International Journal of Modern Physics A

GRIBOV LIGHT QUARK CONFINEMENT SCENARIO

YURI L. DOKSHITZER*

Laboratoire de Physique Théorique et Hautes Energies, Université Paris-VI,
4 place Jussieu, F-75252 Paris, France,
yuri@lpthe.jussieu.fr

Received Day Month Year
Revised Day Month Year

The key ingredients of the Gribov program of attacking quark confinement are reviewed.

Keywords: Quantum Chromodynamics; color confinement; light quarks

PACS numbers: 12.38.Aw, 12.38.Lg, 11.30.Qc, 11.30.Rd

1. Gribov Program of Attacking Confinement

V. N. Gribov became interested in non-Abelian field theories in 1976.[a] Results of his very first apprentice study that were reported at the $12^{\rm th}$ Leningrad Winter School in February 1977 changed forever the non-Abelian landscape.

In the first lecture Ref. 2 "*Instability of non-Abelian gauge fields and impossibility of the choice of the Coulomb gauge*" Gribov gave an elegant physically transparent explanation of the *anti-screening* phenomenon and showed that in non-Abelian gauge theories, the three-dimensional transversality condition

$$(\boldsymbol{\nabla}\cdot\mathbf{A}) \equiv \frac{\partial A_i}{\partial x_i} = 0\,, \qquad i = 1,2,3 \tag{1}$$

(usually imposed on the field potential to describe massless vector particles in the Coulomb gauge) actually *does not* solve the problem of gauge fixing.

This called for revision of the QCD quantization procedure. Gribov addressed this problem in the second lecture Ref. 3 "*Quantization of non-Abelian gauge theories*". The (initially rejected) Nuclear Physics paper Ref. 4 under the same title appeared that was based on the two Winter School lectures and laid foundation for what is known now as the problem of Gribov copies, Gribov horizon, etc.

*On leave of absence from Petersburg Nuclear Physics Institute, 188350 Gatchina, Russia
[a]Gribov works on gauge theories and, in particular, all his papers, talks and lectures devoted to anomalies and the QCD confinement (including the lectures that were translated into English for the first time) were collected and recently published in Ref. 1.

1.1. *Light quarks and confinement of color*

Starting from early 90's, Gribov abandoned pure gluodynamics and formulated for himself the approach to the confinement problem:

- The question of interest is that of **the** confinement — in the real world with two very light (u and d) quarks — rather than **a** confinement. q
- No mechanism for binding massless *bosons* (gluons) seems to exist in QFT, while the Pauli exclusion principle may provide means for binding together massless *fermions* (light quarks).
- The problem of ultraviolet regularization may be more than a technical trick in a QFT with apparently infrared-unstable dynamics: the ultraviolet and infrared regimes of the theory may be closely linked.
- The Feynman diagram technique has to be reconsidered in QCD if one goes beyond trivial perturbative correction effects. Feynman's famous $i\epsilon$ prescription was designed for (and applies only to) the theories with *stable perturbative vacua.*
- The concept of the " Dirac sea" — the picture of the fermionic content of the vacuum — is more than anachronistic model, a sort of Maxwell's mechanical ether, *I'd-rather-do-without.* It plays a crucial rôle in the physics of quantum anomalies and, most probably, of **the** confinement of color.

To understand and describe a physical process in a *confining theory*, it is necessary to take into consideration the response of the vacuum, which leads to essential modifications of the quark and gluon Green functions.

In QED, physical objects — *electrons and photons* — are in one-to-one correspondence with the fundamental fields that one puts into the local Lagrangian of the theory. From this point of view, the rôle of the QED Vacuum may be said to be "trivial": it makes the coupling $\alpha_{\rm e.m.}$ (and the electron mass operator) *run* with momentum scale, but does not affect the *nature* of the interacting fields.

The QCD Vacuum changes the bare fields *beyond recognition.*

1.2. *Supercritical binding of electrons by over-charged nuclei*

A known QFT example of such a violent response of the vacuum — screening of *super-charged ions* with $Z > 137$. The expression for Dirac energy level ϵ of an electron in an external static field created by the point-like electric charge Z contains $\epsilon \propto \sqrt{1-(\alpha_{\rm e.m.}Z)^2}$. For $Z > 137$ the energy becomes *complex.* This means instability. Classically, the electron "*falls onto the centre*". Quantum-mechanically, it also "*falls*", but into the Dirac sea. In QFT the instability develops when the energy ϵ of an empty atomic electron level drops, with increase of Z, below $-m_ec^2$. Then, an e^+e^- pair pops up from the vacuum, with the vacuum electron occupying the level: the super-critically charged ion decays into an "atom" (the ion with the smaller positive charge, $Z-1$) and a real positron:

$$A_Z \Longrightarrow A_{Z-1} + e^+\,, \qquad \text{for } Z > Z_{\rm crit.}$$

Thus, the ion becomes *unstable* and gets rid of an excessive electric charge by emitting a positron.[5]

1.3. *Binding "massless" quarks*

In the QCD context, the increase of the running quark-gluon coupling at large distances replaces the large Z of the QED problem. Gribov generalized the problem of supercritical binding in the field of an infinitely heavy source to the case of two massless fermions interacting via *Coulomb-like exchange*. He found that in this case the supercritical phenomenon develops much earlier.

Namely, a *pair of light fermions* interacting in a Coulomb-like manner develops supercritical behavior if the coupling hits a definite value. With account of the QCD color Casimir operator, this critical value of the coupling above which restructuring of the perturbative vacuum leads to *chiral symmetry breaking* and, likely, to *confinement*, translates into

$$\frac{\alpha_{\rm crit}}{\pi} = C_F^{-1}\left[1-\sqrt{\frac{2}{3}}\right] \simeq 0.137\,; \qquad C_F = \frac{N_c^2-1}{2N_c} = \frac{4}{3}\,. \tag{2}$$

1.4. *Gribov equation*

Gribov developed a new approximation to the Schwinger–Dyson equation for the fermion (quark) Green function that was based upon two simple but powerful observations, one "algebraic", another — dynamical.

The first Gribov's observation was that $1/k^2$ of the boson propagator happens to be *the Green function of the four-dimensional Laplace operator*,

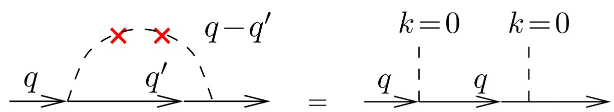
$$\partial_\mu^2 \frac{1}{(q-q')^2+i\epsilon} = -4\pi^2 i\,\delta(q-q'), \qquad \partial_\mu \equiv \frac{\partial}{\partial q_\mu},$$

where ∂_μ denotes the momentum differentiation. Then, considering the first order self-energy diagram $\Sigma_1(q)$ for a fermion (quark/electron) with momentum q virtually decaying into a quark (electron) with momentum q' and a massless vector boson (gluon/photon) with momentum $k=q-q'$,

$$\Sigma_1(q) = [C_F]\frac{\alpha}{\pi}\int \frac{d^4q'}{4\pi^2 i}\left[\,\gamma_\mu\, G_0(q')\,\gamma_\mu\,\right]\, D_0(q-q'), \qquad D_0(k) = \frac{1}{k^2+i\epsilon},$$

(with G and D the fermion and boson propagators, respectively) and differentiating it twice over the external momentum q, one arrives at an algebraic expression

$$\boldsymbol{\partial}^2_{\boldsymbol{\mu}}\ \Sigma_1(q) \quad = $$



Differentiation over external momentum kills the ultraviolet divergences (both linear and logarithmic).

Now comes the *dynamical* observation. In higher orders the fermion Green function and the vertices get dressed: $G_0(q) \Longrightarrow G(q)$, $\gamma_\nu \Longrightarrow \Gamma(q,q,0)$. Invoking the Ward identity,

$$\Gamma(q,q,0) = -\partial_\nu G^{-1}(q),$$

we arrive at the non-linear algebraic equation

$$\boldsymbol{\partial}_\mu^2 G^{-1}(q) \;=\; g \cdot \big(\boldsymbol{\partial}_\nu G^{-1}(q)\big)\; G(q)\; \big(\boldsymbol{\partial}_\nu G^{-1}(q)\big) \;+\; \ldots \qquad \Big(g \equiv [C_F]\frac{\alpha}{\pi}\Big) \tag{3}$$

with ... standing for *less singular* terms of the order $\mathcal{O}\left(g^2\right)$.

The Gribov Equation (3)

- is *local* in the momentum space,
- takes into account the most singular (logarithmically enhanced) *infrared* and *ultraviolet* renormalization effects,
- makes a smart use of the *gauge invariance*,
- retains essential *non-linearity* due to quark-gluon interactions
- and possesses a rich *non-perturbative structure*.

It can be looked upon as a perturbative ("leading logarithmic") approximation that allows one to penetrate into the region of *large anomalous dimensions*, $\gamma = \mathcal{O}(1)$.

Insertions into the *boson line* make the coupling run, $g \to g(\ln\sqrt{q^2})$. In the $g\to 0$ ($|q^2|\to\infty$) limit the general solution of the *free equation* $\partial^2 G^{-1} = 0$ reads

$$G^{-1}(q) = Z_0^{-1}\left[\,(m_0 - \hat{q}) + \frac{\nu_1^3}{q^2} + \frac{\nu_2^4\,\hat{q}}{q^4}\,\right] \qquad (\hat{q} \equiv \gamma_\mu q^\mu)\,. \tag{4}$$

Two new integration constants ν_1 and ν_2 have appeared in addition to the familiar bare mass m_0 and the wave function renormalization constant Z_0. This is due to the fact that, unlike the standard renormalization group (RG) approach, the new equation is the *second order* (matrix) differential equation. These new dimensional parameters can be directly linked with non-perturbative vacuum condensates that were introduced in the context of the famous ITEP sum rules[6]

$$\nu_1^3 \propto \left\langle \bar{\psi}\psi \right\rangle, \qquad \nu_2^4 \propto \left\langle \alpha_s F^a_{\mu\nu} F^a_{\mu\nu} \right\rangle.$$

The new terms in (4) are *singular* at $q^2 \to 0$. In QED we simply drop them as unwanted, thus returning to the RG structure. In so doing, however, we exploit the knowledge that nothing dramatic happens in the *infrared domain*, so that the real electron in the physical spectrum of the theory, whose propagation we seek to describe, is inherently that very same object that we put into the Lagrangian as the fundamental bare field. Not so clear in an infrared unstable theory. In QCD therefore we better keep all four terms, wait and see.

1.5. *Critical coupling, SCSB and confinement*

In the deep (Euclidean) region, $\left|q^2\right| \gg m^2$, the new term constitutes but a small power-suppressed correction, $m(q) \simeq m_0 + \nu_1^3/q^2 \to m_0$. For finite momenta, $\left|q^2\right| \sim m^2$, all four terms are to be treated on the same footing. Then, if in the infrared region the coupling exceeded the *critical value*, a *bifurcation* in the Gribov equation occurs, giving the *non-perturbative solution*. It corresponds to the phase with *spontaneously broken chiral symmetry*. This means that (given a supercritical coupling in the infrared) the quark Green function may possess a non-trivial mass operator even in the chiral limit $m_0 \to 0$.[7]

The dynamical chiral symmetry breaking brings in *pions* as Goldstone bosons. An effective theory of interacting quarks and pions emerges. Given dynamical nature of the pion field, the pion–axial current transition constant f_π is not arbitrary but satisfies an integral equation involving the quark Green function. The pion–quark interaction "constant" is also fixed: $g_\pi = \left\{i\gamma_5, G^{-1}\right\}/f_\pi$. Pions, in turn, affect the propagation of quarks. In his last paper Ref. 8 Gribov argued that these effects are likely to lead to confinement of light quarks and, thus, to confinement of any color states.

2. Gluon Sector

For the Gribov confinement *scenario* to become the *theory*, the gluon sector has to be cracked. Analyzing the quark Green function, Gribov has exploited two hypotheses, namely that 1) the "coupling constant" $\alpha_s(k^2)$ makes sense in the small momentum domain (say, $k < 1$ GeV), and 2) it exceeds there the definite value necessary for super-critical binding.

To fully solve the problem, similar analysis of the *gluon Green function* should be carried out. Constructing an equation for the gluon similar to that for the quark remains an open quest. V. N. Gribov demonstrated how this problem can be solved in an *Abelian* theory, giving rise to formal resolution of the "Landau ghost" problem with the QED photon Green function (in the ultraviolet).[b]

In QCD, separating *running coupling* effects from an unphysical *gauge dependent phase* that are both present in the gluon Green function constitutes a technical complication. (One may think of using "background gauge" or employing the "pinch technique" to address the issue.) From this analysis a consistent picture of the coupling rising above the critical value in the infrared momentum region should emerge.

It is interesting to notice that recent attempts to quantify the QCD interaction strength (α_s) in the infrared domain by looking at non-perturbative effects in *Event Shapes* (thrust, C-parameter, jet-mass, broadening, etc.) in e^+e^- annihilation and Deep Inelastic Scattering consistently pointed at the average value of the coupling

[b] Handwritten notes of "*Quantum Electrodynamics at Short Distances*" (unpublished), deciphered and translated into English by J. Nyiri, can be found in Ref. 1, p. 519.

which (slightly) exceeds $\alpha_{\rm crit}$ (for a recent review see Ref. 9):

$$\alpha_0 \equiv \frac{1}{2\,\mathrm{GeV}} \int_0^{2\,\mathrm{GeV}} dk\, \alpha_s(k^2) \;\simeq\; 0.5 \quad > \quad 0.137\cdot\pi\,. \tag{5}$$

3. Conclusion: Gribov's QCD Heritage

- **Techniques**: quantization of Yang–Mills fields, the origin of asymptotic freedom, the nature of instantons, physics of quantum anomalies, supercritical binding of light fermions and dynamical spontaneous chiral symmetry breaking.
- **Philosophy**: to set up the problem of the confinement of *INFO*s (quarks and gluons = Identified Non-Flying Objects) as that of *light quarks*.
- **Ideology**: to attack the QCD confinement problem *perturbatively*, i.e. using the good old Dyson–Feynman–Keldysh language of Green functions of quarks and gluons.
- **Practice**: Gribov's ideas, being understood and pursued, offer an intriguing possibility to address all the diversity and complexity of the hadron world from within the field theory with a reasonably small effective interaction strength.

For more detailed discussion of the Gribov confinement program see a recent review Ref. 10.

References

1. V. N. Gribov, *Gauge Theories and Quark Confinement. Collection of works* (Phasis Publishing House, Moscow, 2002).
2. V. N. Gribov, Instability of Non-Abelian Gauge Theories and Impossibility of Choice of Coulomb Gauge, in *Proc. XII Winter School*, Vol. 1 *Elementary Particle Physics* (LNPI, Leningrad, 1977), p. 147 (in Russian). Published in English in Ref. 1, p. 258.
3. V. N. Gribov, Quantization of Non-Abelian Gauge Theories, in *Proc. XII Winter School*, Vol. 2 *High Energy Physics* (LNPI, Leningrad, 1977), p. 64 (in Russian). Published in English in Ref. 1, p. 233.
4. V. N. Gribov, *Nucl. Phys.* **B 139** 1 (1978).
5. I. Ya. Pomeranchuk and Ya. A. Smorodinsky, *Journ. Fiz. USSR* **9**, 97 (1945); W. Pieper and W. Greiner, *Z. Phys.* **218**, 327 (1969); Ya. B. Zeldovich and V. S. Popov, *Uspekhi Fiz. Nauk* **105** 4 (1971).
6. M. A. Shifman (ed.), *Vacuum Structure and QCD Sum Rules: Reprints*, Current Physics, Sources and Comments, Vol. 10 (North-Holland, 1992).
7. V. N. Gribov, *Eur. Phys. J.* **C10**, 70 (1999).
8. V. N. Gribov, *Eur. Phys. J.* **C10**, 91 (1999).
9. Yu. L. Dokshitzer, Historical and Futuristic Perturbative and Non-Perturbative Aspects of QCD Jet Physics, to appear in *Acta Physica Polonica* **36** (2005); hep-ph/0411227.
10. Yu. L. Dokshitzer and D. E. Kharzeev, *Ann. Rev. Nucl. Part. Science* **54**, 487 (2004).