quasieikonal approximation:

$$\sigma_{1,2} = (1 \pm \sqrt{\omega_{\sigma}}) \cdot \sigma_{tot}$$

LHC
$$\sigma_1 = 70 \, mb, \, \sigma_2 = 130 \, mb$$

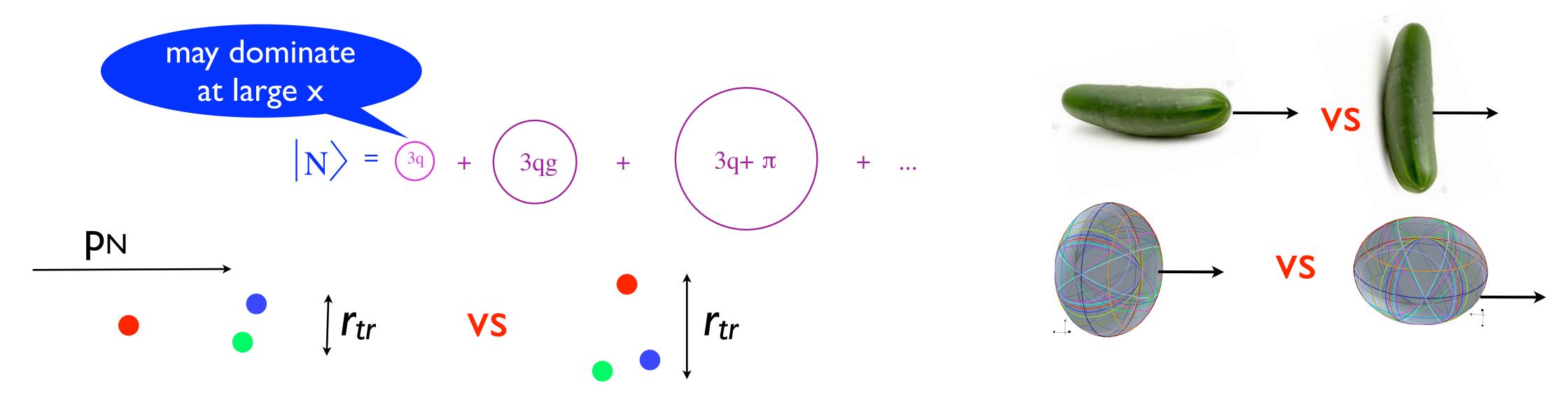
number of wounded nucleons at small b differs by a factor of 2 !!!

Scattering at b=4 fm with probability $\sim 1/2$ generates the same number of wounded nucleons as an average collision at b=0.

Fluctuations lead to broadening of the distribution over V - number of participant nucleons as compared to Glauber model - reported by ATLAS and ALICE.

Large V select configurations with larger than average O

There exist a number of dynamical mechanisms of the fluctuations of the strength of interaction of a fast nucleon/pion: fluctuations of the size, number of valence constituents, orientations



Localization of color certainly plays a role - so we refer to the fluctuations generically as color fluctuations.

Studing effects of CFs in pA aims at

- (i) Mapping 3-dimensional global quark-gluon structure of the nucleon
- (ii) Better understanding of the QCD dynamics of pA and AA collisions

Natural expectation is that there is a correlation between configuration of hard partons in the hadron and strength of interaction of the hadron:

 π (ρ)-meson decay constants: f_{π}, f_{ρ} are determined by configuration with essentially no gluon field and of small transverse size

Operational success of quark counting rules → minimal Fock space configurations dominate at large x. Quarks in these configurations have to be close enough - otherwise generation of Weizsäcker -Williams gluons

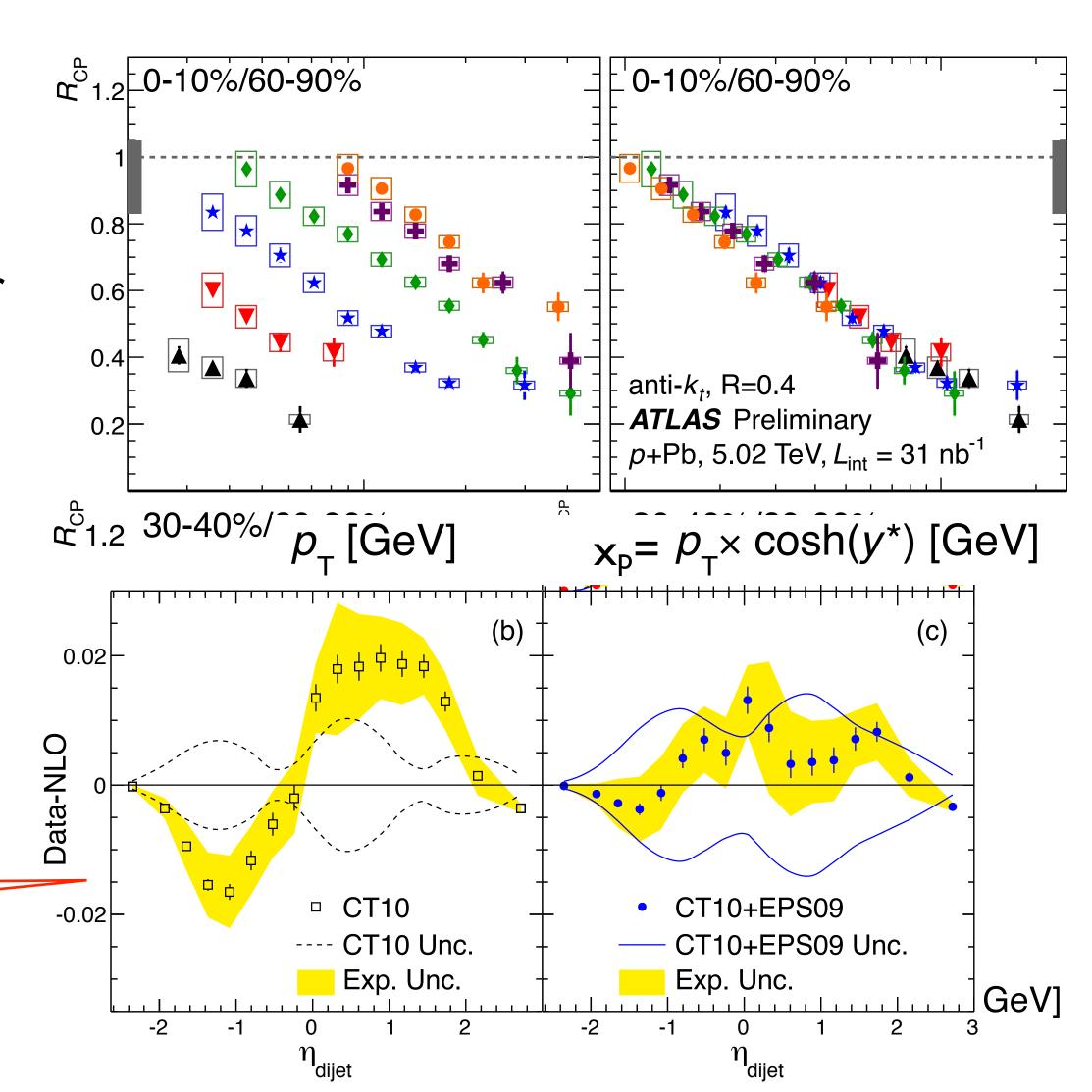


Use the hard trigger (dijet) to determine x of the parton in the proton (x_p) and low p_t hadron activity to measure overall strength of interaction σ_{eff} of configuration in the proton with given x FS83

Expectation: large x ($x \ge 0.5$) correspond to much smaller $\sigma \to drop$ of # of wounded nucleons & overall hadron multiplicity for central collisions

Key relevant observations:

The jet rates for different centrality classes do not match geometric expectations. Discrepancy scales with x of the parton of the proton and maximal for large x_p



pQCD works fine for inclusive production of jets

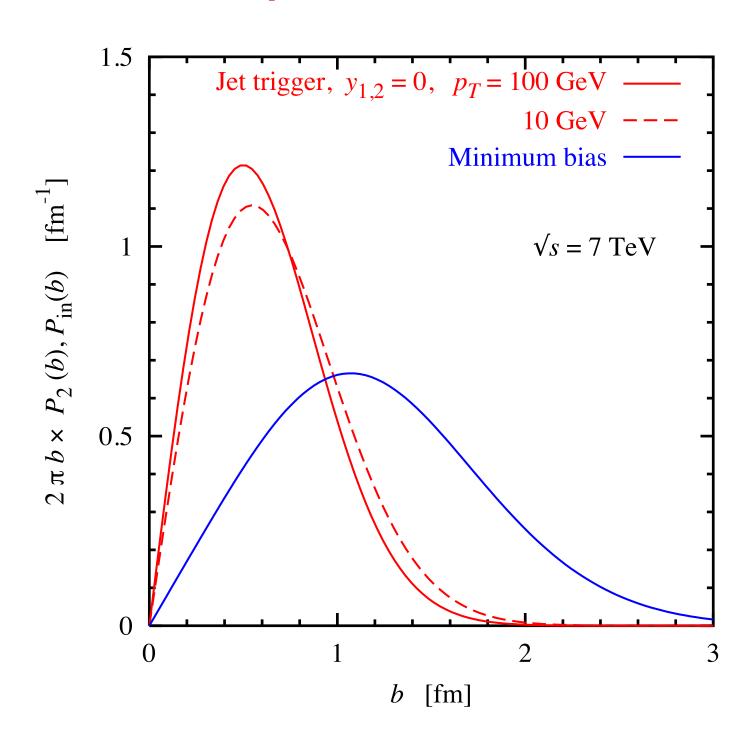
rules out energy loss explanation of the effect

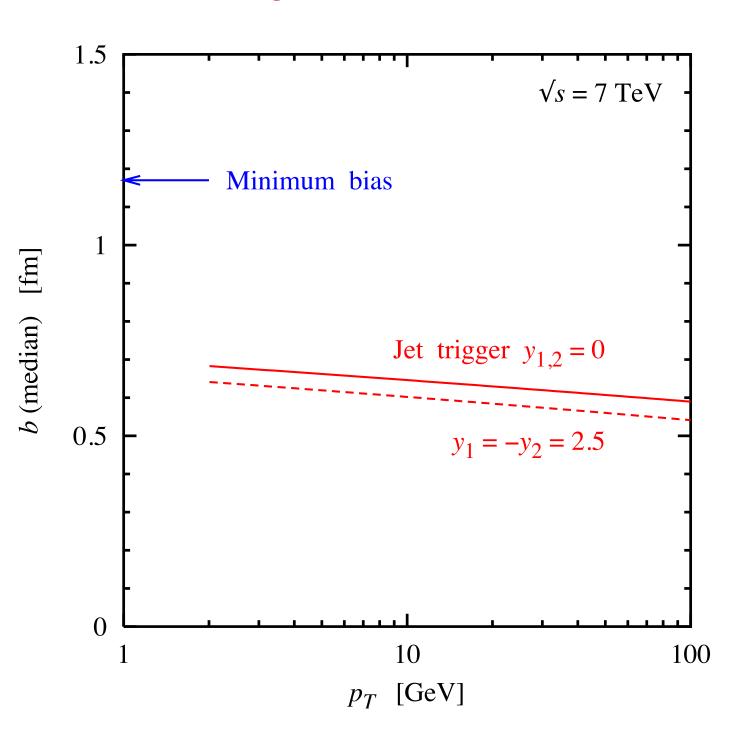
To calculate the expected CF effects accurately it is necessary to take into account grossly different geometry of minimum bias and hard NN collisions

LF, MS, Weiss 03 related hard dynamics in pp and DIS using generalized parton distribution extracted from analysis of exclusive hard processes

Two scale transverse dynamics of pp interactions at LHC -

Comparison of b -distributions for minimum bias and dijet collisions





Transverse area in which most of hard interactions occur in pp scattering is a factor of two smaller than that of minimum bias interactions

DISTRIBUTION OVER THE NUMBER OF COLLISIONS FOR PROCESSES WITH A HARD TRIGGER

M.Alvioli, L.Frankfurt, V.Guzey and M.Strikman, `Revealing nucleon and nucleus flickering in pA collisions at the LHC,' Phys.Rev. C90 (2014) 3, 034914 arXiv 1402.2868

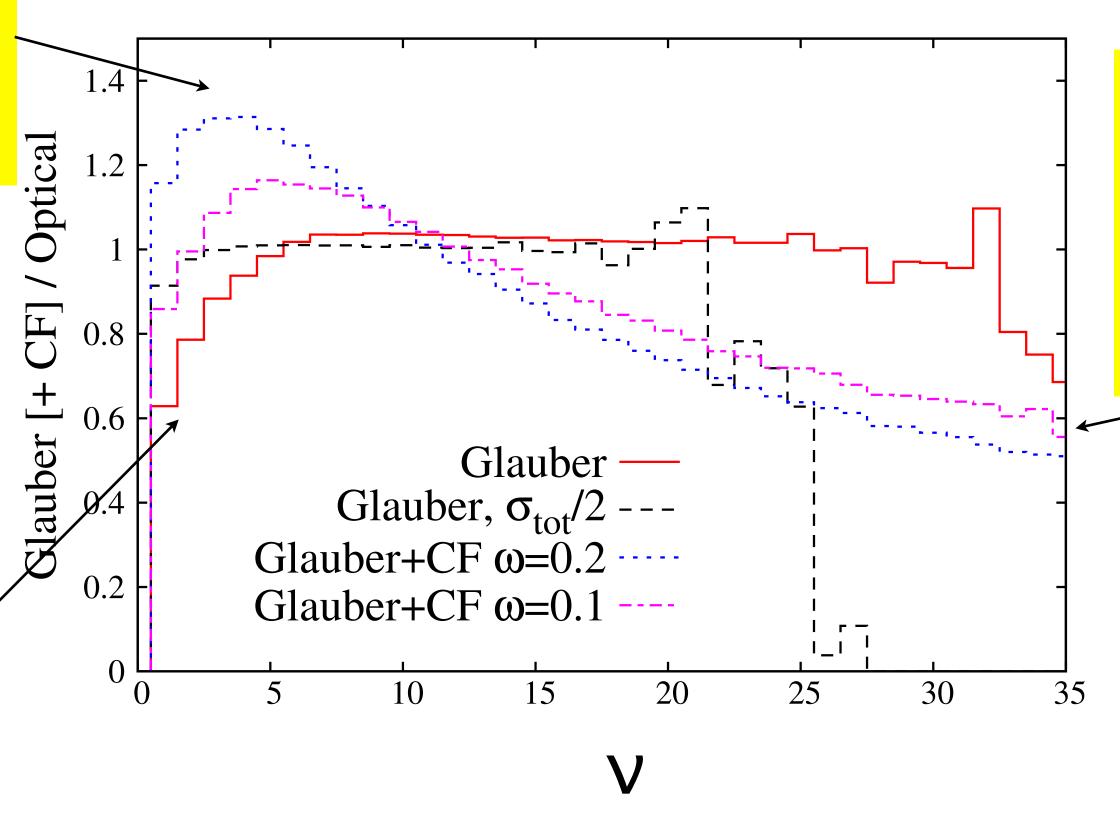
Consider multiplicity of hard events $Mult_{pA}(HT) = \sigma_{pA}(HT+X)/\sigma_{pA}(in)$ as a function of V -- number of collisions

If the radius of strong interaction is small and hard interactions have the same distribution over impact parameters as soft interactions multiplicity of hard events:

$$R_{HT}(\nu) = \frac{Mult_{pA}(HT)}{Mult_{NN}(HT)\nu} = 1$$

Accuracy? Significant corrections due to presence of two transverse scale.

increase due to more central interactions of configurations with $\sigma < \sigma_{tot}$



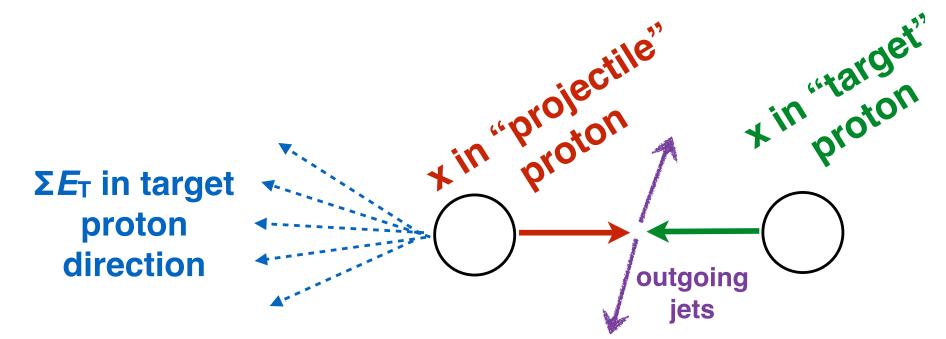
drop due increased role of configurations with $\sigma > \sigma_{tot}$ the cylinder in which interaction occur is larger but local density does not go up as fast in Glauber

drop due to more localized hard interactions

Deviation of $R_{HT}(V)$ from I

In order to compare with the data we need to use a model for the distribution in E_T^{Pb} as a function of V. We use the analysis of ATLAS. Note that E_T^{Pb} was measured at large negative rapidities which minimizes the effects of energy conservation (production of jets with large x_p) suggested as an explanation of centrality dependence

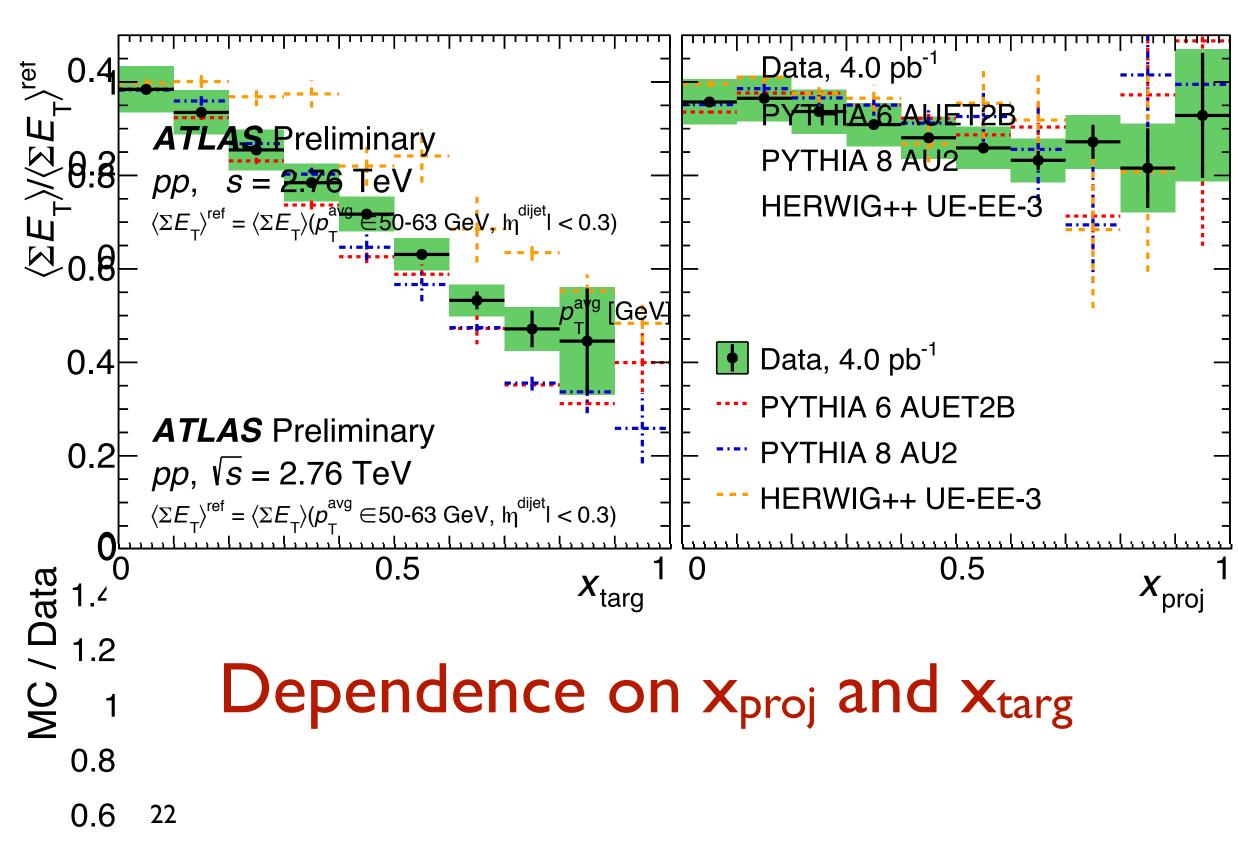
ATLAS-CONF-2015-019 analysis of pp do confirms this expectation of the confirms that the confirms the confirms that the confirmation of the confirmati



Measure ΣΕτ at large pseudorapidity vs.

x in the projectile proton (moving away)

x in the **target** proton (moving towards)



ΣETPb distribution: modeling by ATLAS

Transverse energy distributions in p+p collisions are typically well described by gamma distributions

gamma(x; k, \theta) =
$$\frac{1}{\Gamma(k)} \frac{1}{\theta} \left(\frac{x}{\theta}\right)^{k-1} e^{-x/\theta}$$

gamma distribution has convolution property:

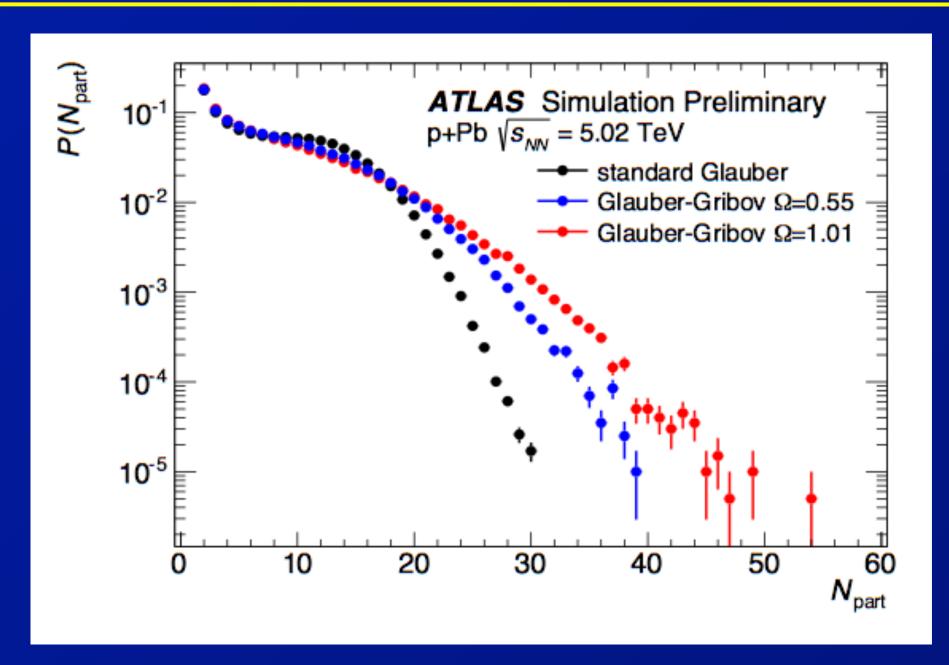
$$k(N_{\text{part}}) = k_0 + k_1 (N_{\text{part}} - 2),$$

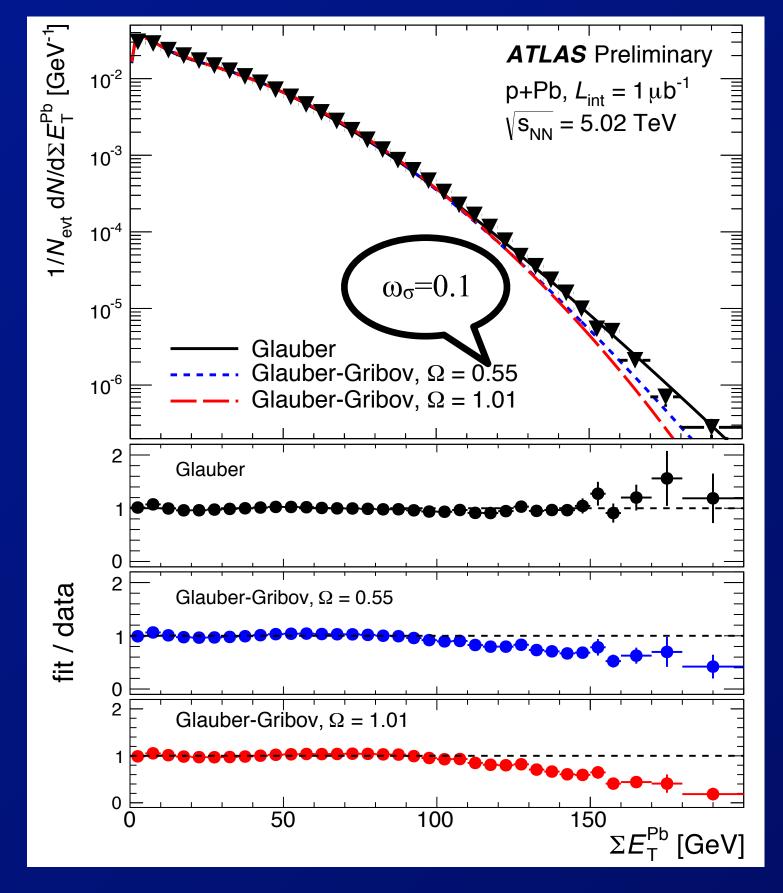
$$\theta(N_{\text{part}}) = \theta_0 + \theta_1 \log(N_{\text{part}} - 1).$$

N-fold conv. of gamma(x,k, θ) = $\frac{1}{\Gamma(Nk)} \frac{1}{\theta} \left(\frac{x}{\theta}\right)^{Nk-1} e^{-x/\theta}$

Note: for k = 1, gamma distribution is exponential, k < 1 is "super-exponential"

Glauber and Glauber-Gribov analysis





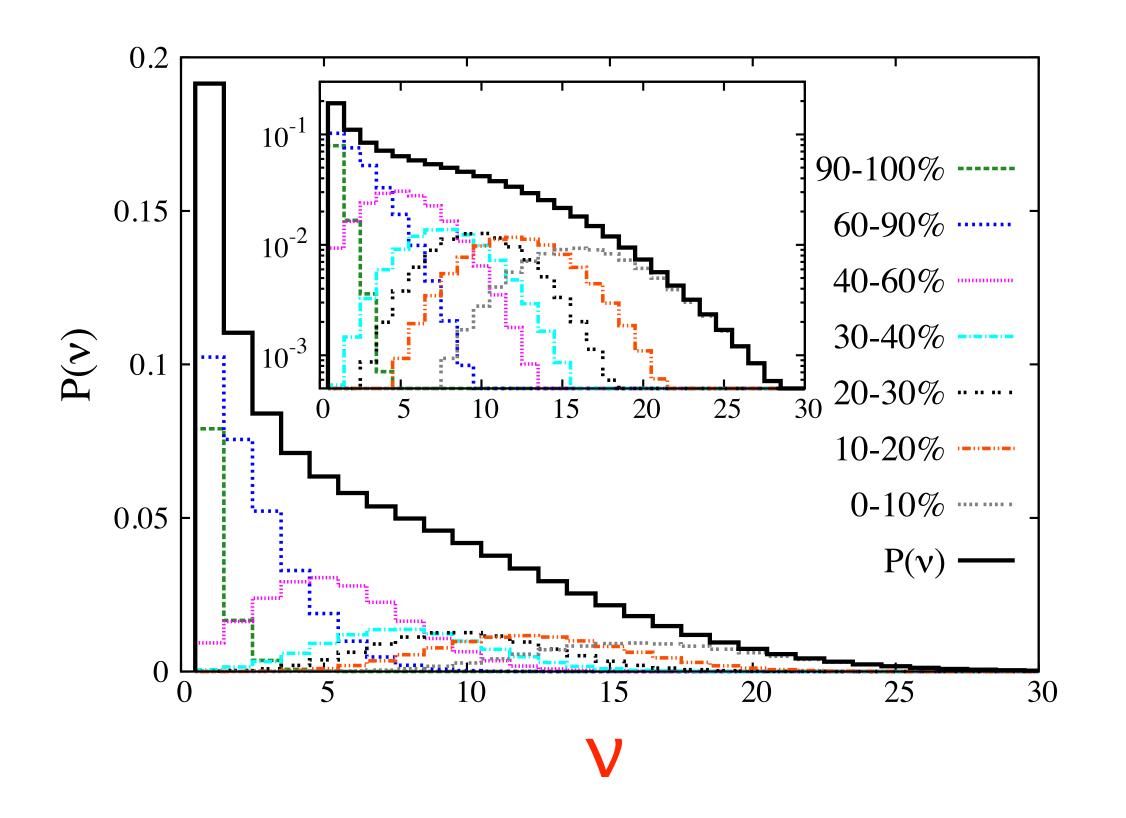
•With Glauber-Gribov

N_{part} distribution, the best fits become more WN-like

-e.g. for Ω = 0.55, k_1 = 0.9 (0.64 k_0), θ_1 = 0.07

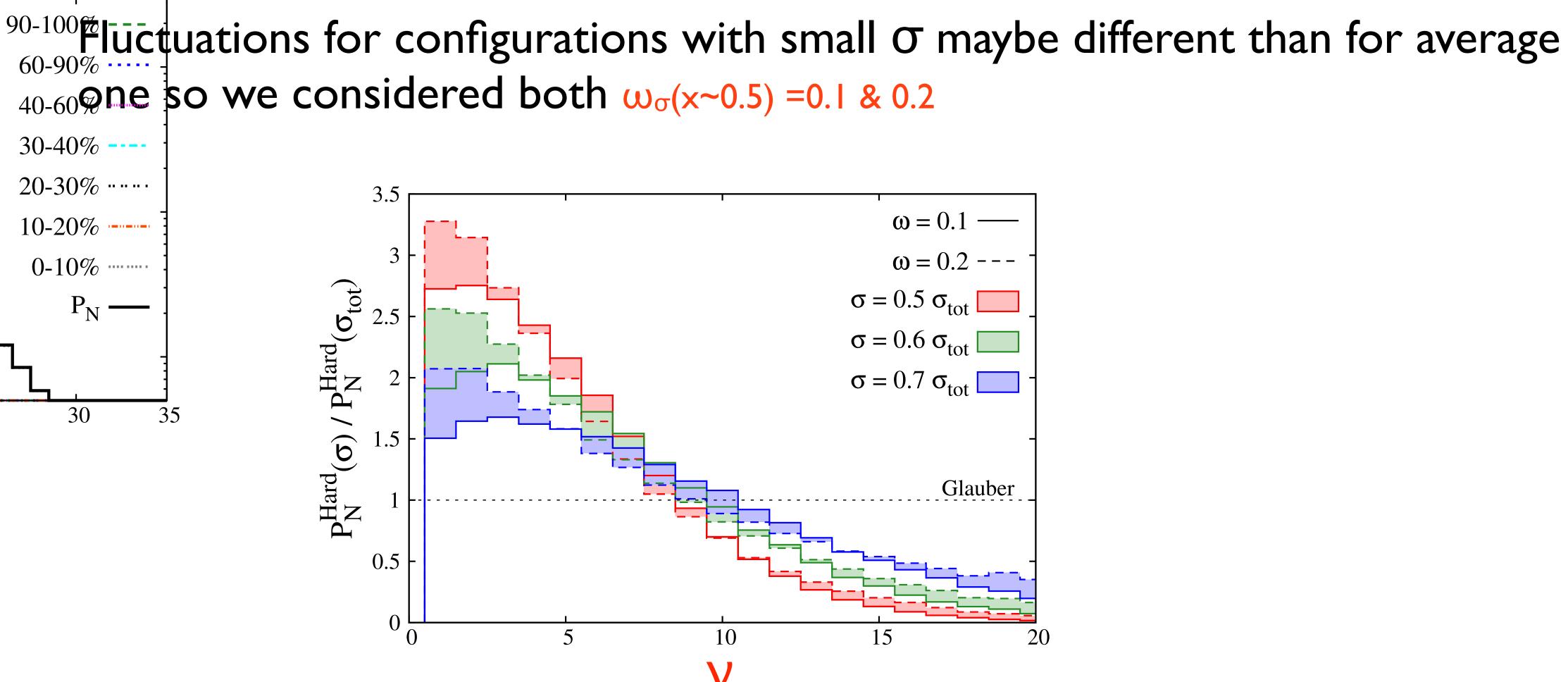
From B.Cole

⇒Glauber-Gribov smooths out the knee in the N_{part} distribution



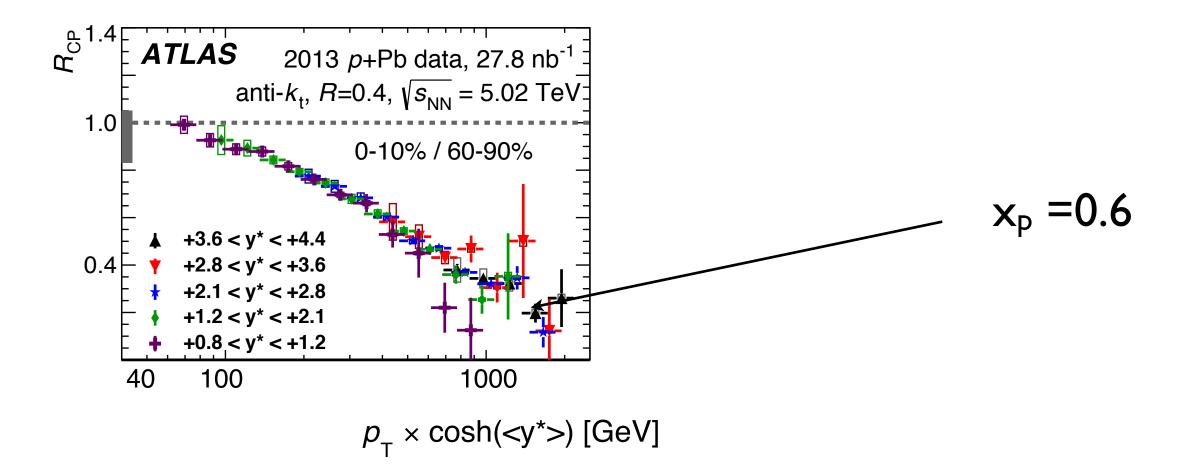
Alvioli, Cole, LF, Perepelitsa, MS, PRC2016

Probability distributions in V proton-nucleus collisions in all pA collisions and in those selected by different ΣE_T , or centrality, ranges. Note that ΣE_T , reasonably tracks V's

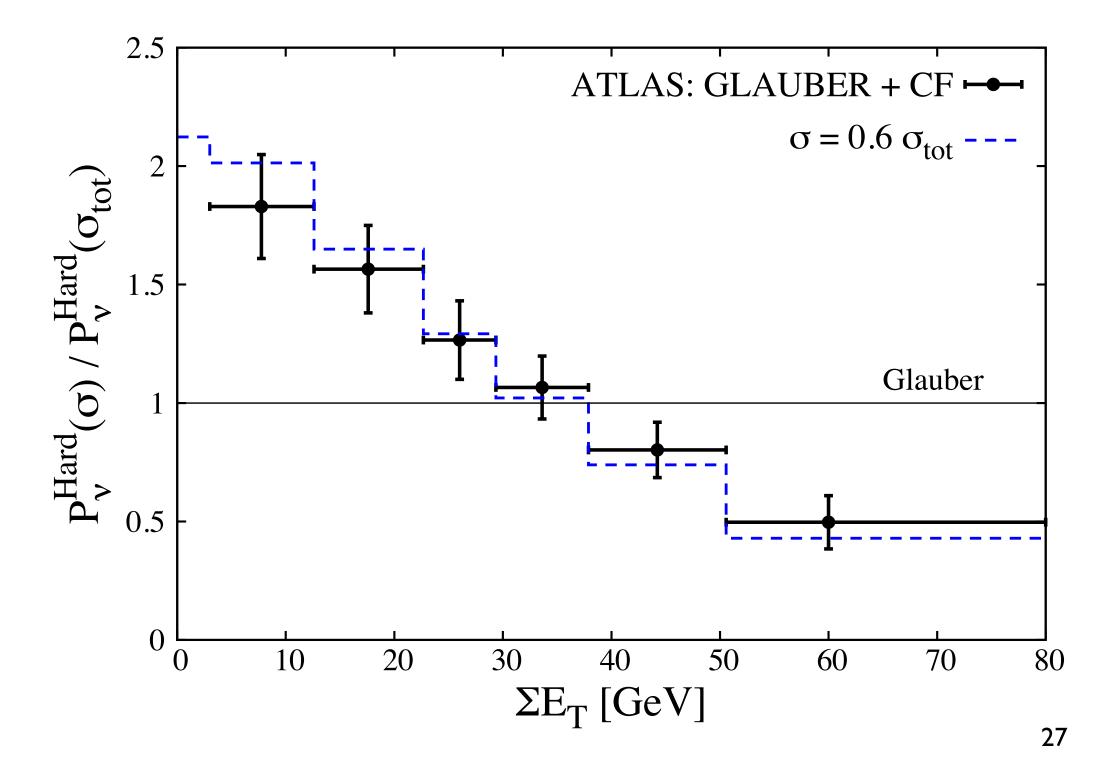


 $\omega = 0.1$

Sensitivity to ω_{σ} is small, so we use ω_{σ} =0.1 for following comparisons

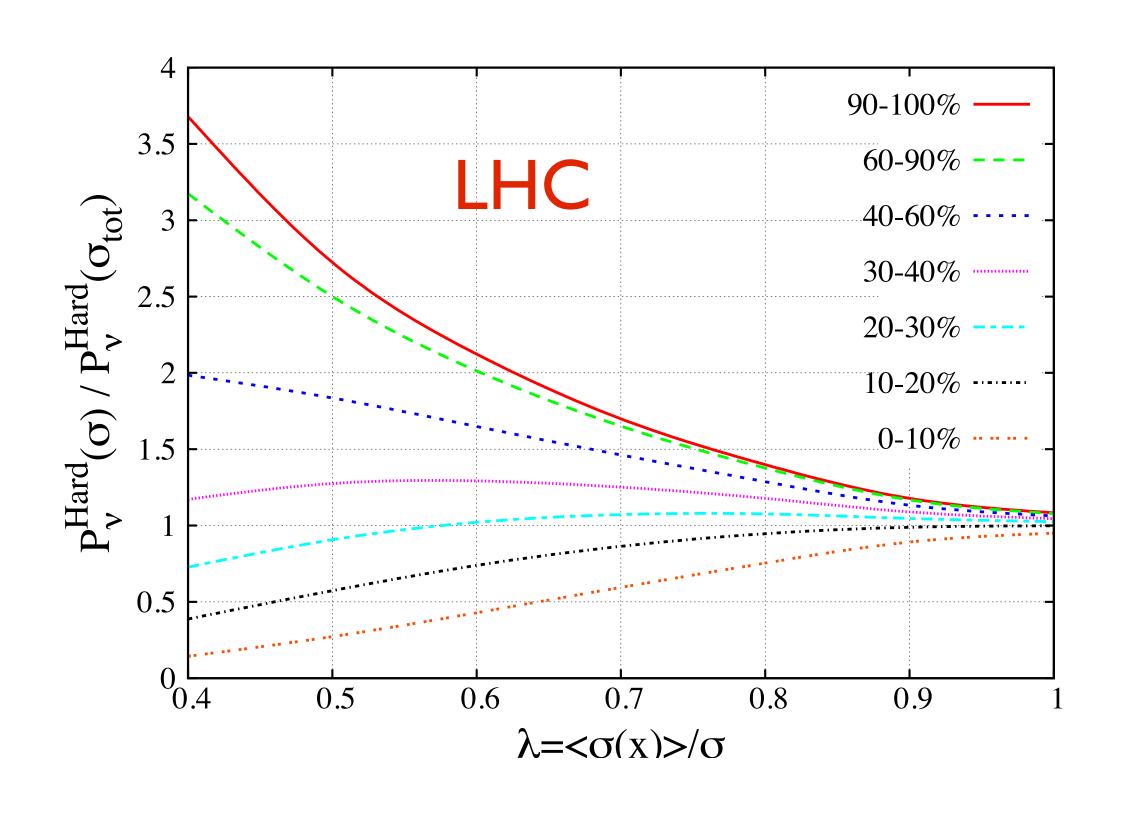


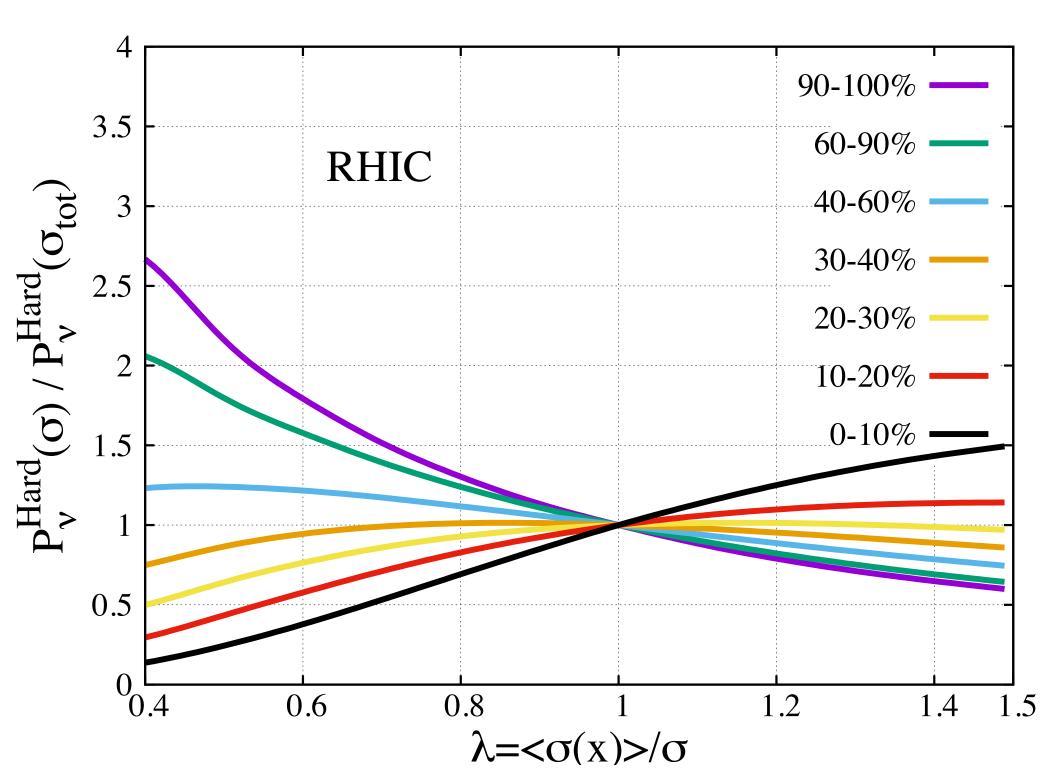
We focus on large x_p where effect is largest and hence corrections for transverse geometry are small (though we do include them)



Rhard for $x = E_{jet}/E_p = 0.6$ for centrality bins extracted from the ATLAS data using V's of the CF model. Errors are combined statistical and systematic errors. The solid line is the Glauber model expectation.

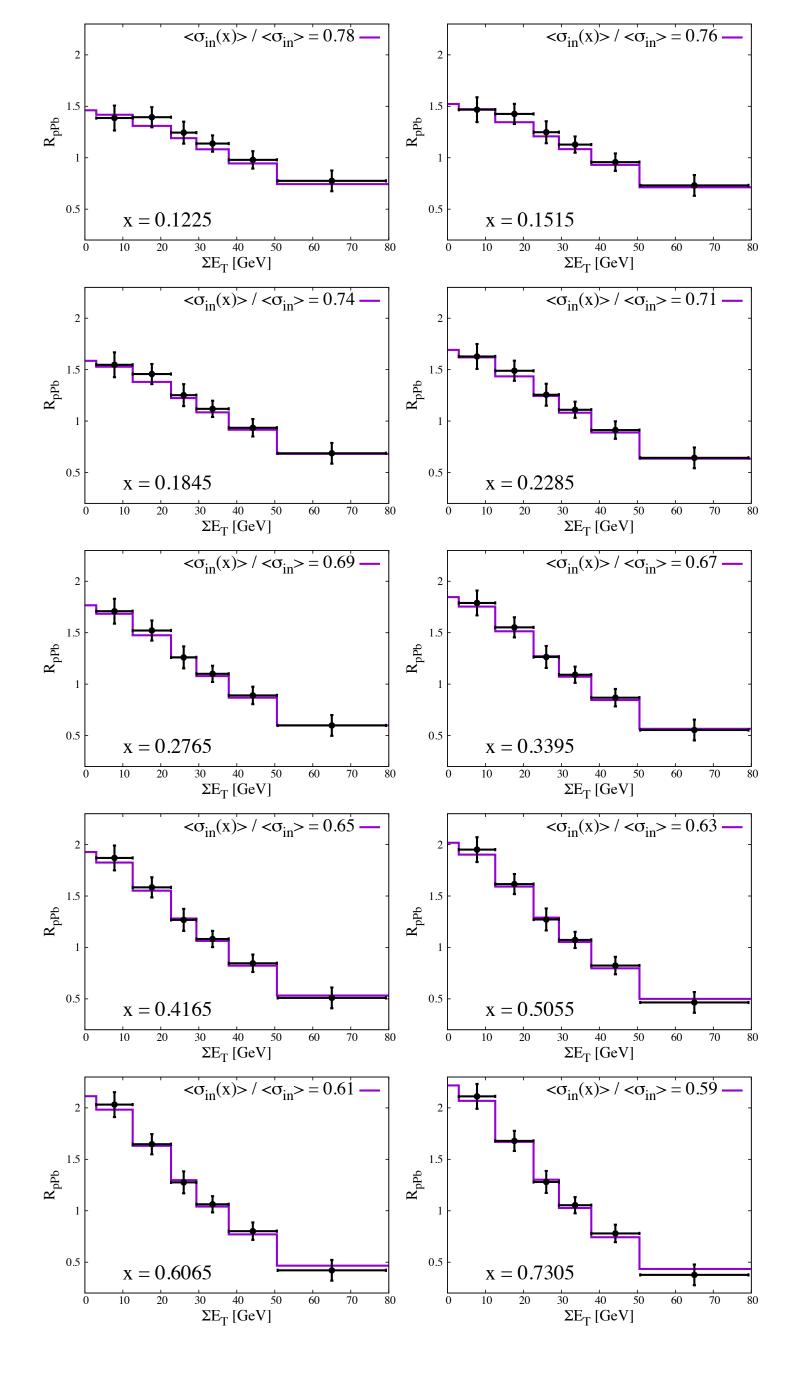
R_{hard} for different centralities is calculated as a function of one x-dependent parameter $\lambda = \sigma(x)/<\sigma>$



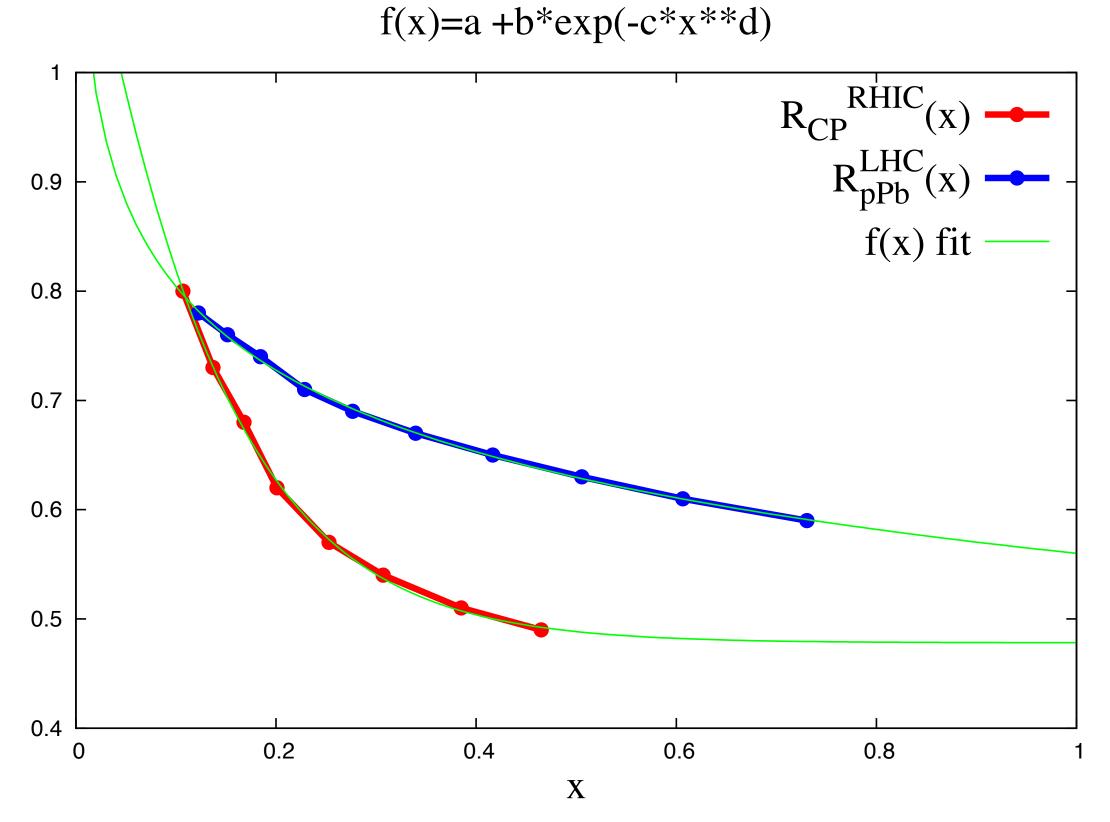


Further analysis of LHC and RHIC data (Alvioli, Perepelitsa, MS)

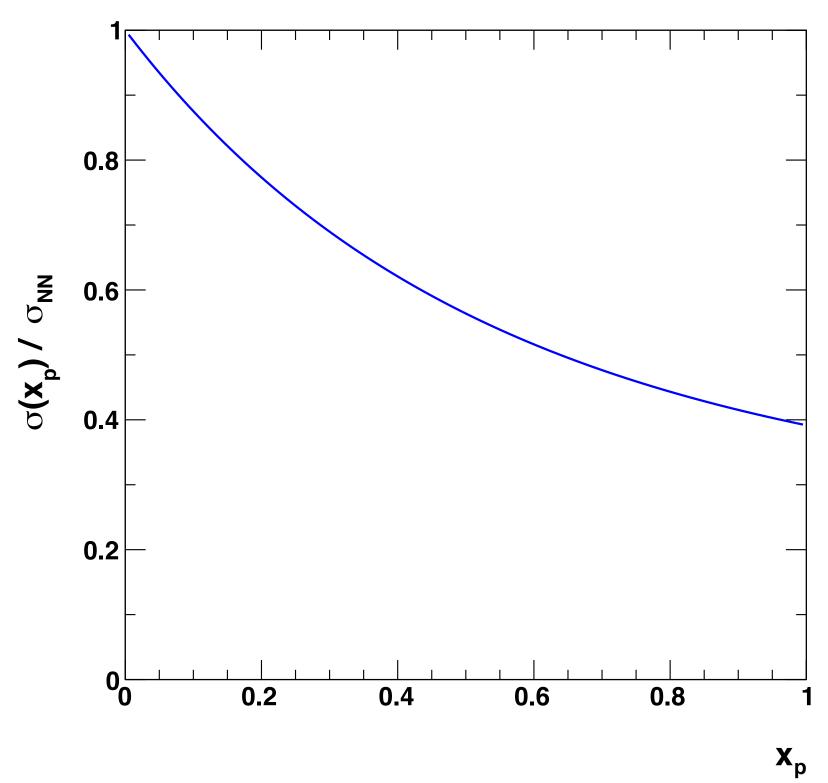
LHC pA



Used fit to correct for spread in x in the x-bins



Somewhat smaller $\lambda(x)$ for lower energies - natural as rate of the increase of cross section with energy is stronger for smaller fluctuations



dA analysis by McGlinchey, Nagle & D.V. Perepelitsa

Relatively small differences in value of $\lambda(x)$ probably due to a simplified treatment of transverse geometry

We can estimate $\sigma(x=0.6)/\sigma_{tot}[fixed target]=1/4$

from probability conservation relation:
$$\int_0^{\sigma(s_1)} P(\sigma, s_1) d\sigma = \int_0^{\sigma(s_2)} P(\sigma, s_2) d\sigma$$

 $x \ge 0.5$ configurations have small transverse size ($\sim r_N/2$)



Small size configurations suppressed in bound nucleons (F83) explanation of the EMC effect

At RHIC suppression is ~ 1.4 stronger than LHC (previous slide our analysis)

Estimate for

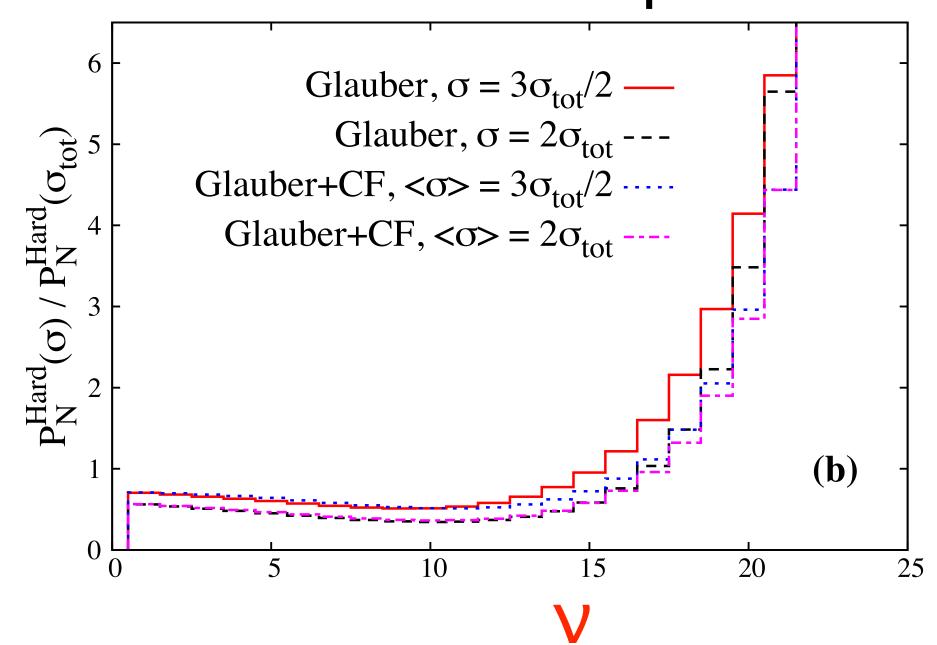
$$\sigma(x=0.2)/<\sigma>=0.7$$

gluon contribution sets in (smaller size than quarks for same x?)

$$\sigma(x=0.2)/<\sigma>=0.7$$

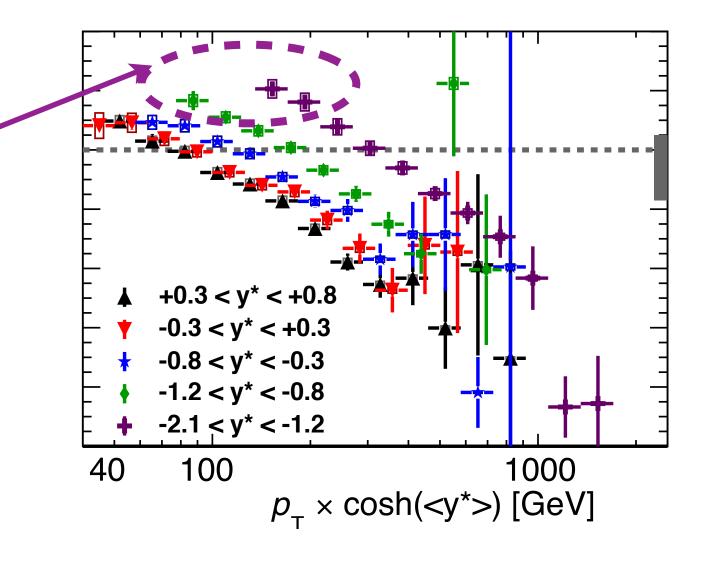
 $\sigma(x=0.1)/<\sigma>=0.8$

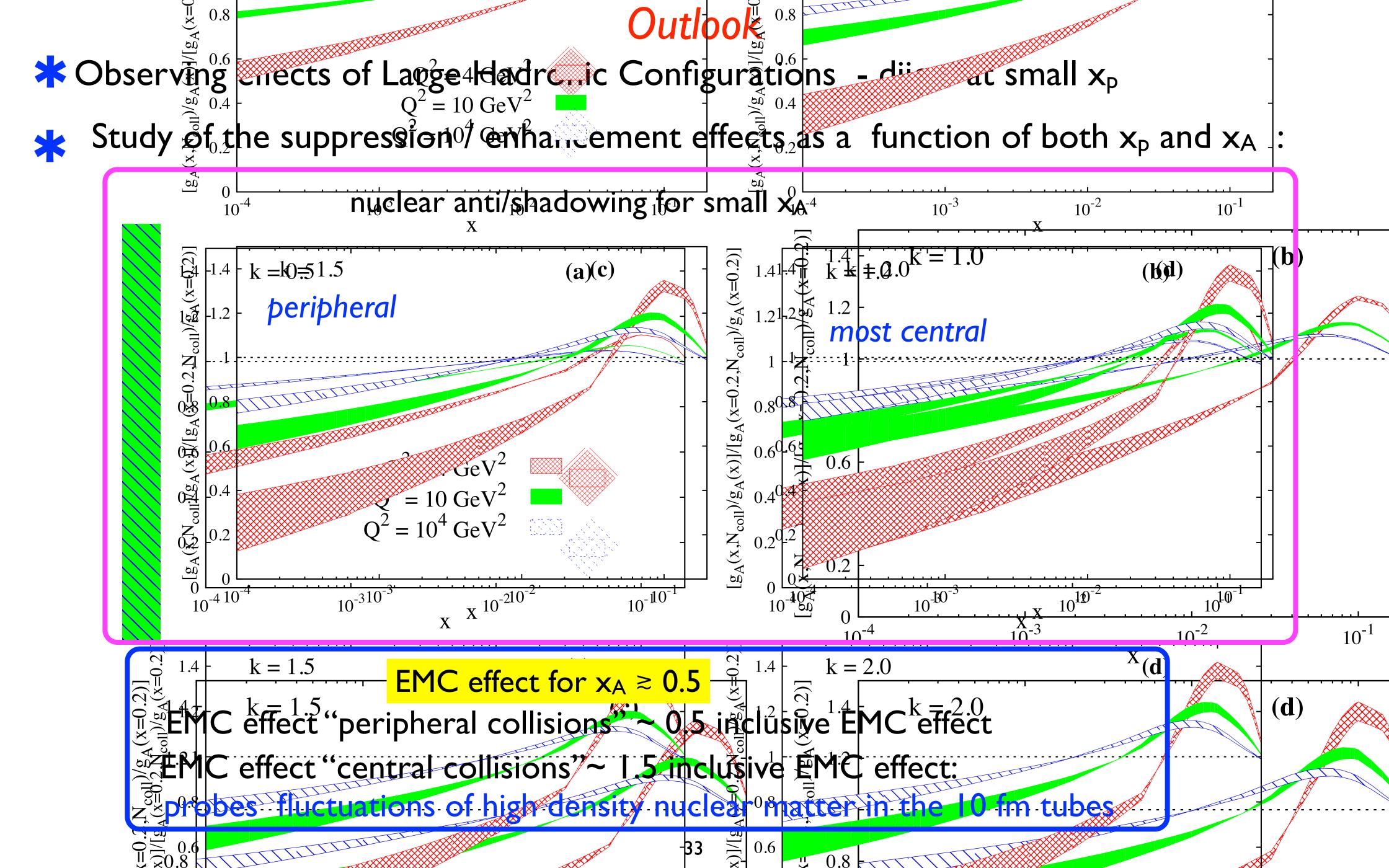
For $\sigma > \langle \sigma \rangle$ dependence on centrality is reversed



Ratio of the probabilities P_N of having V wounded nucleons for scattering of the proton in configurations with different values of $\sigma(x)$ and P_N for $\sigma = \sigma_{tot}$ with CF $(\omega_{\sigma}=0.1)$ and without CF (marked as Glauber)

Transition to dominance of larger than average size - $x < 10^{-1}$?





Several effects (in addition to CF and nuclear pdf effects) which should be included in more detailed modeling of pA with jets:

- Fluctuations of small x gluon strength in nucleons: variance $\omega_g(x=10^{-3}) \sim 0.15$
- Strong dependence of the multiplicity on the impact parameter of the pp collision (Evidence from pp - supplementary slides)
- Influence of CF on impact parameters of the NN interactions in pA.
- Fluctuations of the gluon fields in nuclei Swiss cheese

Experiment:

- Report data in the bins of x_p and x_A
- Study violation of the x_p scaling as a function of jet p_t
- quarks vs gluons for fixed x_p ; u-quarks vs d-quarks (W's)
- LHC vs RHIC for same x_p

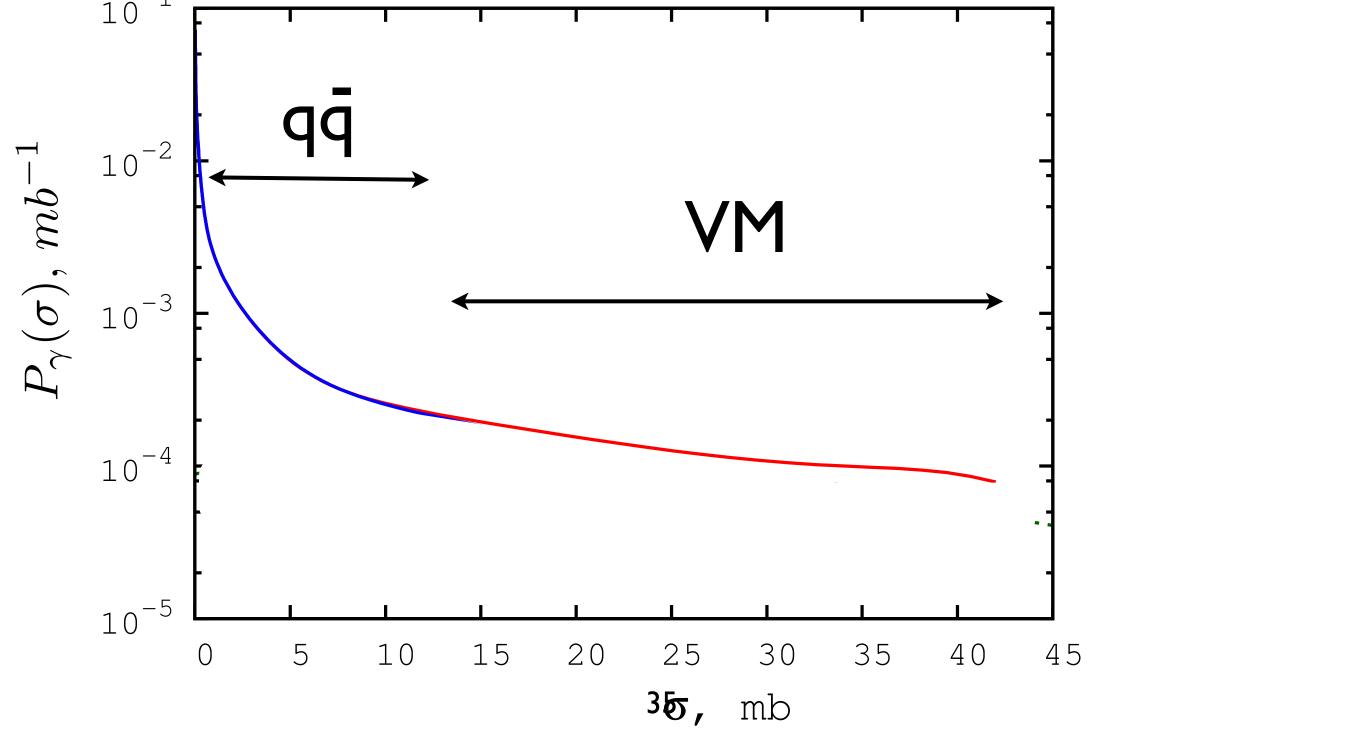
Color fluctuations in photon - nucleus collisions

Ultraperipheral collisions at the LHC - more in Takaki's talk

General statement: YA is more relevant for UH CRs than pA

Photon is a multiscale state:

$$P_{\gamma}(\sigma) \propto 1/\sigma \text{ for } \sigma \ll \sigma(\pi N)$$
 $P_{\gamma}(\sigma) \propto P_{\pi}(\sigma) \text{ for } \sigma > \sigma(\pi N)$



Exclusive processes of vector meson production off nuclei at LHC in ultraperipheral collisions allow to test theoretical expectations for small and large σ

(a) ρ -meson production: $\gamma+A \rightarrow \rho+A$

Expectations:

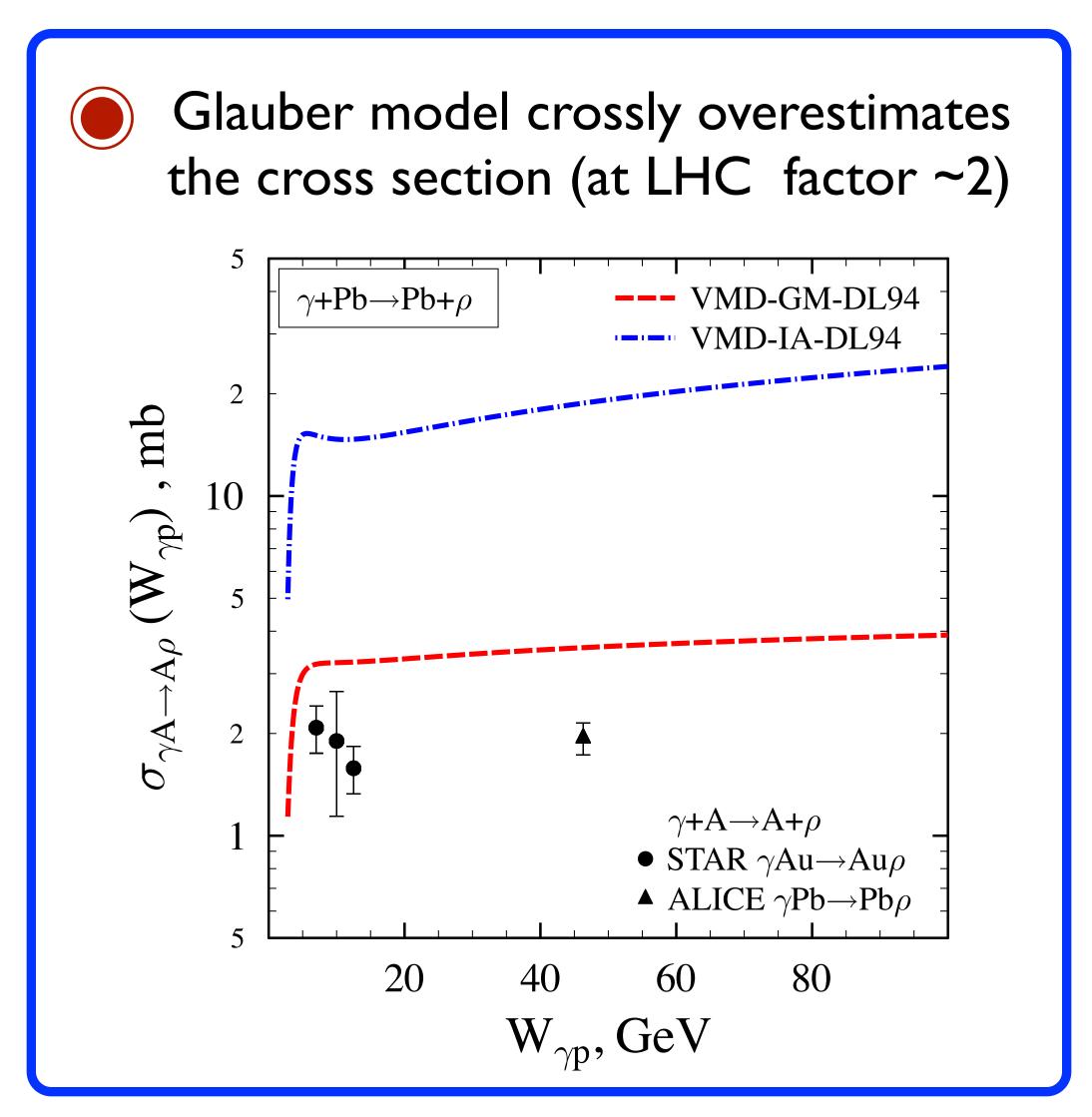
 \Leftrightarrow vector dominance model for scattering off proton $\sigma(\rho N) < \sigma(\pi N)$

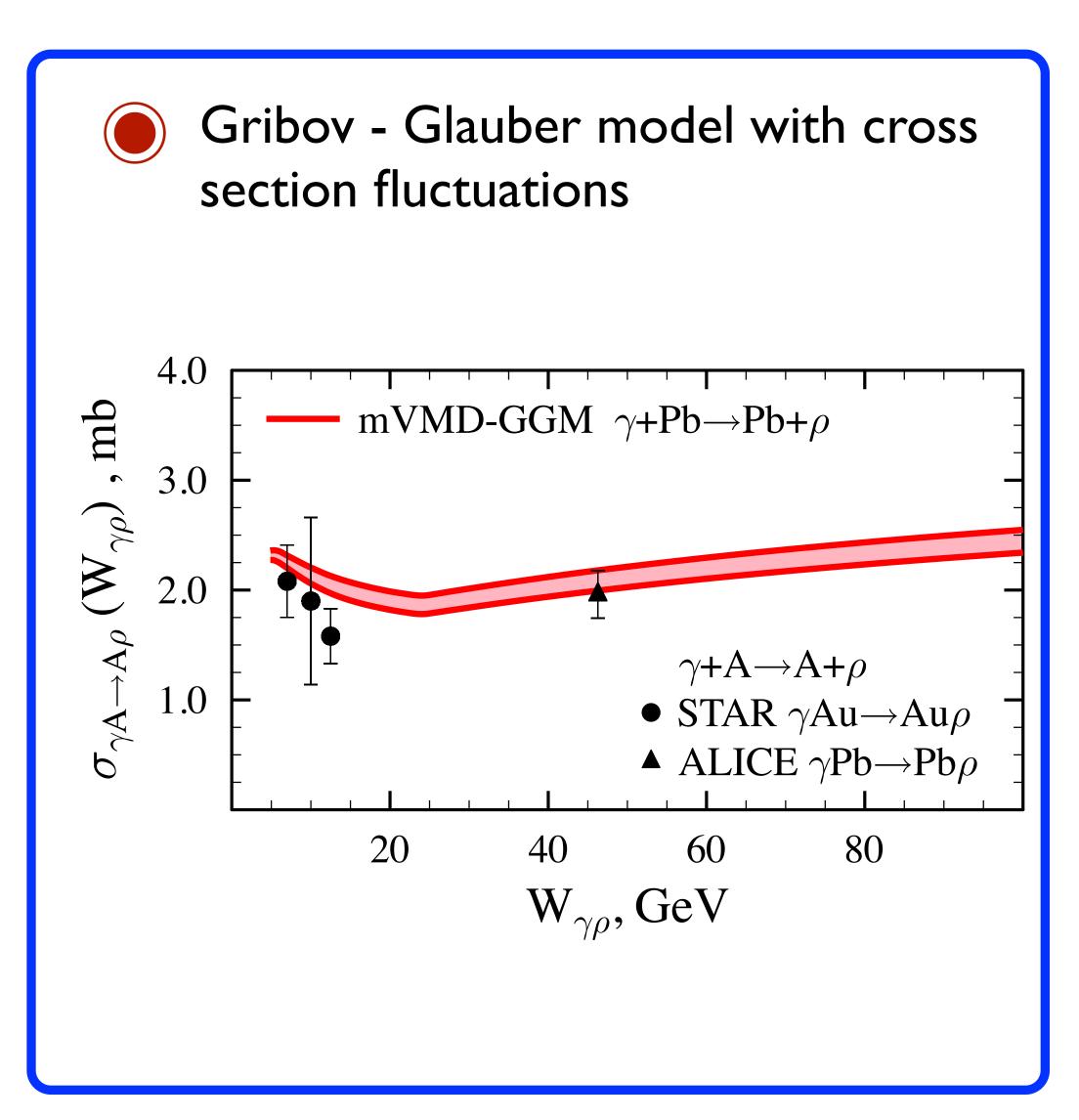
since overlapping integral between γ and ρ suppressed as compared to $\rho \rightarrow \rho$ case

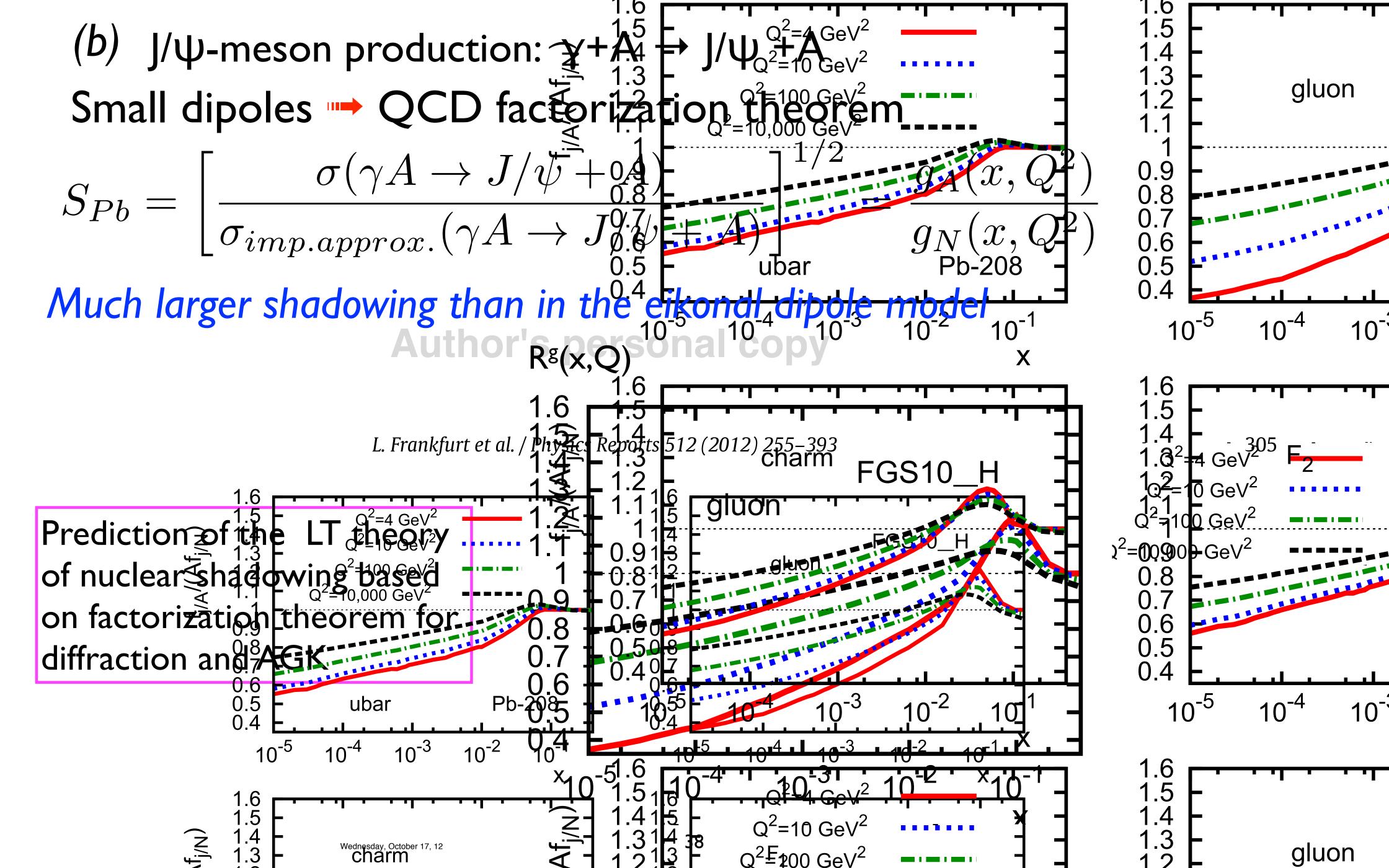
observed at HERA but ignored before our analysis: $\sigma(\rho N)/\sigma(\pi N) \approx 0.85$

Analysis of Guzey, Frankfurt, MS, Zhalov 2015 (1506.07150)

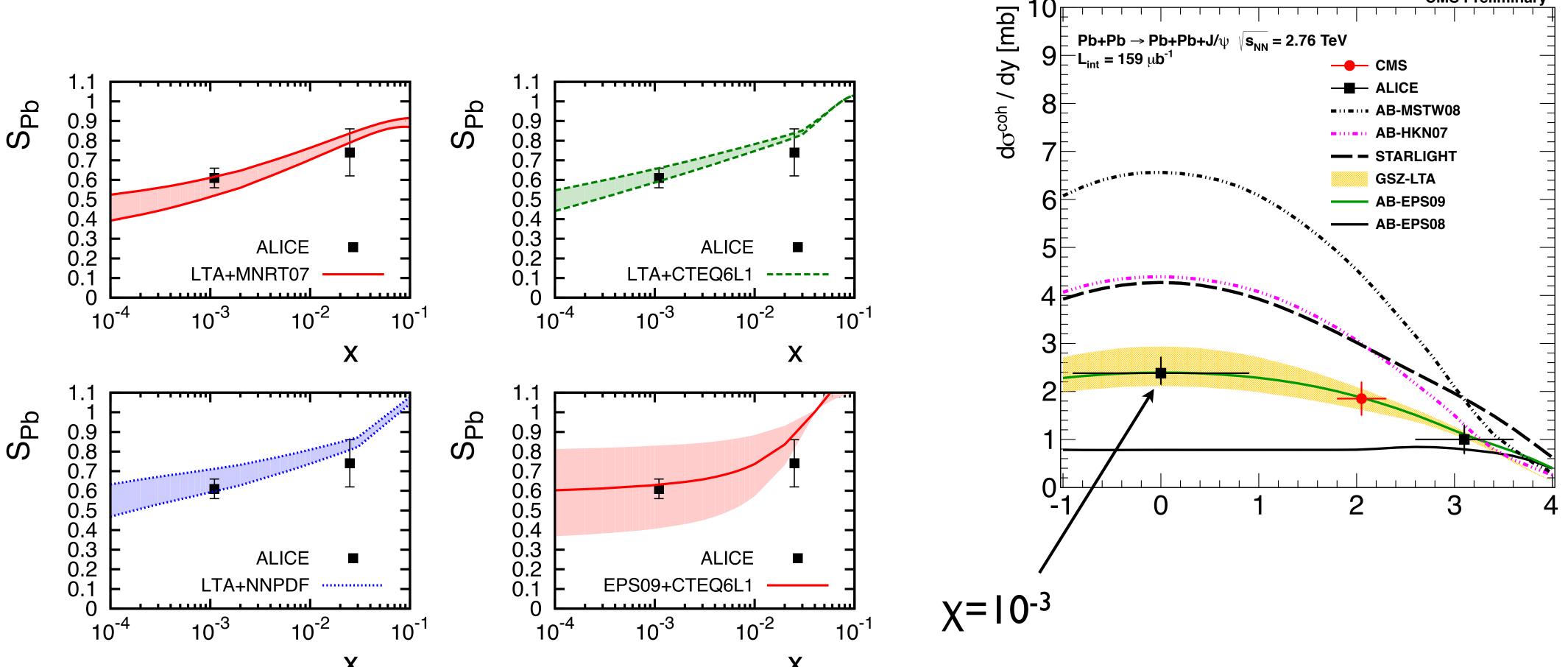
 \clubsuit Gribov inelastic shadowing is enhanced in discussed process - fluctuations grow with decrease of projectile - nucleon cross section. We estimate $\omega_{\gamma \to \rho} \sim 0.5$ and use it to model Also for the same dispersion inelastic shadowing is larger for smaller σ_{tot} . Also effect for coherent cross section is square of that for σ_{tot} .







Test: Strong suppression of coherent J/ ψ production observed by ALICE and CMS confirms our prediction of significant gluon shadowing on the Q² ~ 3 GeV²



Points - experimental values of S extracted by Guzey et al (arXiv:1305.1724) from the ALICE data; Curves - analysis with determination of Q -scale by Guzey and Zhalov arXiv:1307.6689; JHEP 1402 (2014) 046.

Outline of calculation of inelastic YA scattering - distribution over V

• Modeling $P_{\gamma}(\sigma)$

For
$$\sigma > \sigma(\pi N), P_{\gamma}(\sigma) = P_{\gamma \to \rho}(\sigma) + P_{\gamma \to \omega}(\sigma) + P_{\gamma \to \phi}(\sigma)$$

For $\sigma \leq 10mb$ (cross section for a J/ ψ -dipole) use pQCD with

$$\sigma(d, x) = \frac{\pi^2}{3} \alpha_s(Q_{eff}^2) d^2 x G_N(x, Q_{eff}^2)$$

+ smooth interpolation in between



Calculation of distribution over the number of wounded nucleons

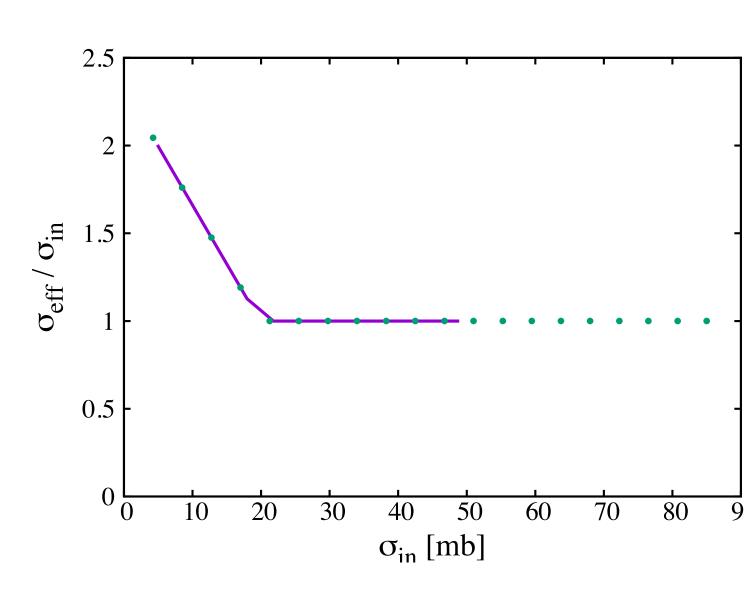
(a) Color fluctuation model

$$\sigma_{\nu} = \int d\sigma P_{\gamma}(\sigma) \begin{pmatrix} A \\ \nu \end{pmatrix} \times \int d\vec{b} \left[\frac{\sigma_{in}(\sigma)T(b)}{A} \right]^{\nu} \left[1 - \frac{\sigma_{in}(\sigma)T(b)}{A} \right]^{A-1}$$
$$p(\nu) = \frac{\sigma_{\nu}}{\sum_{i=1}^{\infty} \sigma_{\nu}}.$$

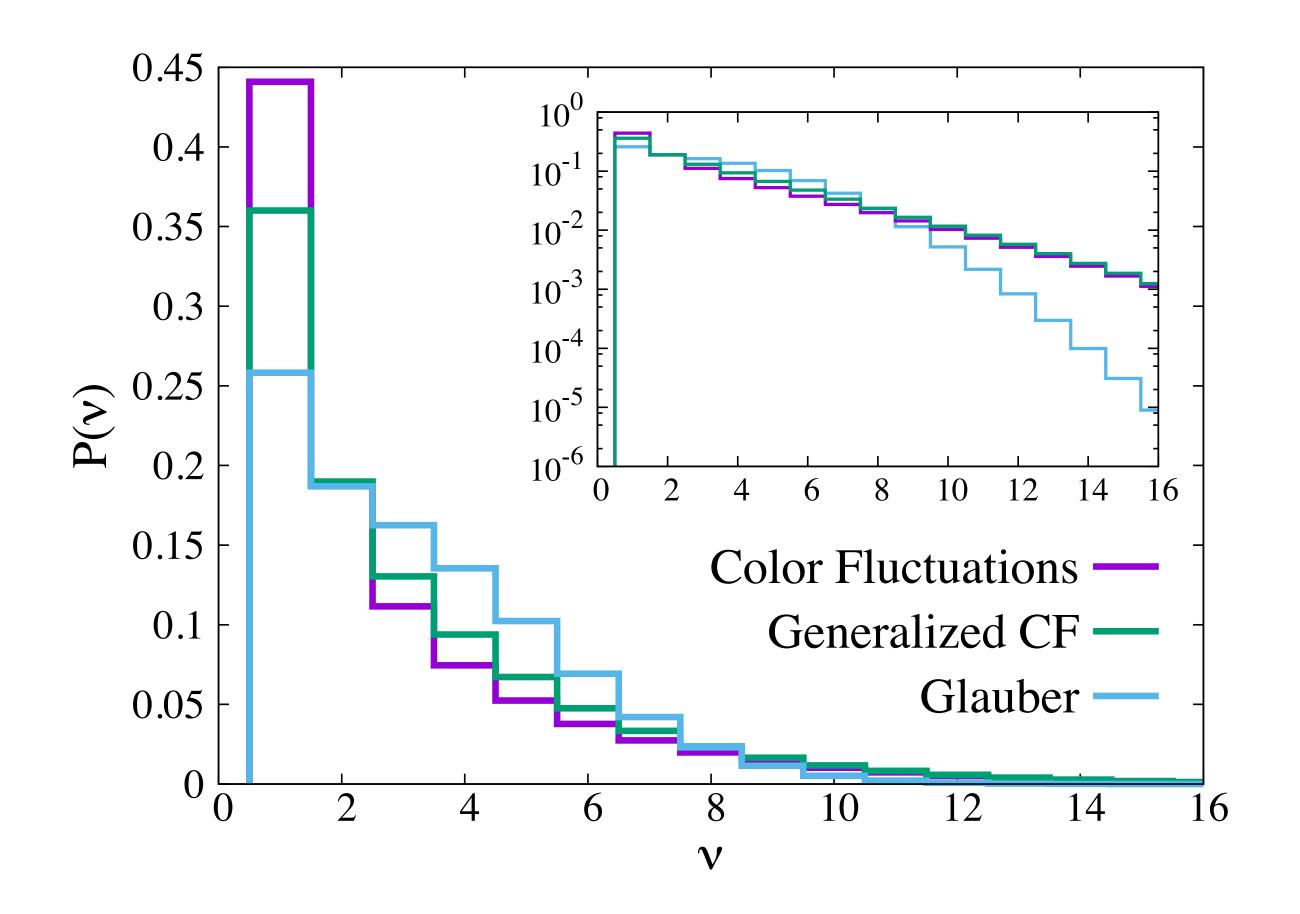
(b) Generalized Color fluctuation model (includes LT shadowing for small σ)

$$P_{\gamma}(\sigma) \begin{pmatrix} A \\ \nu \end{pmatrix} \times \frac{\sigma^{in}}{\sigma_{eff}^{in}} \int d\vec{b} \left[\frac{\sigma_{eff}T(b)}{A} \right]^{\nu} \left[1 - \frac{\sigma_{eff}T(b)}{A} \right]^{A-\nu}$$

 σ_{eff}/σ calculated in the LT nuclear shadowing theory for small σ

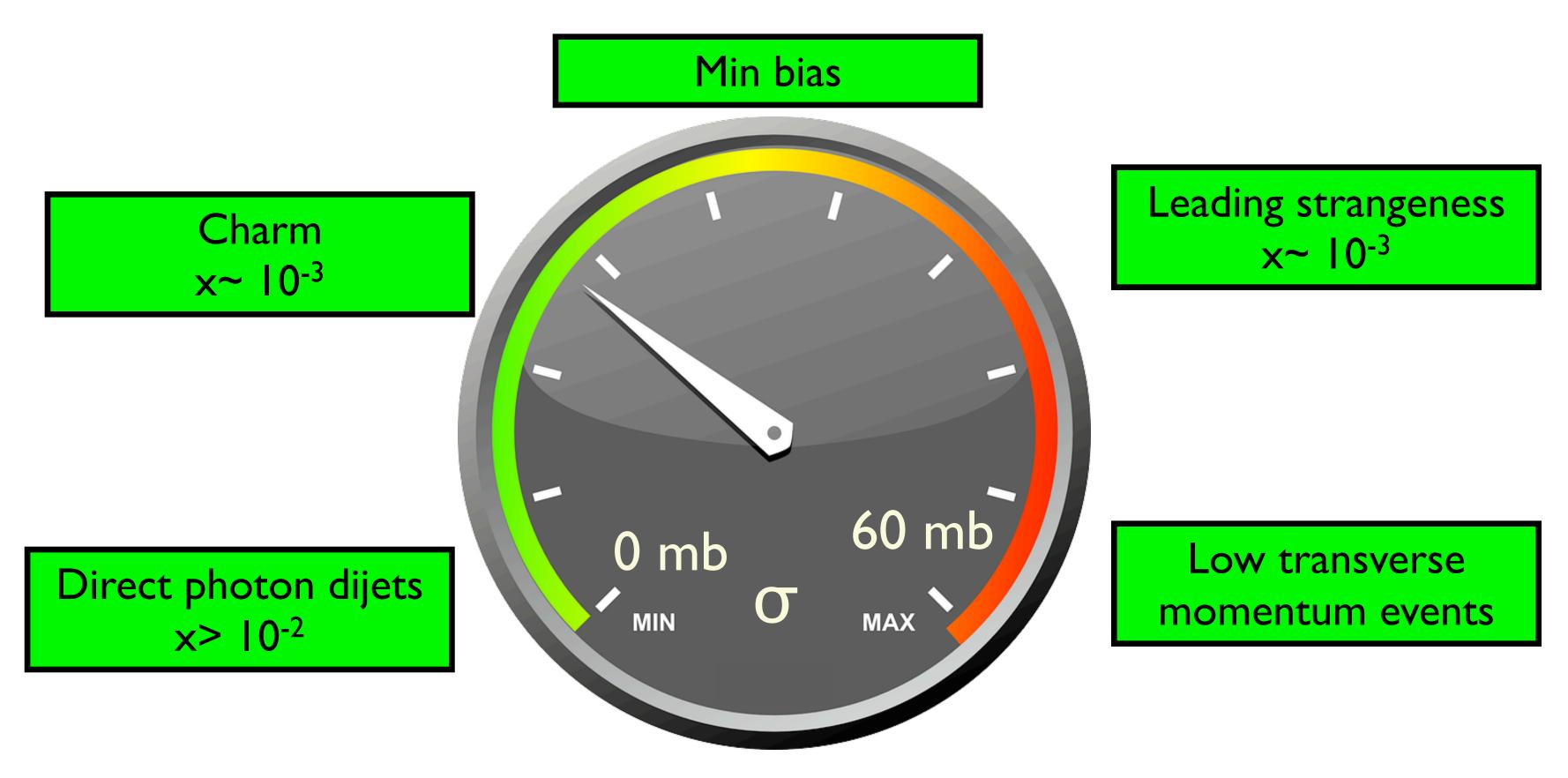


CF broaden very significantly distribution over ν . "pA ATLAS/CMS like analysis" using energy flow at large rapidities would test both presence of configurations with large σ ~40 mb, and weakly interacting configurations.



Ultraperipheral collisions at LHC (W_{YN}< 500 GeV)

Tuning strength of interaction of configurations in photon



"2D strengthonometer" - EIC & LHeC - Q² dependence - decrease of role of "fat" configurations, multinucleon interactions due to LT nuclear shadowing

Novel way to study dynamics of $\gamma \& \gamma^*$ interactions with nuclei

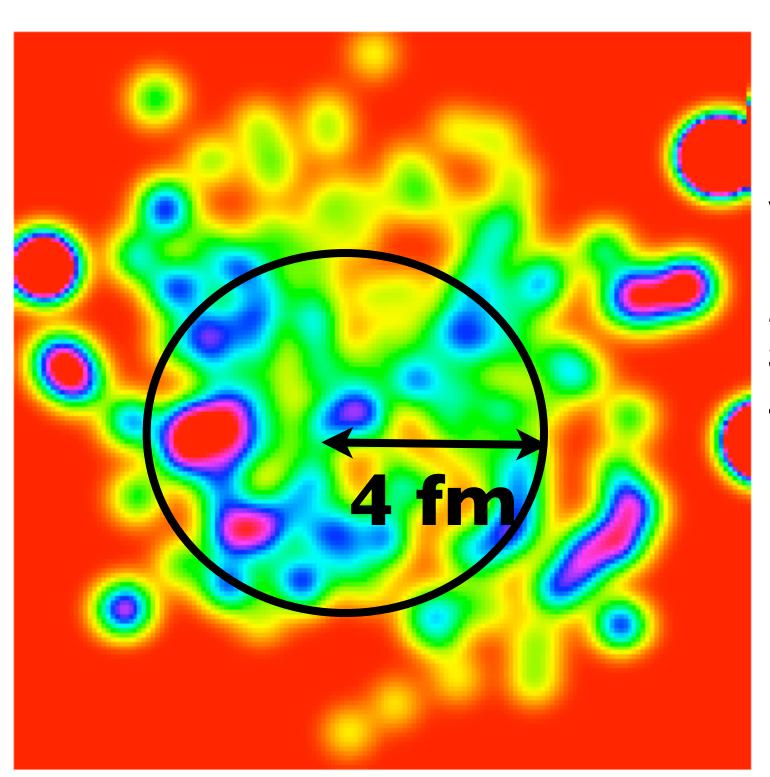
Conclusions

- * Color fluctuations are an important component of high energy dynamics
- * Color fluctuation with large x have smaller size
- * Opportunities for study global 3D structure of nucleon and photon
- * Fruitful to perform parallel studies of YA and pA processes

Proton and Photon-induced nuclear collisions at the LHC Workshop, CERN, 6-8 July 2016 https://indico.cern.ch/event/487649/overview

Slides for discussion & supplementary slides

If two (three) nucleons are at a small relative impact parameter (b < 0.6 fm), the gluon shadowing strongly reduces the overall transverse gluon density. However the thickness of the realistic nuclei is pretty low. So average number of overlapping nucleons is rather small (2.5 for b \sim 0) and hence fluctuations of the gluon transverse density are large



yellow < 1
1 green <2
2 <cyan < 3
3 <blue <4
4 < magenta < 5
5 < red

Heavy nuclei are not large enough to suppress fluctuations - A=200 nucleus for gluons with x > 10⁻² is like a thin slice of Swiss cheese.

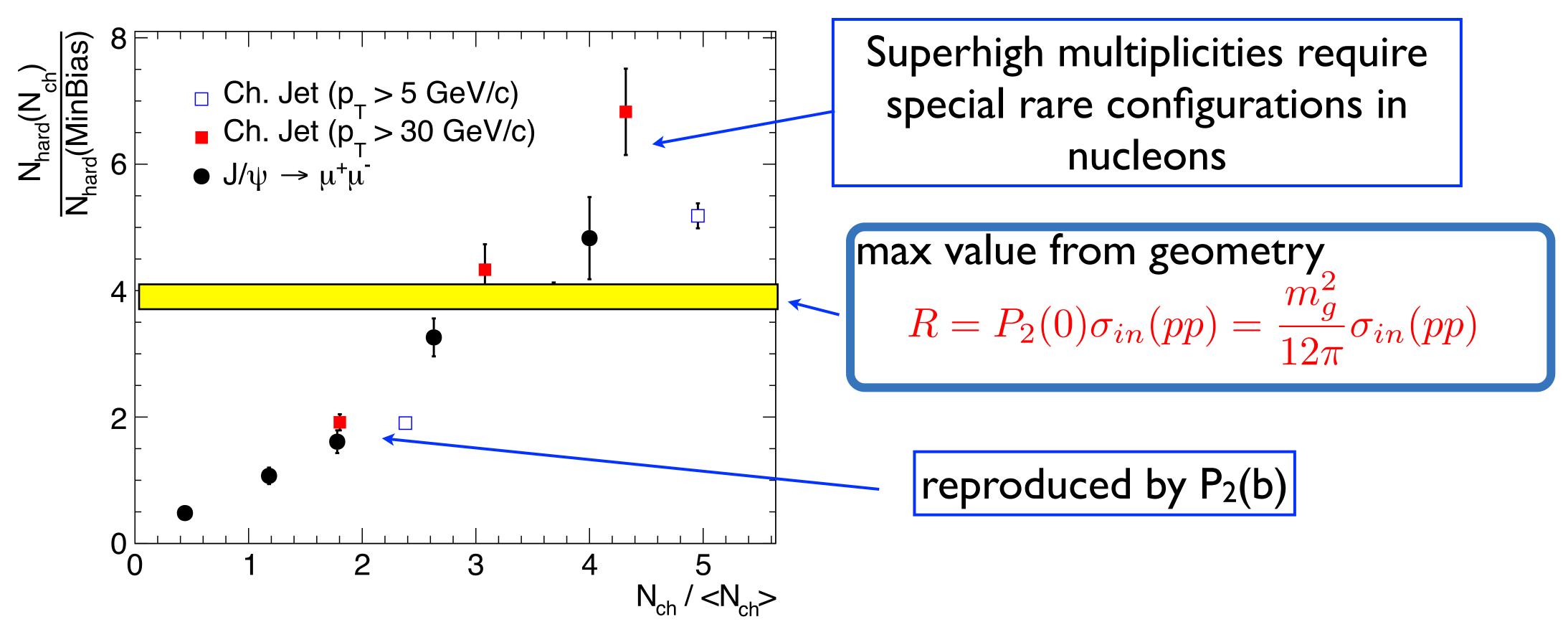
Far from the $A \rightarrow \infty$ limit.

Fluctuations of transverse density of gluons in Pb on event by event basis (Alvioli and MS 09) for x outside the shadowing region

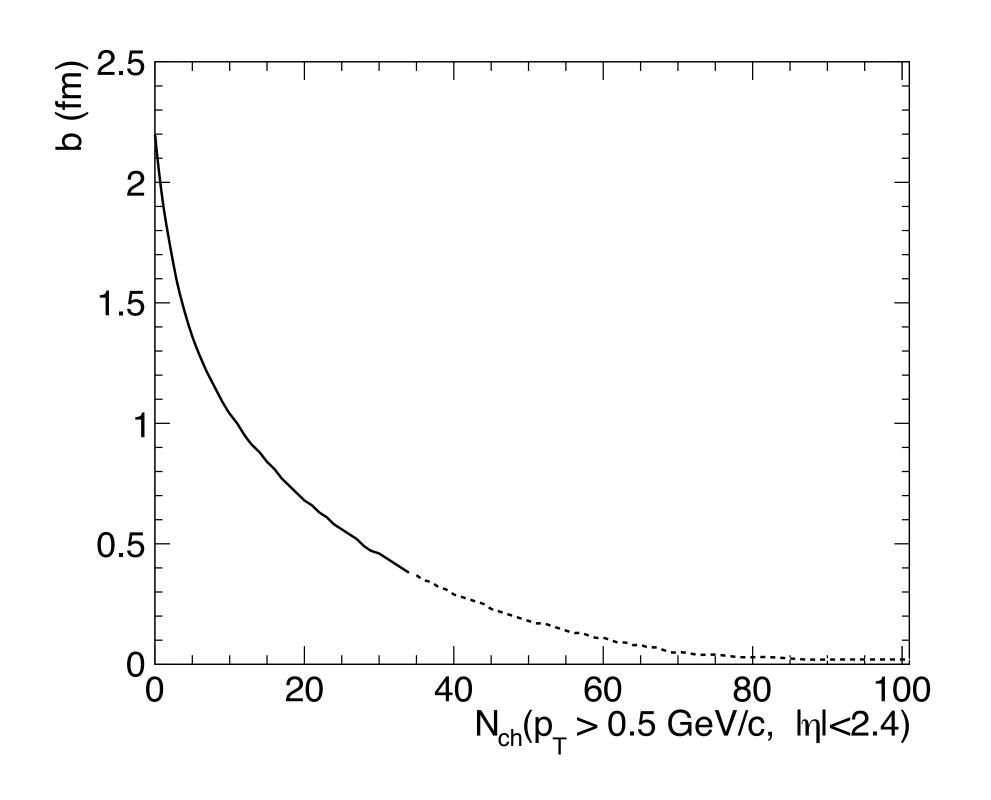
Leading twist shadowing observed at LHC does suppress some of fluctuations but new types of fluctuations

Universal relationship of soft and hard multiplicity

(Azarkin, Dremin, MS, 14)

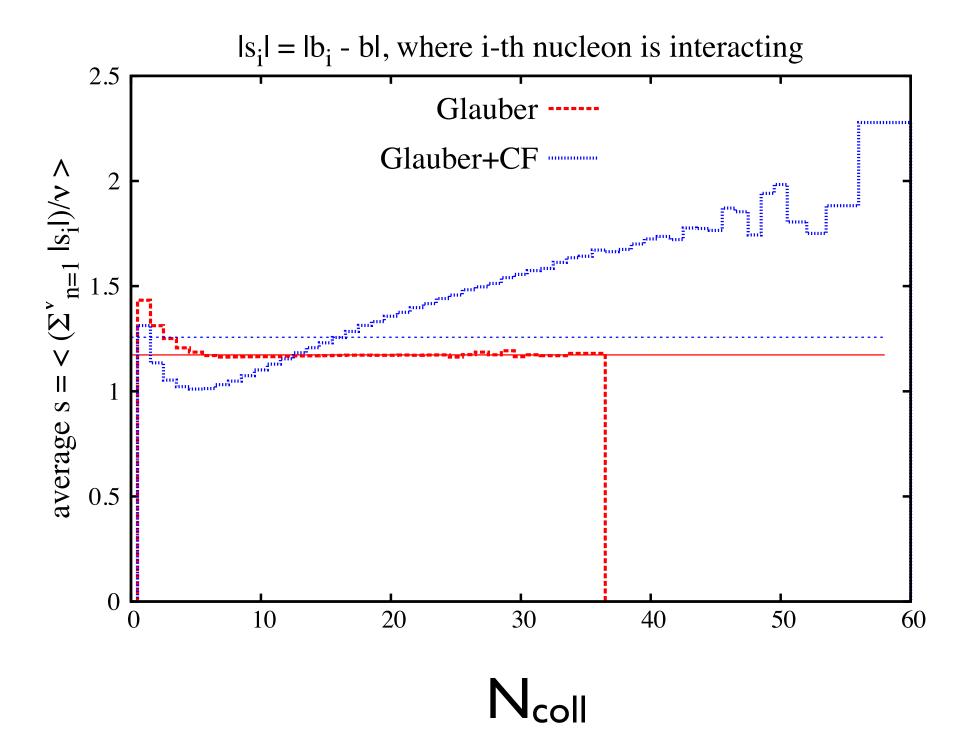


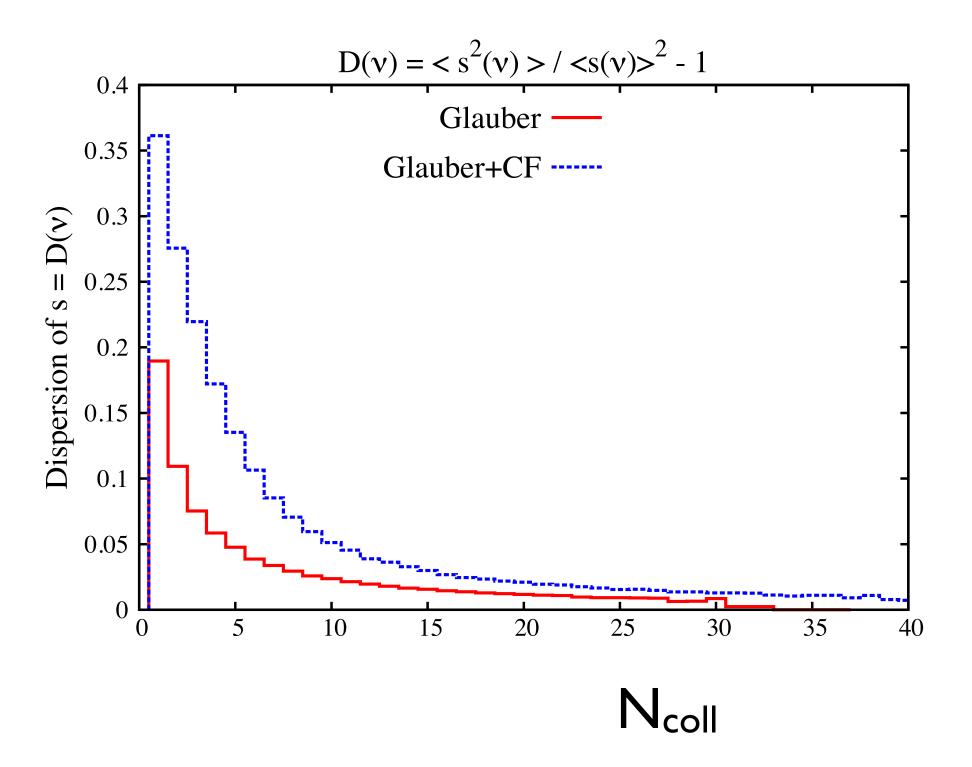
Universality of scaling of for hard processes scales with multiplicity: simple trigger - dijets(CMS) & direct J/ ψ , D and B-mesons (Alice)



Correspondence between impact parameter and N_{ch} . N_{ch} is defined here as a number of charged particles with $|\eta|$ < 2.4 and p_T > 0.5 GeV/c. Since events with N_{ch} > 35 are effectively central, the correspondence is not valid there.

- $b(N_{ch}/<N_{ch}>\sim 2)\sim 0.7 \text{ fm}$
- $b(N_{ch}/<N_{ch}>\sim 3)\sim 0.5 \text{ fm}$
- For $N_{ch}/< N_{ch}> \approx 4$ gluon fluctuations are important: jet multiplicity otherwise too high & probability of $N_{ch}/< N_{ch}> > 4$ events is much smaller than given by $P_2(b)$.





Average impact parameter, s, for pN interactions as a function of number of wounded nucleons and its dispersion. Average s traces average σ .

Reminder -in the limit of small inelastic diffraction and neglecting radius of NN interaction as compared to internucleon distance, Gribov - Glauber model leads to

$$\sigma_{\text{in}}^{\text{hA}} = \int d\vec{b} \left[1 - (1 - x)^A \right] = \sum_{n=1}^A \frac{(-1)^{n+1}A!}{(A - n)!n!} \int d\vec{b} \, x^n$$
where
$$x = \sigma_{\text{in}}^{\text{hN}} T(\mathbf{b})/A \qquad \int d\vec{b} T(b) = A$$

Series can be rewritten as sum of positive terms corresponding to cross sections σ_n of exactly one, two ... inelastic interactions Bertocchi, Treleani, 1976

$$\sigma_{\text{in}}^{\text{hA}} = \sum_{n=1}^{A} \sigma_n, \quad \sigma_n = \frac{A!}{(A-n)! \, n!} \int d\vec{b} \, x^n (1-x)^{A-n}$$

$$\langle N \rangle = \sum_{n=1}^{A} n \sigma_n / \sum_{n=1}^{A} \sigma_n = \frac{\sigma_{\text{in}}^{\text{hN}}}{\sigma_{\text{in}}^{\text{hA}}} \int d^2b \sum_{n=1}^{A} \frac{A!}{(A-n)!(n-1)!} x^n (1-x)^{A-n}$$

$$= \frac{\sigma_{\text{in}}^{\text{hN}}}{\sigma_{\text{in}}^{\text{hA}}} \int d^2b A T(\mathbf{b}) = \frac{A\sigma_{\text{in}}^{\text{hN}}}{\sigma_{\text{in}}^{\text{hA}}}, \quad \text{Simple geometric interpretation}$$

Can use $P(\sigma)$ to implement Gribov- Glauber dynamics of inelastic pA interactions. Baym et al 91-93

$$\sigma_{\text{in}}^{\text{NA}} = \int d\sigma_{in} P(\sigma_{in}) \int d\vec{b} \left[1 - (1 - x)^A \right]$$

$$\sigma_n = \int d\sigma_{in} P(\sigma_{in}) \frac{A!}{(A-n)! \, n!} \int d\vec{b} \, x^n (1-x)^{A-n} \, .$$

Probability of exactly n interactions is $P_n = \sigma_n/\sigma_{in}^{hA}$

New experimental observation relevant for color fluctuation phenomenon: coherent photoproduction of ρ -meson in ultraperipheral heavy ion collisions at LHC (ALICE): $\gamma + A \rightarrow \rho + A$

Analysis of Guzey, Frankfurt, MS, Zhalov 2015 (1506.07150)

