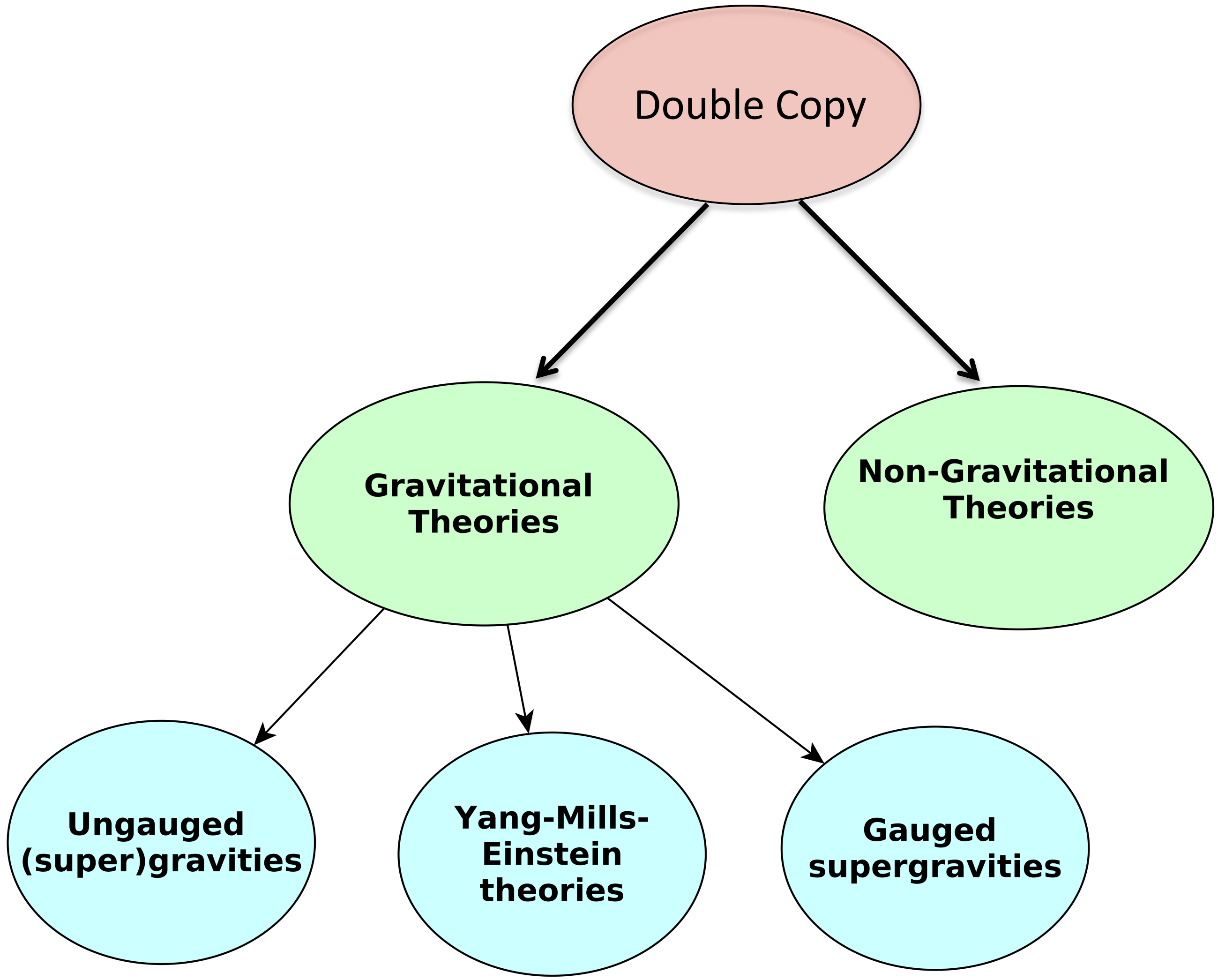
Double Copy
Gravitational Theories
Non-Gravitational Theories
Ungauged (super)gravities
Yang-Mills-Einstein theories
Gauged supergravities

Gravity	Gauge theories	Variants and notes
$\mathcal{N} > 4$ supergravity	• $\mathcal{N} = 4$ sYM theory • sYM theory ($\mathcal{N} = 1, 2, 4$)	• truncations to theories with reduced supersymmetry
$\mathcal{N} = 4$ supergravity with vector multiplets	• $\mathcal{N} = 4$ sYM theory • YM-scalar theory from dimensional reduction	• $\mathcal{N} = 2 \times \mathcal{N} = 2$ construction is also possible
pure $\mathcal{N} < 4$ supergravity	• (s)YM theory with ghosts • (s)YM theory with ghosts	• ghost fields in fundamental rep
$\mathcal{N} = 2$ Maxwell-Einstein supergravities (generic family)	• $\mathcal{N} = 2$ sYM theory • YM-scalar theory from dimensional reduction	• truncations to $\mathcal{N} = 1, 0$ • only adjoint fields
$\mathcal{N} = 2$ Maxwell-Einstein supergravities (homogeneous theories)	• $\mathcal{N} = 2$ sYM theory with half hypermultiplet • YM-scalar theory from dimensional reduction with matter fermions	• fields in pseudo-real reps • includes "magical" supergravities • truncations with $\mathcal{N} = 1, 0$
$\mathcal{N} = 2$ supergravities with hypermultiplets	• $\mathcal{N} = 2$ sYM theory with half hypermultiplet • YM-scalar theory from dimensional reduction with extra matter scalars	• fields in matter representations • construction known in particular cases • truncations/massive deformations possible
$\mathcal{N} = 2$ supergravities with vector/hypermultiplets	• $\mathcal{N} = 1$ sYM theory with chiral multiplets • $\mathcal{N} = 1$ sYM-scalar with chiral multiplets	• construction known in particular cases
$\mathcal{N} = 1$ supergravities with vector multiplets	• $\mathcal{N} = 1$ sYM theory with chiral multiplets • YM-scalar theory with fermions	• fields in matter representations • construction known in particular cases
$\mathcal{N} = 1$ supergravities with chiral multiplets	• $\mathcal{N} = 1$ sYM theory with chiral multiplets • YM-scalar with extra matter scalars	• fields in matter representations • construction known in particular cases

Einstein gravity with matter	• YM theory with matter • YM theory with matter	• construction known in particular cases
Einstein gravity $+ \phi R^2 + R^3$	• YM theory $+ F^3 + \ldots$ • YM theory $+ F^3 + \ldots$	
Conformal (super)gravity	• DF^2 theory • (s)YM theory	• up to $\mathcal{N} = 4$ supersymmetry • involves specific gauge theory with dimension-six operators
$3D$ maximal supergravity	• BLG theory • BLG theory	• $3D$ only

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Gravity	Gauge theories	Notes
Yang-Mills-Einstein supergravities	• sYM theory • YM + ϕ^3 theory	• trilinear scalar couplings • $\mathcal{N} = 0, 1, 2, 4$ possible
Higgsed supergravities	• sYM theory (Coulomb branch) • YM + ϕ^3 theory with extra massive scalars	• $\mathcal{N} = 0, 1, 2, 4$ possible • massive fields in supergravity
$U(1)_R$ gauged supergravities	• sYM theory (Coulomb branch) • YM theory with supersymmetry broken by fermion masses	• $0 \leq \mathcal{N} \leq 8$ possible • supersymmetry is spontaneously broken

Are all supergravities double copies?

Color/kinematics duality

Bern, Carrasco, Johansson

$$\mathcal{A}_m^{L-\mathrm{loop}} = i^L g^{m-2+2L} \sum_{i\in\mathcal{G}_3} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{n_i C_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

$$n_i = n_i(p_\alpha \cdot p_\beta, \epsilon \cdot p_\alpha, \ldots)$$

$$C_i = \ldots f^{a_1bc} f^{ca_2d} \ldots$$

n_i are not gauge-invariants

C_i and n_i have the same symmetries

Whenever gauge invariance requires: $C_i + C_j + C_j = 0$

kinematic numerators obey: $n_i + n_j + n_j = 0$

The double copy

Bern, Carrasco, Johansson

$$\mathcal{M}_m^{L-\mathrm{loop}} = i^{L+1} \left(\frac{\kappa}{2}\right)^{m-2+2L} \sum_{i\in\mathcal{G}_3} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{n_i \tilde{n}_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

A brief example of $\mathcal{N}$=2 SGs and of designing the single copies

Maxwell/Einstein 5d supergravity theories

Gunaydin, Sierra, Townsend

$$e^{-1}\mathcal{L} = -\frac{R}{2} - \frac{1}{4}\mathring{a}_{IJ}F^I_{\mu\nu}F^{J\mu\nu} - \frac{1}{2}g_{xy}\partial_\mu\varphi^x\partial^\mu\varphi^y + \frac{e^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda$$

$$C^{IJK}C_{J(MN}C_{PQ)K} = \delta^I_{(M}C_{NPQ)}$$

(whenever C related to Jordan algebra)

Everything is determined by a prepotential

Describe $\mathcal{M} = \frac{SO(n-1,1)}{SO(n-1)} \times SO(1,1)$ as hypersurface in ambient space

$$N(\xi) = \left(\tfrac{2}{3}\right)^{3/2} C_{IJK}\xi^I\xi^J\xi^K \qquad \xi = (\phi^x, \rho) \qquad a_{IJ} = -\tfrac{1}{2}\partial_I\partial_J \ln N(\xi)$$

$$\mathring{a}_{IJ}(\varphi) = a_{IJ}\big|_{N(\xi)=1} \qquad\qquad g_{xy}(\varphi) = \mathring{a}_{IJ}\partial_x\xi^I\partial_y\xi^J$$

$\mathring{a}_{IJ}$ in terms of vielbeine on the scalar manifold:

$$\mathring{a}_{IJ} = h_Ih_J + h^a_Ih^a_J \qquad h^Ih^J\mathring{a}_{IJ} = 1 \qquad h^I_ah^J_b\mathring{a}_{IJ} = \delta_{ab} \qquad h^Ih_{Ia} = h_Ih^{Ia} = 0$$

- canonical basis for C : $C_{000} = 1 \quad C_{00i} = 0 \quad C_{0ij} = -\frac{1}{2}\delta_{ij} \quad i,j = 1,\ldots,n$
  all other entries C_{ijk} arbitrary

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- Generic Jordan family: $N(\xi) = \sqrt{2}\xi^0\Big((\xi^1)^2 - (\xi^2)^2 - \ldots - (\xi^n)^2\Big)$
  (natural basis)
- Particular examples can be found by truncating $\mathcal{N}$=8 supergravity

The 4d generic Jordan family Maxwell-Einstein supergravities

$$\mathcal{L}_{\mathcal{N}=2} = \mathrm{Tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D^\mu\bar{\varphi})(D_\mu\varphi) - \frac{g^2}{4}[\varphi,\bar{\varphi}]^2 - i\bar{\lambda}D_\mu\bar{\sigma}^\mu\lambda + \sqrt{2}g\lambda^\alpha[\varphi,\lambda^\beta]\epsilon_{\alpha\beta} + \sqrt{2}g\bar{\lambda}^\alpha[\bar{\varphi},\bar{\lambda}^\beta]\epsilon_{\alpha\beta}\Big]$$

$$\mathcal{L}_{\mathcal{N}=0} = \mathrm{Tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(D_\mu\phi^{\hat{A}})(D^\mu\phi^{\hat{B}})\delta_{\hat{A}\hat{B}} + \underline{\frac{g^2}{4}[\phi^{\hat{B}},\phi^{\hat{C}}][\phi^{\hat{B}'},\phi^{\hat{C}'}]\delta_{\hat{B}\hat{B}'}\delta_{\hat{C}\hat{C}'}}\Big]$$

- Component double-copy spectrum: *I = (0, A) ; + = (-1+0) ; - = (-1 -0)*

spin $= 2$:	$h_{++} = A_{1+} \otimes A_{2+}$	$h_{--} = A_{1-} \otimes A_{2-}$
spin $= 3/2$:	$\psi^\alpha_+ = A_{1+} \otimes \lambda^\alpha_{2+}$	$\psi^\alpha_- = A_{1-} \otimes \lambda^\alpha_{2-}$
spin $= 1$:	$V_+ = A_{1+} \otimes \varphi_2$	$\tilde{V}_- = A_{1-} \otimes \varphi_2$
	$\tilde{V}_+ = A_{1+} \otimes \bar{\varphi}_2$	$V_- = A_{1-} \otimes \bar{\varphi}_2$
	$V^{\hat{A}}_+ = \phi^{\hat{A}}_1 \otimes A_{2+}$	$V^{\hat{A}}_- = \phi^{\hat{A}}_1 \otimes A_{2-}$
spin $= 1/2$:	$\tilde{\zeta}^\alpha_+ = A_{1+} \otimes \lambda^\alpha_{2-}$	$\tilde{\zeta}^\alpha_- = A_{1-} \otimes \lambda^\alpha_{2+}$
	$\zeta^{\hat{A}\alpha}_+ = \phi^{\hat{A}}_1 \otimes \lambda^\alpha_{2+}$	$\zeta^{\hat{A}\alpha}_- = \phi^{\hat{A}}_1 \otimes \lambda^\alpha_{2-}$
spin $= 0$:	$S_{+-} = A_{1+} \otimes A_{2-}$	$S_{-+} = A_{1-} \otimes A_{2+}$
	$S^{\hat{A}} = \phi^{\hat{A}}_1 \otimes \varphi_2$	$\bar{S}^{\hat{A}} = \phi^{\hat{A}}_1 \otimes \bar{\varphi}_2$

Three-point amplitudes agree between Lagrangian and double copy

Gauging in d=5:

Gunaydin, Sierra, Townsent

- Covariantization of derivatives and field strengths:

$$\partial_\mu \varphi^x \to \partial_\mu \varphi^x + g A_\mu^s K_s^x$$

$$\nabla_\mu \lambda^{ia} \to \nabla_\mu \lambda^{ia} + g L_t^{ab} A_\mu^t \lambda^{ib}$$

$$F_{\mu\nu}^I \to \mathcal{F}_{\mu\nu}^I = \partial_\mu A_\nu^I - \partial_\nu A_\mu^I + g f^I{}_{JK} A_\mu^J A_\nu^K$$

- Modifications to the vector triple coupling:

$$\frac{e^{-1}}{6\sqrt{6}} C_{IJK} \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu}^I F_{\rho\sigma}^J A_\lambda^K \longrightarrow$$

$$\frac{e^{-1}}{6\sqrt{6}} C_{IJK} \epsilon^{\mu\nu\rho\sigma\lambda} \left\{ F_{\mu\nu}^I F_{\rho\sigma}^J A_\lambda^K + \frac{3}{2} g f^K{}_{J'K'} F_{\mu\nu}^I A_\rho^J A_\sigma^{J'} A_\lambda^{K'} + \frac{3}{5} g^2 A_\mu^I f^J{}_{I'J'} A_\nu^{I'} A_\rho^{J'} f^K{}_{K'L'} A_\sigma^{K'} A_\lambda^{L'} \right\}$$

- No potential in 5d ⟶ Minkowski ground state with unbroken susy
- Reduction to 4d: - potential from nonabelian field strength;
  - zero minimum energy ⟶ Minkowski ground state
  - choose symplectic frame (depending on purpose);
    part of the definition of the theory

Designer's gauge theories

Modify the MESGT double-copy construction to include non-abelian couplings

- Minimal couplings with spin-0 and spin-1/2 fields

$$\mathcal{L}^{\rm YMESGT} \sim \ldots f_{rst} V_\mu^r (\phi^s \partial_\mu \phi^t - \phi^t \partial_\mu \phi^s) \oplus f_{rst} V_\mu^r \bar{\psi}^s \sigma^\mu \psi^t \ldots$$

⟶ standard 3-point S-matrix elements

$$[M_3] = 1$$

- Require that this can be factorized…

$$M_3 = A_3 A_3'$$

… and that A_3 and A_3' are Lorentz-invariant

$$[A_3] = 0 \quad \& \quad [A_3'] = 1$$

from dim 3 operator (4d counting) (↗ $[A_3] = 0$)

from standard dim 4 operator (4d counting) (↖ $[A_3'] = 1$)

⟶ Unique local option: trilinear scalar operator

The 4d generic Jordan family of YME $\mathcal{N}$=2 SGs

$$\mathcal{L}_{\mathcal{N}=2} = \mathrm{Tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D^\mu\bar\varphi)(D_\mu\varphi) - \frac{g^2}{4}[\varphi,\bar\varphi]^2 - i\bar\lambda D_\mu\bar\sigma^\mu\lambda + \sqrt{2}g\lambda^\alpha[\varphi,\lambda^\beta]\epsilon_{\alpha\beta} + \sqrt{2}g\bar\lambda^\alpha[\bar\varphi,\bar\lambda^\beta]\epsilon_{\alpha\beta}\Big]$$

$$\mathcal{L}_{\mathcal{N}=0} = \mathrm{Tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(D_\mu\phi^{\hat A})(D^\mu\phi^{\hat B})\delta_{\hat A\hat B} + \frac{g^2}{4}[\phi^{\hat B},\phi^{\hat C}][\phi^{\hat B'},\phi^{\hat C'}]\delta_{\hat B\hat B'}\delta_{\hat C\hat C'} + \frac{gg'}{3!}F_{\hat A\hat B\hat C}\phi^{\hat A}[\phi^{\hat B},\phi^{\hat C}]\Big]$$

Check existence of color/kinematics with g':
- should hold order by order in g'

Chiodaroli, Jin, RR;
Chiodaroli, Gunaydin, Johansson, RR

- Manifest for highest power of g'
- Lower powers of g': use BCJ amplitudes relations. E.g. 5pt:

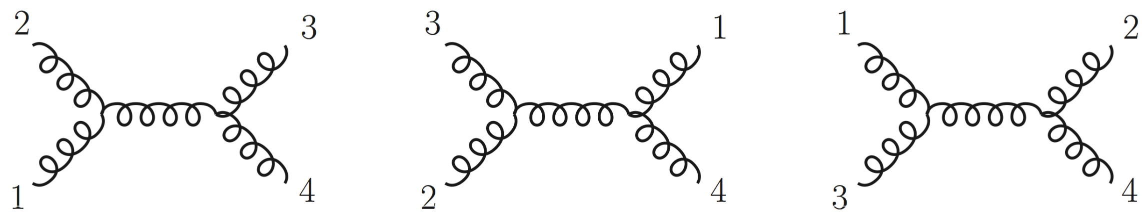
$$s_{24}A_5^{(0)}(1,2,4,3,5) = A_5^{(0)}(1,2,3,4,5)(s_{14}+s_{45}) + A_5^{(0)}(1,2,3,5,4)s_{14}$$

$$\begin{aligned}
&\mathcal{A}_5^{(0)}(1^{\phi^{\hat A_1}}2^{\phi^{\hat A_2}}3^{\phi^{\hat A_3}}4^{\phi^{\hat A_3}}5^{\phi^{\hat A_3}})\big|_{g'}\\
&\quad= \frac{1}{2}g^3g'F^{\hat A_1\hat A_2\hat A_3}\Bigg[\bigg(\frac{k_{12\bar 3}\cdot k_{4\bar 5}}{s_{12}s_{45}}\mathrm{f}^{a_1a_2b}\mathrm{f}^{ba_3c} + \frac{k_{\bar 123}\cdot k_{4\bar 5}}{s_{23}s_{45}}\mathrm{f}^{a_2a_3b}\mathrm{f}^{ba_1c}\\
&\qquad\qquad + \frac{k_{1\bar 23}\cdot k_{4\bar 5}}{s_{13}s_{45}}\mathrm{f}^{a_3a_1b}\mathrm{f}^{ba_2c}\bigg)\mathrm{f}^{ca_4a_5} + (3\leftrightarrow 4) + (3\leftrightarrow 5)\Bigg]\\
&\quad+ \frac{1}{2}g^3g'F^{\hat A_1\hat A_2\hat A_3}\Bigg[-\left(\frac{1}{s_{13}}+\frac{1}{s_{24}}\right)\mathrm{f}^{a_1a_3b}\mathrm{f}^{ba_5c}\mathrm{f}^{ca_2a_4} - \left(\frac{1}{s_{13}}+\frac{1}{s_{25}}\right)\mathrm{f}^{a_1a_3b}\mathrm{f}^{ba_4c}\mathrm{f}^{ca_2a_5}\\
&\qquad\qquad + (3\leftrightarrow 4) + (3\leftrightarrow 5)\Bigg]
\end{aligned}$$

Simplest check: 3-point amplitudes. Needs more notation

Next-simplest double-copy example: 4-vector scattering in SG, highest power of g'

(SG vectors) = ($\mathcal{N}$=2 YM vectors) x ($\mathcal{N}$=0 scalars)



$$\mathcal{A}_4^{\text{tree},YM} = \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u} \qquad c_s = -2 f^{a_1 a_2 b} f^{b a_3 a_4}$$

$$n_s = \frac{i}{2}\Big\{\Big[(\varepsilon_1\cdot\varepsilon_2)p_1^\mu + 2(\varepsilon_1\cdot p_2)\varepsilon_2^\mu - (1\leftrightarrow 2)\Big]\Big[(\varepsilon_3\cdot\varepsilon_4)p_{3\mu} + 2(\varepsilon_3\cdot p_4)\varepsilon_{4\mu} - (3\leftrightarrow 4)\Big] + s\Big[(\varepsilon_1\cdot\varepsilon_3)(\varepsilon_2\cdot\varepsilon_4) - (\varepsilon_1\cdot\varepsilon_4)(\varepsilon_2\cdot\varepsilon_3)\Big]\Big\},$$

$$\mathcal{A}_4^{\text{tree},scalar} = \frac{N_s c_s}{s} + \frac{N_t c_t}{t} + \frac{N_u c_u}{u} \qquad c_s = -2 f^{a_1 a_2 b} f^{b a_3 a_4} \qquad N_s = -\frac{1}{2} g' F^{A_1 A_2 B} F^{B A_3 A_4}$$

$$\mathcal{M}_4^{\text{tree},YM} = \left(\frac{\kappa}{4}\right)^2 \left[\frac{n_s N_s}{s} + \frac{n_t N_t}{t} + \frac{n_u N_u}{u}\right]$$

Other amplitudes also check out; can find explicit expressions for $\langle h^n A^m \rangle$, loop-level analogs of BCJ amplitudes relations, etc.

An application: EYM trees & Loop-level amplitudes relations

Tree-level EYM amplitudes from 2-copy: single-trace gluon-graviton amplitudes

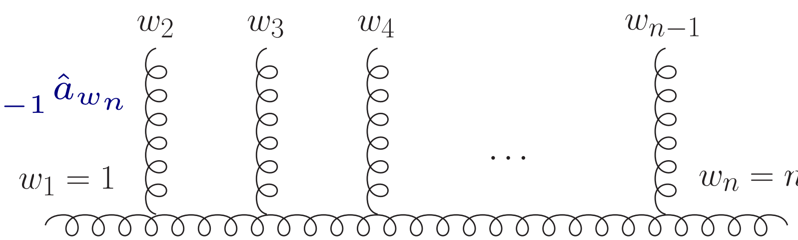
- can be expressed in terms of YM color-ordered amplitudes
- gauge/diffeomorphism invariance helps bypass need of manifest c/k rep.

$$\mathcal{A}_n^{\rm YM}(1,\ldots,n) = -ig^{n-2}\sum_{i\in\text{cubic}}\frac{c_i\, n_i^{\rm YM}}{D_i}$$

Del Duca, Dixon, Maltoni

$$= -ig^{n-2}\sum_{w\in S_{n-2}} C^{\rm DDM}(1,w_2,\ldots,w_{n-1},n)\, A_n^{\rm YM}(1,w_2,\ldots,w_{n-1},n)$$

$$C^{\rm DDM}(w) = i^{n-2} f^{\hat a_{w_1}\hat a_{w_2}\hat x_1} f^{\hat x_1 \hat a_{w_3}\hat x_2}\ldots f^{\hat x_{n-3}\hat a_{w_{n-1}}\hat a_{w_n}}$$



Uses only Jacobi relations

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Del Duca, Dixon, Maltoni

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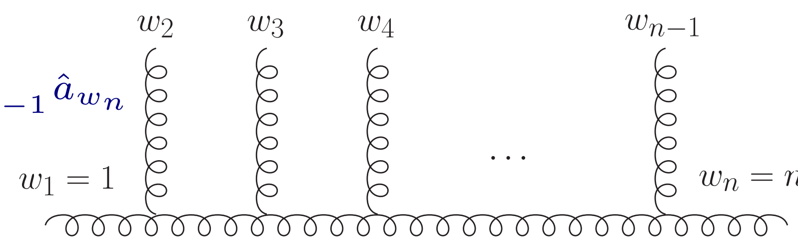


Uses only Jacobi relations

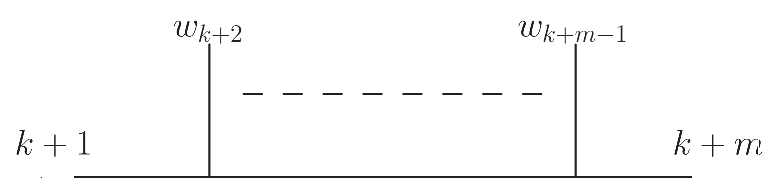
Similar structure in YMESG theories; slight twist is that there exist multi-ladder terms

$$\mathcal{M}_{k,m}^{\rm EYM}(1,\ldots,k|k+1\ldots k+m)$$

$$= \sum_{w\in S_{m-2}} \widetilde{C}^{\rm DDM}(k+1,w_{k+2},\ldots,w_{k+m-1},k+m)\, M_{k,m}^{\rm YME}(1,\ldots,k|k+1\ldots k+m) + \text{multi-ladder}$$

$$\widetilde{C}^{\rm DDM}(k+1,\ldots,k+m)$$

$$= i^{m-2} F^{A_{k+1}A_{k+2}X_1} F^{X_1 A_{k+3} X_2}\ldots F^{X_{m-3}A_{k+m-1}A_{k+m}}$$



➡ Need DDM form of YM-scalar wrt flavor group; exists because of c/k duality

Putting together DDM YM color decomposition and YM+scalar flavor decomposition:

$$\mathcal{M}^{\text{EYM}}_{k,m}(1,\ldots,k|k+1\ldots k+m)$$
$$= \sum_{w\in S_{m-2}} \widetilde{C}^{\text{DDM}}(k+1,w_{k+2},\ldots,w_{k+m-1},k+m)\, M^{\text{YME}}_{k,m}(1,\ldots,k|k+1\ldots k+m) + \text{multi-ladder}$$

$$M^{\text{YME}}_{k,m}(1,\ldots,k\,|\,k+1,\ldots,k+m) = \sum_{w\in\sigma_{12\ldots k}} N_k(w) A^{\text{YM}}_{k+m}(w) + \text{Perm}(1,\ldots,k)$$

$$N_k(w) = \prod_{i=1}^{k} 2(\varepsilon_i\cdot z_i(w)) + \text{contact terms}$$

$N_k(w)$ /contact terms should/could be determined from:

1) Massage the c/k dual YM-scalar amplitude into flavor-DDM form

2) Generalized double-copy from any YM-scalar amplitude put in flavor-DDM form

3) ….

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$$M^{\rm YME}_{k,m}(1,\ldots,k\,|\,k+1,\ldots,k+m) = \sum_{w\in\sigma_{12\ldots k}} N_k(w) A^{\rm YM}_{k+m}(w) + {\rm Perm}(1,\ldots,k)$$

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$N_k(w)$ /contact terms should/could be determined from:

1) Massage the c/k dual YM-scalar amplitude into flavor-DDM form

2) Generalized double-copy from any YM-scalar amplitude put in flavor-DDM form

3) Diff invariance with known leading term and BCJ amplitude relations for YM amps

Putting together DDM YM color decomposition and YM+scalar flavor decomposition:

$$\mathcal{M}^{\rm EYM}_{k,m}(1,\dots,k|k+1\dots k+m)$$
$$=\sum_{w\in S_{m-2}} \widetilde{C}^{\rm DDM}(k+1,w_{k+2},\dots,w_{k+m-1},k+m)\, M^{\rm YME}_{k,m}(1,\dots,k|k+1\dots k+m)+\text{multi-ladder}$$

$$M^{\rm YME}_{k,m}(1,\dots,k\,|\,k+1,\dots,k+m)=\sum_{w\in\sigma_{12\dots k}} N_k(w) A^{\rm YM}_{k+m}(w)+{\rm Perm}(1,\dots,k)$$

$$N_k(w)=\prod_{i=1}^{k} 2(\varepsilon_i\cdot z_i(w))+\text{contact terms}$$

$N_k(w)$ from diff/gauge invariance with known leading term:

- Degree k in momenta; Degree k in polarization vectors; each of them enters once
- Built from $\left\{\,(\varepsilon_i z_i)\,,\quad (p_i z_i)\,,\quad (\varepsilon_i\varepsilon_j)\,,\quad (\varepsilon_i p_j)\,,\quad (p_i p_j)\,\right\}, \qquad i,j=1,\dots,k$
- Scalar momenta implicit through z; ansatz is independent of nr. of scalars ordering enters only through $z(w)$
- Vanishes upon $\epsilon_i(p_i)\to p_i\ (\forall)\ i=1,\dots,k$ and use of the BCJ amplitude relations
- BCJ amp rels remove remaining freedom from $M^{\rm YME}_{k,m}(1,\dots,k\,|\,k+1,\dots,k+m)$ but can be used to find nice expressions for $N_k(w)$
- semi-recursive

Explicit tree-level EYM amplitudes from 2-copy Chiodaroli, Gunaydin, Johansson, RR

- One graviton, (m-1) gluons, single-trace

$$A_m^{\rm YME}(1,\ldots,m-1;p_1) = \sum_{i=1}^{k-1} 2\varepsilon_1\cdot z_1\, A^{\rm YM}(1,\ldots,\overset{i}{p_1},\ldots,m-1)\,, \qquad z_1=\sum_{l=1}^{i} k_l$$

- Two gravitons, (m-2) gluons, single-trace

$$A_m^{\rm YME}(1,\ldots,m-2;p_1,p_2)=\sum_{j=3}^{m-1}\sum_{i=2}^{j-1} n_{ij}A(1,\ldots,\overset{i}{p_1},\ldots,\overset{j}{p_2},\ldots,m-2)+{\rm perm}(p_1,p_2)$$

$$n_{ij} = -4(\varepsilon_1\cdot z_1)\,(\varepsilon_2\cdot z_2)+\frac{1}{2}(\varepsilon_1\cdot\varepsilon_2)(z_1\cdot p_1 - z_2\cdot p_2) \qquad z_2=\sum_{l=1}^{j} k_l + p_1$$

- Three gravitons, (m-3) gluons, single-trace

$$A_m^{\rm YME}(1,\ldots,m-3;p_1,p_2,p_3)=\sum_{k=4}^{m-1}\sum_{j=3}^{k-1}\sum_{i=2}^{j-1} n_{ijk}A(1,\ldots,\overset{i}{p_1},\ldots,\overset{j}{p_2},\ldots,\overset{k}{p_3},\ldots,m-3)$$
$$+\,{\rm perm}(p_1,p_2,p_3)$$

$$n_{ijk}=4(\varepsilon_1\cdot z_1)(\varepsilon_2\cdot z_2)(\varepsilon_3\cdot z_3)-\Big(2(p_1\cdot z_1)(\varepsilon_1\cdot\varepsilon_2)(\varepsilon_3\cdot z_3-\frac{1}{2}\varepsilon_3\cdot p_2+\frac{1}{2}\varepsilon_3\cdot p_1)-(1\leftrightarrow 3)\Big)$$

- Four & five gravitons – not too bad either
- Simpler expressions than from scattering eqs e.g. Nandan, Plefka, Schlotterer, Wen
- Recursive construction using these variables Teng, Feng

Double-copy, YMESGTs and constraints on single-trace (s)YM loop integrands

Chiodaroli, Gunaydin, Johansson, RR

Tree-level: BCJ amplitude rel's → existence of color/kinematics-dual rep's

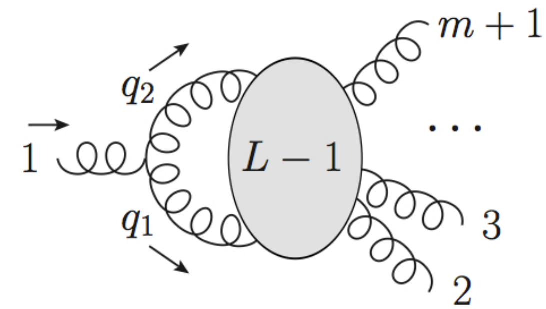
What are analogous necessary relations at loop level?

- earlier string theory-based work at 1 loop and 2 loops Vanhove, Tourkine; + Ochirov; Hohenegger, Stieberger

Here: diff invariance of EYM amplitudes

Observation: L-loop YM-ϕ^3 amplitude with 1 gluon and highest power of g' is c/k dual

- All cubic graphs with aligned color and flavor
- QED-type coupling for the sole gluon
- No possible contact terms



➡ $M^{\rm YME}_{1,m,L}(1\,|\,2,3,\dots,m+1) =$

$$= A^{{\rm YM},(L)}_{\text{2-trace}}(1\,|\,2,\dots,m+1)[2\,\varepsilon_1\cdot q_1] + \sum_{\text{cyclic}(2,\dots,m+1)} A^{{\rm YM},(L)}_{\text{1-trace}}(1,2,\dots,m+1)[2\,\varepsilon_1\cdot q_1]$$

diff inv.

➡ $$0 = A^{{\rm YM},(L)}_{\text{2-trace}}(1\,|\,2,\dots,m+1)[p_1\cdot q_1] - \sum_{\text{cyclic}(2,\dots,m+1)} A^{{\rm YM},(L)}_{\text{1-trace}}(1,2,\dots,m+1)[p_1\cdot q_1]$$

U(1) decoupling

➡ $$0 = A^{{\rm YM},(L)}_{\text{2-trace}}(1\,|\,2,\dots,m+1)[1] + \sum_{\text{cyclic}(2,\dots,m+1)} A^{{\rm YM},(L)}_{\text{1-trace}}(1,2,\dots,m+1)[1]$$

Double-copy, YMESGTs and constraints on single-trace (s)YM loop integrands

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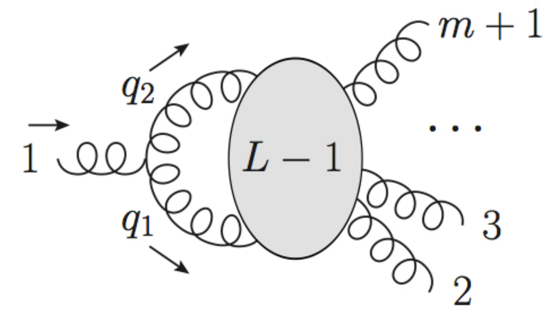

➡ $$M^{\rm YME}_{1,m,L}(1\,|\,2,3,\dots,m+1) = \\ = A^{{\rm YM},(L)}_{\text{2-trace}}(1\,|\,2,\dots,m+1)[2\,\varepsilon_1\cdot q_1] + \sum_{\text{cyclic}(2,\dots,m+1)} A^{{\rm YM},(L)}_{\text{1-trace}}(1,2,\dots,m+1)[2\,\varepsilon_1\cdot q_1]$$

$$\sum_{\text{cyclic}(2,\dots,m+1)} A^{{\rm YM},(L)}_{\text{1-trace}}(1,2,\dots,m+1)[p_1\cdot q_1] = 0$$

➡ Reproduces earlier low loop results