Closed Strings on the Milne Universe, and Electric Fields

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LPTHE and LPTENS, Paris

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slides available from

`http://www.lpthe.jussieu.fr/~pioline/seminars.html`

Introduction

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- Target space supersymmetry is presumably incompatible with time dependence.
- First quantized string theory requires an Euclidean worldsheet, hence Euclidean target space. The analytic continuation may be ambiguous or ill-defined, Lorentzian observables may be very different from their Euclidean counterparts.
- Worse, String theory is not content on a finite time interval, and one is frequently forced into Big Bang / Big Crunch singularities, CTC in the process of maximally extending the geometry.

String theory and spacelike singularities

- Cosmological singularities occur for generic initial data in classical Einstein's gravity. Can the no-bounce theorem be avoided in string theory ?

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- String theory has a variety of extended objects that may become light at a space-like singularity, could their exchange (or condensation ?) dominate the dynamics and lead to finite amplitudes ?
- Assuming that the singularity persists, do spatially separated points still decouple near $T = 0$? What remains of the classical chaotic billiard motion under string and quantum corrections ? What boundary conditions should one impose at $T = 0$ and how ?

Damour Henneaux

Toy-models for cosmological singularities

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- The Lorentzian orbifold, quotient of flat $R^{1,1}$ Minkowski space by a boost $J : X^{\pm} \sim e^{\pm\beta} X^{\pm}$, describes a locally flat cosmology with a Kasner singularity of type $(1, 0, 0, \ldots)$:

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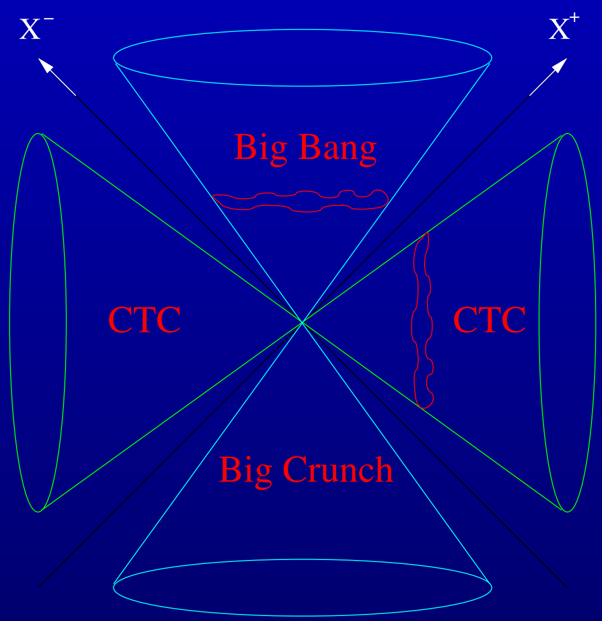

Milne Universe region:

$$ds^2 = -dT^2 + T^2 d\theta^2$$
$$X^{\pm} = T e^{\pm\theta}/\sqrt{2}, \quad \theta \equiv \theta + 2\pi\beta$$

"Whiskers" with CTC:

$$ds^2 = -r^2 d\eta^2 + dr^2$$
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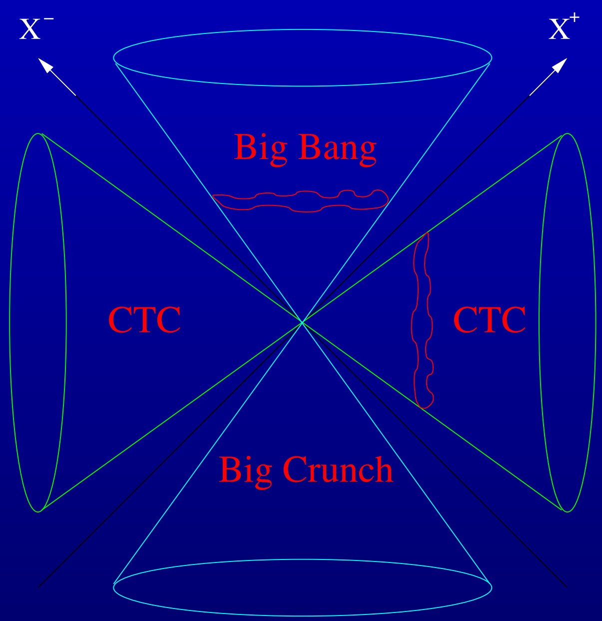

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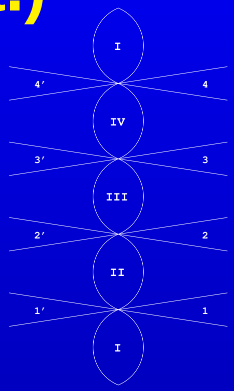
- The singularity may be resolved by combining the boost with a translation on an extra spatial direction: the CTC are now shielded by a cosmological horizon, and may possibly be excised by orientifold planes.

Cornalba Costa

Toy-models for cosmological singularities (cont.)



- The gauged WZW model $Sl(2) \times Sl(2)/U(1) \times U(1)$ describes a bouncing 4-dimensional Universe, locally isometric to the Lorentzian orbifold at the singularities. Singularities may be resolved by switching on an electric field.

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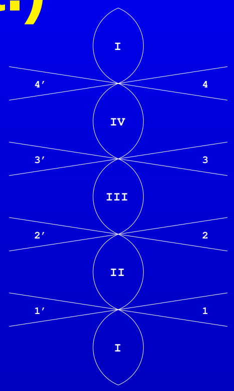

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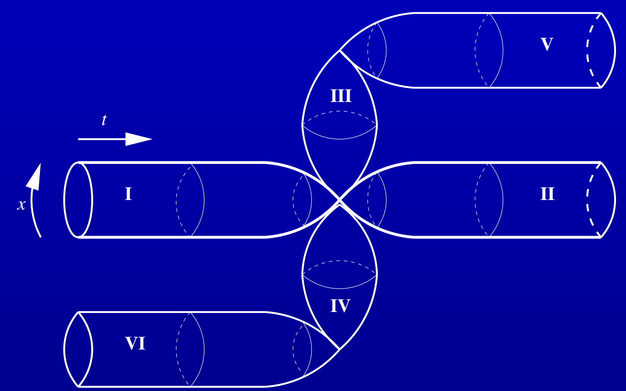

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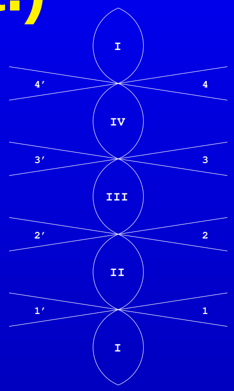

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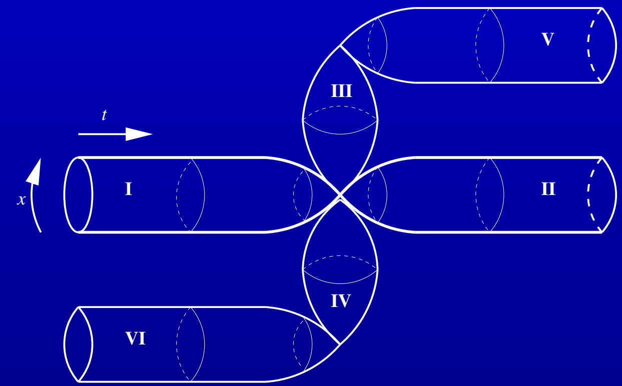

- The Milne singularity is a very non-generic case of Kasner singularities. Can more general singularities be studied ? Are whiskers generic ? What is the fate of the cosmological singularity and CTC under string corrections ?
- We will be focusing on the dynamics of the twisted strings, wrapping the Milne circle and/or the CTCs.

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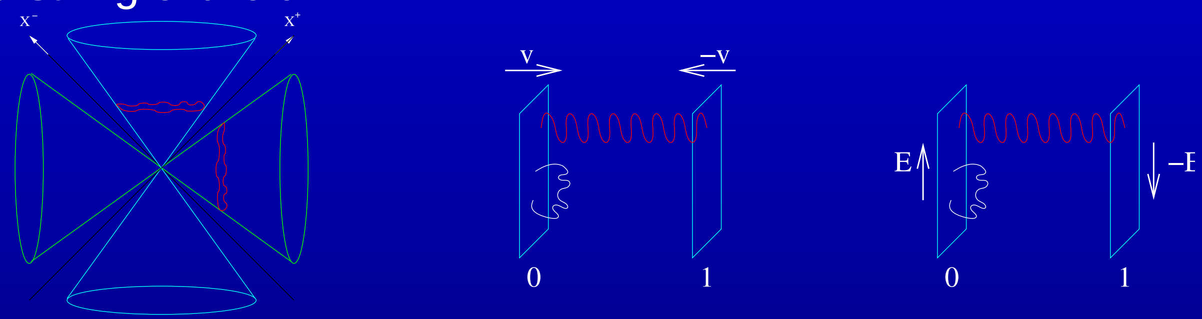

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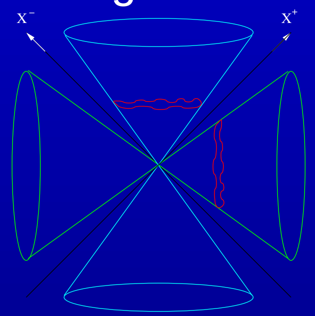

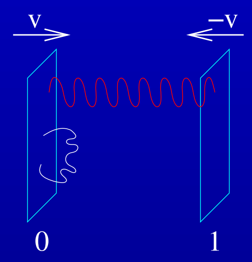

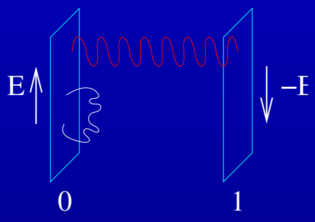

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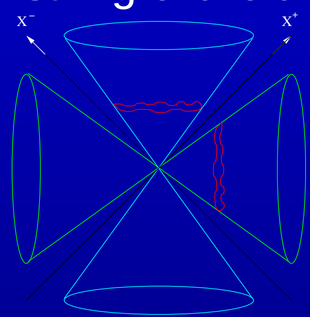

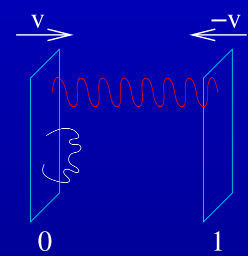

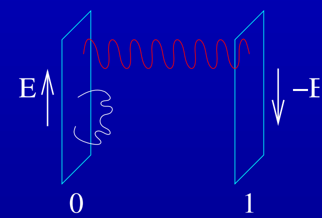

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- As we shall see, the analogy is quite precise, under identifying $w\beta \sim$ ArcTan (E): can some analogue of the Schwinger effect cause the boost parameter to go to zero ?

Outline of the talk

1. Introduction

2. First quantization: first pass

Bachas Porrati; Nekrasov

3. First quantization: second pass

Berkooz BP

4. Second quantization: zeroth pass

Berkooz BP Rozali

5. Conclusions, speculations

Open strings in constant electromag. field vs orbifolds

- Open strings couple to an electromagnetic field through their boundary only. The embedding coordinates are free bosons on the Minkowskian strip $0 < \sigma < \pi, \quad \tau \in R$,

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- Twisted closed strings in orbifolds satisfy

$$X^\mu(\sigma+2\pi,\tau) = R^\mu_\nu\, X^\nu(\sigma,\tau) \quad \Rightarrow \quad e^{-2\pi i\omega_n} = R^\mu_\nu$$

- Twisted closed strings and charged open strings have the same eigenfrequencies when $R = (1+F)/(1-F)$. For $R = e^{\pm\beta}$, this is $w\beta = \text{Arctanh}E$.

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$$\omega_n = n \pm \nu \ , \quad \pi\nu = \text{ArcTan}\, b_1 - \text{ArcTan}\, b_0$$

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  This instability is due to Schwinger production of charged pairs.

Open string mode expansion

- The light-cone embedding coordinates may be expanded in orthonormal modes,

$$X^{\pm} = x_0^{\pm} + i \sum_{n=-\infty}^{+\infty} (-)^n (n \pm i\nu)^{-1} a_n^{\pm} e^{-i(n \pm i\nu)\tau} \cos[(n \pm i\nu)\sigma \mp i \,\text{arcth}(\pi e_0)]$$

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- The world-sheet Hamiltonian, normal ordered with respect to this vacuum, takes the form

$$L_0^{l.c.} = -\sum_{m=0}^{\infty} a_{-m}^+ a_m^- - \sum_{m=1}^{\infty} a_{-m}^- a_m^+ + \frac{i\nu}{2}(1 - i\nu) - \frac{1}{12}$$

One-loop amplitude and Schwinger pair production

- Using this quantization scheme, the one-loop (Euclidean worldsheet, Minkowskian target) vacuum free energy reads

$$A_{bos} = \frac{i\pi V_{26}(e_0+e_1)}{2} \int_0^\infty \frac{dt}{(4\pi^2 t)^{13}} \frac{e^{-\pi\nu^2 t/2}}{\eta^{21}(it/2)\ \theta_1(t\nu/2; it/2)}$$

where θ_1 is the Jacobi theta function,

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- In particular, the contribution of the zero-modes is

$$\frac{1}{2i\sin\pi t\nu/2} = e^{-\pi t\nu/2}(1+e^{-\pi t\nu}+e^{-2\pi t\nu}+\dots)$$

consistent with the quantization scheme $a_0^+|0\rangle = 0$

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$$\frac{1}{2i \sin \pi t\nu/2} = e^{-\pi t\nu/2}(1 + e^{-\pi t\nu} + e^{-2\pi t\nu} + \dots)$$

consistent with the quantization scheme $a_0^+|0\rangle = 0$

- Each of the poles at $t = 2k/\nu$ contributes to the imaginary part, yielding the rate for charged string pair production,

Bachas Porrati

$$\mathcal{W} = \frac{1}{2(2\pi)^{25}} \frac{(e_0+e_1)}{\nu} \sum_{k=1}^{\infty} (-)^{k+1} \left(\frac{|\nu|}{k}\right)^{13} \sum_{N=-1}^{\infty} c_b(N) \exp\left(-2\pi k \frac{N}{|\nu|} - 2\pi k|\nu|\right)$$

Closed string mode expansion

- Eigenmodes of closed strings in the twisted sector of order w are free fields satisfying

$$X^{\pm}(\sigma + 2\pi, \tau) = e^{\pm\nu} X^{\pm}(\sigma, \tau) \;, \quad \nu = w\beta$$

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hence the normal mode expansion:

$$\begin{aligned} X_R^{\pm}(\tau - \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha_n^{\pm} e^{-i(n \pm i\nu)(\tau - \sigma)} \\ X_L^{\pm}(\tau + \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \mp i\nu)^{-1} \tilde{\alpha}_n^{\pm} e^{-i(n \mp i\nu)(\tau + \sigma)} \end{aligned}$$

with canonical commutation relations

$$[\alpha_m^+, \alpha_n^-] = -(m + i\nu)\delta_{m+n} \quad , \quad [\tilde{\alpha}_m^+, \tilde{\alpha}_n^-] = -(m - i\nu)\delta_{m+n}$$
$$(\alpha_m^{\pm})^* = \alpha_{-m}^{\pm} \quad , \quad (\tilde{\alpha}_m^{\pm})^* = \tilde{\alpha}_{-m}^{\pm}$$

- In particular, zero-modes are isomorphic to the open string case:

$$[\alpha_0^+, \alpha_0^-] = -i\nu \ , \quad [\tilde{\alpha}_0^+, \tilde{\alpha}_0^-] = i\nu$$

Vacuum energy and physical states (absence thereof)

- Representing these oscillators on a Fock space with vacuum $|0\rangle$ annihilated by all $\alpha^{\pm}_{n>0}$ and by α^-_0, the normal ordered worldsheet Hamiltonian reads

$$L_0^{l.c.} = -\sum_{n=0}^{\infty} (\alpha_n^+)^* \alpha_n^- - \sum_{n=1}^{\infty} (\alpha_n^-)^* \alpha_n^+ + \frac{1}{2} i\nu(1-i\nu) - 1 + L_{int}$$

with a similar answer for $\tilde{L}_0$.

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$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=0}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho) x\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

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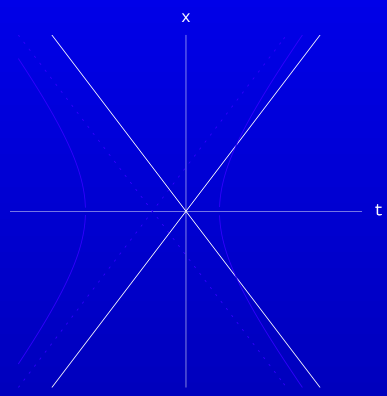
Nekrasov

- Much as in the case of the thermal BTZ black hole, the integrand has poles in the bulk of the moduli space.

Ooguri Maldacena

Open string zero-modes



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$$X^{\pm} = x_0^{\pm} \pm \frac{1}{\nu} a_0^{\pm} e^{\pm \nu \tau} \cosh \nu \sigma$$

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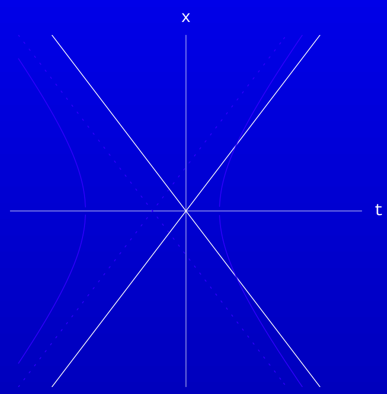

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- For $\sigma = 0$, this is just the trajectory of a charged particle in an electric field,

$$L = \frac{1}{2} m \left(-2 \partial_\tau X^+ \partial_\tau X^- + (\partial_\tau X^i)^2 \right) + \frac{e}{2} \left(X^+ \partial_\tau X^- - X^- \partial_\tau X^+ \right)$$

The canonical momenta

$$\pi^{\pm} = m\, \partial_\tau X^{\pm} \mp \frac{e}{2} X^{\pm} = \mp \frac{e}{2} x_0^{\pm} + \frac{1}{2} a_0^{\pm} e^{\pm e\tau/m} \,, \quad \pi^i = m\, \partial_\tau X^i = p^i$$

satisfy the usual equal-time commutation rules

$$[\pi^+, x^-] = [\pi^-, x^+] = i \,, \quad [\pi^i, x^j] = i \delta_{ij}$$

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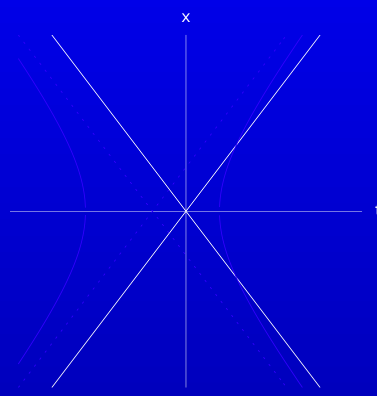

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- At $\tau = 0$, one can thus express

$$a_0^\pm = \pi^\pm \pm \frac{\nu}{2} x^\pm \ , \quad x_0^\pm = \mp \frac{1}{\nu} \left(\pi^\pm \mp \frac{\nu}{2} x^\pm \right)$$

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- Quantum mechanically, one may represent $\pi^\pm = i\partial/\partial x^\mp$ so that $a_0^\pm$ become covariant derivatives in the electric field ν.
- The zero-mode piece of L_0, including the evil $\frac{i\nu}{2}$,

$$L_0^{(0)} = -a_0^+ a_0^- + \frac{i\nu}{2} = -\frac{1}{2}(a_0^+ a_0^- + a_0^- a_0^+)$$

is just the Klein-Gordon operator of a particle of 2D mass $M^2 = -2L_0^{(0)}$ and charge ν.

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^{\pm} = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator just becomes an inverted harmonic oscillator:

$$M^2 = \alpha_0^+ \alpha_0^- + \alpha_0^- \alpha_0^+ = -\frac{1}{2}(P^2 - Q^2)$$

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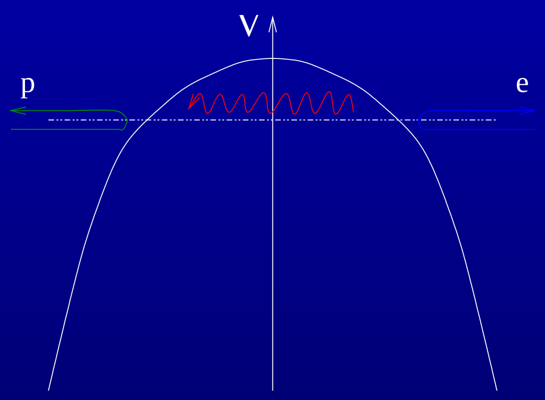
- The latter admits a respectable delta-normalizable spectrum of scattering states, in terms of parabolic cylinder functions, e.g:

$$\phi_{in}^+ = D_{-\frac{1}{2}+i\frac{M^2}{2\nu}}(e^{-\frac{3i\pi}{4}}u)e^{-i\tilde{p}t}e^{i\nu xt/2}$$



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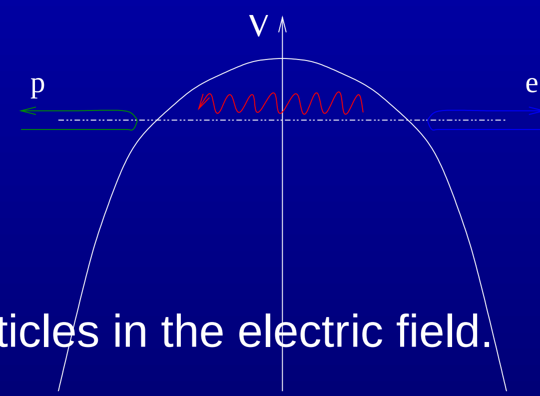

- These correspond to non-compact trajectories of charged particles in the electric field. Tunnelling is just (stimulated) Schwinger pair creation,

$$e^- \longrightarrow (1+\eta)\, e^- + \eta\, e^+ \,, \quad \eta \sim e^{-\pi M^2/\nu}$$

Brezin Itzykson; Brout Massar Parentani Spindel

Lorentzian vs Euclidean states

- Analytic continuation $X^0 \longrightarrow e^{-i\pi/2} X^0$, $\nu \longrightarrow e^{i\pi/2}\nu$ takes us from an electric field in $R^{1,1}$ to a magnetic field in R^2. At the same time, one should Wick rotate the worldsheet time.

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$$\frac{1}{2i\sin(\nu t/2)} = \sum_{n=1}^{\infty} e^{-i(n+\frac{1}{2})\nu t} = \int dM^2 \rho(M^2) e^{-M^2 t/2}$$

where the density of states is obtained from the reflection phase shift,

$$\rho(M^2) = \frac{1}{\nu}\log\Lambda - \frac{1}{2\pi i}\frac{d}{dM^2}\log\frac{\Gamma\left(\frac{1}{2}+i\frac{M^2}{2\nu}\right)}{\Gamma\left(\frac{1}{2}-i\frac{M^2}{2\nu}\right)}$$

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- The physical spectrum can be explicitely worked out at low levels, and is free of ghosts: a tachyon at level 0, a transverse gauge boson at level 1, . . .

Physical spectrum at low level

- The ground state tachyon

$$|T\rangle = \phi(x^+, x^-)|0_{ex}, k\rangle$$

should satisfy the Virasoro constraint

$$L_0|T\rangle = \left[-\frac{1}{2}\left(\alpha_0^+\alpha_0^- + \alpha_0^-\alpha_0^+\right) + \frac{1}{2}\nu^2 - 1 + \frac{1}{2}k_i^2\right]|T\rangle$$

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- Despite the non-vanishing two-dimensional mass $k_i^2 - \nu^2$, the spurious state $L_{-1}\phi|0\rangle$ is still physical, eliminating an extra polarization. One thus has $D-2$ transverse degrees of freedom, ie a massless gauge boson in D dimensions.

Charged particle in Rindler space

- For applications to the Milne universe, one should diagonalize the boost momentum J, ie consider an accelerated observer.

Gabriel Spindel; Mottola Cooper

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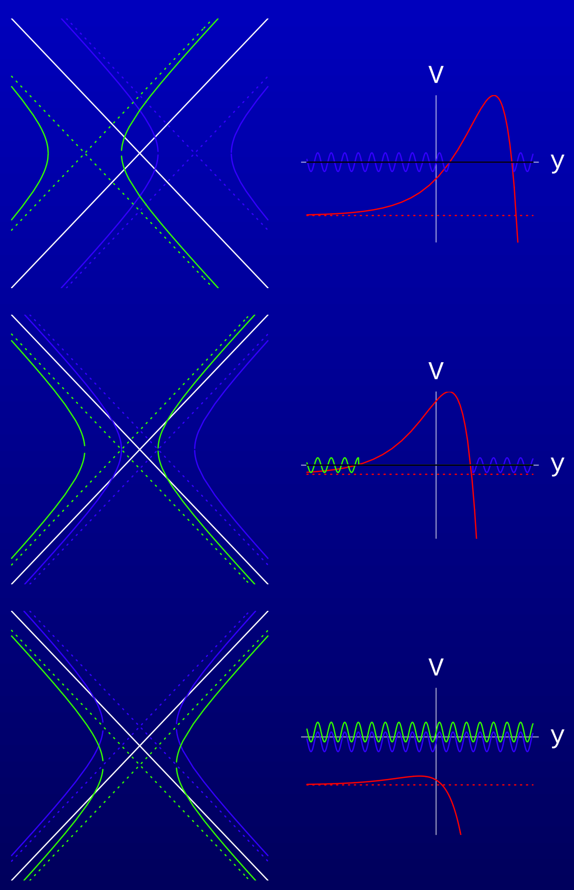

- In the Rindler patch R, letting $f(r,\eta) = e^{-iJ\eta}f_J(r)$ and $r = e^y$, one gets a Schrodinger equation for a particle in a potential

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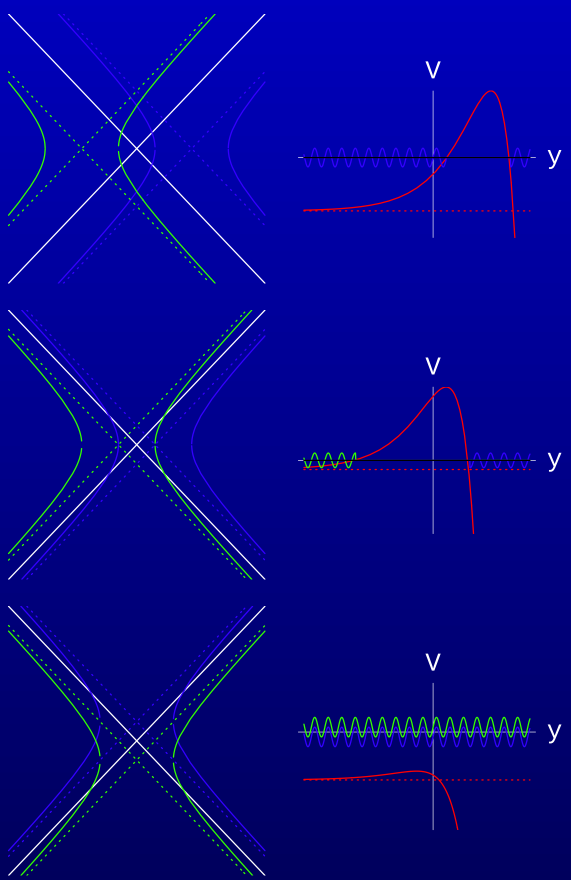

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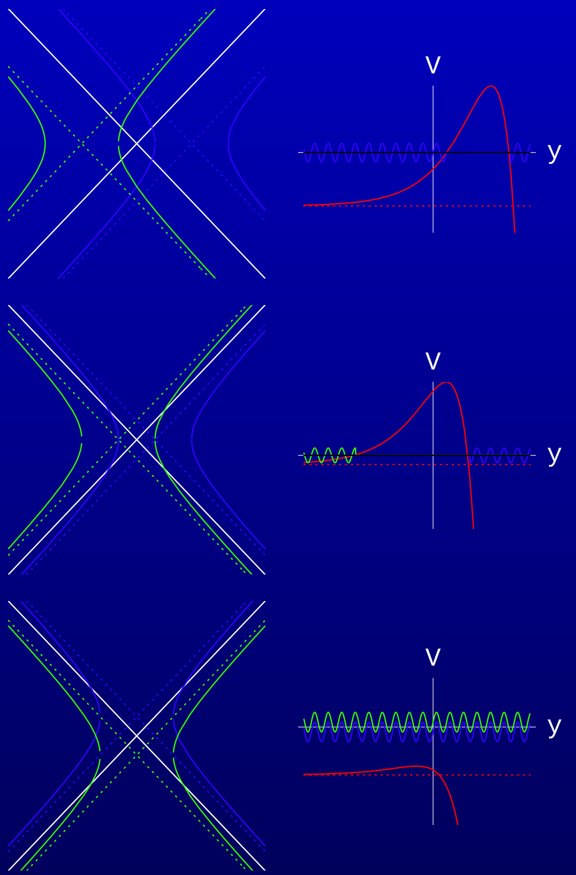

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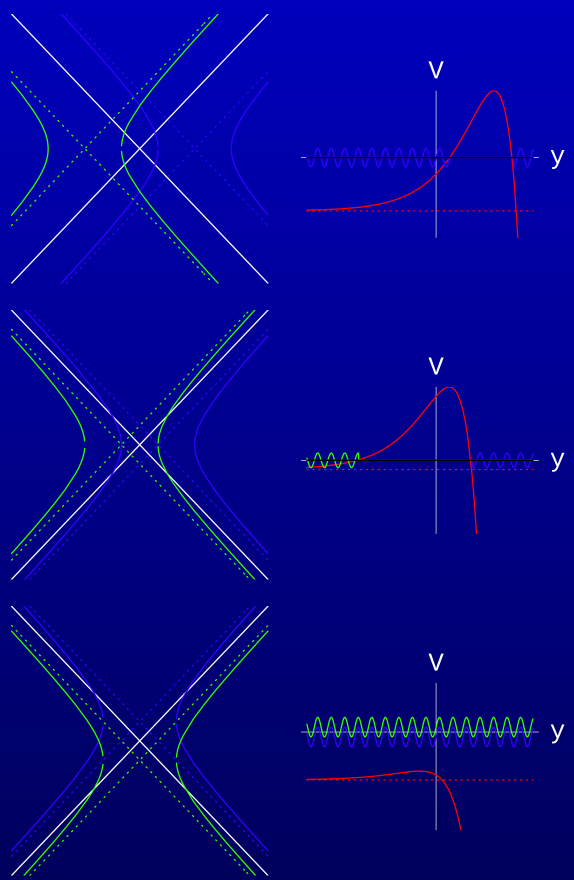

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- If $j > M^2/(2\nu)$, the electron branches extend in the Milne regions. There is no tunelling, but partial reflection amounts to a combination of Schwinger and Hawking emission.

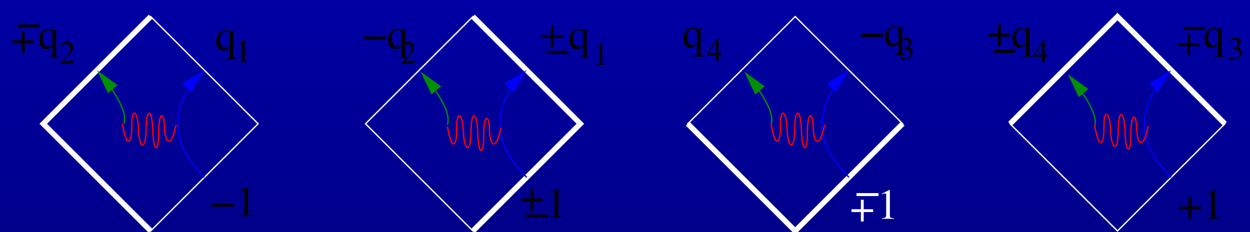
Rindler modes

- Solutions are expressable in terms of parabolic cylinder functions:
  Incoming modes from Rindler infinity I_R^- read

$$\mathcal{V}_{in,R}^{j} = e^{-ij\eta} r^{-1} M_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),-\frac{ij}{2}}(i\nu r^2/2)$$

Incoming modes from the Rindler horizon H_R^- read

$$\mathcal{U}_{in,R}^{j} = e^{-ij\eta} r^{-1} W_{i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}(-i\nu r^2/2)$$



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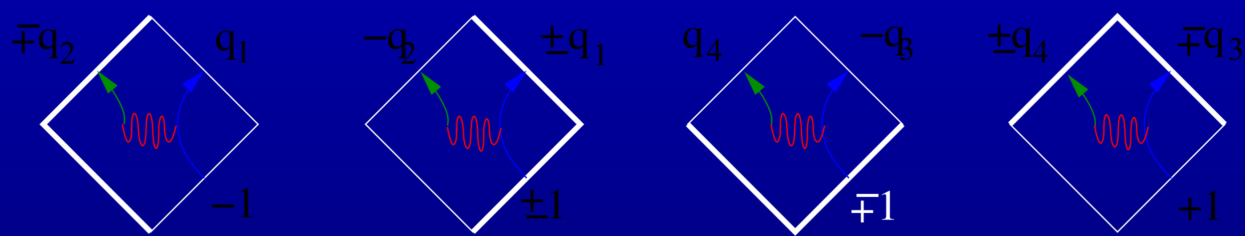

- The reflection coefficients can be computed:

$$q_1 = e^{-\pi j} \frac{\cosh\left[\pi \frac{M^2}{2\nu}\right]}{\cosh\left[\pi\left(j - \frac{M^2}{2\nu}\right)\right]}, \quad q_3 = e^{\pi\left(j-\frac{M^2}{2\nu}\right)} \frac{\cosh\left[\pi\frac{M^2}{2\nu}\right]}{|\sinh \pi j|}$$

and $q_2 = 1 - q_1$, $q_4 = q_3 - 1$, by charge conservation.

Global Charged Unruh Modes

- Global Unruh modes may be defined by patching together Rindler modes, ie by analytic continuation across the horizons:

$$\Omega^j_{in,+} = \mathcal{V}^j_{in,P} = W_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}(-i\nu X^+X^-)[X^+/X^-]^{-ij/2}$$

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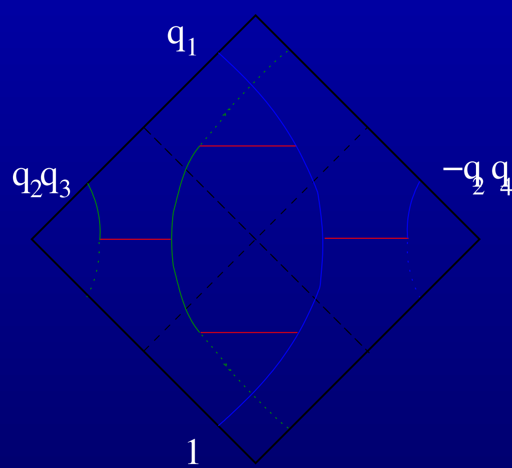
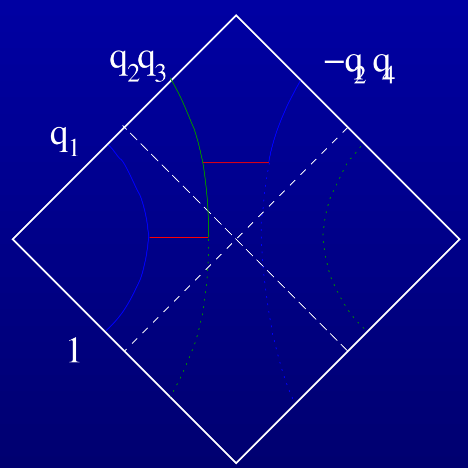
- There are two types of modes, involving 2 or 4 tunelling events:





Closed string zero-modes

- Let us analyze the classical solutions for the closed string zero modes

$$X^{\pm}(\tau,\sigma) = \pm\frac{1}{2\nu}\alpha_0^{\pm}e^{\pm\nu(\tau-\sigma)} \mp \frac{1}{2\nu}\tilde{\alpha}_0^{\pm}e^{\mp\nu(\tau+\sigma)}\ , \quad \alpha_0^{\pm}, \tilde{\alpha}_0^{\pm} \in R$$

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- The behavior at early/late proper time now depends on $\epsilon\tilde{\epsilon}$: For $\epsilon\tilde{\epsilon} = 1$, the string begin/ends in the Milne regions. For $\epsilon\tilde{\epsilon} = -1$, the string begin/ends in the Rindler regions.

Short and long strings ($j = 0$)

- $\epsilon = 1$, $\tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma, \tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma} \,, \quad T = \frac{M}{\nu} \sinh(\nu\tau) \,, \quad \theta = \nu\sigma$$

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- For $|j| > 0$, the short string in the Milne region attaches to a short string in the Rindler region stretching from $r = 0$ to $r_0 = |j|/(M + \tilde{M})$ and back. The induced worldsheet metric is of Misner type at the light-cone:

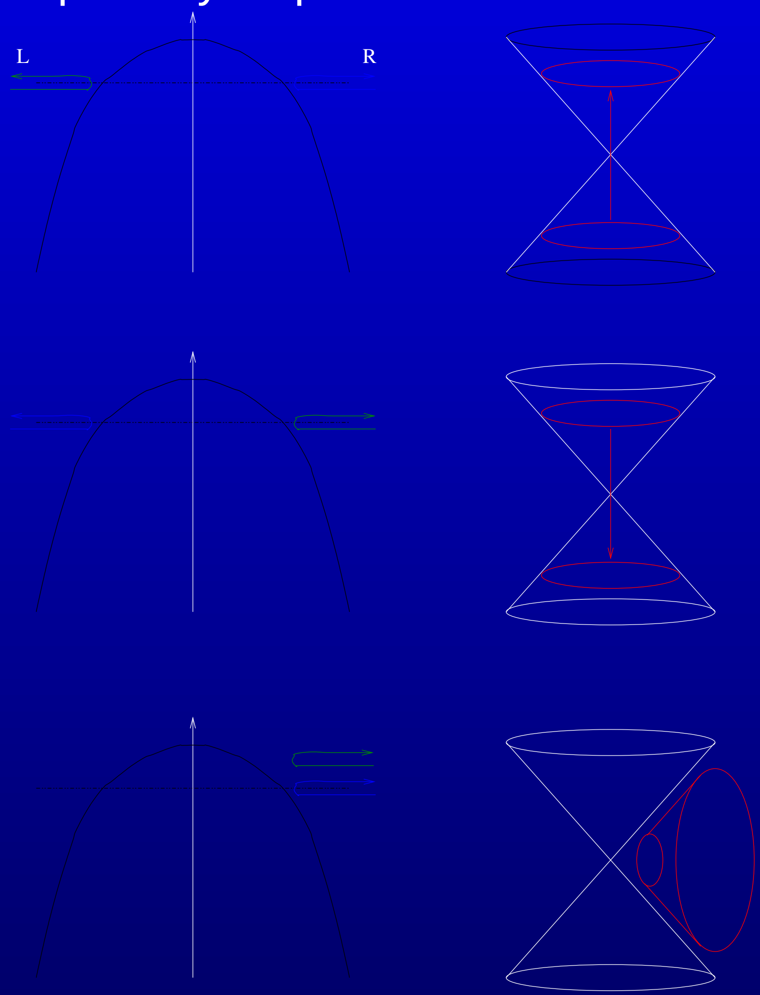
$$-2dX^+dX^- = -\nu j d\tau d\sigma + \nu|j|(\tau - \tau_0)d\sigma^2 - \frac{1}{2}(M^2 + \tilde{M}^2)d\tau^2$$

much like long strings or supertubes in Gödel Universe.

Drukker Fiol Simon; Israel

Short and long strings (static modes)

Just as in the open string case, we may now quantize the left and right-moving zero-modes separately as particles in inverted harmonic oscillator:



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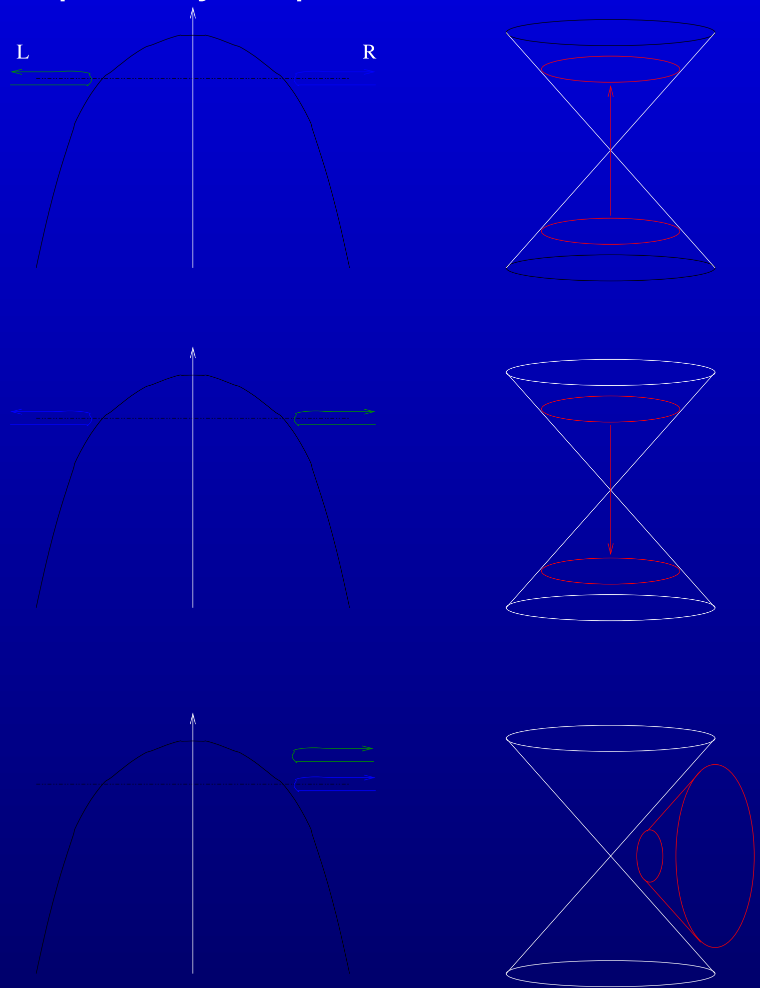

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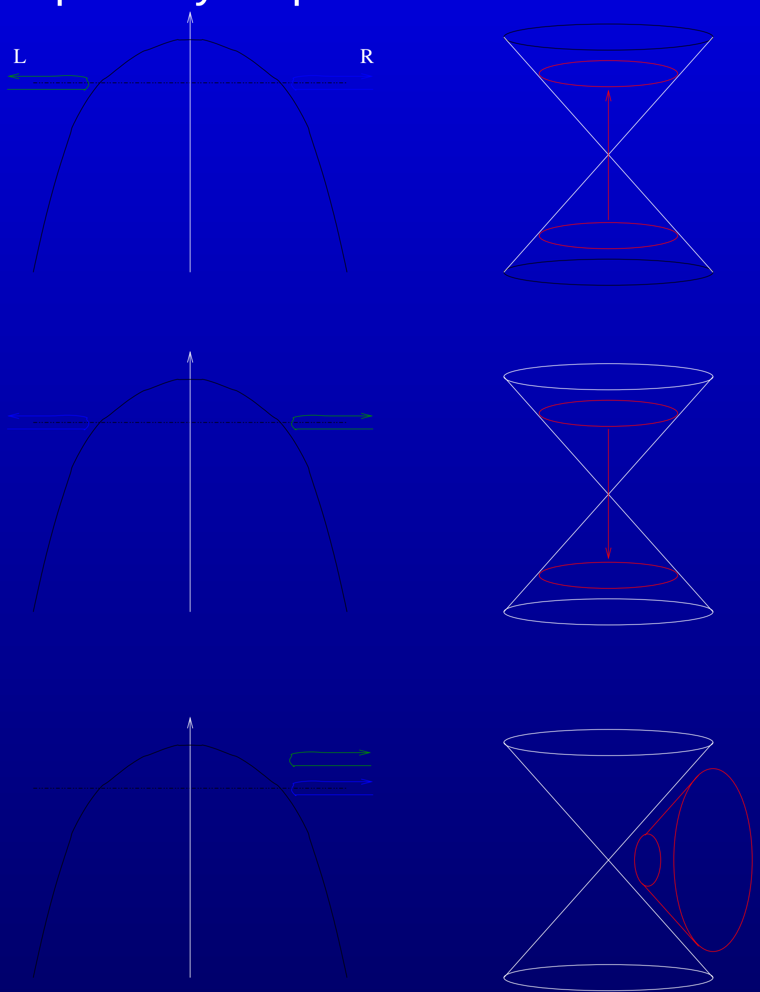

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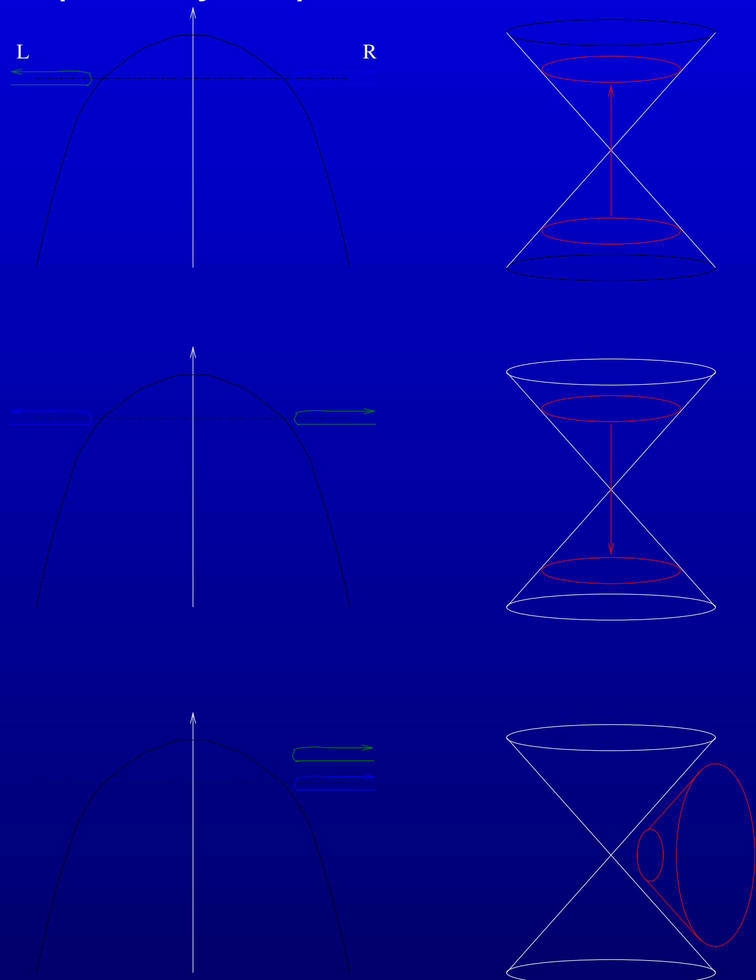

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Short and long strings, Unruh modes

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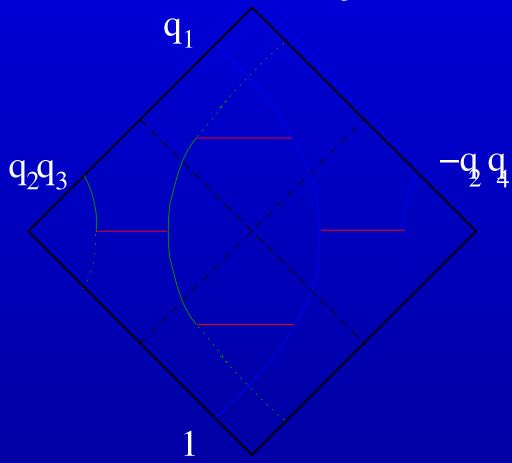

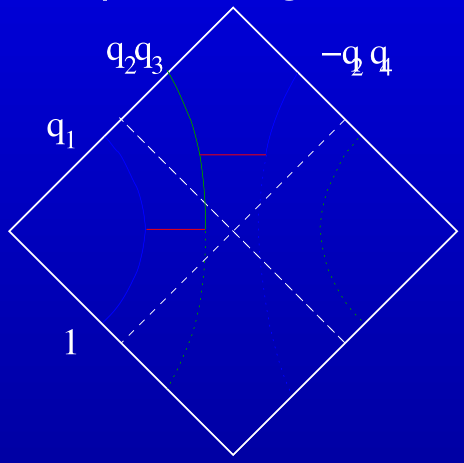

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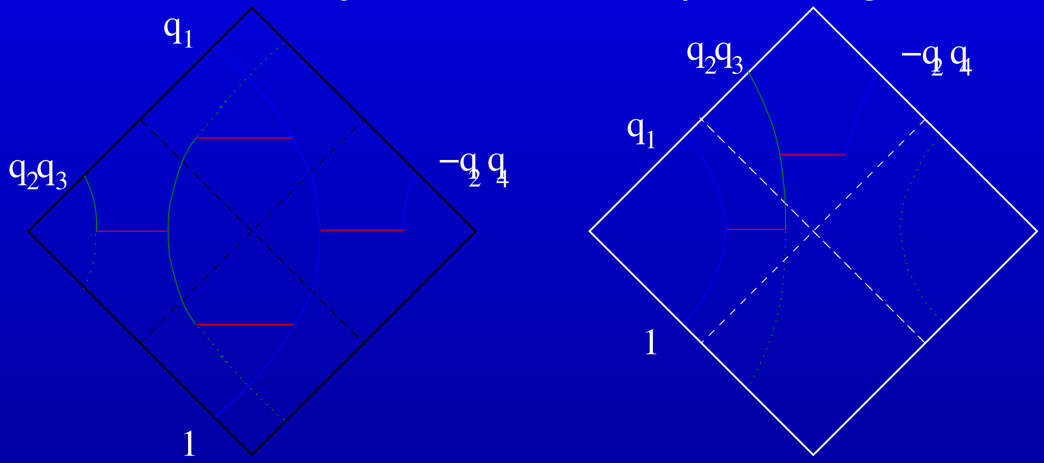

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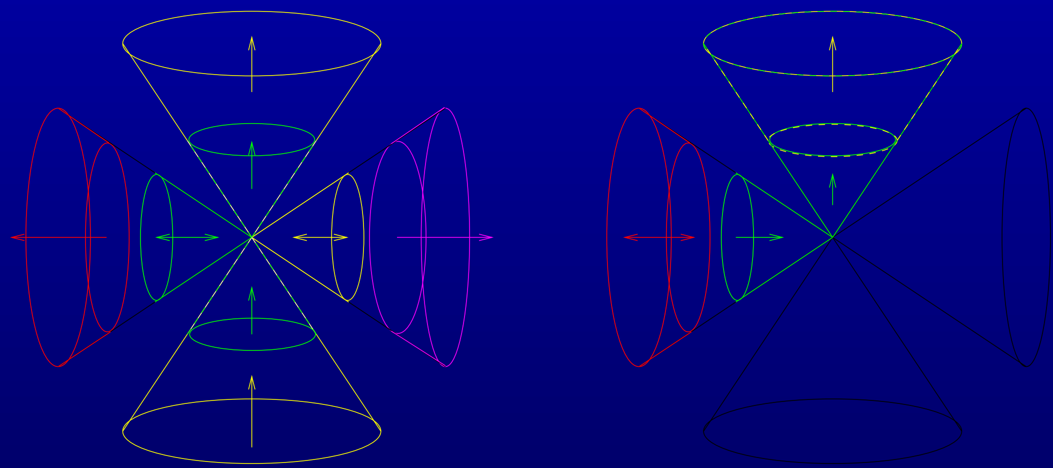

- The probability amplitude of winding strings at $T = +\infty$, *assuming that there are no stretched pairs in the whiskers*, is q_1 times the incoming amplitude at $T = -\infty$.

The one-loop amplitude again

- Recall the (Euclidean ws, Minkowskian target) one-loop amplitude:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=0}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{\left|\eta^{21}(\rho)\ \theta_1(i\beta(l+w\rho);\rho)\right|^2}$$

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  leading to logarithmic divergences, $\int d\rho d\bar{\rho}/|\rho-\rho_0|^2 \sim log\epsilon$.
- These can be traced to the existence of infinite families of periodic orbits, where all but one 4-uple $(\alpha^+_{\pm n}, \tilde{a}_{\pm n^+})$ (or its X^- counterpart) vanishes:

$$X^+ = \frac{i}{2}(n+i\nu)^{-1}\alpha^+_n e^{-i(n+i\nu)(\tau-\sigma)} + \frac{i}{2}(n-i\nu)^{-1}\tilde{\alpha}^\pm_n e^{-i(n-i\nu)(\tau+\sigma)}$$

  is periodic under $(\sigma,\tau) \to (\sigma+\rho_1, \tau+i\rho_2)$. These states are localized on the light-cone (currently under investigation)

Conclusions - speculations

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Cooper, Eisenberg, Kluger, Mottola and Svetitsky

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- Twisted sector states are produced in correlated pairs, i.e. squeezed states, whose condensation should involve non-local deformations of the worldsheet.

Aharony Berkooz Silverstein

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- As a less ambitious goal, can one compute scattering amplitudes of twisted states, and check if they are better behaved than untwisted states. For this, the relation with negative level $Sl(2)/U(1)$ and double analytic continuation of the Nappi-Witten plane wave may be useful.

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- The closed string orbifold we have discussed are highly non-generic trajectories on the cosmological billiard: Do whiskers feature also for more general Kasner-like singularities ?

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Open Strings in Electric Fields,
and the Milne Universe

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