Open bosonic strings and D-branes

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1. Classical open strings

Just like closed strings were described by maps $S^1 \times R \longrightarrow M$
(infinite cylinder)

open strings are described by maps $I \times \mathbb{R} \longrightarrow M$
(infinite strip)

$$\sigma \in [0,\pi],\ \tau \in \mathbb{R} \ \rightarrow \ X^\mu(\sigma,\tau)$$

as well as world sheet metric $\gamma_{\alpha\beta}$, with action

$$S_p = -\frac{T}{2}\int d\sigma\, d\tau\, \sqrt{-\gamma}\ \gamma^{\alpha\beta}\,\partial_\alpha X^\mu\,\partial_\beta X^\nu\ g_{\mu\nu}(X)$$

The action is still invariant under Diff $\times$ Weyl, so we can impose conformal gauge $\gamma_{\alpha\beta} = \begin{pmatrix} -1 & \\ & 1 \end{pmatrix}$

Local equations of motion are same as for closed strings:

$$T_{\alpha\beta} = 0 \qquad ; \qquad \partial_\alpha\, g_{\mu\nu}(X)\,\partial^\alpha X^\nu = 0$$

but boundary conditions differ: requiring stationarity of S_p,

$$\delta S_p = -T\int d\sigma\, d\tau\ \delta\partial_\alpha X^\mu \cdot g_{\mu\nu}\,\partial^\alpha X^\nu$$

$$= T\int d\sigma\, d\tau\ \delta X^\mu\left(\partial_\alpha\, g_{\mu\nu}\,\partial^\alpha X^\nu\right)$$

$$- T\int d\tau\ \Big[\delta X^\mu \cdot g_{\mu\nu}\,\partial_\sigma X^\nu\Big]_0^\pi$$

so we require $\delta X^\mu \cdot g_{\mu\nu}\,\partial_\sigma X^\nu = 0$ at $\sigma = 0,\pi$, $\forall\tau$

* One way to solve this condition is to enforce Neumann boundary conditions:

$$\partial_\sigma X^\mu = 0 \quad \text{at } \sigma = 0, \pi .$$

Another possibility are Dirichlet b.c

$$\delta X^\mu = 0 \quad \text{at } \sigma = 0, \pi$$

but this breaks translation invariance: fixes ends of the open string on some trajectories $X^\mu(0,\tau)$, $X^\mu(\pi,\tau)$.
Classically, any choice is fine, but quantum mechanically, conformal invariance severely requires possible choices.
$X^\mu(0,\tau) = x_0^\mu$, $X^\mu(0,\tau) = x_1^\mu$ in flat space are OK.

It is possible to choose Neuman for some directions, Dirichlet for others.
$X^0 \dots X^p$ $\quad$ $X^{p+1} \dots X^{D-1}$

This describes an open string whose ends are stuck on some dimension $(p+1)$ defect: Dp-brane

One could also choose different defects at the two ends:
open string stretched between Dp and Dp' branes
(can be parallel, orthogonal or at some angle)

Rk: * "All Neuman" is a special case of "space filling D_3-brane"

*, one could also add some new boundary term to the action, e.g $\int A_\mu(X) \dfrac{dX^\mu}{d\tau} d\tau$: as an exercise, show that bc becomes "mixed Dirichlet Neumann", $\partial_\sigma X^\mu + F^{\mu\nu}_{(X)} \partial_\tau X_\nu = 0$ at $\sigma = 0, \pi$.

* Dirichlet condition along time $X^0 \longrightarrow$ "S brane" in Lorentzian signature, "Euclidean D-brane" in Euclidean sig.

* In the directions with Neumann conditions at both ends:

$$\begin{aligned} \partial_\sigma X^\mu(\sigma=0) &= 0 \\ \partial_\sigma X^\mu(\sigma=\pi) &= 0 \end{aligned} \quad \text{requires} \quad \alpha_k^\mu = \tilde{\alpha}_k^\mu$$

so that the general mode expansion is

$$X^\mu(\tau,\sigma) = x^\mu + 2 l_s^2 p^\mu \tau + i\sqrt{2}\, l_s \sum_{k\neq 0} \frac{\alpha_k^\mu}{k} \cos k\sigma \, e^{-ik\tau}$$

factor of 2 such that

$$p^\mu = T \int_0^\pi d\sigma \, \dot{X}^\mu = \underbrace{T.\pi.2l_s^2}_{=1} p^\mu$$

* In the directions with Dirichlet conditions at both ends:

$$\begin{aligned} \partial_\tau X^I(\sigma=0) &= 0 \\ \partial_\tau X^I(\sigma=\pi) &= 0 \end{aligned} \quad \text{requires} \quad \alpha_k^I = -\tilde{\alpha}_k^I$$

but the zeromode is also different:

$$X^I(\tau,\sigma) = x_0^I + (x_1^I - x_0^I)\frac{\sigma}{\pi} - \sqrt{2}\, l_s \sum_{k\neq 0} \frac{\alpha_k^I}{k} \sin k\sigma \, e^{-ik\tau}$$

* There is now only one set of Virasoro constraints:

$$L_m = \frac{1}{2} \sum_{n\in\mathbb{Z}} \left(\alpha_{m-n}^\mu \alpha_n^\mu + \alpha_{m-n}^I \alpha_n^I \right) + l_s^2 (x_1^I - x_0^I)^2$$

with $\alpha_0^\mu = \sqrt{2}\, l_s\, p^\mu$, $\alpha_0^I = 0$

no momentum transverse to D-brane

The Poisson algebra is identical as before after $[\,] \to \frac{1}{i}\{\,\}$:

$$[\alpha_m, \alpha_n] = m\, \delta_{m+n}$$

$$[x^\mu, p^\nu] = i\eta^{\mu\nu}$$

$$[L_m, L_n] = (m-n) L_{m+n} \quad \text{(up to possible anomalies)}$$

* Light cone quantization

As for the bosonic closed string, one can fix the gauge completely by requiring

$$X^+ = x^+ + 2 l_s^2 p^+ \tau + 0$$

where $X^\pm = X^0 \pm X^1$ are Neumann directions

(this fails for a D_p brane with $p \geq D-2$)

The Virasoro constraints $L_m = \frac{1}{2} \sum_{n \in \mathbb{Z}} \left(-\alpha^+_{m-n} \alpha^-_n + \dots \right)$

$$= -\frac{1}{2} \sqrt{2}\, l_s\, p^+ \alpha^-_m + \dots$$

allow to solve for $\alpha^-_m = \frac{\sqrt{2}}{2 l_s p^+} \left(\sum_{n \in \mathbb{Z}} \left(\underbrace{\alpha^i_{m-n} \alpha^i_n}_{\text{transverse Neumann}} + \underbrace{\alpha^I_{m-n} \alpha^I_n \big) + l_s^2 \Delta x^2}_{\text{Dirichlet}} \right)$

For $m=0$, one obtains the mass-shell condition [omit Δx in 1st pass]

$$\frac{2 l_s}{\sqrt{2}} p^- = \frac{\sqrt{2}}{2 l_s p^+} \left(\frac{4 l_s^2}{2} p_i^2 + l_s^2 \Delta x^2 + \sum_{m \neq 0} \left(\alpha^i_{-n} \alpha^i_n + \alpha^I_{-n} \alpha^I_n \right) \right)$$

$$l_s^2 (p^- p^+ - p_i^2) = \frac{2}{4} \sum_{m=1}^{\infty} \left(l_s^2 \Delta x^2 + 2 \sum_{n=1}^{\infty} \left(\alpha^i_{-n} \alpha^i_n + \alpha^I_{-n} \alpha^I_n \right) - a \right)$$

$$l_s^2 M^2 = N + \frac{1}{2} l_s^2 \Delta x^2 - a$$

Unlike the bosonic string, there is no level matching condition. Since the momentum p^I vanishes, excitations propagate along D-brane world volume only, and are classified by

$SO(p-1) \times SO(D-p-1)$ for massless modes

$SO(p) \times SO(D-p-1)$ for massive modes.

Take $\Delta x = 0$ for now

$i = 1 \dots p-1$
$I = 1 \dots D-p-1$

- The ground state $|0\rangle$ has mass $m^2 = -a/\ell_s^2$

- The first level has mass $m^2 = (1-a)/\ell_s^2$

  $\alpha_{-1}^{i}|0\rangle$: vector of $SO(p-1)$, singlet of $SO(D-p-1)$

  $\alpha_{-1}^{I}|0\rangle$: singlet " , vector of "

  Since it cannot be cast in $SO(p)$ reps, it must be massless:

  $a = 1 \Rightarrow D = 26$, like in closed bosonic string

  $\Rightarrow$ 1 massless $U(1)$ gauge field

  + $D-p-1$ scalars : can be understood as transverse fluctuations of Dp brane

- Second level:

$$\alpha_{-1}^{i}\alpha_{-1}^{j}|0\rangle \qquad \alpha_{-2}^{i}|0\rangle \qquad \frac{(p-1)p}{2} + p-1 = \frac{p(p+1)}{2} - 1$$

$$\alpha_{-1}^{i}\alpha_{-1}^{J}|0\rangle \qquad \alpha_{-2}^{I}|0\rangle$$

$$\alpha_{-1}^{I}\alpha_{-1}^{J}|0\rangle$$

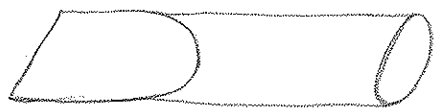
can be cast as symmetric, traceless massive tensor $f_{\mu\nu}$ propagating on Dp brane, + $(D-p-1)$ massive vectors f_μ^I
+ $(D-p-1)(D-p)/2$ massive scalars

- The number of states grows at the same rate as for closed strings:
  $\Omega(M) \sim \exp(4\pi\sqrt{N}) \sim \exp(4\pi\sqrt{\ell_s^2 M^2})$: same Hagedorn T

- At level N, the state with maximal spin (x_2, x_3) plane (inside Neumann directions) is $(\alpha_{-1}^2 + i\alpha_{-1}^3)^N|0\rangle$, with $J = N$
  thus $J_{max} = \ell_s^2 M^2 + 1$: 'leading Regge trajectory'

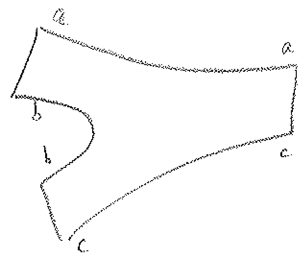
More remarks:

* The fact that the critical dimension / Hagedorn temperature is the same for open and closed bosonic strings is in agreement with the fact that open string interactions will lead to closed strings:



- The same diagram  can be read in two ways...
- D-brane interactions

* Since open strings interact by joining their ends, it is possible to decorate each end of a string by a "color", or Chan Paton index $\alpha \in \{1..N$ and require that only ends with the same color can join:



N^2 species of open strings.

At massless level: N^2 massless gauge bosons with cubic interaction

$\rightarrow$ $U(N)$ Yang Mills theory!
+ adjoint scalars

α can be understood as labelling (possibly space filling) D-branes.

If the ends of an open string are on two parallel D-branes with separation Δx, the mass spectrum is shifted by $M^2 \rightarrow M^2 + (T\Delta x)^2$

$T = \frac{1}{2\pi l_s^2}$

For the 1st excited level, this is just the Higgs mechanism: D-brane coordinates are noncommuting

$U(2) \rightarrow U(1) \times U(1)$ by adjoint scalar

* One could also consider non oriented open strings (Möbius strip), or non oriented closed strings (Klein bottle).

String Theory — Exercises 2.

* Consider classical solution of open string in conformal gauge

$$X^0 = B\tau$$

$$X^1 = B\cos\tau\cos\sigma$$

$$X^2 = B\sin\tau\cos\sigma$$

$$X^i = 0 \qquad i>2$$

Check that it satisfies the equations of motion and Neumann b.c.
Compute energy $E = p^0$ and angular momentum J, and check that $|J| = 4\, l_s^2\, E^2$

* Determine mode expansion for open strings with Neumann bc at $\sigma = 0$ and Dirichlet condition at $\sigma = \pi$

* Show that adding a minimal coupling $\int A_\mu \frac{dX^\mu}{d\tau} d\tau$ on the boundary of an open string worldsheet changes the Neumann boundary condition to $\partial_\sigma X^\mu + 2\alpha l_s^2 F^\mu_{\ \nu}\, \partial_\tau X^\nu = 0$