Closed Strings in the Misner Universe

Boris Pioline
LPTHE, Paris

séminaire du GreCo
12 février 2004

based on hep-th/0307280 w/ M. Berkooz
and work in progress w/ M. Berkooz, B. Durin, D. Reichmann, M. Rozali

slides available from
`http://www.lpthe.jussieu.fr/~pioline/seminars.html`

Motivational string cosmology

- Observational Cosmology is now challenging string theory with high-precision data:

$$\Omega_{baryon} = 0.047\ , \quad \Omega_{darkm} = 0.243\ , \quad \Omega_{\Lambda} = 0.71\ , \quad w = -0.98 \pm .12\ , \ldots$$

Motivational string cosmology

- Observational Cosmology is now challenging string theory with high-precision data:

$$\Omega_{baryon} = 0.047 \; , \quad \Omega_{darkm} = 0.243 \; , \quad \Omega_{\Lambda} = 0.71 \; , \quad w = -0.98 \pm .12 \; , \ldots$$

- These results are so far (very) well explained by effective field theoretical inflationary models, yet their validity is not fool-proof: large potential energy density, transplankian fluctuations...

Motivational string cosmology

- Observational Cosmology is now challenging string theory with high-precision data:

$$\Omega_{baryon} = 0.047 \ , \quad \Omega_{darkm} = 0.243 \ , \quad \Omega_\Lambda = 0.71 \ , \quad w = -0.98 \pm .12 \ , \ldots$$

- These results are so far (very) well explained by effective field theoretical inflationary models, yet their validity is not fool-proof: large potential energy density, transplankian fluctuations...

- With the expected improved accuracy of cosmological measurements, it is conceivable that distinctive features of string theory may reveal themselves:
  1. UV softness, Regge behavior
  2. exponentially large density of states, limiting Hagedorn temperature $T_H \sim 1/l_s$
  3. existence of topological excitations, minimal length $R \geq l_s$ or rather $R_1 R_2 R_3 \geq l_M^3$
  4. holographic entropy bounds...

Motivational string cosmology

- Observational Cosmology is now challenging string theory with high-precision data:

$$\Omega_{baryon} = 0.047\ , \quad \Omega_{darkm} = 0.243\ , \quad \Omega_{\Lambda} = 0.71\ , \quad w = -0.98 \pm .12\ , \ldots$$

- These results are so far (very) well explained by effective field theoretical inflationary models, yet their validity is not fool-proof: large potential energy density, transplankian fluctuations...

- With the expected improved accuracy of cosmological measurements, it is conceivable that distinctive features of string theory may reveal themselves:
  1. UV softness, Regge behavior
  2. exponentially large density of states, limiting Hagedorn temperature $T_H \sim 1/l_s$
  3. existence of topological excitations, minimal length $R \geq l_s$ or rather $R_1 R_2 R_3 \geq l_M^3$
  4. holographic entropy bounds...

- With LHC still far in the future, understanding StringY Cosmology may be the only way to make contact with reality...

Time dependence in string theory

Attempts to discuss time dependent backgrounds in string theory immediately face difficulties:

Time dependence in string theory

Attempts to discuss time dependent backgrounds in string theory immediately face difficulties:

- First-quantized string theory is well suited for particle physics S-matrix computations around an asymptotically flat coherent background, with a unique stable vacuum.

Time dependence in string theory

Attempts to discuss time dependent backgrounds in string theory immediately face difficulties:

- First-quantized string theory is well suited for particle physics S-matrix computations around an asymptotically flat coherent background, with a unique stable vacuum.
- In contrast, time dependent backgrounds have no canonical vacuum state, due to particle production. On-shell S-matrix elements are replaced by off-shell transition amplitudes.

Time dependence in string theory

Attempts to discuss time dependent backgrounds in string theory immediately face difficulties:

- First-quantized string theory is well suited for particle physics S-matrix computations around an asymptotically flat coherent background, with a unique stable vacuum.
- In contrast, time dependent backgrounds have no canonical vacuum state, due to particle production. On-shell S-matrix elements are replaced by off-shell transition amplitudes.
- Closed string field theory would be the natural framework to address these questions, unfortunately it has remained untractable to this day. Can Bogolioubov transformations between vacua still be implemented in a first-quantized formalism ?

Time dependence in string theory

Attempts to discuss time dependent backgrounds in string theory immediately face difficulties:

- First-quantized string theory is well suited for particle physics S-matrix computations around an asymptotically flat coherent background, with a unique stable vacuum.
- In contrast, time dependent backgrounds have no canonical vacuum state, due to particle production. On-shell S-matrix elements are replaced by off-shell transition amplitudes.
- Closed string field theory would be the natural framework to address these questions, unfortunately it has remained untractable to this day. Can Bogolioubov transformations between vacua still be implemented in a first-quantized formalism ?
- Perturbative string theory requires an Euclidean worldsheet, hence Euclidean target space. The analytic continuation may be ambiguous or ill-defined, Lorentzian observables may be very different from their Euclidean counterparts.

Time dependence in string theory

Attempts to discuss time dependent backgrounds in string theory immediately face difficulties:

- First-quantized string theory is well suited for particle physics S-matrix computations around an asymptotically flat coherent background, with a unique stable vacuum.
- In contrast, time dependent backgrounds have no canonical vacuum state, due to particle production. On-shell S-matrix elements are replaced by off-shell transition amplitudes.
- Closed string field theory would be the natural framework to address these questions, unfortunately it has remained untractable to this day. Can Bogolioubov transformations between vacua still be implemented in a first-quantized formalism ?
- Perturbative string theory requires an Euclidean worldsheet, hence Euclidean target space. The analytic continuation may be ambiguous or ill-defined, Lorentzian observables may be very different from their Euclidean counterparts.
- String theory is not content on a finite time interval, and one is frequently forced into Big Bang / Big Crunch singularities, CTC in the process of maximally extending the geometry.

String theory and cosmological singularities

- Spacelike singularities occur for generic initial data and matter (with appropriate energy conditions) in classical gravity, can string theory avoid / resolve them ?

String theory and cosmological singularities

- Spacelike singularities occur for generic initial data and matter (with appropriate energy conditions) in classical gravity, can string theory avoid / resolve them ?
- Scattering amplitudes in gravity typically diverge due to large graviton exchange at high blue-shift, can the softer UV behavior of string theory tame these divergences ?

Liu Moore Seiberg; Berkooz Craps Kutasov Rajesh; Horowitz Polchinski

String theory and cosmological singularities

- Spacelike singularities occur for generic initial data and matter (with appropriate energy conditions) in classical gravity, can string theory avoid / resolve them ?
- Scattering amplitudes in gravity typically diverge due to large graviton exchange at high blue-shift, can the softer UV behavior of string theory tame these divergences ?

Liu Moore Seiberg; Berkooz Craps Kutasov Rajesh; Horowitz Polchinski

- String theory has a variety of extended objects that may become light at a space-like singularity, could their exchange dominate the dynamics and lead to finite amplitudes ?

String theory and cosmological singularities

- Spacelike singularities occur for generic initial data and matter (with appropriate energy conditions) in classical gravity, can string theory avoid / resolve them ?
- Scattering amplitudes in gravity typically diverge due to large graviton exchange at high blue-shift, can the softer UV behavior of string theory tame these divergences ?

Liu Moore Seiberg; Berkooz Craps Kutasov Rajesh; Horowitz Polchinski

- String theory has a variety of extended objects that may become light at a space-like singularity, could their exchange dominate the dynamics and lead to finite amplitudes ?
- Usual Euclidean (static) orbifolds can often be resolved by condensation of twisted sector fields. Does a similar process take place for spacelike singularities ?

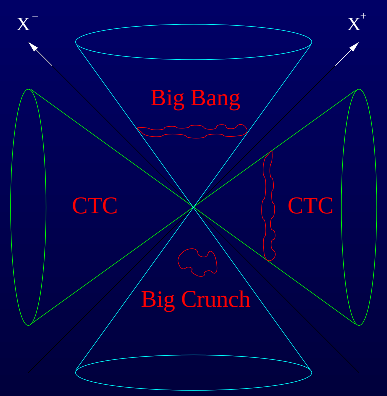
String theory and cosmological singularities

- Spacelike singularities occur for generic initial data and matter (with appropriate energy conditions) in classical gravity, can string theory avoid / resolve them ?
- Scattering amplitudes in gravity typically diverge due to large graviton exchange at high blue-shift, can the softer UV behavior of string theory tame these divergences ?

Liu Moore Seiberg; Berkooz Craps Kutasov Rajesh; Horowitz Polchinski

- String theory has a variety of extended objects that may become light at a space-like singularity, could their exchange dominate the dynamics and lead to finite amplitudes ?
- Usual Euclidean (static) orbifolds can often be resolved by condensation of twisted sector fields. Does a similar process take place for spacelike singularities ?

- In this talk, we shall discuss the “Lorentzian” orbifold of flat Minkowski space by a discret boost, as a toy model of a singular cosmological universe where string theory can in principle be solved explicitely.

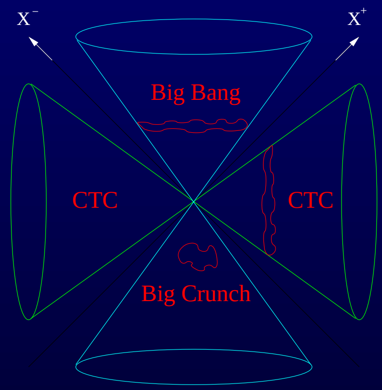


String theory and cosmological singularities

- Spacelike singularities occur for generic initial data and matter (with appropriate energy conditions) in classical gravity, can string theory avoid / resolve them ?
- Scattering amplitudes in gravity typically diverge due to large graviton exchange at high blue-shift, can the softer UV behavior of string theory tame these divergences ?

Liu Moore Seiberg; Berkooz Craps Kutasov Rajesh; Horowitz Polchinski

- String theory has a variety of extended objects that may become light at a space-like singularity, could their exchange dominate the dynamics and lead to finite amplitudes ?
- Usual Euclidean (static) orbifolds can often be resolved by condensation of twisted sector fields. Does a similar process take place for spacelike singularities ?
- In this talk, we shall discuss the "Lorentzian" orbifold of flat Minkowski space by a discret boost, as a toy model of a singular cosmological universe where string theory can in principle be solved explicitely.



- We shall focus in particular on the topological excitations which wind around the collapsing dimension: can the production of winding states resolve the singularity ?

Outline of the talk

1. Introduction
2. The Lorentzian orbifold and its avatars

   Misner, Taub-NUT, Grant...
3. Closed strings in Misner space: first pass

   Nekrasov
3. A detour: Open strings in electric fields

   Berkooz BP
4. Closed strings in Misner space: second pass

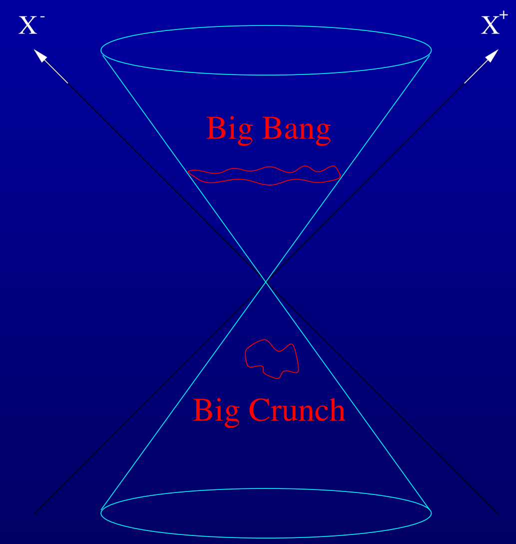
   Berkooz BP; Berkooz Durin BP Reichmann Rozali
5. Conclusions, speculations

The Lorentzian orbifold

- One of the simplest examples of space-like singularities is the quotient of flat Minkowski space by a discrete boost, also known as Misner space (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^\pm \sim e^{\pm 2\pi\beta} X^\pm$$

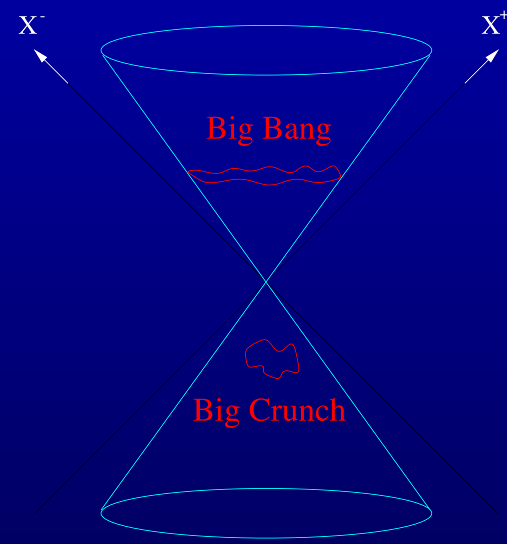


The Lorentzian orbifold

- One of the simplest examples of space-like singularities is the quotient of flat Minkowski space by a discrete boost, also known as Misner space (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^\pm \sim e^{\pm 2\pi\beta} X^\pm$$



- The future (past) regions $X^+X^- > 0$ describes a cosmological universe often known as the Milne universe (1932), linearly expanding (contracting) away from a Big Bang singularity (toward a Big Crunch singularity):

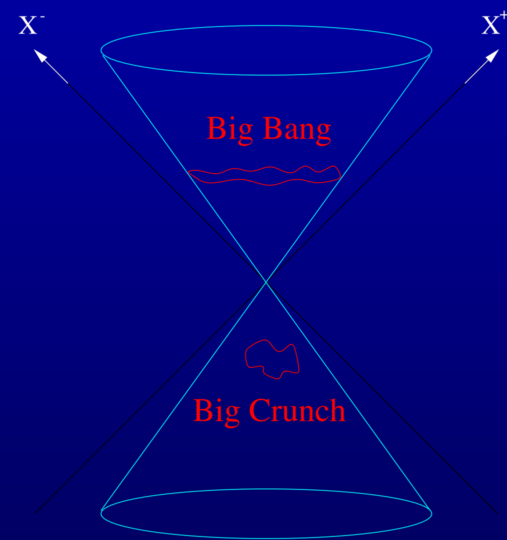
$$ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + (dX^i)^2\ , \quad \theta \equiv \theta + 2\pi\ , \quad X^\pm = T e^{\pm\beta\theta}/\sqrt{2}$$

The Lorentzian orbifold

- One of the simplest examples of space-like singularities is the quotient of flat Minkowski space by a discrete boost, also known as Misner space (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^{\pm} \sim e^{\pm 2\pi\beta} X^{\pm}$$



- The future (past) regions $X^+X^- > 0$ describes a cosmological universe often known as the Milne universe (1932), linearly expanding (contracting) away from a Big Bang singularity (toward a Big Crunch singularity):

$$ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + (dX^i)^2 \,, \quad \theta \equiv \theta + 2\pi \,, \quad X^{\pm} = Te^{\pm\beta\theta}/\sqrt{2}$$

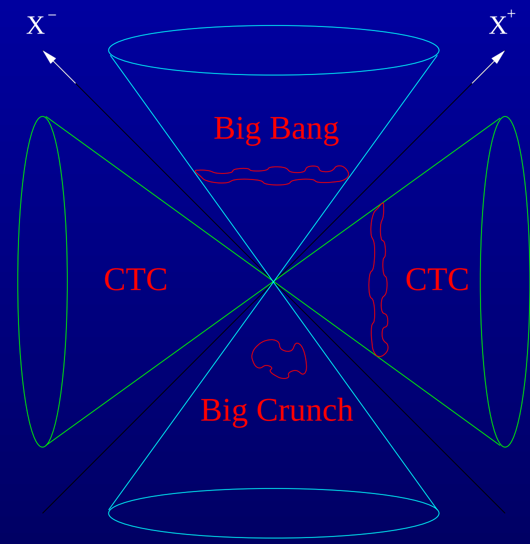
This is a (degenerate) Kasner singularity, everywhere flat, but for a delta-function curvature at $T = 0$.

The Lorentzian orbifold

- One of the simplest examples of space-like singularities is the quotient of flat Minkowski space by a discrete boost, also known as Misner space (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^\pm \sim e^{\pm 2\pi\beta} X^\pm$$

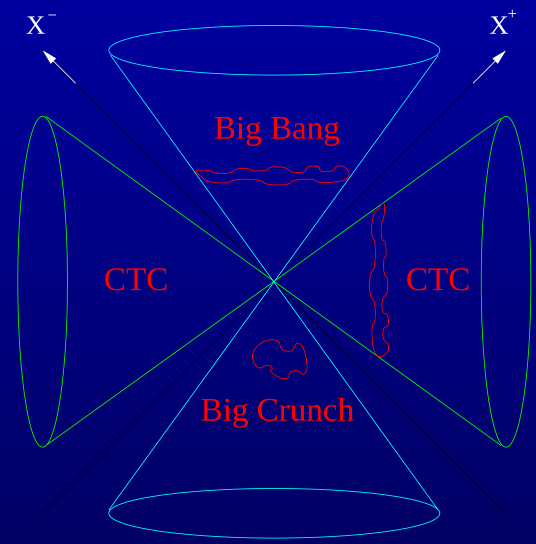


The Lorentzian orbifold

- One of the simplest examples of space-like singularities is the quotient of flat Minkowski space by a discrete boost, also known as Misner space (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^\pm \sim e^{\pm 2\pi\beta} X^\pm$$



- In addition, the spacelike regions $X^+X^- < 0$ describe two Rindler wedges with compact time, often known as whiskers, leading to closed time-like curves:

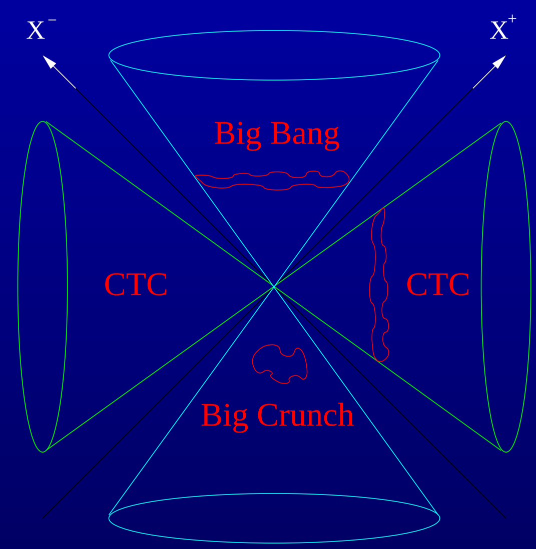
$$ds^2 = dr^2 - \beta^2 r^2 d\eta^2 + (dX^i)^2 \quad , \eta \equiv \eta + 2\pi \ , \quad X^\pm = \pm r e^{\pm\beta\eta}/\sqrt{2}$$

The Lorentzian orbifold

- One of the simplest examples of space-like singularities is the quotient of flat Minkowski space by a discrete boost, also known as Misner space (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^\pm \sim e^{\pm 2\pi\beta} X^\pm$$



- In addition, the spacelike regions $X^+X^- < 0$ describe two Rindler wedges with compact time, often known as whiskers, leading to closed time-like curves:

$$ds^2 = dr^2 - \beta^2 r^2 d\eta^2 + (dX^i)^2 \quad , \eta \equiv \eta + 2\pi \, , \quad X^\pm = \pm r e^{\pm\beta\eta}/\sqrt{2}$$

- Finally, the lightcone $X^+X^- = 0$ gives rise to non-Hausdorff sets with a degenerate metric, attached to the singularities.

Close relatives of the Misner Universe

- Misner space was first introduced as a local model of Lorentzian Taub-NUT space:

$$ds^2 = 4l^2 U(t)\sigma_3^2 + 4l\sigma_3 dt + (t^2 + l^2)(\sigma_1^2 + \sigma_2^2) \ , \quad U(t) = -1 + \frac{2mt + l^2}{t^2 + l^2}$$

A bouncing universe, isomorphic to $R^{1,1}/boost \times S^2$ around each singularity.

Close relatives of the Misner Universe

- Misner space was first introduced as a local model of Lorentzian Taub-NUT space:

$$ds^2 = 4l^2 U(t)\sigma_3^2 + 4l\sigma_3 dt + (t^2 + l^2)(\sigma_1^2 + \sigma_2^2)\ , \quad U(t) = -1 + \frac{2mt + l^2}{t^2 + l^2}$$

A bouncing universe, isomorphic to $R^{1,1}/boost \times S^2$ around each singularity.

- A close variant of Misner space is the quotient of flat space by the combination of a discrete boost and a translation on an extra direction, often known as the Grant space:

$$ds^2 = -2dX^+ dX^- + dX^2 + (dX^i)^2\ , \quad (X^\pm, X) \sim (e^{\pm 2\pi\beta} X^\pm, X + 2\pi R)$$

This describes the space away from two moving cosmic strings. Due to the absence of fixed point, the cosmological singularity is smoothed out.

Gott 91, Grant 93; Cornalba, Costa, Kounnas

Close relatives of the Misner Universe

- Misner space was first introduced as a local model of Lorentzian Taub-NUT space:

$$ds^2 = 4l^2 U(t)\sigma_3^2 + 4l\sigma_3 dt + (t^2 + l^2)(\sigma_1^2 + \sigma_2^2)\ , \quad U(t) = -1 + \frac{2mt + l^2}{t^2 + l^2}$$

A bouncing universe, isomorphic to $R^{1,1}/boost \times S^2$ around each singularity.

- A close variant of Misner space is the quotient of flat space by the combination of a discrete boost and a translation on an extra direction, often known as the Grant space:

$$ds^2 = -2dX^+ dX^- + dX^2 + (dX^i)^2\ , \quad (X^\pm, X) \sim (e^{\pm 2\pi\beta} X^\pm, X + 2\pi R)$$

This describes the space away from two moving cosmic strings. Due to the absence of fixed point, the cosmological singularity is smoothed out.

Gott 91, Grant 93; Cornalba, Costa, Kounnas

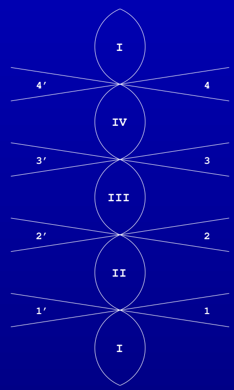
- The Misner geometry arose again more recently as the M-theory lift of a simple (ekpyrotic) cosmological solution of Einstein-dilaton gravity with no potential.

Khoury Ovrut Seiberg Steinhard Turok

Close relatives of the Misner Universe (cont)

- The gauged WZW model $Sl(2) \times Sl(2)/U(1) \times U(1)$ describes a bouncing 4-dimensional Universe, with singularities analogous to the Lorentzian orbifold.

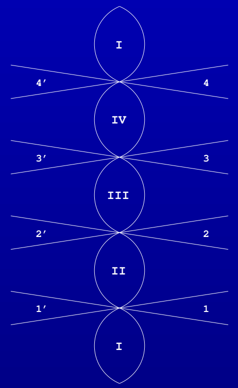
Nappi Witten; Elitzur Giveon Kutasov Rabinovici



Close relatives of the Misner Universe (cont)

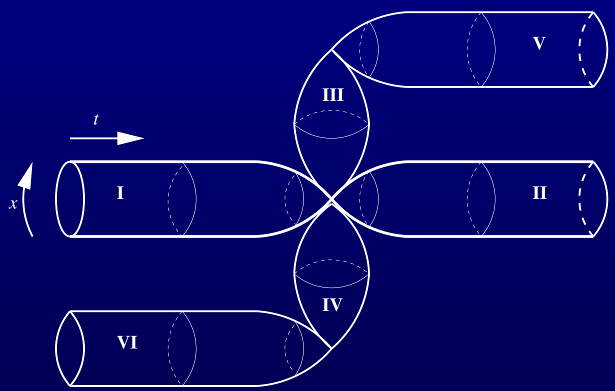
- The gauged WZW model $Sl(2) \times Sl(2)/U(1) \times U(1)$ describes a bouncing 4-dimensional Universe, with singularities analogous to the Lorentzian orbifold.

  Nappi Witten; Elitzur Giveon Kutasov Rabinovici



- The gauged WZW model $Sl(2)/U(1)$ at negative level orbifolded by a boost J describes two parallel Universes with a curvature and a Milne singularity, and compact whiskers.

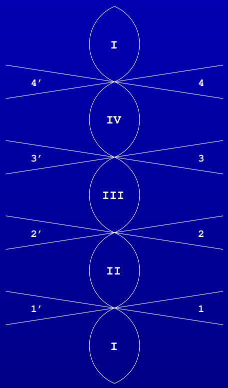
  Tseytlin Vafa; Craps Kutasov Rajesh; Craps Ovrut



Close relatives of the Misner Universe (cont)

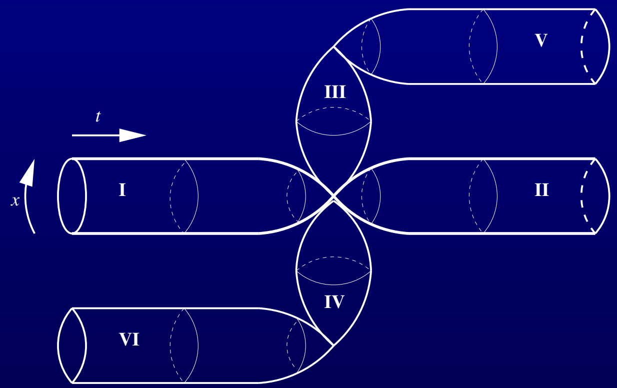
- The gauged WZW model $Sl(2) \times Sl(2)/U(1) \times U(1)$ describes a bouncing 4-dimensional Universe, with singularities analogous to the Lorentzian orbifold.

  Nappi Witten; Elitzur Giveon Kutasov Rabinovici



- The gauged WZW model $Sl(2)/U(1)$ at negative level orbifolded by a boost J describes two parallel Universes with a curvature and a Milne singularity, and compact whiskers.

  Tseytlin Vafa; Craps Kutasov Rajesh; Craps Ovrut

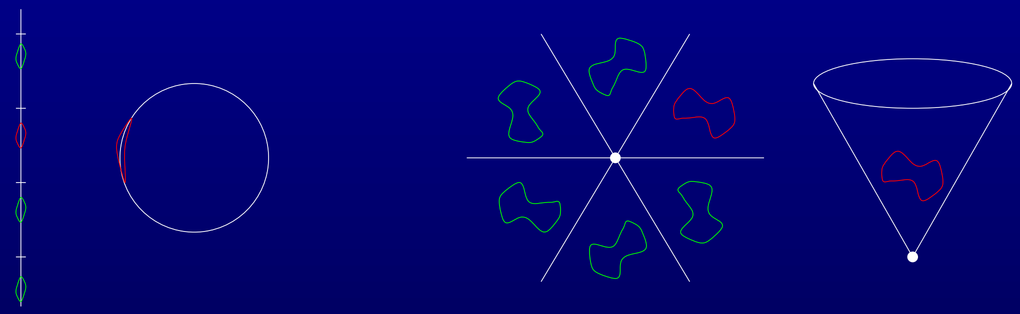


- The Lorentzian orientifold $IIB/[(-)^F boost]/[\Omega(-)^{F_L}]$ was also recently argued to describe orientifolds of non-supersymmetric strings with non-vanishing Neveu-Schwarz tadpoles.

  Dudas Mourad Timirgaziu

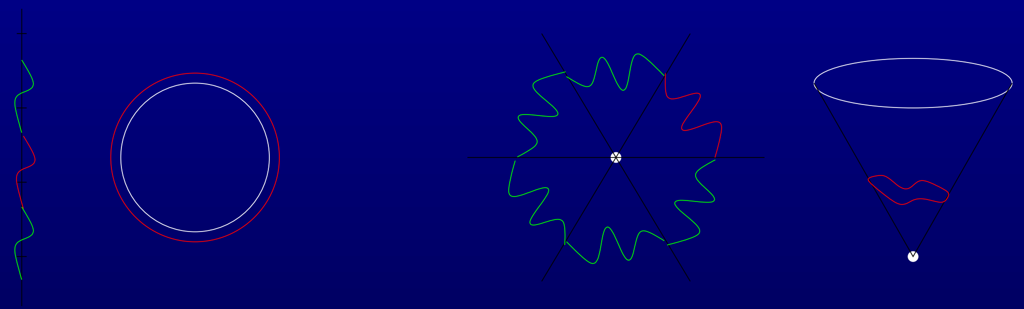
Strings on Euclidean orbifolds - untwisted states

- One way to obtain non-trivial yet solvable backgrounds in string theory is the orbifold construction: to a CFT with a discrete global symmetry G, associate a CFT' with only G-invariant states. Simple examples are the circle, R/Z, and the rotation orbifold R^2/Z_k.
- The spectrum of the quotient theory contains closed string states of the parent theory which are invariant under G: untwisted states.



Strings on Euclidean orbifolds - twisted states

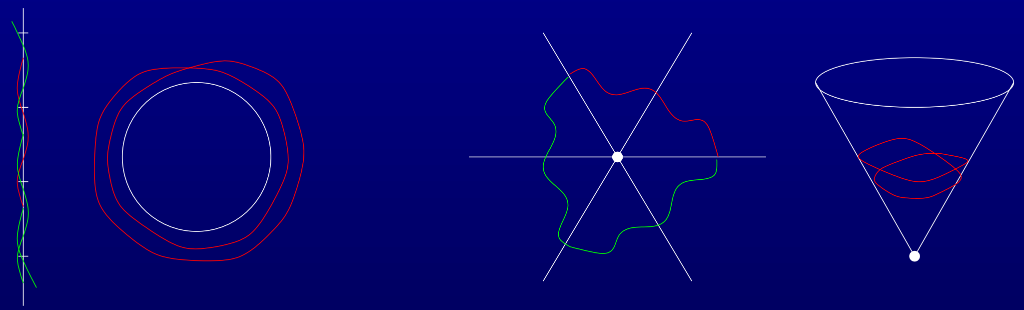
- One way to obtain non-trivial yet solvable backgrounds in string theory is the orbifold construction: to a CFT with a discrete global symmetry G, associate a CFT' with only G-invariant states. Simple examples are the circle, R/Z, and the rotation orbifold R^2/Z_k.
- Modular invariance requires that the spectrum should also include closed strings in the quotient theory which close up to the action of G in the parent theory: twisted states.



Strings on Euclidean orbifolds - twisted sectors

- One way to obtain non-trivial yet solvable backgrounds in string theory is the orbifold construction: to a CFT with a discrete global symmetry G, associate a CFT' with only G-invariant states. Simple examples are the circle, R/Z, and the rotation orbifold R^2/Z_k.
- Twisted sectors are labelled by conjugacy classes of G. Higher twisted sectors correspond to multiply wound states

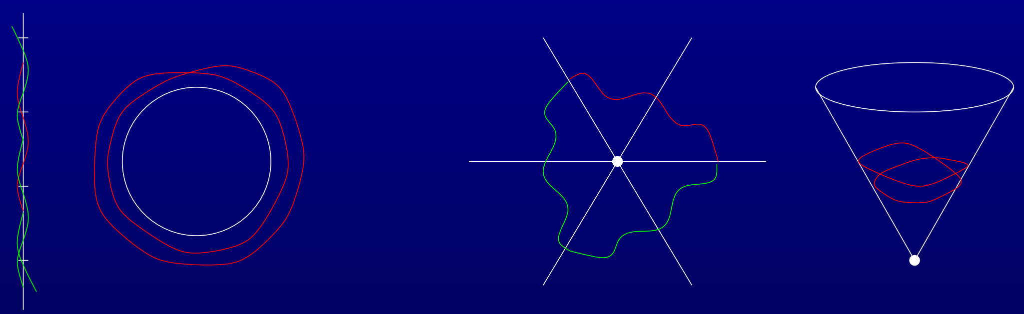
.



Strings on Euclidean orbifolds - twisted sectors

- One way to obtain non-trivial yet solvable backgrounds in string theory is the orbifold construction: to a CFT with a discrete global symmetry G, associate a CFT' with only G-invariant states. Simple examples are the circle, R/Z, and the rotation orbifold R^2/Z_k.
- Twisted sectors are labelled by conjugacy classes of G. Higher twisted sectors correspond to multiply wound states

.

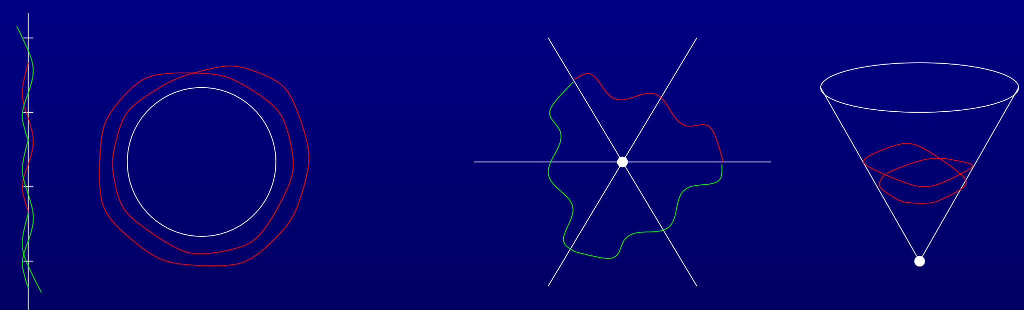


- When G acts non-freely, the twisted sector states are localized at the fixed points. They yield new localized degrees of freedom, which ensure the consistency of the background: anomaly free, divergence free...

Strings on Euclidean orbifolds - twisted sectors

- One way to obtain non-trivial yet solvable backgrounds in string theory is the orbifold construction: to a CFT with a discrete global symmetry G, associate a CFT' with only G-invariant states. Simple examples are the circle, R/Z, and the rotation orbifold R^2/Z_k.
- Twisted sectors are labelled by conjugacy classes of G. Higher twisted sectors correspond to multiply wound states

.



- When G acts non-freely, the twisted sector states are localized at the fixed points. They yield new localized degrees of freedom, which ensure the consistency of the background: anomaly free, divergence free...
- The condensation of these twisted states changes the vacuum, and effectively resolves the singularity: $R^2/Z_k \to R^2/Z_{k-1} \to \ldots$ (tachyon), $R^4/Z_k \to$ multi-centered Eguchi-Hanson (massless mode).

Closed strings in Misner space - untwisted states

- As usual in standard orbifolds, part of the spectrum involves closed strings on Minkowski covering space, which are invariant under the orbifold projection. In conformal gauge,

$$X^{\pm}(\sigma + 2\pi, \tau) = X^{\pm}(\sigma, \tau) \ , \quad (\partial_\tau^2 - \partial_\sigma^2) X^{\pm} = 0$$

satisfying the Virasoro (physical state) condition $(\dot{X}^\mu \pm X'^\mu)^2 = 0$.

Closed strings in Misner space - untwisted states

- As usual in standard orbifolds, part of the spectrum involves closed strings on Minkowski covering space, which are invariant under the orbifold projection. In conformal gauge,

$$X^{\pm}(\sigma + 2\pi, \tau) = X^{\pm}(\sigma, \tau) \ , \quad (\partial_\tau^2 - \partial_\sigma^2) X^{\pm} = 0$$

satisfying the Virasoro (physical state) condition $(\dot{X}^\mu \pm X'^\mu)^2 = 0$.

- Vertex operators (or states) can be obtained by (infinite) sum over images, e.g.

$$\sum_{n=-\infty}^{\infty} \partial X^+ \bar{\partial} X^- \exp\left(ik^+ X^- e^{-2\pi\beta n} + ik^- X^+ e^{2\pi\beta n} + ik_i x^i \right)$$

with the physical state condition $2k^+ k^- = M^2$.

Closed strings in Misner space - untwisted states

- As usual in standard orbifolds, part of the spectrum involves closed strings on Minkowski covering space, which are invariant under the orbifold projection. In conformal gauge,

$$X^{\pm}(\sigma + 2\pi, \tau) = X^{\pm}(\sigma, \tau) \ , \quad (\partial_\tau^2 - \partial_\sigma^2) X^{\pm} = 0$$

  satisfying the Virasoro (physical state) condition $(\dot{X}^\mu \pm X'^\mu)^2 = 0$.
- Vertex operators (or states) can be obtained by (infinite) sum over images, e.g.

$$\sum_{n=-\infty}^{\infty} \partial X^+ \bar{\partial} X^- \exp\left(ik^+ X^- e^{-2\pi\beta n} + ik^- X^+ e^{2\pi\beta n} + ik_i x^i \right)$$

  with the physical state condition $2k^+ k^- = M^2$.
- Equivalently, after Poisson resummation over n, this is a superposition of states with integer boost momentum $j = x^+ \partial_+ - x^- \partial_-$,

$$(\sum_{j=-\infty}^{\infty}) \partial X^+ \bar{\partial} X^- \int_{-\infty}^{\infty} dv \exp\left(+ik^+ X^- e^{-2\pi\beta v} + ik^- X^+ e^{2\pi\beta v} + ik_i X^i + 2\pi i v j \right)$$

Closed strings in Misner space - untwisted states

- As usual in standard orbifolds, part of the spectrum involves closed strings on Minkowski covering space, which are invariant under the orbifold projection. In conformal gauge,

$$X^{\pm}(\sigma+2\pi,\tau)=X^{\pm}(\sigma,\tau)\ ,\quad (\partial_\tau^2-\partial_\sigma^2)X^{\pm}=0$$

satisfying the Virasoro (physical state) condition $(\dot{X}^\mu \pm X'^\mu)^2=0$.

- Vertex operators (or states) can be obtained by (infinite) sum over images, e.g.

$$\sum_{n=-\infty}^{\infty} \partial X^+ \bar\partial X^- \exp\left(ik^+ X^- e^{-2\pi\beta n} + ik^- X^+ e^{2\pi\beta n} + ik_i x^i \right)$$

with the physical state condition $2k^+k^-=M^2$.

- Equivalently, after Poisson resummation over n, this is a superposition of states with integer boost momentum $j=x^+\partial_+ - x^-\partial_-$,

$$(\sum_{j=-\infty}^{\infty})\partial X^+ \bar\partial X^- \int_{-\infty}^{\infty} dv \exp\left(+ik^+ X^- e^{-2\pi\beta v} + ik^- X^+ e^{2\pi\beta v} + ik_i X^i + 2\pi i v j \right)$$

- The resulting eigenfunctions describe closed strings traveling around the Milne circle with integer momentum j.

Vacua of Misner space

As in any time-dependent background, there is no canonical choice of vacuum state:

Vacua of Misner space

As in any time-dependent background, there is no canonical choice of vacuum state:

- At $T \to +\infty$, positive energy solutions arise from superpositions of $k_+ > 0, k_- > 0$ plane waves on the covering space:

$$H^{(1)}_{-ij}(mT)e^{-ij\theta} \sim e^{-ij\theta - imT}/\sqrt{T}$$

  They annihilate the *out* adiabatic vacuum. They are also exponentially decreasing in the Rindler wedges. j is now the (quantized) Rindler energy.

Vacua of Misner space

As in any time-dependent background, there is no canonical choice of vacuum state:

- At $T \to +\infty$, positive energy solutions arise from superpositions of $k_+ > 0, k_- > 0$ plane waves on the covering space:

$$H^{(1)}_{-ij}(mT)e^{-ij\theta} \sim e^{-ij\theta - imT}/\sqrt{T}$$

  They annihilate the *out* adiabatic vacuum. They are also exponentially decreasing in the Rindler wedges. j is now the (quantized) Rindler energy.
- As $T \to -\infty$, positive energy solutions arise from superpositions of $k_+ < 0, k_- < 0$ plane waves on the covering space:

$$H^{(2)}_{-ij}(mT)e^{-ij\theta} \sim e^{-ij\theta - imT}/\sqrt{T}$$

  They follow by analytical continuation $T \to e^{i\pi}T$ from those at $T \to \infty$. There is thus no particle production between the adiabatic *in* and *out* vacua.

Vacua of Misner space

As in any time-dependent background, there is no canonical choice of vacuum state:

- At $T \longrightarrow +\infty$, positive energy solutions arise from superpositions of $k_+ > 0, k_- > 0$ plane waves on the covering space:

$$H^{(1)}_{-ij}(mT)e^{-ij\theta} \sim e^{-ij\theta - imT}/\sqrt{T}$$

  They annihilate the *out* adiabatic vacuum. They are also exponentially decreasing in the Rindler wedges. j is now the (quantized) Rindler energy.

- As $T \to -\infty$, positive energy solutions arise from superpositions of $k_+ < 0, k_- < 0$ plane waves on the covering space:

$$H^{(2)}_{-ij}(mT)e^{-ij\theta} \sim e^{-ij\theta - imT}/\sqrt{T}$$

  They follow by analytical continuation $T \longrightarrow e^{i\pi}T$ from those at $T \longrightarrow \infty$. There is thus no particle production between the adiabatic *in* and *out* vacua.

- As $T \longrightarrow 0$, solutions become independent of the mass, and define the conformal vacuum basis,

$$J_{ij}(mT) = H^{(1)}_{-ij}(mT) + H^{(2)}_{-ij}(mT) \sim e^{-ij\theta + ij\log(mT)}$$

Vacua of Misner space

As in any time-dependent background, there is no canonical choice of vacuum state:

- At $T \to +\infty$, positive energy solutions arise from superpositions of $k_+ > 0, k_- > 0$ plane waves on the covering space:

$$H^{(1)}_{-ij}(mT)e^{-ij\theta} \sim e^{-ij\theta - imT}/\sqrt{T}$$

  They annihilate the *out* adiabatic vacuum. They are also exponentially decreasing in the Rindler wedges. j is now the (quantized) Rindler energy.

- As $T \to -\infty$, positive energy solutions arise from superpositions of $k_+ < 0, k_- < 0$ plane waves on the covering space:

$$H^{(2)}_{-ij}(mT)e^{-ij\theta} \sim e^{-ij\theta - imT}/\sqrt{T}$$

  They follow by analytical continuation $T \to e^{i\pi}T$ from those at $T \to \infty$. There is thus no particle production between the adiabatic *in* and *out* vacua.

- As $T \to 0$, solutions become independent of the mass, and define the conformal vacuum basis,

$$J_{ij}(mT) = H^{(1)}_{-ij}(mT) + H^{(2)}_{-ij}(mT) \sim e^{-ij\theta + ij\log(mT)}$$

- The adiabatic and conformal vacua are related by a non-trivial Bogolioubov transformation.

Vacua of Misner space

As in any time-dependent background, there is no canonical choice of vacuum state:

- At $T \to +\infty$, positive energy solutions arise from superpositions of $k_+ > 0, k_- > 0$ plane waves on the covering space:

$$H^{(1)}_{-ij}(mT) e^{-ij\theta} \sim e^{-ij\theta - imT}/\sqrt{T}$$

  They annihilate the *out* adiabatic vacuum. They are also exponentially decreasing in the Rindler wedges. j is now the (quantized) Rindler energy.

- As $T \to -\infty$, positive energy solutions arise from superpositions of $k_+ < 0, k_- < 0$ plane waves on the covering space:

$$H^{(2)}_{-ij}(mT) e^{-ij\theta} \sim e^{-ij\theta - imT}/\sqrt{T}$$

  They follow by analytical continuation $T \to e^{i\pi} T$ from those at $T \to \infty$. There is thus no particle production between the adiabatic *in* and *out* vacua.

- As $T \to 0$, solutions become independent of the mass, and define the conformal vacuum basis,

$$J_{ij}(mT) = H^{(1)}_{-ij}(mT) + H^{(2)}_{-ij}(mT) \sim e^{-ij\theta + ij\log(mT)}$$

- The adiabatic and conformal vacua are related by a non-trivial Bogolioubov transformation. The Minkowski vacuum on the covering space descends to the adiabatic vacuum on the quotient.

Vacua of Misner space

As in any time-dependent background, there is no canonical choice of vacuum state:

- At $T \to +\infty$, positive energy solutions arise from superpositions of $k_+ > 0, k_- > 0$ plane waves on the covering space:

$$H^{(1)}_{-ij}(mT) e^{-ij\theta} \sim e^{-ij\theta - imT}/\sqrt{T}$$

  They annihilate the *out* adiabatic vacuum. They are also exponentially decreasing in the Rindler wedges. j is now the (quantized) Rindler energy.

- As $T \to -\infty$, positive energy solutions arise from superpositions of $k_+ < 0, k_- < 0$ plane waves on the covering space:

$$H^{(2)}_{-ij}(mT) e^{-ij\theta} \sim e^{-ij\theta - imT}/\sqrt{T}$$

  They follow by analytical continuation $T \to e^{i\pi} T$ from those at $T \to \infty$. There is thus no particle production between the adiabatic *in* and *out* vacua.

- As $T \to 0$, solutions become independent of the mass, and define the conformal vacuum basis,

$$J_{ij}(mT) = H^{(1)}_{-ij}(mT) + H^{(2)}_{-ij}(mT) \sim e^{-ij\theta + ij\log(mT)}$$

- The adiabatic and conformal vacua are related by a non-trivial Bogolioubov transformation. The Minkowski vacuum on the covering space descends to the adiabatic vacuum on the quotient.

Quantum fluctuations and backreaction

- In the Minkowski vacuum (inherited from the covering space), the renormalized propagator can be obtained as a sum over images, e.g in D=4

$$G(x;x') = \sum_{n=-\infty, n\neq 0}^{\infty} [-2(X^+ - e^{2\pi\beta n} X^{+'})(X^- - e^{2\pi\beta n} X^{-'}) + (X^i - X^{i'})^2]^{-1}$$

Quantum fluctuations and backreaction

- In the Minkowski vacuum (inherited from the covering space), the renormalized propagator can be obtained as a sum over images, e.g in D=4

$$G(x;x') = \sum_{n=-\infty, n\neq 0}^{\infty} [-2(X^{+} - e^{2\pi\beta n} X^{+'})(X^{-} - e^{2\pi\beta n} X^{-'}) + (X^{i} - X^{i'})^2]^{-1}$$

- The one-loop stress-energy tensor follows from $G(x,x)$, e.g for a conformally coupled scalar,

$$\langle T_{ab} \rangle = \lim_{x\to x'} \left[(1-2\xi)\nabla_a \nabla'_b - 2\xi \nabla_a \nabla_b + (2\xi - \frac{1}{2}) g_{ab} \nabla_c \nabla'^{c} \right] G(x,x')$$

Quantum fluctuations and backreaction

- In the Minkowski vacuum (inherited from the covering space), the renormalized propagator can be obtained as a sum over images, e.g in D=4

$$G(x;x') = \sum_{n=-\infty, n\neq 0}^{\infty} [-2(X^+ - e^{2\pi\beta n} X^{+'})(X^- - e^{2\pi\beta n} X^{-'}) + (X^i - X^{i'})^2]^{-1}$$

- The one-loop stress-energy tensor follows from $G(x,x)$, e.g for a conformally coupled scalar,

$$\langle T_{ab} \rangle = \lim_{x\to x'} \left[(1-2\xi)\nabla_a \nabla'_b - 2\xi \nabla_a \nabla_b + (2\xi - \frac{1}{2}) g_{ab} \nabla_c \nabla'^c \right] G(x,x')$$

leading to a divergent quantum backreaction:

$$\langle T_\mu^\nu \rangle = \frac{K}{12\pi^2} T^{-4} \text{diag}(1,-3,1,1) \ , \quad K = \sum_{n=1}^{\infty} \frac{2+\cosh 2\pi n\beta}{[\cosh 2\pi n \beta - 1]^2}$$

Hiscock Konkowski 82

Quantum fluctuations and backreaction

- In the Minkowski vacuum (inherited from the covering space), the renormalized propagator can be obtained as a sum over images, e.g in D=4

$$G(x;x') = \sum_{n=-\infty, n\neq 0}^{\infty} [-2(X^+ - e^{2\pi\beta n} X^{+'})(X^- - e^{2\pi\beta n} X^{-'}) + (X^i - X^{i'})^2]^{-1}$$

- The one-loop stress-energy tensor follows from $G(x,x)$, e.g for a conformally coupled scalar,

$$\langle T_{ab} \rangle = \lim_{x\to x'} \left[(1-2\xi)\nabla_a \nabla'_b - 2\xi \nabla_a \nabla_b + (2\xi - \frac{1}{2}) g_{ab} \nabla_c \nabla'^c \right] G(x,x')$$

leading to a divergent quantum backreaction:

$$\langle T_\mu^\nu \rangle = \frac{K}{12\pi^2} T^{-4} \text{diag}(1,-3,1,1) \; , \quad K = \sum_{n=1}^{\infty} \frac{2+\cosh 2\pi n\beta}{[\cosh 2\pi n\beta - 1]^2}$$

Hiscock Konkowski 82

- In the case of the Grant space, the one-loop energy momentum tensor diverges as $1/(R^2T^2)$ on the chronological horizon, and $1/(T-T_n)^3$ on the polarized hypersurfaces. This is at the basis of Hawking's chronology protection conjecture.

Scattering of untwisted states

- Scattering amplitudes of untwisted sector states can be computed from those in flat space by the inheritance principle,

$$\langle V(j_1,k_1)\dots V(j_n,k_n)\rangle_{Misner} = \int dv_1\dots dv_n\, e^{i(j_1 v_1+\dots+j_n v_n)}$$
$$\langle V(e^{\beta v_1}k_1^+, e^{-\beta v_1}k_1^-, k_1^i)\dots V(e^{\beta v_n}k_n^+, e^{-\beta v_n}k_n^-, k_n^i)\rangle_{Minkowski}$$

Scattering of untwisted states

- Scattering amplitudes of untwisted sector states can be computed from those in flat space by the inheritance principle,

$$\langle V(j_1,k_1)\dots V(j_n,k_n)\rangle_{Misner} = \int dv_1 \dots dv_n \, e^{i(j_1 v_1 + \cdots + j_n v_n)}$$
$$\langle V(e^{\beta v_1}k_1^+, e^{-\beta v_1}k_1^-, k_1^i)\dots V(e^{\beta v_n}k_n^+, e^{-\beta v_n}k_n^-, k_n^i)\rangle_{Minkowski}$$

- The integral diverges due to Regge behavior in the large momentum, fixed angle regime. E.g, the four-tachyon scattering amplitude in bosonic string leads to

$$\int dv \; v^{-\frac{1}{2}(k_1^i - k_3^i)^2 + i(j_2 - j_4)}$$

which diverges if $(k_1^i - k_3^i)^2 \leq 2$. This can be understood as large graviton exchange near the cosmological singularity.

Berkooz Craps Rajesh Kutasov

Scattering of untwisted states

- Scattering amplitudes of untwisted sector states can be computed from those in flat space by the inheritance principle,

$$\langle V(j_1,k_1)\dots V(j_n,k_n)\rangle_{Misner} = \int dv_1 \dots dv_n\; e^{i(j_1 v_1+\dots+j_n v_n)}$$
$$\langle V(e^{\beta v_1}k_1^+, e^{-\beta v_1}k_1^-, k_1^i)\dots V(e^{\beta v_n}k_n^+, e^{-\beta v_n}k_n^-, k_n^i)\rangle_{Minkowski}$$

- The integral diverges due to Regge behavior in the large momentum, fixed angle regime. E.g, the four-tachyon scattering amplitude in bosonic string leads to

$$\int dv\; v^{-\frac{1}{2}(k_1^i-k_3^i)^2+i(j_2-j_4)}$$

which diverges if $(k_1^i - k_3^i)^2 \leq 2$. This can be understood as large graviton exchange near the cosmological singularity.

Berkooz Craps Rajesh Kutasov

- The divergence disappears for the Grant space, except for a localized contribution at $k_1^i = k_3^i$. The amplitude is also finite for transverse gravitons in type II superstring on Misner space, but reappears for longitudinal gravitons.

Berkooz Durin Pioline Reichmann, unpublished

Closed string in Misner space - twisted sectors

- In addition, there is an infinite set of twisted sectors, corresponding to strings on the covering space that close up to the action of the orbifold group:

$$X^{\pm}(\sigma + 2\pi, \tau) = e^{\pm\nu} X^{\pm}(\sigma, \tau) \ , \quad \nu = 2\pi w \beta$$

Closed string in Misner space - twisted sectors

- In addition, there is an infinite set of twisted sectors, corresponding to strings on the covering space that close up to the action of the orbifold group:

$$X^{\pm}(\sigma + 2\pi, \tau) = e^{\pm\nu} X^{\pm}(\sigma, \tau) \ , \quad \nu = 2\pi w \beta$$

- They have a normal mode expansion:

$$\begin{aligned} X_R^{\pm}(\tau - \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha_n^{\pm} e^{-i(n \pm i\nu)(\tau - \sigma)} \\ X_L^{\pm}(\tau + \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \mp i\nu)^{-1} \tilde{\alpha}_n^{\pm} e^{-i(n \mp i\nu)(\tau + \sigma)} \end{aligned}$$

Closed string in Misner space - twisted sectors

- In addition, there is an infinite set of twisted sectors, corresponding to strings on the covering space that close up to the action of the orbifold group:

$$X^{\pm}(\sigma + 2\pi, \tau) = e^{\pm\nu} X^{\pm}(\sigma, \tau) \ , \quad \nu = 2\pi w \beta$$

- They have a normal mode expansion:

$$\begin{aligned} X_R^{\pm}(\tau - \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha_n^{\pm} e^{-i(n \pm i\nu)(\tau - \sigma)} \\ X_L^{\pm}(\tau + \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \mp i\nu)^{-1} \tilde{\alpha}_n^{\pm} e^{-i(n \mp i\nu)(\tau + \sigma)} \end{aligned}$$

with canonical commutation relations

$$[\alpha_m^+, \alpha_n^-] = -(m + i\nu)\delta_{m+n} \quad , \quad [\tilde{\alpha}_m^+, \tilde{\alpha}_n^-] = -(m - i\nu)\delta_{m+n}$$
$$(\alpha_m^{\pm})^* = \alpha_{-m}^{\pm} \quad , \quad (\tilde{\alpha}_m^{\pm})^* = \tilde{\alpha}_{-m}^{\pm}$$

Closed string in Misner space - twisted sectors

- In addition, there is an infinite set of twisted sectors, corresponding to strings on the covering space that close up to the action of the orbifold group:

$$X^{\pm}(\sigma + 2\pi, \tau) = e^{\pm\nu} X^{\pm}(\sigma, \tau) \ , \quad \nu = 2\pi w \beta$$

- They have a normal mode expansion:

$$\begin{aligned} X^{\pm}_R(\tau - \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha^{\pm}_n e^{-i(n \pm i\nu)(\tau - \sigma)} \\ X^{\pm}_L(\tau + \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \mp i\nu)^{-1} \tilde{\alpha}^{\pm}_n e^{-i(n \mp i\nu)(\tau + \sigma)} \end{aligned}$$

with canonical commutation relations

$$[\alpha^+_m, \alpha^-_n] = -(m + i\nu)\delta_{m+n} \quad , \quad [\tilde{\alpha}^+_m, \tilde{\alpha}^-_n] = -(m - i\nu)\delta_{m+n}$$
$$(\alpha^{\pm}_m)^* = \alpha^{\pm}_{-m} \quad , \quad (\tilde{\alpha}^{\pm}_m)^* = \tilde{\alpha}^{\pm}_{-m}$$

- There are no translational zero-modes, instead two pairs of quasi zero-modes which are canonically conjugate real operators:

$$[\alpha^+_0, \alpha^-_0] = -i\nu \ , \quad [\tilde{\alpha}^+_0, \tilde{\alpha}^-_0] = i\nu$$

Physical states (absence thereof)

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated, e.g., by

$$\alpha^{\pm}_{n>0}\ , \quad \tilde{\alpha}^{\pm}_{n>0}\ , \qquad \alpha^{-}_{0}\ , \quad \tilde{\alpha}^{+}_{0}$$

Physical states (absence thereof)

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated, e.g., by

$$\alpha^{\pm}_{n>0}\,, \quad \tilde{\alpha}^{\pm}_{n>0}\,, \qquad \alpha^{-}_{0}\,, \quad \tilde{\alpha}^{+}_{0}$$

- The worldsheet Hamiltonian, normal-ordered wrt to this vacuum, reads

$$L_0^{l.c.} = -\sum_{n=0}^{\infty} (\alpha_n^+)^* \alpha_n^- - \sum_{n=1}^{\infty} (\alpha_n^-)^* \alpha_n^+ + \frac{1}{2} i\nu(1-i\nu) - 1 + L_{int}$$

with a similar answer for $\tilde{L}_0$.

Physical states (absence thereof)

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated, e.g., by

$$\alpha^{\pm}_{n>0}\ , \quad \tilde{\alpha}^{\pm}_{n>0}\ , \qquad \alpha^{-}_{0}\ , \quad \tilde{\alpha}^{+}_{0}$$

- The worldsheet Hamiltonian, normal-ordered wrt to this vacuum, reads

$$L_0^{l.c.} = -\sum_{n=0}^{\infty} (\alpha_n^+)^* \alpha_n^- \ - \sum_{n=1}^{\infty} (\alpha_n^-)^* \alpha_n^+ + \frac{1}{2} i\nu(1-i\nu) - 1 + L_{int}$$

with a similar answer for $\tilde{L}_0$.

- This is the familiar result for the vacuum energy $\frac{1}{2}\theta(1-\theta)$ a rotation orbifold, after analytic continuation $\theta \to i\nu$...

Physical states (absence thereof)

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated, e.g., by

$$\alpha^{\pm}_{n>0}\ ,\quad \tilde\alpha^{\pm}_{n>0}\ ,\qquad \alpha^{-}_0\ ,\quad \tilde\alpha^{+}_0$$

- The worldsheet Hamiltonian, normal-ordered wrt to this vacuum, reads

$$L_0^{l.c.} = -\sum_{n=0}^{\infty}(\alpha_n^+)^*\alpha_n^- \ -\sum_{n=1}^{\infty}(\alpha_n^-)^*\alpha_n^+ + \frac{1}{2}i\nu(1-i\nu) - 1 + L_{int}$$

  with a similar answer for $\tilde L_0$.
- This is the familiar result for the vacuum energy $\frac{1}{2}\theta(1-\theta)$ a rotation orbifold, after analytic continuation $\theta \to i\nu$...
- Due to the $i\nu/2$ term in the ground state energy, all states obtained by acting on $|0\rangle$ by creation operators $\alpha^{\pm}_{n<0}$ and by α_0^+ will have imaginary energy, hence the physical state condition $L_0 = 0$ has no solutions.

Nekrasov

One-loop amplitude

- Independently of this fact, one may compute the one-loop (Euclidean ws, Minkowskian target) free energy using path integral methods:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho)x\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

where θ_1 is the Jacobi theta function,

$$\theta_1(v;\rho) = 2q^{1/8} \sin \pi v \prod_{n=1}^{\infty} (1 - e^{2\pi i v} q^n)(1 - q^n)(1 - e^{-2\pi i v} q^n)\ , \quad q = e^{2\pi i \rho}$$

One-loop amplitude

- Independently of this fact, one may compute the one-loop (Euclidean ws, Minkowskian target) free energy using path integral methods:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho)x\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

where θ_1 is the Jacobi theta function,

$$\theta_1(v;\rho) = 2q^{1/8} \sin \pi v \prod_{n=1}^{\infty} (1 - e^{2\pi i v} q^n)(1 - q^n)(1 - e^{-2\pi i v} q^n) , \quad q = e^{2\pi i \rho}$$

- In the untwisted sector, this reproduces the integrated vacuum free energy found by the method of images:

$$\int dx^+ dx^- G(x,x) = \sum_{l=-\infty}^{+\infty} \int_0^{\infty} \frac{d\rho}{\rho^{D/2}} \frac{e^{-m^2\rho}}{\sinh^2(\pi\beta l)}$$

One-loop amplitude, twisted sector

- Independently of this fact, one may compute the one-loop (Euclidean ws, Minkowskian target) free energy using path integral methods:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2 \rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho)x\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

where θ_1 is the Jacobi theta function,

$$\theta_1(v;\rho) = 2q^{1/8} \sin \pi v \prod_{n=1}^{\infty} (1-e^{2\pi i v} q^n)(1-q^n)(1-e^{-2\pi i v} q^n) \ , \quad q = e^{2\pi i \rho}$$

One-loop amplitude, twisted sector

- Independently of this fact, one may compute the one-loop (Euclidean ws, Minkowskian target) free energy using path integral methods:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho)x\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

where θ_1 is the Jacobi theta function,

$$\theta_1(v;\rho) = 2q^{1/8} \sin\pi v \prod_{n=1}^{\infty} (1-e^{2\pi i v}q^n)(1-q^n)(1-e^{-2\pi i v}q^n)\ , \quad q = e^{2\pi i \rho}$$

- In the twisted sector, the left-moving zero-modes contribute

$$\frac{1}{2\sinh(\beta w \rho)]} = \sum_{n=1}^{\infty} q^{i(n+\frac{1}{2})\beta w}$$

in accordance with the quantization scheme based on a Fock vacuum annihilated by α_0^-.

One-loop amplitude, twisted sector

- Independently of this fact, one may compute the one-loop (Euclidean ws, Minkowskian target) free energy using path integral methods:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho)x\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

where θ_1 is the Jacobi theta function,

$$\theta_1(v;\rho) = 2q^{1/8}\sin\pi v \prod_{n=1}^{\infty}(1-e^{2\pi i v}q^n)(1-q^n)(1-e^{-2\pi i v}q^n)\ , \quad q = e^{2\pi i \rho}$$

- In the twisted sector, the left-moving zero-modes contribute

$$\frac{1}{2\sinh(\beta w \rho)]} = \sum_{n=1}^{\infty} q^{i(n+\frac{1}{2})\beta w}$$

in accordance with the quantization scheme based on a Fock vacuum annihilated by α_0^-.

- The absence of physical twisted states crushes our hopes for resolving the singularity... yet it is hard to swallow.

One-loop amplitude, twisted sector

- Independently of this fact, one may compute the one-loop (Euclidean ws, Minkowskian target) free energy using path integral methods:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar\rho}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho) x\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

where θ_1 is the Jacobi theta function,

$$\theta_1(v;\rho) = 2q^{1/8}\sin\pi v \prod_{n=1}^{\infty}(1-e^{2\pi i v}q^n)(1-q^n)(1-e^{-2\pi i v}q^n)\ , \quad q=e^{2\pi i\rho}$$

- In the twisted sector, the left-moving zero-modes contribute

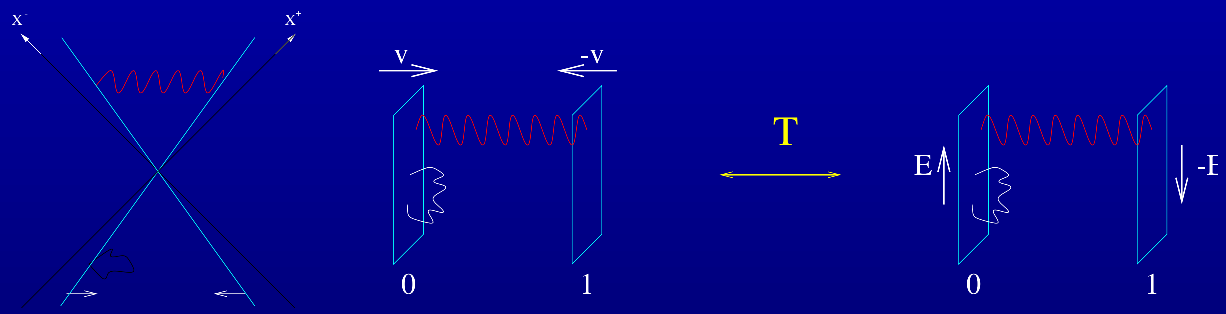
$$\frac{1}{2\sinh(\beta w\rho)]} = \sum_{n=1}^{\infty} q^{i(n+\frac{1}{2})\beta w}$$

in accordance with the quantization scheme based on a Fock vacuum annihilated by α_0^-.

- The absence of physical twisted states crushes our hopes for resolving the singularity... yet it is hard to swallow. In particular, α_0^+ and α_0^- are not hermitian conjugate to each other, but rather self-hermitian...

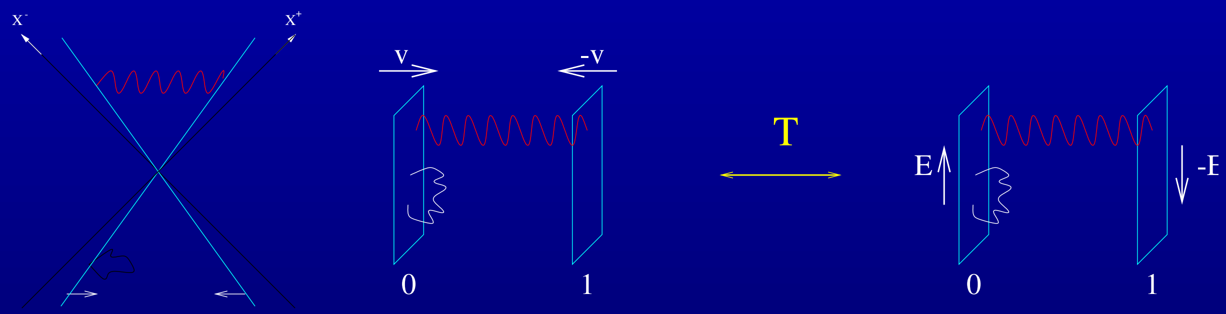
Open strings in electric field vs Lorentzian orbifold

- A very similar puzzle is faced in the case of colliding D-branes, or in the T-dual process of charged open strings in a constant electric field:



Open strings in electric field vs Lorentzian orbifold

- A very similar puzzle is faced in the case of colliding D-branes, or in the T-dual process of charged open strings in a constant electric field:



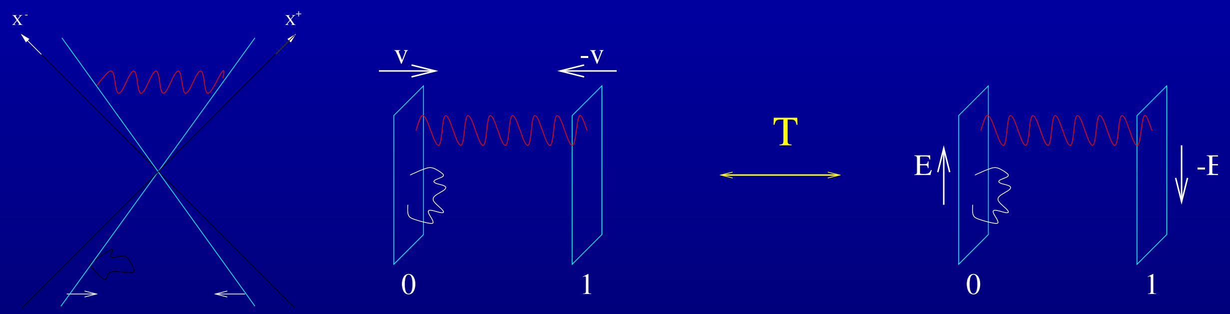
- Recall that for open strings stretched between two D-branes with electromagnetic fields F_0 and F_1, proper frequencies satisfy

$$e^{-2\pi i\omega_n} = \frac{1+F_0}{1-F_0} \cdot \frac{1-F_1}{1+F_1}$$

For $F_0 \neq F_1$, the open string carries a net electric charge, and the motion of its center of motion is that of a charged particle.

Open strings in electric field vs Lorentzian orbifold

- A very similar puzzle is faced in the case of colliding D-branes, or in the T-dual process of charged open strings in a constant electric field:



- Recall that for open strings stretched between two D-branes with electromagnetic fields F_0 and F_1, proper frequencies satisfy

$$e^{-2\pi i\omega_n} = \frac{1+F_0}{1-F_0}\cdot\frac{1-F_1}{1+F_1}$$

For $F_0 \neq F_1$, the open string carries a net electric charge, and the motion of its center of motion is that of a charged particle.

- In the case of an electric field $F_1 = E dx^+ \wedge dx^-$, $F_1 = 0$, the resulting spectrum is

$$\omega_n = n + i\nu\ , \qquad \nu := \text{Arctanh} E = w\beta$$

just as in the Lorentzian orbifold case. More precisely, the charged open string has half as many excited modes than the twisted closed strings, and isomorphic quasi-zero modes.

Open string mode expansion

- The light-cone embedding coordinates have the normal mode expansion

$$X^{\pm} = x_0^{\pm} + i \sum_{n=-\infty}^{+\infty} (-)^n (n \pm i\nu)^{-1} a_n^{\pm} e^{-i(n \pm i\nu)\tau} \cos[(n \pm i\nu)\sigma]$$

with reality conditions $(a_n^{\pm})^* = a_{-n}^{\pm}\ , \quad (x_0^{\pm})^* = x_0^{\pm}$

Open string mode expansion

- The light-cone embedding coordinates have the normal mode expansion

$$X^{\pm} = x_0^{\pm} + i \sum_{n=-\infty}^{+\infty} (-)^n (n \pm i\nu)^{-1} a_n^{\pm} e^{-i(n \pm i\nu)\tau} \cos[(n \pm i\nu)\sigma]$$

with reality conditions $(a_n^{\pm})^* = a_{-n}^{\pm}\ , \quad (x_0^{\pm})^* = x_0^{\pm}$

- The canonical commutation relations read

$$[a_m^+, a_n^-] = -(m + i\nu)\delta_{m+n}\ , \quad [x_0^+, x_0^-] = -\frac{i}{E}$$

Open string mode expansion

- The light-cone embedding coordinates have the normal mode expansion

$$X^{\pm} = x_0^{\pm} + i \sum_{n=-\infty}^{+\infty} (-)^n (n \pm i\nu)^{-1} a_n^{\pm} e^{-i(n\pm i\nu)\tau} \cos[(n \pm i\nu)\sigma]$$

with reality conditions $(a_n^{\pm})^* = a_{-n}^{\pm} \ , \quad (x_0^{\pm})^* = x_0^{\pm}$

- The canonical commutation relations read

$$[a_m^+, a_n^-] = -(m + i\nu)\delta_{m+n} \ , \quad [x_0^+, x_0^-] = -\frac{i}{E}$$

- In particular, the open and closed strings have isomorphic (quasi) zero-mode structures, with $\alpha_0^{\pm} \equiv \alpha_0^{\pm}$ and $\tilde{\alpha}_0^{\pm} \equiv \pm\sqrt{\nu E} x_0^{\pm}$.

Open string mode expansion

- The light-cone embedding coordinates have the normal mode expansion

$$X^{\pm} = x_0^{\pm} + i \sum_{n=-\infty}^{+\infty} (-)^n (n \pm i\nu)^{-1} a_n^{\pm} e^{-i(n \pm i\nu)\tau} \cos[(n \pm i\nu)\sigma]$$

with reality conditions $(a_n^{\pm})^* = a_{-n}^{\pm}$, $\quad (x_0^{\pm})^* = x_0^{\pm}$

- The canonical commutation relations read

$$[a_m^+, a_n^-] = -(m + i\nu)\delta_{m+n} \; , \quad [x_0^+, x_0^-] = -\frac{i}{E}$$

- In particular, the open and closed strings have isomorphic (quasi) zero-mode structures, with $\alpha_0^{\pm} \equiv \alpha_0^{\pm}$ and $\tilde{\alpha}_0^{\pm} \equiv \pm\sqrt{\nu E} x_0^{\pm}$.
- The world-sheet Hamiltonian, normal ordered with respect to the vacuum annihilated by $a_{n>0}^+, a_{n>0}^-$ and a_0^+, takes the form

$$L_0^{l.c.} = -\sum_{m=0}^{\infty} a_{-m}^+ a_m^- - \sum_{m=1}^{\infty} a_{-m}^- a_m^+ + \frac{i\nu}{2}(1 - i\nu) - \frac{1}{12}$$

Open string mode expansion

- The light-cone embedding coordinates have the normal mode expansion

$$X^{\pm} = x_0^{\pm} + i \sum_{n=-\infty}^{+\infty} (-)^n (n \pm i\nu)^{-1} a_n^{\pm} e^{-i(n \pm i\nu)\tau} \cos[(n \pm i\nu)\sigma]$$

  with reality conditions $(a_n^{\pm})^* = a_{-n}^{\pm}\ , \quad (x_0^{\pm})^* = x_0^{\pm}$

- The canonical commutation relations read

$$[a_m^+, a_n^-] = -(m + i\nu)\delta_{m+n}\ , \quad [x_0^+, x_0^-] = -\frac{i}{E}$$

- In particular, the open and closed strings have isomorphic (quasi) zero-mode structures, with $\alpha_0^{\pm} \equiv \alpha_0^{\pm}$ and $\tilde{\alpha}_0^{\pm} \equiv \pm\sqrt{\nu E}x_0^{\pm}$.
- The world-sheet Hamiltonian, normal ordered with respect to the vacuum annihilated by $a_{n>0}^+, a_{n>0}^-$ and a_0^+, takes the form

$$L_0^{l.c.} = -\sum_{m=0}^{\infty} a_{-m}^+ a_m^- - \sum_{m=1}^{\infty} a_{-m}^- a_m^+ + \frac{i\nu}{2}(1 - i\nu) - \frac{1}{12}$$

- By the same token, charged open strings should have no physical states...

One-loop amplitude and Schwinger pair production

- Using this quantization scheme, the one-loop (Euclidean worldsheet, Minkowskian target) vacuum free energy reads

$$A_{bos} = \frac{i\pi V_{26}(e_0+e_1)}{2}\int_0^\infty \frac{dt}{(4\pi^2 t)^{13}}\frac{e^{-\pi\nu^2 t/2}}{\eta^{21}(it/2)\;\theta_1(t\nu/2;it/2)}$$

One-loop amplitude and Schwinger pair production

- Using this quantization scheme, the one-loop (Euclidean worldsheet, Minkowskian target) vacuum free energy reads

$$A_{bos} = \frac{i\pi V_{26}(e_0+e_1)}{2} \int_0^\infty \frac{dt}{(4\pi^2 t)^{13}} \frac{e^{-\pi\nu^2 t/2}}{\eta^{21}(it/2)\ \theta_1(t\nu/2; it/2)}$$

- In particular, the contribution of the zero-modes is consistent with $a_0^+|0\rangle = 0$,

$$\frac{1}{2\sinh \pi t\nu/2} = e^{-\pi t\nu/2}(1 + e^{-\pi t\nu} + e^{-2\pi t\nu} + \dots)$$

One-loop amplitude and Schwinger pair production

- Using this quantization scheme, the one-loop (Euclidean worldsheet, Minkowskian target) vacuum free energy reads

$$A_{bos} = \frac{i\pi V_{26}(e_0+e_1)}{2} \int_0^\infty \frac{dt}{(4\pi^2 t)^{13}} \frac{e^{-\pi\nu^2 t/2}}{\eta^{21}(it/2)\ \theta_1(t\nu/2; it/2)}$$

- In particular, the contribution of the zero-modes is consistent with $a_0^+|0\rangle = 0$,

$$\frac{1}{2\sinh \pi t\nu/2} = e^{-\pi t\nu/2}(1 + e^{-\pi t\nu} + e^{-2\pi t\nu} + \dots)$$

- Each of the poles at $t = 2k/\nu$ contributes to the imaginary part, yielding the production rate of charged open strings,

$$\mathcal{W} = \frac{1}{2(2\pi)^{25}} \frac{(e_0+e_1)}{\nu} \sum_{k=1}^{\infty} (-)^{k+1} \left(\frac{|\nu|}{k}\right)^{13} \sum_{N=-1}^{\infty} c_b(N) \exp\left(-2\pi k \frac{N}{|\nu|} - 2\pi k|\nu|\right)$$

Bachas Porrati

where $\eta^{-24}(q) = \sum_{N=-1}^{\infty} c_b(N) q^N$. This can be viewed as the sum of the Schwinger production rates for each state in the spectrum, of mass $m^2 = 2N + \nu^2$.

One-loop amplitude and Schwinger pair production

- Using this quantization scheme, the one-loop (Euclidean worldsheet, Minkowskian target) vacuum free energy reads

$$A_{bos} = \frac{i\pi V_{26}(e_0+e_1)}{2} \int_0^\infty \frac{dt}{(4\pi^2 t)^{13}} \frac{e^{-\pi\nu^2 t/2}}{\eta^{21}(it/2)\ \theta_1(t\nu/2; it/2)}$$

- In particular, the contribution of the zero-modes is consistent with $a_0^+|0\rangle = 0$,

$$\frac{1}{2\sinh \pi t\nu/2} = e^{-\pi t\nu/2}(1 + e^{-\pi t\nu} + e^{-2\pi t\nu} + \dots)$$

- Each of the poles at $t = 2k/\nu$ contributes to the imaginary part, yielding the production rate of charged open strings,

$$\mathcal{W} = \frac{1}{2(2\pi)^{25}} \frac{(e_0+e_1)}{\nu} \sum_{k=1}^{\infty} (-)^{k+1} \left(\frac{|\nu|}{k}\right)^{13} \sum_{N=-1}^{\infty} c_b(N) \exp\left(-2\pi k \frac{N}{|\nu|} - 2\pi k|\nu|\right)$$

Bachas Porrati

where $\eta^{-24}(q) = \sum_{N=-1}^{\infty} c_b(N) q^N$. This can be viewed as the sum of the Schwinger production rates for each state in the spectrum, of mass $m^2 = 2N + \nu^2$.

- This seems to support the quantization scheme based on a vacuum, hence the absence of physical states.

One-loop amplitude and Schwinger pair production

- Using this quantization scheme, the one-loop (Euclidean worldsheet, Minkowskian target) vacuum free energy reads

$$A_{bos} = \frac{i\pi V_{26}(e_0+e_1)}{2}\int_0^\infty \frac{dt}{(4\pi^2 t)^{13}}\frac{e^{-\pi\nu^2 t/2}}{\eta^{21}(it/2)\ \theta_1(t\nu/2;it/2)}$$

- In particular, the contribution of the zero-modes is consistent with $a_0^+|0\rangle = 0$,

$$\frac{1}{2\sinh \pi t\nu/2} = e^{-\pi t\nu/2}(1+e^{-\pi t\nu}+e^{-2\pi t\nu}+\dots)$$

- Each of the poles at $t = 2k/\nu$ contributes to the imaginary part, yielding the production rate of charged open strings,

$$\mathcal{W} = \frac{1}{2(2\pi)^{25}}\frac{(e_0+e_1)}{\nu}\sum_{k=1}^{\infty}(-)^{k+1}\left(\frac{|\nu|}{k}\right)^{13}\sum_{N=-1}^{\infty}c_b(N)\exp\left(-2\pi k\frac{N}{|\nu|}-2\pi k|\nu|\right)$$

Bachas Porrati

where $\eta^{-24}(q) = \sum_{N=-1}^{\infty} c_b(N) q^N$. This can be viewed as the sum of the Schwinger production rates for each state in the spectrum, of mass $m^2 = 2N+\nu^2$.

- This seems to support the quantization scheme based on a vacuum, hence the absence of physical states. But physical states do exist classically, how could quantization make them disappear altogether ?

Charged particle and open string zero-modes

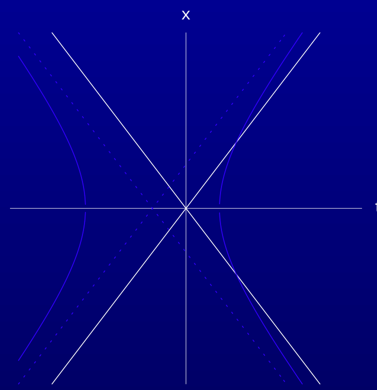
- Let us recall the quantization of a charged particle in an electric field:

$$L = \frac{1}{2} m \left(-2\partial_\tau X^+ \partial_\tau X^- + (\partial_\tau X^i)^2 \right) + \frac{e}{2} \left(X^+ \partial_\tau X^- - X^- \partial_\tau X^+ \right)$$

Charged particle and open string zero-modes

- Let us recall the quantization of a charged particle in an electric field:

$$L = \frac{1}{2}m\left(-2\partial_\tau X^+\partial_\tau X^- + (\partial_\tau X^i)^2\right) + \frac{e}{2}\left(X^+\partial_\tau X^- - X^-\partial_\tau X^+\right)$$



- The classical trajectories are identical to the open string zero-mode:

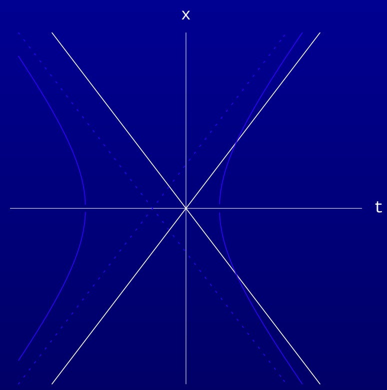
$$X^\pm = x_0^\pm \pm \frac{1}{\nu}a_0^\pm e^{\pm\nu\tau}$$

$\pm e x_0^\pm$ is the conserved linear momentum, and $a_0^\pm$ the velocity.

Charged particle and open string zero-modes

- Let us recall the quantization of a charged particle in an electric field:

$$L = \frac{1}{2} m \left(-2\partial_\tau X^+ \partial_\tau X^- + (\partial_\tau X^i)^2 \right) + \frac{e}{2} \left(X^+ \partial_\tau X^- - X^- \partial_\tau X^+ \right)$$



- The classical trajectories are identical to the open string zero-mode:

$$X^\pm = x_0^\pm \pm \frac{1}{\nu} a_0^\pm e^{\pm \nu \tau}$$

$\pm e x_0^\pm$ is the conserved linear momentum, and $a_0^\pm$ the velocity.

- Starting from the canonical equal-time commutation rules

$$[\pi^+, x^-] = [\pi^-, x^+] = i \ , \quad [\pi^i, x^j] = i\delta_{ij}$$

one obtains the open string zero-mode commutation relations ($\nu = e$),

$$[a_0^+, a_0^-] = -i\nu \ , \quad [x_0^+, x_0^-] = -\frac{i}{\nu}$$

Charged particle and ppen string zero-modes

- Quantum mechanically, one may represent $\pi^{\pm} = i\partial/\partial x^{\mp}$ hence obtain $a_0^{\pm}, x_0^{\pm}$ as covariant derivatives

$$a_0^{\pm} = i\partial_{\mp} \pm \frac{\nu}{2} x^{\pm} \, , \quad x_0^{\pm} = \mp \frac{1}{\nu} \left(i\partial_{\mp} \mp \frac{\nu}{2} x^{\pm} \right)$$

acting on wave functions $f(x^+, x^-)$.

Charged particle and ppen string zero-modes

- Quantum mechanically, one may represent $\pi^\pm = i\partial/\partial x^\mp$ hence obtain $a_0^\pm, x_0^\pm$ as covariant derivatives

$$a_0^\pm = i\partial_\mp \pm \frac{\nu}{2} x^\pm \,, \quad x_0^\pm = \mp \frac{1}{\nu}\left(i\partial_\mp \mp \frac{\nu}{2} x^\pm \right)$$

  acting on wave functions $f(x^+, x^-)$.

- The zero-mode piece of L_0, including the evil $\frac{i\nu}{2}$,

$$L_0^{(0)} = -a_0^+ a_0^- + \frac{i\nu}{2} = -\frac{1}{2}(\nabla_0^+\nabla_0^- + \nabla_0^-\nabla_0^+)$$

  is just the Klein-Gordon operator of a particle of charge ν.

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^{\pm} = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator can be rewritten as an inverted harmonic oscillator:

$$M^2 = a_0^+ a_0^- + a_0^- a_0^+ = -\frac{1}{2}(P^2 - Q^2)$$

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^{\pm} = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator can be rewritten as an inverted harmonic oscillator:

$$M^2 = a_0^+ a_0^- + a_0^- a_0^+ = -\frac{1}{2}(P^2 - Q^2)$$

- More explicitely, in terms of $u = (\tilde{p} + \nu x)\sqrt{2/\nu}$,

$$\left(-\partial_u^2 - \frac{1}{4}u^2 + \frac{M^2}{2\nu} \right) \psi_{\tilde{p}}(u) = 0$$

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^{\pm} = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator can be rewritten as an inverted harmonic oscillator:

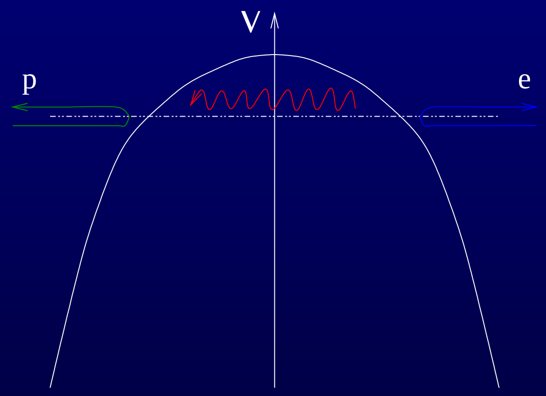
$$M^2 = a_0^+ a_0^- + a_0^- a_0^+ = -\frac{1}{2}(P^2 - Q^2)$$

- More explicitely, in terms of $u = (\tilde{p} + \nu x)\sqrt{2/\nu}$,

$$\left(-\partial_u^2 - \frac{1}{4}u^2 + \frac{M^2}{2\nu} \right) \psi_{\tilde{p}}(u) = 0$$

- The latter admits a respectable delta-normalizable spectrum of scattering states, in terms of parabolic cylinder functions, e.g:

$$\phi_{in}^+ = D_{-\frac{1}{2}+i\frac{M^2}{2\nu}}(e^{-\frac{3i\pi}{4}}u)e^{-i\tilde{p}t}e^{i\nu xt/2}$$



Klein-Gordon and the inverted harmonic oscillator

- Defining $a_0^\pm = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator can be rewritten as an inverted harmonic oscillator:

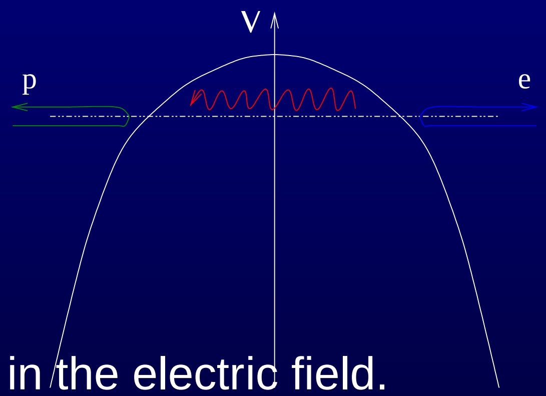
$$M^2 = a_0^+ a_0^- + a_0^- a_0^+ = -\frac{1}{2}(P^2 - Q^2)$$

- More explicitely, in terms of $u = (\tilde{p} + \nu x)\sqrt{2/\nu}$,

$$\left(-\partial_u^2 - \frac{1}{4}u^2 + \frac{M^2}{2\nu} \right) \psi_{\tilde{p}}(u) = 0$$

- The latter admits a respectable delta-normalizable spectrum of scattering states, in terms of parabolic cylinder functions, e.g:

$$\phi_{in}^+ = D_{-\frac{1}{2}+i\frac{M^2}{2\nu}}(e^{-\frac{3i\pi}{4}} u) e^{-i\tilde{p}t} e^{i\nu x t/2}$$



- These correspond to non-compact trajectories of charged particles in the electric field. Tunnelling is just (stimulated) Schwinger pair creation,

$$e^- \longrightarrow (1+\eta)\, e^- + \eta\, e^+ \,, \quad \eta \sim e^{-\pi M^2/\nu}$$

Brezin Itzykson; Brout Massar Parentani Spindel

Lorentzian vs Euclidean states

- Analytic continuation $X^0 \to -iX^0$, $\nu \to i\nu$ turns an electric field in $R^{1,1}$ into a magnetic field in R^2. At the same time, one should Wick rotate the worldsheet time.

Lorentzian vs Euclidean states

- Analytic continuation $X^0 \longrightarrow -iX^0$, $\nu \longrightarrow i\nu$ turns an electric field in $R^{1,1}$ into a magnetic field in R^2. At the same time, one should Wick rotate the worldsheet time.
- The discrete spectrum with complex energy comes by analytic continuation of the normalizable (Landau) states of the (stable) harmonic oscillator.

Lorentzian vs Euclidean states

- Analytic continuation $X^0 \to -iX^0$, $\nu \to i\nu$ turns an electric field in $R^{1,1}$ into a magnetic field in R^2. At the same time, one should Wick rotate the worldsheet time.
- The discrete spectrum with complex energy comes by analytic continuation of the normalizable (Landau) states of the (stable) harmonic oscillator.
- Conversely, the physical continuous scattering states of the inverted harmonic oscillator continue to non-normalizable states of the stable harmonic oscillator.

Lorentzian vs Euclidean states

- Analytic continuation $X^0 \to -iX^0$, $\nu \to i\nu$ turns an electric field in $R^{1,1}$ into a magnetic field in R^2. At the same time, one should Wick rotate the worldsheet time.
- The discrete spectrum with complex energy comes by analytic continuation of the normalizable (Landau) states of the (stable) harmonic oscillator.
- Conversely, the physical continuous scattering states of the inverted harmonic oscillator continue to non-normalizable states of the stable harmonic oscillator.
- The contribution of zero-modes to the one-loop amplitude can be interpreted either way,

$$\frac{1}{2i\sin(\nu t/2)} = \sum_{n=1}^{\infty} e^{-i(n+\frac{1}{2})\nu t} = \int dM^2 \rho(M^2) e^{-M^2 t/2}$$

The density of states is obtained from the reflection phase shift,

$$\rho(M^2) = \frac{1}{\nu}\log\Lambda - \frac{1}{2\pi i}\frac{d}{dM^2}\log\frac{\Gamma\left(\frac{1}{2}+i\frac{M^2}{2\nu}\right)}{\Gamma\left(\frac{1}{2}-i\frac{M^2}{2\nu}\right)}$$

Lorentzian vs Euclidean states

- Analytic continuation $X^0 \to -iX^0$, $\nu \to i\nu$ turns an electric field in $R^{1,1}$ into a magnetic field in R^2. At the same time, one should Wick rotate the worldsheet time.
- The discrete spectrum with complex energy comes by analytic continuation of the normalizable (Landau) states of the (stable) harmonic oscillator.
- Conversely, the physical continuous scattering states of the inverted harmonic oscillator continue to non-normalizable states of the stable harmonic oscillator.
- The contribution of zero-modes to the one-loop amplitude can be interpreted either way,

$$\frac{1}{2i\sin(\nu t/2)} = \sum_{n=1}^{\infty} e^{-i(n+\frac{1}{2})\nu t} = \int dM^2 \rho(M^2) e^{-M^2 t/2}$$

The density of states is obtained from the reflection phase shift,

$$\rho(M^2) = \frac{1}{\nu}\log\Lambda - \frac{1}{2\pi i}\frac{d}{dM^2}\log\frac{\Gamma\left(\frac{1}{2}+i\frac{M^2}{2\nu}\right)}{\Gamma\left(\frac{1}{2}-i\frac{M^2}{2\nu}\right)}$$

- The physical spectrum can be explicitely worked out at low levels, and is free of ghosts: a tachyon at level 0, a transverse gauge boson at level 1, . . .

Physical spectrum at low level

- The ground state tachyon

$$|T\rangle = \phi(x^+, x^-)|0_{ex}, k\rangle$$

should satisfy the Virasoro constraint

$$L_0|T\rangle = \left[-\frac{1}{2}\left(a_0^+ a_0^- + a_0^- a_0^+\right) + \frac{1}{2}\nu^2 - 1 + \frac{1}{2}k_i^2\right]|T\rangle$$

which is the two-dimensional KG equation.

Physical spectrum at low level

- The ground state tachyon

$$|T\rangle = \phi(x^+, x^-)|0_{ex}, k\rangle$$

should satisfy the Virasoro constraint

$$L_0|T\rangle = \left[-\frac{1}{2}\left(a_0^+ a_0^- + a_0^- a_0^+\right) + \frac{1}{2}\nu^2 - 1 + \frac{1}{2}k_i^2\right]|T\rangle$$

which is the two-dimensional KG equation.

- Level 1 states consist of

$$|A\rangle = \left(-f^+ \, a_{-1}^- - f^- \, a_{-1}^+ + f^i \, a_{-1}^i\right)|0_{ex}, k\rangle$$

with the mass shell conditions

$$[M^2 - k_i^2 - \nu^2]f^i = 0\ , \quad [M^2 - k_i^2 - \nu^2 \mp 2i\nu]f^\pm = 0$$

- The L_1 Virasoro constraint eliminates one polarization.

Physical spectrum at low level

- The ground state tachyon

$$|T\rangle = \phi(x^+, x^-)|0_{ex}, k\rangle$$

should satisfy the Virasoro constraint

$$L_0|T\rangle = \left[-\frac{1}{2}\left(a_0^+ a_0^- + a_0^- a_0^+\right) + \frac{1}{2}\nu^2 - 1 + \frac{1}{2}k_i^2\right]|T\rangle$$

which is the two-dimensional KG equation.

- Level 1 states consist of

$$|A\rangle = \left(-f^+\, a_{-1}^- - f^-\, a_{-1}^+ + f^i\, a_{-1}^i\right)|0_{ex}, k\rangle$$

with the mass shell conditions

$$[M^2 - k_i^2 - \nu^2]f^i = 0\ , \quad [M^2 - k_i^2 - \nu^2 \mp 2i\nu]f^\pm = 0$$

- The L_1 Virasoro constraint eliminates one polarization. Despite the non-vanishing two-dimensional mass $k_i^2 - \nu^2$, the spurious state $L_{-1}\phi|0\rangle$ is still physical, eliminating an extra polarization.
- One thus has $D-2$ transverse degrees of freedom, ie a massless gauge boson in D dimensions.

Charged particle in Rindler space

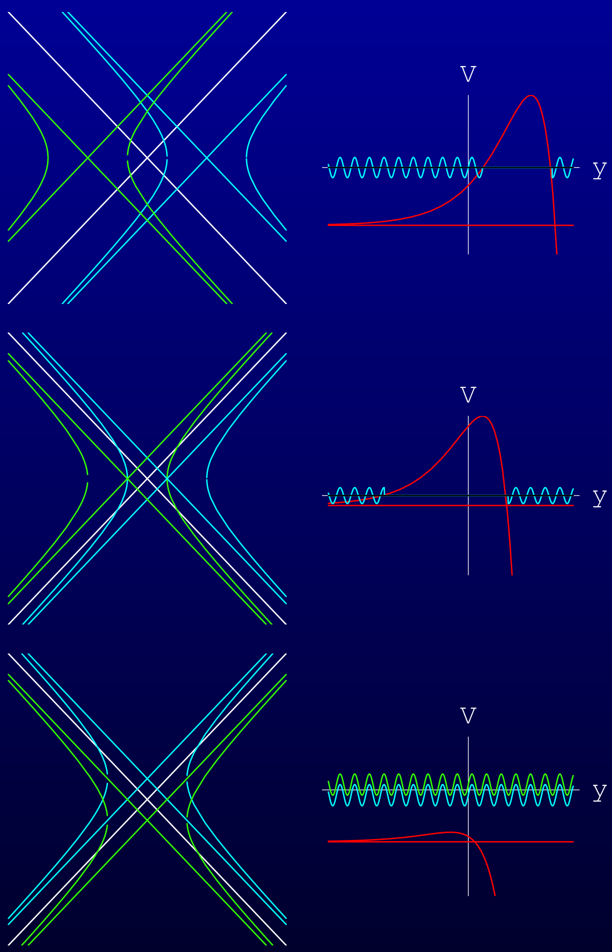
- For applications to the Milne universe, one should diagonalize the boost momentum J, ie consider an accelerated observer.

Gabriel Spindel; Mottola Cooper

Charged particle in Rindler space

- For applications to the Milne universe, one should diagonalize the boost momentum J, ie consider an accelerated observer.

Gabriel Spindel; Mottola Cooper



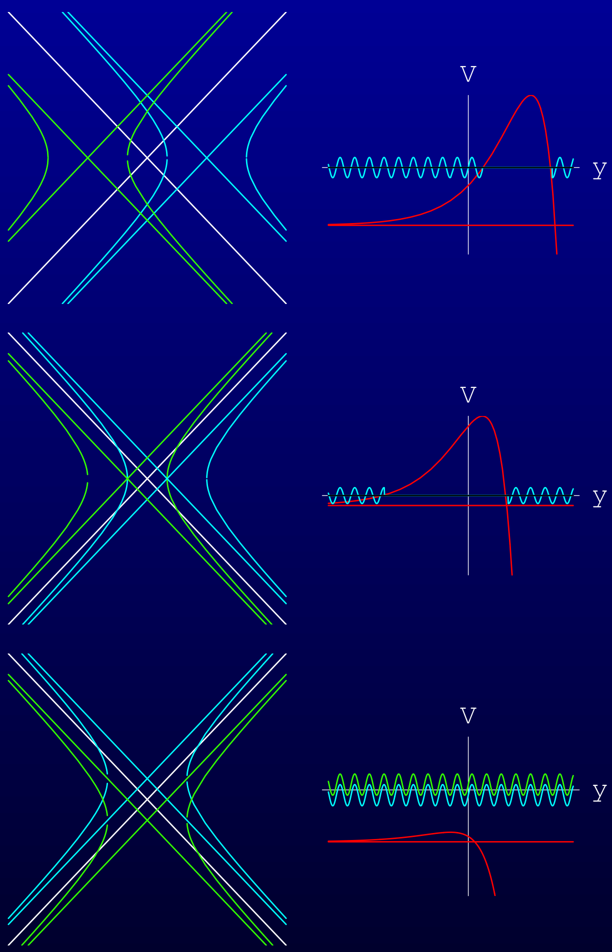
- In the Rindler patch R, letting $f(r,\eta) = e^{-iJ\eta} f_J(r)$ and $r = e^y$, one gets a Schrödinger equation for a particle in a potential

$$V(y) = M^2 e^{2y} - (J + \frac{1}{2}\nu\ e^{2y})^2$$

Charged particle in Rindler space

- For applications to the Milne universe, one should diagonalize the boost momentum J, ie consider an accelerated observer.

Gabriel Spindel; Mottola Cooper



- In the Rindler patch R, letting $f(r,\eta) = e^{-iJ\eta} f_J(r)$ and $r = e^y$, one gets a Schrödinger equation for a particle in a potential

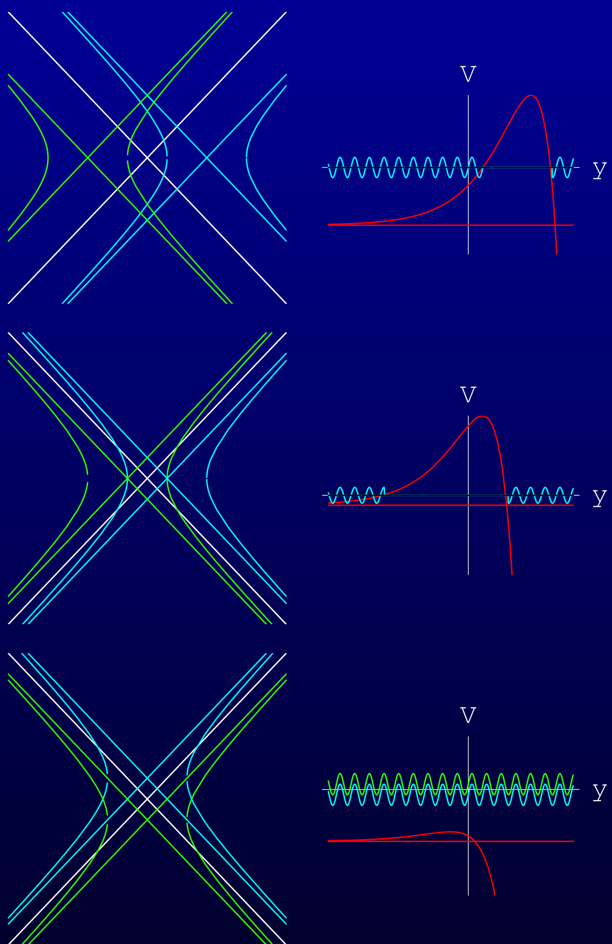
$$V(y) = M^2 e^{2y} - (J + \frac{1}{2}\nu\, e^{2y})^2$$

- If $j < 0$, the electron and positron branches are in the same Rindler quadrant. Tunneling corresponds to Schwinger particle production.

Charged particle in Rindler space

- For applications to the Milne universe, one should diagonalize the boost momentum J, ie consider an accelerated observer.

Gabriel Spindel; Mottola Cooper



- In the Rindler patch R, letting $f(r, \eta) = e^{-iJ\eta} f_J(r)$ and $r = e^y$, one gets a Schrödinger equation for a particle in a potential

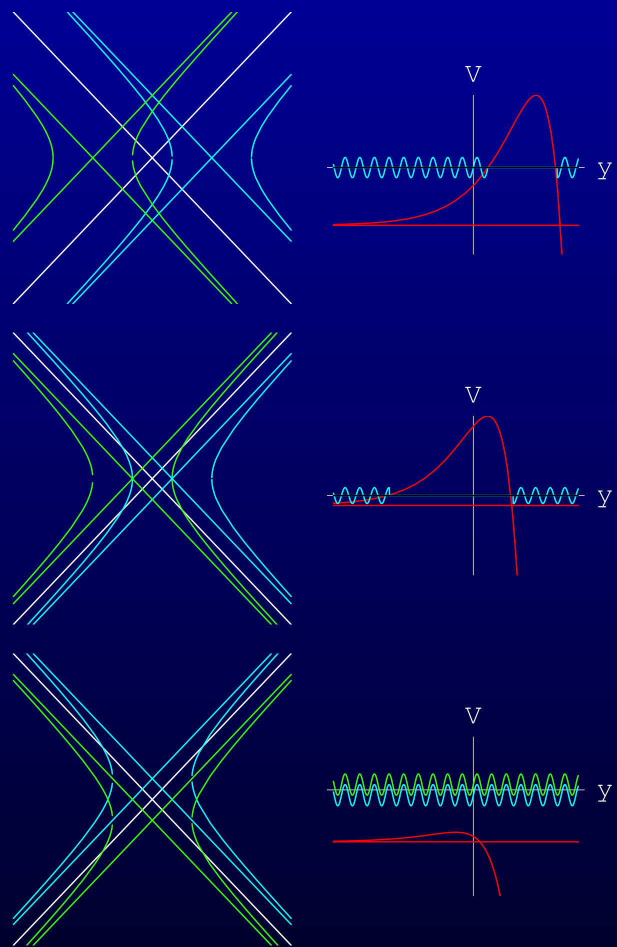
$$V(y) = M^2 e^{2y} - (J + \frac{1}{2}\nu \; e^{2y})^2$$

- If $j < 0$, the electron and positron branches are in the same Rindler quadrant. Tunneling corresponds to Schwinger particle production.
- If $0 < j < M^2/(2\nu)$, the two electron branches are in the same Rindler quadrant. Tunneling corresponds to Hawking radiation.

Charged particle in Rindler space

- For applications to the Milne universe, one should diagonalize the boost momentum J, ie consider an accelerated observer.

Gabriel Spindel; Mottola Cooper



- In the Rindler patch R, letting $f(r, \eta) = e^{-iJ\eta} f_J(r)$ and $r = e^y$, one gets a Schrödinger equation for a particle in a potential

$$V(y) = M^2 e^{2y} - (J + \frac{1}{2}\nu\, e^{2y})^2$$

- If $j < 0$, the electron and positron branches are in the same Rindler quadrant. Tunneling corresponds to Schwinger particle production.
- If $0 < j < M^2/(2\nu)$, the two electron branches are in the same Rindler quadrant. Tunneling corresponds to Hawking radiation.
- If $j > M^2/(2\nu)$, the electron branches extend in the Milne regions. There is no tunneling, but partial reflection amounts to a combination of Schwinger and Hawking emission.

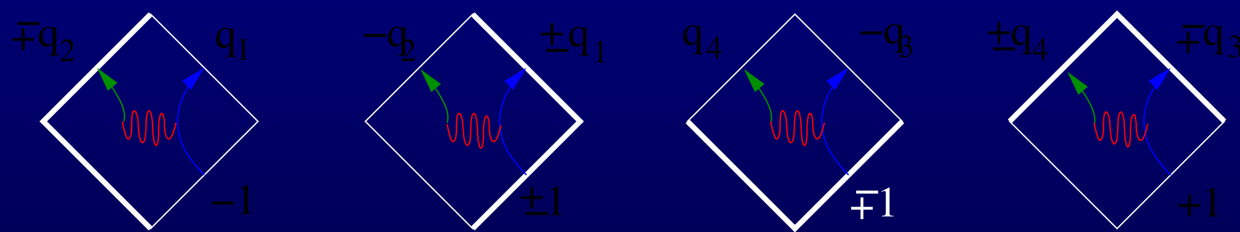
Rindler modes

- Incoming modes from Rindler infinity I_R^- read, in terms of parabolic cylinder functions:

$$\mathcal{V}^j_{in,R} = e^{-ij\eta} r^{-1} M_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),-\frac{ij}{2}}(i\nu r^2/2)$$

Incoming modes from the Rindler horizon H_R^- read

$$\mathcal{U}^j_{in,R} = e^{-ij\eta} r^{-1} W_{i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}(-i\nu r^2/2)$$



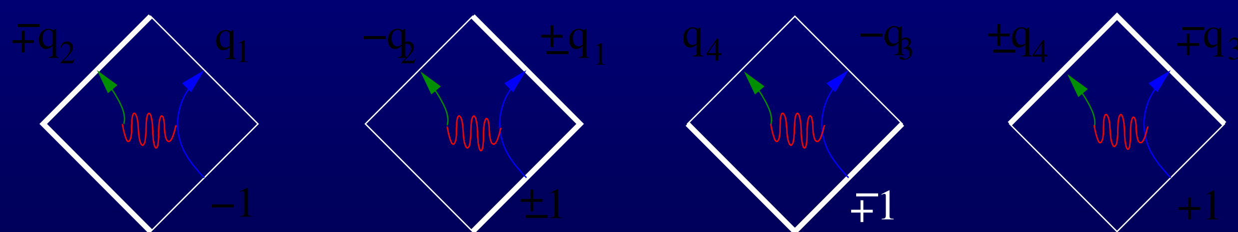
Rindler modes

- Incoming modes from Rindler infinity I_R^- read, in terms of parabolic cylinder functions:

$$\mathcal{V}^j_{in,R} = e^{-ij\eta} r^{-1} M_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),-\frac{ij}{2}}(i\nu r^2/2)$$

Incoming modes from the Rindler horizon H_R^- read

$$\mathcal{U}^j_{in,R} = e^{-ij\eta} r^{-1} W_{i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}(-i\nu r^2/2)$$



- The reflection coefficients can be computed:

$$q_1 = e^{-\pi j}\frac{\cosh\left[\pi\frac{M^2}{2\nu}\right]}{\cosh\left[\pi\left(j-\frac{M^2}{2\nu}\right)\right]}\,, \quad q_3 = e^{\pi\left(j-\frac{M^2}{2\nu}\right)}\frac{\cosh\left[\pi\frac{M^2}{2\nu}\right]}{|\sinh \pi j|}$$

and $q_2 = 1 - q_1$, $q_4 = q_3 - 1$, by charge conservation.

Global Charged Unruh Modes

- Global modes may be defined by patching together Rindler modes, ie by analytic continuation across the horizons. Unruh modes are those which are superposition of positive energy Minkowski modes,

$$\begin{aligned} \Omega^j_{in,+} &= \mathcal{V}^j_{in,P} = (-i\nu X^+X^-)[X^+/X^-]^{-ij/2} W_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}} \\ \omega^j_{in,-} &= \mathcal{U}^j_{in,P} = (i\nu X^+X^-)[X^+/X^-]^{-ij/2} M_{i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}} \end{aligned}$$

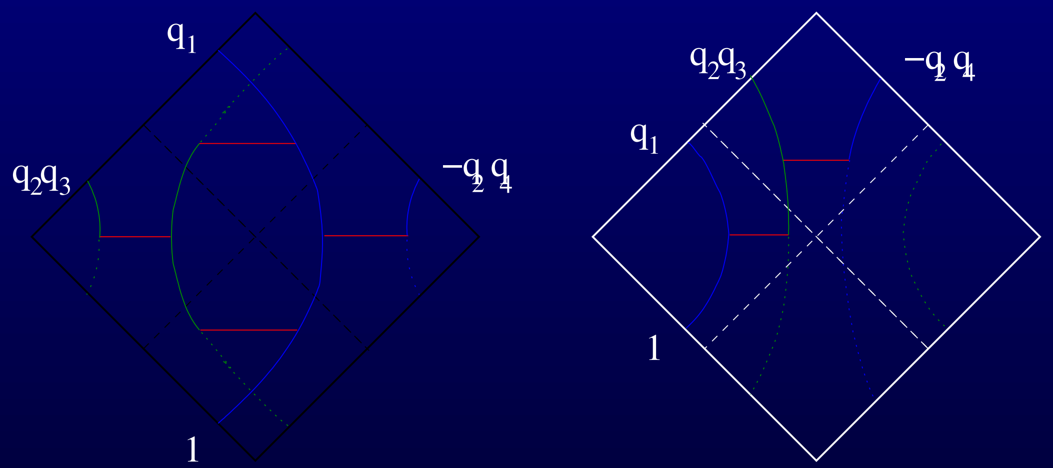
Global Charged Unruh Modes

- Global modes may be defined by patching together Rindler modes, ie by analytic continuation across the horizons. Unruh modes are those which are superposition of positive energy Minkowski modes,

$$\Omega^j_{in,+} = \mathcal{V}^j_{in,P} = (-i\nu X^+X^-)[X^+/X^-]^{-ij/2} W_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}$$

$$\omega^j_{in,-} = \mathcal{U}^j_{in,P} = (i\nu X^+X^-)[X^+/X^-]^{-ij/2} M_{i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}$$

- There are two types of Unruh modes, involving 2 or 4 tunelling events:



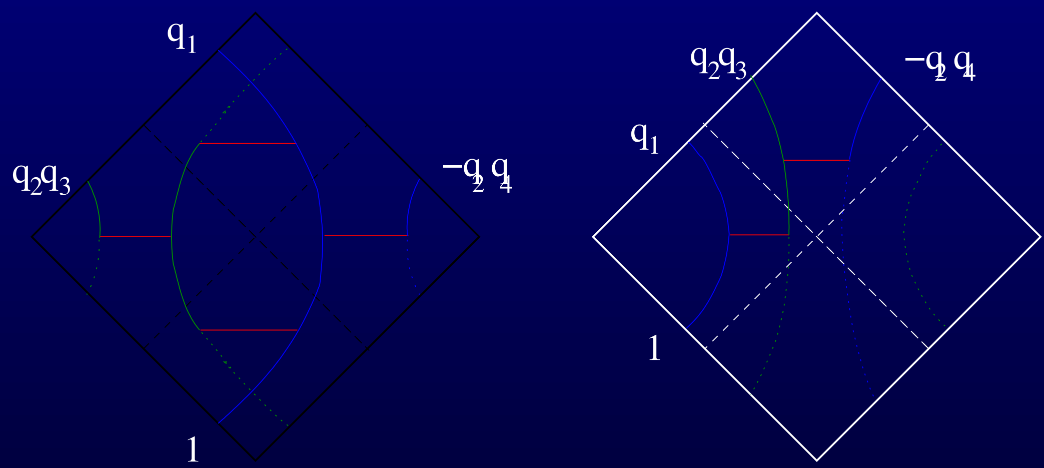
Global Charged Unruh Modes

- Global modes may be defined by patching together Rindler modes, ie by analytic continuation across the horizons. Unruh modes are those which are superposition of positive energy Minkowski modes,

$$\Omega^j_{in,+} = \mathcal{V}^j_{in,P} = (-i\nu X^+ X^-)[X^+/X^-]^{-ij/2} W_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}$$

$$\omega^j_{in,-} = \mathcal{U}^j_{in,P} = (i\nu X^+ X^-)[X^+/X^-]^{-ij/2} M_{i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}$$

- There are two types of Unruh modes, involving 2 or 4 tunelling events:



- Any state in Minkowski space can be represented as a state in the tensor product of the Hilbert spaces of the left and right Rindler patches. In contrast to neutral fields in Rindler space, Boulware-Fulling modes that vanish in L or R have positive Minkowski energy.

Closed string zero-modes

- Let us analyze the classical solutions for the closed string zero modes

$$X^{\pm}(\tau,\sigma) = e^{\mp\nu\sigma}\left[\pm\frac{1}{2\nu}\alpha_0^{\pm}e^{\pm\nu\tau} \mp \frac{1}{2\nu}\tilde{\alpha}_0^{\pm}e^{\mp\nu\tau}\right] \ , \quad \alpha_0^{\pm}, \tilde{\alpha}_0^{\pm} \in R$$

Closed string zero-modes

- Let us analyze the classical solutions for the closed string zero modes

$$X^{\pm}(\tau,\sigma) = e^{\mp\nu\sigma}\left[\pm\frac{1}{2\nu}\alpha_0^{\pm}e^{\pm\nu\tau} \mp \frac{1}{2\nu}\tilde{\alpha}_0^{\pm}e^{\mp\nu\tau}\right] \ , \quad \alpha_0^{\pm}, \tilde{\alpha}_0^{\pm} \in R$$

- The Milne time, or Rindler radius, is independent of σ:

$$4\nu^2 X^+ X^- = \alpha_0^+\tilde{\alpha}_0^- e^{2\nu\tau} + \alpha_0^-\tilde{\alpha}_0^+ e^{-2\nu\tau} - \alpha_0^+\alpha_0^- - \tilde{\alpha}_0^+\tilde{\alpha}_0^-$$

We may thus follow the motion of a single point $\sigma = \sigma_0$ and obtain the rest of the worldsheet by acting with the boost.

Closed string zero-modes

- Let us analyze the classical solutions for the closed string zero modes

$$X^{\pm}(\tau,\sigma) = e^{\mp\nu\sigma}\left[\pm\frac{1}{2\nu}\alpha_0^{\pm}e^{\pm\nu\tau} \mp \frac{1}{2\nu}\tilde{\alpha}_0^{\pm}e^{\mp\nu\tau}\right] \,, \quad \alpha_0^{\pm}, \tilde{\alpha}_0^{\pm} \in R$$

- The Milne time, or Rindler radius, is independent of σ:

$$4\nu^2 X^+ X^- = \alpha_0^+\tilde{\alpha}_0^- e^{2\nu\tau} + \alpha_0^-\tilde{\alpha}_0^+ e^{-2\nu\tau} - \alpha_0^+\alpha_0^- - \tilde{\alpha}_0^+\tilde{\alpha}_0^-$$

  We may thus follow the motion of a single point $\sigma = \sigma_0$ and obtain the rest of the worldsheet by acting with the boost.

- Up to a shift of τ and σ, the physical state conditions require

$$\alpha_0^+ = \alpha_0^- = \epsilon\frac{M}{\sqrt{2}} \,, \quad \tilde{\alpha}_0^+ = \tilde{\alpha}_0^- = \tilde{\epsilon}\frac{\tilde{M}}{\sqrt{2}} \,, \quad M^2 - \tilde{M}^2 = 2\nu j \in Z$$

Closed string zero-modes

- Let us analyze the classical solutions for the closed string zero modes

$$X^{\pm}(\tau,\sigma) = e^{\mp\nu\sigma}\left[\pm\frac{1}{2\nu}\alpha_0^{\pm}e^{\pm\nu\tau} \mp \frac{1}{2\nu}\tilde{\alpha}_0^{\pm}e^{\mp\nu\tau}\right] \ , \quad \alpha_0^{\pm}, \tilde{\alpha}_0^{\pm} \in R$$

- The Milne time, or Rindler radius, is independent of σ:

$$4\nu^2 X^+ X^- = \alpha_0^+\tilde{\alpha}_0^- e^{2\nu\tau} + \alpha_0^-\tilde{\alpha}_0^+ e^{-2\nu\tau} - \alpha_0^+\alpha_0^- - \tilde{\alpha}_0^+\tilde{\alpha}_0^-$$

  We may thus follow the motion of a single point $\sigma = \sigma_0$ and obtain the rest of the worldsheet by acting with the boost.

- Up to a shift of τ and σ, the physical state conditions require

$$\alpha_0^+ = \alpha_0^- = \epsilon\frac{M}{\sqrt{2}} \ , \quad \tilde{\alpha}_0^+ = \tilde{\alpha}_0^- = \tilde{\epsilon}\frac{\tilde{M}}{\sqrt{2}} \ , \quad M^2 - \tilde{M}^2 = 2\nu j \in Z$$

- The behavior at early/late proper time now depends on $\epsilon\tilde{\epsilon}$: For $\epsilon\tilde{\epsilon} = 1$, the string begin/ends in the Milne regions. For $\epsilon\tilde{\epsilon} = -1$, the string begin/ends in the Rindler regions.

Short and long strings ($j = 0$)

Choosing $j = 0$ for simplicity, we have 4 solutions:

- $\epsilon = 1$, $\tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma, \tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma} \ , \quad T = \frac{M}{\nu} \sinh(\nu\tau) \ , \quad \theta = \nu\sigma$$

  is a short string winding around the Milne circle from $T = -\infty$ to $T = +\infty$.

Short and long strings ($j = 0$)

Choosing $j = 0$ for simplicity, we have 4 solutions:

- $\epsilon = 1$, $\tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma, \tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma} \,, \quad T = \frac{M}{\nu} \sinh(\nu\tau) \,, \quad \theta = \nu\sigma$$

is a short string winding around the Milne circle from $T = -\infty$ to $T = +\infty$.

$\epsilon = -1$, $\tilde{\epsilon} = -1$ is just the time reversal of this process.

Short and long strings ($j = 0$)

Choosing $j = 0$ for simplicity, we have 4 solutions:

- $\epsilon = 1$, $\tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma, \tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma} \,, \quad T = \frac{M}{\nu} \sinh(\nu\tau) \,, \quad \theta = \nu\sigma$$

is a short string winding around the Milne circle from $T = -\infty$ to $T = +\infty$.

$\epsilon = -1$, $\tilde{\epsilon} = -1$ is just the time reversal of this process.

- $\epsilon = 1$, $\tilde{\epsilon} = -1$:

$$X^{\pm}(\sigma, \tau) = \pm\frac{M}{\nu\sqrt{2}} \cosh(\nu\tau) e^{\pm\nu\sigma} \,, \quad r = \frac{M}{\nu} \cosh(\nu\tau) \,, \quad \eta = \nu\sigma$$

is a long string stretched in the right Rindler patch, from $r = \infty$ to $r = M/\nu$ and back to $r = \infty$; σ is now the proper time direction in the induced metric.

Short and long strings ($j = 0$)

Choosing $j = 0$ for simplicity, we have 4 solutions:

- $\epsilon = 1$, $\tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma,\tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma} \, , \quad T = \frac{M}{\nu} \sinh(\nu\tau) \, , \quad \theta = \nu\sigma$$

is a short string winding around the Milne circle from $T = -\infty$ to $T = +\infty$.

$\epsilon = -1$, $\tilde{\epsilon} = -1$ is just the time reversal of this process.

- $\epsilon = 1$, $\tilde{\epsilon} = -1$:

$$X^{\pm}(\sigma,\tau) = \pm \frac{M}{\nu\sqrt{2}} \cosh(\nu\tau) e^{\pm\nu\sigma} \, , \quad r = \frac{M}{\nu} \cosh(\nu\tau) \, , \quad \eta = \nu\sigma$$

is a long string stretched in the right Rindler patch, from $r = \infty$ to $r = M/\nu$ and back to $r = \infty$; σ is now the proper time direction in the induced metric.

$\epsilon = -1$, $\tilde{\epsilon} = 1$ is the analogue in the left Rindler patch.

Short and long strings ($j = 0$)

Choosing $j = 0$ for simplicity, we have 4 solutions:

- $\epsilon = 1$, $\tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma,\tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma} \ , \quad T = \frac{M}{\nu} \sinh(\nu\tau) \ , \quad \theta = \nu\sigma$$

is a short string winding around the Milne circle from $T = -\infty$ to $T = +\infty$.

$\epsilon = -1$, $\tilde{\epsilon} = -1$ is just the time reversal of this process.

- $\epsilon = 1$, $\tilde{\epsilon} = -1$:

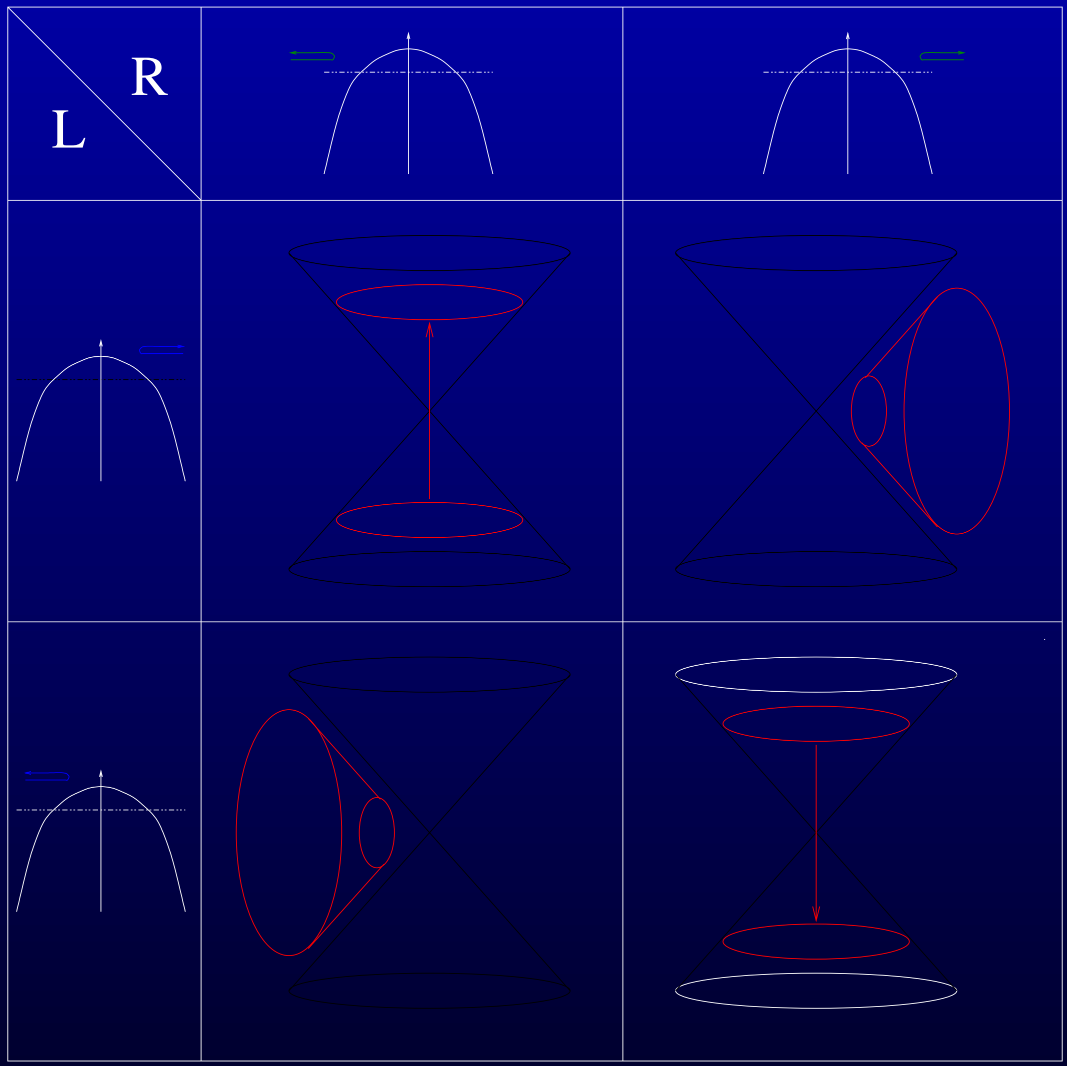
$$X^{\pm}(\sigma,\tau) = \pm\frac{M}{\nu\sqrt{2}} \cosh(\nu\tau) e^{\pm\nu\sigma} \ , \quad r = \frac{M}{\nu} \cosh(\nu\tau) \ , \quad \eta = \nu\sigma$$

is a long string stretched in the right Rindler patch, from $r = \infty$ to $r = M/\nu$ and back to $r = \infty$; σ is now the proper time direction in the induced metric.

$\epsilon = -1$, $\tilde{\epsilon} = 1$ is the analogue in the left Rindler patch.

Short and long strings

Closed string trajectories are thus generated by the motion of two decoupled particles in inverted harmonic oscillators:



Relation to open string modes

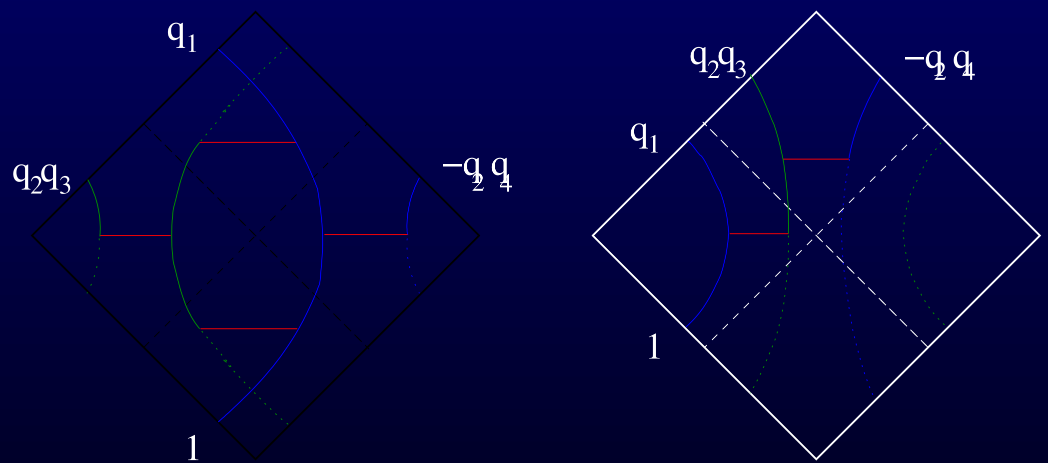
- Instead of following the motion of a point at fixed σ, one may consider instead a point a fixed $\sigma + \tau$: this is precisely the trajectory of the open string zero-mode.
- Using the covariant derivative representation

$$\alpha_0^{\pm} = i\partial_{\mp} \pm \frac{\nu}{2} x^{\pm} \,, \quad \tilde{\alpha}_0^{\pm} = i\partial_{\mp} \mp \frac{\nu}{2} x^{\pm}$$

we observe that $x^{\pm}$ is the Heisenberg operator corresponding to the location of the closed string (at $\sigma = 0$):

$$X_0^{\pm}(\sigma, \tau) = e^{\mp\nu\sigma} \left[\cosh(\nu\tau)\, x^{\pm} + i \sinh(\nu\tau)\, \partial_{\mp} \right]$$

- The open string global wave functions. . .



Relation to open string modes

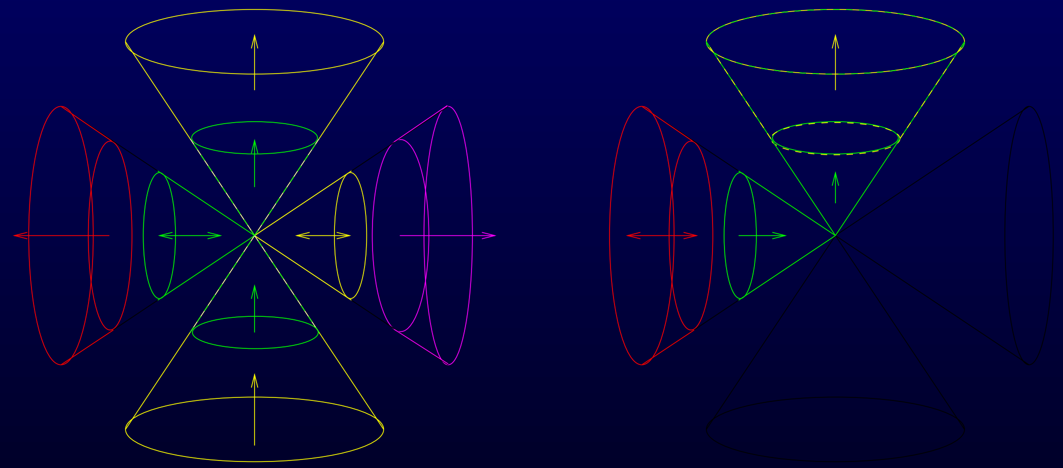
- Instead of following the motion of a point at fixed σ, one may consider instead fixed $\sigma+\tau$: these are the trajectories of the open string zero-mode, in Rindler coordinates.
- Using the covariant derivative representation

$$\alpha_0^{\pm} = i\partial_{\mp} \pm \frac{\nu}{2} x^{\pm} \,, \quad \tilde{\alpha}_0^{\pm} = i\partial_{\mp} \mp \frac{\nu}{2} x^{\pm}$$

we observe that $x^{\pm}$ is the Heisenberg operator corresponding to the location of the closed string (at $\sigma = 0$):

$$X_0^{\pm}(\sigma,\tau) = e^{\mp\nu\sigma} \left[\cosh(\nu\tau)\, x^{\pm} + i \sinh(\nu\tau)\, \partial_{\mp} \right]$$

- The open string global wave functions are also the closed string wave functions. . .



Quantization in the Rindler patch

- For long strings in conformal gauge, the worldsheet time τ is in fact a spacelike coordinate wrt to the induced metric. This is also true for short strings: as they wander in the Rindler patch, the induced metric undergoes a signature flip.
- If so we should quantize the string with respect to the "time" coordinate σ rather than τ. The canonical generator of time translations

$$E = -\int_{-\infty}^{\infty} d\tau \left(X^+ \partial_\sigma X^- - X^- \partial_\sigma X^+ \right) = \int_{-\infty}^{\infty} d\tau r^2 \partial_\sigma \eta$$

  is infinite: long strings carry an infinite Rindler energy.
- Introducing a cut-off $-T \leq \tau < T$, the Rindler energy

$$E_T \sim -\frac{e^{2\nu T}}{4\nu^2} \left(\tilde{\alpha}_0^+ \alpha_0^- + \tilde{\alpha}_0^- \alpha_0^+ \right)$$

  can be understood as the tensive energy of the static stretched string.
- The Rindler energy spectrum is unbounded: long strings ($\epsilon = -1$) have $E_T > e^{2\nu T} M \tilde{M} / (4\nu^2)$ unbounded from below, while the short strings ($\epsilon = 1$) have $E_T < -e^{2\nu T} M \tilde{M} / (4\nu^2)$ unbounded from above.

The one-loop amplitude again

- Recall the (Euclidean ws, Minkowskian target) one-loop amplitude:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=0}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{\left|\eta^{21}(\rho)\ \theta_1(i\beta(l+w\rho);\rho)\right|^2}$$

The one-loop amplitude again

- Recall the (Euclidean ws, Minkowskian target) one-loop amplitude:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=0}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho)\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

- As in the open string case, the zero mode contribution $1/\sinh^2(\pi\beta(l+w\rho))$ may be interpreted either as a sum over (Euclidean) discrete states, or a continuous integral over the continuous (Lorentzian) modes: there are physical states at each level.

The one-loop amplitude again

- Recall the (Euclidean ws, Minkowskian target) one-loop amplitude:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=0}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{\left|\eta^{21}(\rho)\ \theta_1(i\beta(l+w\rho);\rho)\right|^2}$$

- As in the open string case, the zero mode contribution $1/\sinh^2(\pi\beta(l+w\rho))$ may be interpreted either as a sum over (Euclidean) discrete states, or a continuous integral over the continuous (Lorentzian) modes: there are physical states at each level.
- In addition, there are poles in the bulk of the moduli space, for

$$i\beta(l+w\rho) = m + n\rho\ , \quad (l,w,m,n) \in Z$$

leading to logarithmic divergences $\int d\rho d\bar{\rho}/|\rho-\rho_0|^2 \sim log\epsilon$, analogous to the long strings in AdS_3.

Maldacena Ooguri

The one-loop amplitude again

- Recall the (Euclidean ws, Minkowskian target) one-loop amplitude:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=0}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho)\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

- As in the open string case, the zero mode contribution $1/\sinh^2(\pi\beta(l+w\rho))$ may be interpreted either as a sum over (Euclidean) discrete states, or a continuous integral over the continuous (Lorentzian) modes: there are physical states at each level.
- In addition, there are poles in the bulk of the moduli space, for

$$i\beta(l+w\rho) = m + n\rho\ , \quad (l,w,m,n) \in Z$$

leading to logarithmic divergences $\int d\rho d\bar{\rho}/|\rho-\rho_0|^2 \sim log\epsilon$, analogous to the long strings in AdS_3.

Maldacena Ooguri

- In contrast to the open string case, these poles do not yield an imaginary part: the overall cosmological particle production seems to vanish. *This is not to say that there is no particle production at intermediate stages !*

Wick rotation to a rotation orbifold

- Note first that the (future) Milne region $ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + dx_i^2$ cannot be directly Wick-rotated to Euclidean.

Wick rotation to a rotation orbifold

- Note first that the (future) Milne region $ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + dx_i^2$ cannot be directly Wick-rotated to Euclidean. Instead, the analytical continuation $T = ir$, $\theta = \eta + i\pi/(2\beta)$ leads to the (right) Rindler wedge.

Wick rotation to a rotation orbifold

- Note first that the (future) Milne region $ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + dx_i^2$ cannot be directly Wick-rotated to Euclidean. Instead, the analytical continuation $T = ir$, $\theta = \eta + i\pi/(2\beta)$ leads to the (right) Rindler wedge.
- Rotating $\beta = i\mu$, the Rindler region becomes get indeed an Euclidean metric,

$$ds^2 = dr^2 + \mu^2 r^2 d\eta^2 + (dX^i)^2 = 2\, dZ d\bar{Z} + (dX^i)^2$$

$$Z = X^+ = re^{i\mu\eta}\,, \quad \bar{Z} = -X^- = re^{-i\mu\eta} \qquad (r > 0)$$

- The Milne identification $\eta \equiv \eta + 2\pi$ amounts to a rotation identification $Z \to e^{2\pi i\mu} Z$.

Wick rotation to a rotation orbifold

- Note first that the (future) Milne region $ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + dx_i^2$ cannot be directly Wick-rotated to Euclidean. Instead, the analytical continuation $T = ir$, $\theta = \eta + i\pi/(2\beta)$ leads to the (right) Rindler wedge.
- Rotating $\beta = i\mu$, the Rindler region becomes get indeed an Euclidean metric,

$$ds^2 = dr^2 + \mu^2 r^2 d\eta^2 + (dX^i)^2 = 2\, dZ d\bar{Z} + (dX^i)^2$$

$$Z = X^+ = re^{i\mu\eta}\,, \quad \bar{Z} = -X^- = re^{-i\mu\eta} \qquad (r > 0)$$

- The Milne identification $\eta \equiv \eta + 2\pi$ amounts to a rotation identification $Z \to e^{2\pi i\mu} Z$.
- This cannot be the usual rotation orbifold however, because this would imply that the physics depends on β being rational or not.

Wick rotation to a rotation orbifold

- Note first that the (future) Milne region $ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + dx_i^2$ cannot be directly Wick-rotated to Euclidean. Instead, the analytical continuation $T = ir$, $\theta = \eta + i\pi/(2\beta)$ leads to the (right) Rindler wedge.
- Rotating $\beta = i\mu$, the Rindler region becomes get indeed an Euclidean metric,

$$ds^2 = dr^2 + \mu^2 r^2 d\eta^2 + (dX^i)^2 = 2\, dZ d\bar{Z} + (dX^i)^2$$

$$Z = X^+ = re^{i\mu\eta}\,, \quad \bar{Z} = -X^- = re^{-i\mu\eta} \qquad (r > 0)$$

- The Milne identification $\eta \equiv \eta + 2\pi$ amounts to a rotation identification $Z \to e^{2\pi i\mu} Z$.
- This cannot be the usual rotation orbifold however, because this would imply that the physics depends on β being rational or not. In other words, Z and $e^{2\pi i}Z$ should not identified: $(Z, \bar{Z})$ really take value in $\widetilde{R^2\backslash\{0\}}$.

Wick rotation to a rotation orbifold

- Note first that the (future) Milne region $ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + dx_i^2$ cannot be directly Wick-rotated to Euclidean. Instead, the analytical continuation $T = ir$, $\theta = \eta + i\pi/(2\beta)$ leads to the (right) Rindler wedge.
- Rotating $\beta = i\mu$, the Rindler region becomes get indeed an Euclidean metric,

$$ds^2 = dr^2 + \mu^2 r^2 d\eta^2 + (dX^i)^2 = 2\, dZ d\bar{Z} + (dX^i)^2$$

$$Z = X^+ = re^{i\mu\eta}\,, \quad \bar{Z} = -X^- = re^{-i\mu\eta} \qquad (r > 0)$$

- The Milne identification $\eta \equiv \eta + 2\pi$ amounts to a rotation identification $Z \longrightarrow e^{2\pi i\mu} Z$.
- This cannot be the usual rotation orbifold however, because this would imply that the physics depends on β being rational or not. In other words, Z and $e^{2\pi i} Z$ should not identified: $(Z, \bar{Z})$ really take value in $\widetilde{R^2 \backslash \{0\}}$.
- By the same token, the left Rindler wedge rotates to another copy of the Euclidean plane with the origin removed: the complete analytic continuation of Misner space is therefore

$$\widetilde{R^2 \backslash \{0\}}_L / e^{i\mu} \backslash \widetilde{R^2 \backslash \{0\}}_R$$

and states of interest are non-normalizable !

D'Appolonio Kiritsis

Conclusions - speculations

- Winding states in the Milne Universe behave in close analogy with open strings in an electric field. Using intuition from open strings, we have found that physical states do exist in the twisted sector of the Lorentzian orbifold, and can be pair produced.

Conclusions - speculations

- Winding states in the Milne Universe behave in close analogy with open strings in an electric field. Using intuition from open strings, we have found that physical states do exist in the twisted sector of the Lorentzian orbifold, and can be pair produced.
- In view of this analogy, it is natural to ask if the same mechanism (Schwinger production) which leads in the open string case to the decay of the electric field could “relax the boost parameter”.

Cooper, Eisenberg, Kluger, Mottola and Svetitsky

Conclusions - speculations

- Winding states in the Milne Universe behave in close analogy with open strings in an electric field. Using intuition from open strings, we have found that physical states do exist in the twisted sector of the Lorentzian orbifold, and can be pair produced.
- In view of this analogy, it is natural to ask if the same mechanism (Schwinger production) which leads in the open string case to the decay of the electric field could “relax the boost parameter”.

Cooper, Eisenberg, Kluger, Mottola and Svetitsky

- In particular, since winding strings are spontaneously produced near the singularity, they contribute an energy proportional to the radius, hence akin to a two-dimensional positive cosmological constant: it seems plausible that the resulting transcient inflation may smooth out the singularity.

Conclusions - speculations

- Winding states in the Milne Universe behave in close analogy with open strings in an electric field. Using intuition from open strings, we have found that physical states do exist in the twisted sector of the Lorentzian orbifold, and can be pair produced.
- In view of this analogy, it is natural to ask if the same mechanism (Schwinger production) which leads in the open string case to the decay of the electric field could “relax the boost parameter”.

Cooper, Eisenberg, Kluger, Mottola and Svetitsky

- In particular, since winding strings are spontaneously produced near the singularity, they contribute an energy proportional to the radius, hence akin to a two-dimensional positive cosmological constant: it seems plausible that the resulting transcient inflation may smooth out the singularity.
- To demonstrate this, one should take into account the production of (an infinite number) of twisted sector states are produced in correlated pairs, i.e. squeezed states:

Conclusions - speculations

- Winding states in the Milne Universe behave in close analogy with open strings in an electric field. Using intuition from open strings, we have found that physical states do exist in the twisted sector of the Lorentzian orbifold, and can be pair produced.
- In view of this analogy, it is natural to ask if the same mechanism (Schwinger production) which leads in the open string case to the decay of the electric field could “relax the boost parameter”.

Cooper, Eisenberg, Kluger, Mottola and Svetitsky

- In particular, since winding strings are spontaneously produced near the singularity, they contribute an energy proportional to the radius, hence akin to a two-dimensional positive cosmological constant: it seems plausible that the resulting transcient inflation may smooth out the singularity.
- To demonstrate this, one should take into account the production of (an infinite number) of twisted sector states are produced in correlated pairs, i.e. squeezed states: non-local deformations of the worldsheet ? string field theory ?

Conclusions - speculations (cont.)

- As a less ambitious goal, can one compute scattering amplitudes of twisted states, and check if they are better behaved than untwisted states. For this, the relation with negative level $Sl(2)/U(1)$ and double analytic continuation of the Nappi-Witten plane wave may be useful.

D'Appollonio, Kiritsis; B. Durin, BP

Conclusions - speculations (cont.)

- As a less ambitious goal, can one compute scattering amplitudes of twisted states, and check if they are better behaved than untwisted states. For this, the relation with negative level $Sl(2)/U(1)$ and double analytic continuation of the Nappi-Witten plane wave may be useful.

D'Appollonio, Kiritsis; B. Durin, BP

- The "dynamics" of the long strings living in the whiskers is still unclear: what is the proper way of quantizing them ? Could they perhaps provide a dual holographic dynamics to the bulk ? Or do CTC make them unredeemable ?

Conclusions - speculations (cont.)

- As a less ambitious goal, can one compute scattering amplitudes of twisted states, and check if they are better behaved than untwisted states. For this, the relation with negative level $Sl(2)/U(1)$ and double analytic continuation of the Nappi-Witten plane wave may be useful.

  D'Appollonio, Kiritsis; B. Durin, BP

- The "dynamics" of the long strings living in the whiskers is still unclear: what is the proper way of quantizing them ? Could they perhaps provide a dual holographic dynamics to the bulk ? Or do CTC make them unredeemable ?

- The closed string orbifold we have discussed are highly non-generic trajectories on the cosmological billiard: Do whiskers feature also for more general Kasner-like singularities ?

  Damour, Henneaux, Nicolai, Julia, . . .

Conclusions - speculations (cont.)

- As a less ambitious goal, can one compute scattering amplitudes of twisted states, and check if they are better behaved than untwisted states. For this, the relation with negative level $Sl(2)/U(1)$ and double analytic continuation of the Nappi-Witten plane wave may be useful.

D'Appollonio, Kiritsis; B. Durin, BP

- The "dynamics" of the long strings living in the whiskers is still unclear: what is the proper way of quantizing them ? Could they perhaps provide a dual holographic dynamics to the bulk ? Or do CTC make them unredeemable ?

- The closed string orbifold we have discussed are highly non-generic trajectories on the cosmological billiard: Do whiskers feature also for more general Kasner-like singularities ?

Damour, Henneaux, Nicolai, Julia, . . .

- More generally, we still lack a framework to compute the production of closed strings in cosmological backgrounds. Those however are likely to lead to large departures from FRW cosmology, and possibly spectacular effects: cosmological bounce, Hagedorn phase transition...

Lawrence Martinec, Gubser