Soutenance de thèse d'habilitation

Université Pierre et Marie Curie

Contributions à la théorie dite des supercordes

Aspects de la théorie des cordes en champ de fond dépendant du temps

Boris Pioline

LPTHE, Paris

19 septembre 2003

transparents disponibles sur
`http://www.lpthe.jussieu.fr/~pioline/seminars.html`

Remerciements

α. Thèse de doctorat: *Aspects non-perturbatifs de la théorie des cordes*

direction I. Antoniadis, avril 1998

1. Dualité, amplitudes non perturbatives et instantons

   Kiritsis, Obers, Hanany, Partouche

2. Théories de jauge non commutatives

   Schwarz, Elitzur, Rabinovici

3. Sur la correspondance AdS/CFT

   D'Hoker, Bachas, Hoppe, Kiritsis

4. Supercordes à basse énergie

   Antoniadis, Bachas

5. Formes automorphes et la supermembrane

   Nicolai, Plefka, Waldron, Kazhdan

6. Champs de fond dépendant du temps

   Waldron, Durin, Berkooz, Gutperle

Pour des contraintes dépendantes du temps, je me restreindrai à ce dernier thème.

Plan de l'exposé

1. Introduction

2. Modèles-jouets de champs de fond dépendant du temps

3. Cordes ouvertes dans une onde électromagnétique

   Durin BP; Gutperle BP

4. Cordes ouvertes en champ électrique et l'univers de Milne

   Berkooz BP

5. Conclusions et spéculations

   Waldron BP

Introduction

- Initialement introduite pour décrire les interactions fortes, la théorie des cordes a pour principale vocation la description quantique de la gravitation. Accessoirement, elle décrit aussi les interactions de jauge du Modèle Standard dans un cadre unifié.

Introduction

- Initialement introduite pour décrire les interactions fortes, la théorie des cordes a pour principale vocation la description quantique de la gravitation. Accessoirement, elle décrit aussi les interactions de jauge du Modèle Standard dans un cadre unifié.

- Grâce aux propriétés uniques des théories conformes bidimensionnelles, la théorie des cordes conduit à des amplitudes de diffusion de gravitons finies dans l'UV, tout au moins au voisinage de l'espace plat.

Introduction

- Initialement introduite pour décrire les interactions fortes, la théorie des cordes a pour principale vocation la description quantique de la gravitation. Accessoirement, elle décrit aussi les interactions de jauge du Modèle Standard dans un cadre unifié.

- Grâce aux propriétés uniques des théories conformes bidimensionnelles, la théorie des cordes conduit à des amplitudes de diffusion de gravitons finies dans l'UV, tout au moins au voisinage de l'espace plat.

- Au contraire d'autres approches plus “canoniques” à la gravité quantique, la théorie des cordes est définie par rapport à un champ de fond de référence, par exemple l'espace de Minkowski à 10 dimensions, et l'indépendance vis-à-vis du champ de fond est non manifeste.

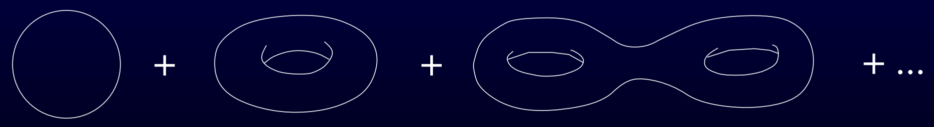
Introduction

- Initialement introduite pour décrire les interactions fortes, la théorie des cordes a pour principale vocation la description quantique de la gravitation. Accessoirement, elle décrit aussi les interactions de jauge du Modèle Standard dans un cadre unifié.

- Grâce aux propriétés uniques des théories conformes bidimensionnelles, la théorie des cordes conduit à des amplitudes de diffusion de gravitons finies dans l'UV, tout au moins au voisinage de l'espace plat.

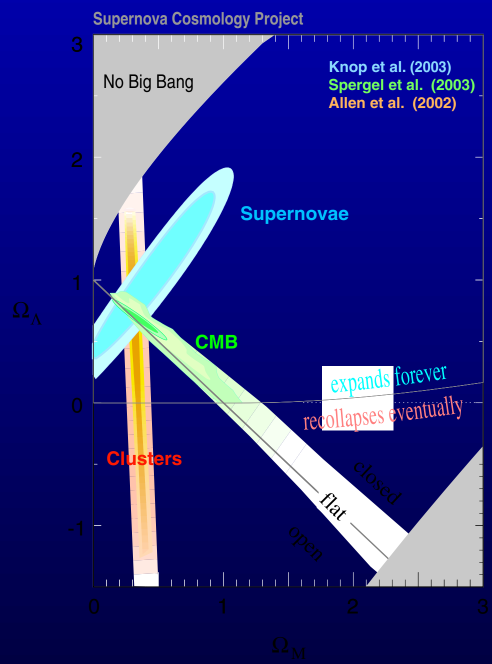
- Au contraire d'autres approches plus “canoniques” à la gravité quantique, la théorie des cordes est définie par rapport à un champ de fond de référence, par exemple l'espace de Minkowski à 10 dimensions, et l'indépendance vis-à-vis du champ de fond est non manifeste.

- La formulation perturbative actuelle de la théorie des cordes est bien adaptée au calcul d'éléments de matrice S en espace asymptotiquement plat, fort utile pour la physique des particules mais moins pour la cosmologie.



Introduction

- Alors que les confirmations expérimentales en physique des particules sont suspendues à la prochaine génération d'accélérateurs de particules, la cosmologie observationnelle connaît un essor impressionnant, conduisant à des mesures des paramètres cosmologiques à une précision inégalée:

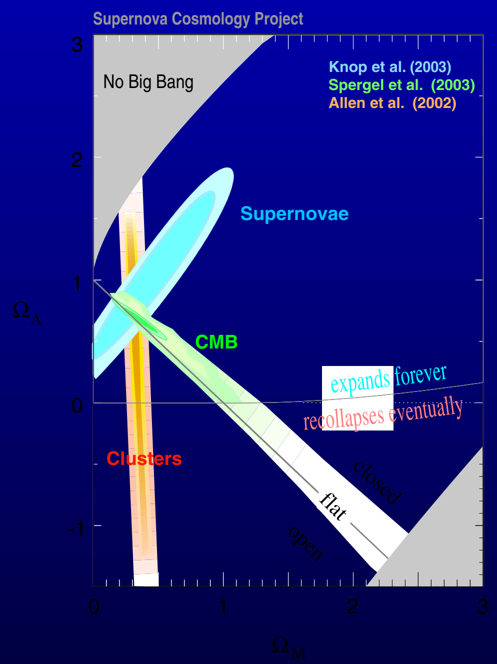


$$\Omega_\Lambda = 71.0\%$$
$$\Omega_{baryon} = 4.7\%$$
$$\Omega_{dark} = 24.3\%$$

$$w = -0.98 \pm .12$$
$$h = 0.72 \pm .05$$

Introduction

- Alors que les confirmations expérimentales en physique des particules sont suspendues à la prochaine génération d'accélérateurs de particules, la cosmologie observationnelle connaît un essor impressionnant, conduisant à des mesures des paramètres cosmologiques à une précision inégalée:



$$\Omega_\Lambda = 71.0\%$$
$$\Omega_{baryon} = 4.7\%$$
$$\Omega_{dark} = 24.3\%$$

$$w = -0.98 \pm .12$$
$$h = 0.72 \pm .05$$

- Si la théorie effective de supergravité suffit quelquefois à décrire les modèles cosmologiques, éventuellement inspirés par la théorie des cordes (e.g. univers branaires), la compréhension de la théorie des cordes en champs de fond dépendant du temps reste très embryonnaire.

Strings in time-dependent backgrounds

De nombreuses questions se posent lorsqu'on cherche à décrire la théorie des cordes en champ de fond dépendant du temps:

- La définition perturbative des amplitudes de genre plus élevé impose de considérer une surface d'univers euclidienne, et donc un espace-cible euclidien. La continuation analytique de l'espace-cible est en général ambiguë et complexe.

Strings in time-dependent backgrounds

De nombreuses questions se posent lorsqu'on cherche à décrire la théorie des cordes en champ de fond dépendant du temps:

- La définition perturbative des amplitudes de genre plus élevé impose de considérer une surface d'univers euclidienne, et donc un espace-cible euclidien. La continuation analytique de l'espace-cible est en général ambiguë et complexe.
- Différents états du vide sont en général possibles, comment implémenter une transformation de Bogolioubov d'un vide à l'autre ? Théorie des champs de cordes ou surface d'univers non locales ?

Strings in time-dependent backgrounds

De nombreuses questions se posent lorsqu'on cherche à décrire la théorie des cordes en champ de fond dépendant du temps:

- La définition perturbative des amplitudes de genre plus élevé impose de considérer une surface d'univers euclidienne, et donc un espace-cible euclidien. La continuation analytique de l'espace-cible est en général ambiguë et complexe.
- Différents états du vide sont en général possibles, comment implémenter une transformation de Bogolioubov d'un vide à l'autre ? Théorie des champs de cordes ou surface d'univers non locales ?
- Le choix des observables est obscur en l'absence de région asymptotique plate, dans le cas d'univers fermés, ou présentant des régions asymptotiques avec courbes causales fermées.

Strings in time-dependent backgrounds

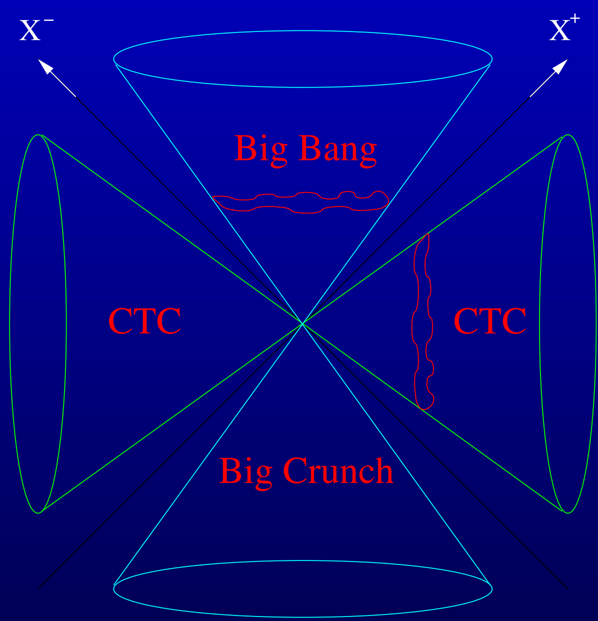
De nombreuses questions se posent lorsqu'on cherche à décrire la théorie des cordes en champ de fond dépendant du temps:

- La définition perturbative des amplitudes de genre plus élevé impose de considérer une surface d'univers euclidienne, et donc un espace-cible euclidien. La continuation analytique de l'espace-cible est en général ambiguë et complexe.
- Différents états du vide sont en général possibles, comment implémenter une transformation de Bogolioubov d'un vide à l'autre ? Théorie des champs de cordes ou surface d'univers non locales ?
- Le choix des observables est obscur en l'absence de région asymptotique plate, dans le cas d'univers fermés, ou présentant des régions asymptotiques avec courbes causales fermées.
- L'impossibilité d'imposer un cut-off infrarouge compatible avec l'invariance conforme conduit classiquement à des singularités de Big Bang ou de Big Crunch... Que deviennent elles sous l'effet des corrections quantiques / de cordes ?

Lorentzian orbifold

- The Lorentzian orbifold, quotient of flat $R^{1,1}$ Minkowski space by a boost J is a toy example of a cosmological singularity:

Horowitz Steif; Seiberg; Nekrasov



Milne Universe region:

$$ds^2 = -dT^2 + T^2 d\theta^2$$
$$\theta \equiv \theta + 2\pi$$

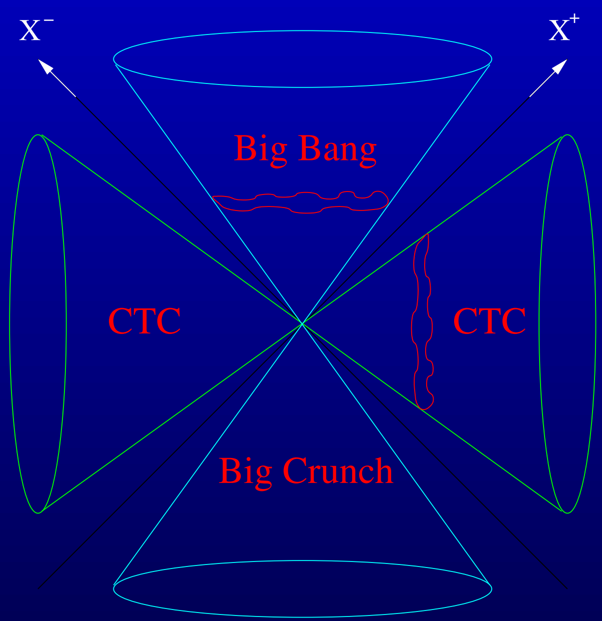
"Whiskers" with CTC:

$$ds^2 = -r^2 d\eta^2 + dr^2$$
$$\eta \equiv \eta + 2\pi$$

Lorentzian orbifold

- The Lorentzian orbifold, quotient of flat $R^{1,1}$ Minkowski space by a boost J is a toy example of a cosmological singularity:

Horowitz Steif; Seiberg; Nekrasov



Milne Universe region:

$$ds^2 = -dT^2 + T^2 d\theta^2$$
$$\theta \equiv \theta + 2\pi$$

"Whiskers" with CTC:

$$ds^2 = -r^2 d\eta^2 + dr^2$$
$$\eta \equiv \eta + 2\pi$$

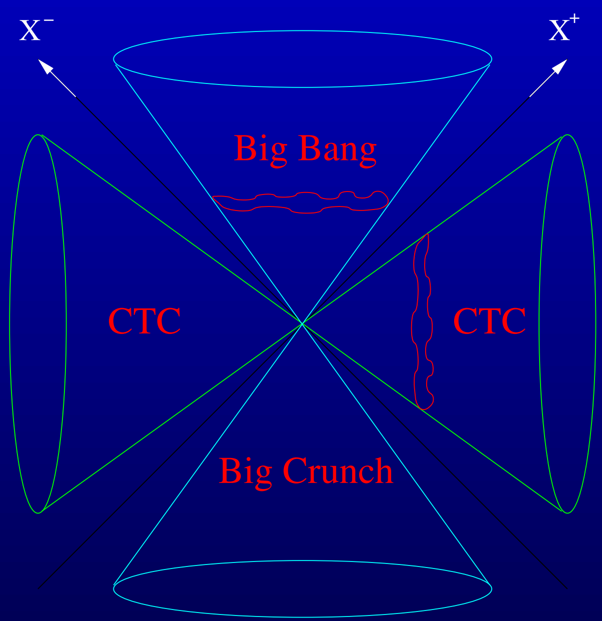
- This is a non-generic example of a Kasner singularity, arising locally in other models such as gauged WZW models. *Are whiskers and CTC generic ?*

Cornalba Costa; Craps Kutasov Rajesh; Elitzur Giveon Kutasov Rabinovici

Lorentzian orbifold

- The Lorentzian orbifold, quotient of flat $R^{1,1}$ Minkowski space by a boost J is a toy example of a cosmological singularity:

Horowitz Steif; Seiberg; Nekrasov



Milne Universe region:

$$\begin{aligned} ds^2 &= -dT^2 + T^2 d\theta^2 \\ \theta &\equiv \theta + 2\pi \end{aligned}$$

"Whiskers" with CTC:

$$\begin{aligned} ds^2 &= -r^2 d\eta^2 + dr^2 \\ \eta &\equiv \eta + 2\pi \end{aligned}$$

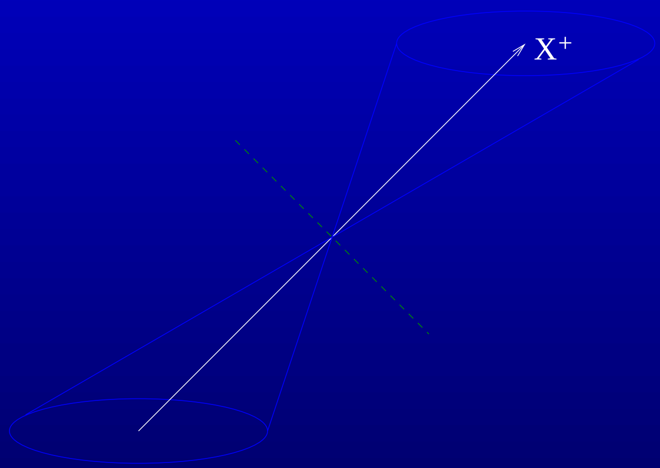
- This is a non-generic example of a Kasner singularity, arising locally in other models such as gauged WZW models. *Are whiskers and CTC generic ?*

Cornalba Costa; Craps Kutasov Rajesh; Elitzur Giveon Kutasov Rabinovici

- *What is the fate of the cosmological singularity and CTC under string corrections ? Can twisted states resolve the singularity ?*

Parabolic orbifold

- The parabolic orbifold is a quotient of flat $R^{1,2}$ Minkowski space by a null boost, ie the product of a boost J_{01} and a rotation R_{12},

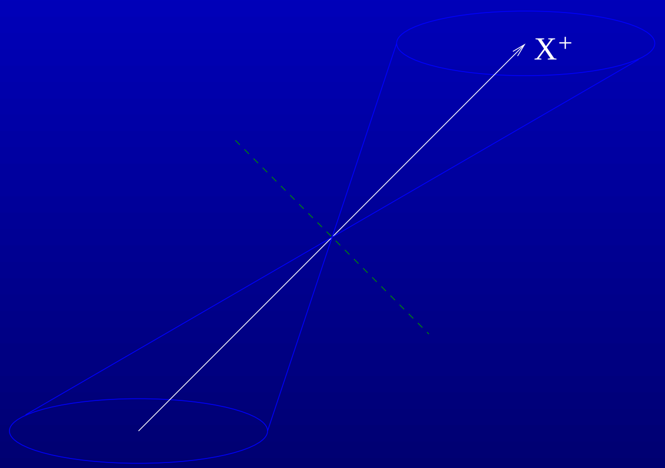


$$\begin{aligned} ds^2 &= -2dy^+dy^- + (y^+)^2 dy^2 \\ y &\equiv y + 2\pi \end{aligned}$$

Simon; Liu Moore Seiberg

Parabolic orbifold

- The parabolic orbifold is a quotient of flat $R^{1,2}$ Minkowski space by a null boost, ie the product of a boost J_{01} and a rotation R_{12},



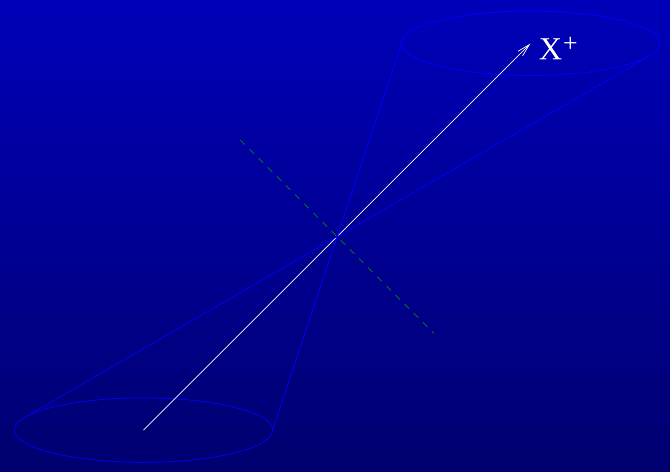
$$ds^2 = -2dy^+dy^- + (y^+)^2dy^2$$
$$y \equiv y + 2\pi$$

Simon; Liu Moore Seiberg

- This background has no CTC and preserves half of the supersymmetries.

Parabolic orbifold

- The parabolic orbifold is a quotient of flat $R^{1,2}$ Minkowski space by a null boost, ie the product of a boost J_{01} and a rotation R_{12},



$$ds^2 = -2dy^+dy^- + (y^+)^2dy^2$$
$$y \equiv y + 2\pi$$

Simon; Liu Moore Seiberg

- This background has no CTC and preserves half of the supersymmetries.
- Yet it has Closed Null Curves and a non-Hausdorff line of singularity. It is more relevant to the physics of null singularities and gravitational wave than to cosmology.

Toys are broken

- Either of these examples seem to be plagued with perturbative divergences, related to large graviton exchange at the singularity, implying large backreaction.

Liu Moore Seiberg

Berkooz Craps Kutasov Rajesh

Toys are broken

- Either of these examples seem to be plagued with perturbative divergences, related to large graviton exchange at the singularity, implying large backreaction.

Liu Moore Seiberg

Berkooz Craps Kutasov Rajesh

- Due to high blue-shift towards the singularity, the images of the particles on the covering space may non-perturbatively form a large black hole, that eats up the space.

Horowitz Polchinski

Toys are broken

- Either of these examples seem to be plagued with perturbative divergences, related to large graviton exchange at the singularity, implying large backreaction.

Liu Moore Seiberg

Berkooz Craps Kutasov Rajesh

- Due to high blue-shift towards the singularity, the images of the particles on the covering space may non-perturbatively form a large black hole, that eats up the space.

Horowitz Polchinski

Can backreaction and particle production resolve these divergences ?

Open strings in time dep. backgrounds

In order to disentangle gravitational instabilities from time-dependence, it may be simpler to consider time-dependent open-string configurations: moving D-branes or electric fields:

- Backreaction in the closed string sector may be neglected as $g_s \longrightarrow 0$. Yet general issues such as choice of vacua and production of open strings are retained.

Open strings in time dep. backgrounds

In order to disentangle gravitational instabilities from time-dependence, it may be simpler to consider time-dependent open-string configurations: moving D-branes or electric fields:

- Backreaction in the closed string sector may be neglected as $g_s \longrightarrow 0$. Yet general issues such as choice of vacua and production of open strings are retained.
- More powerful techniques are available for open strings: boundary states, string field theory, gauge theory . . .

Open strings in time dep. backgrounds

In order to disentangle gravitational instabilities from time-dependence, it may be simpler to consider time-dependent open-string configurations: moving D-branes or electric fields:

- Backreaction in the closed string sector may be neglected as $g_s \longrightarrow 0$. Yet general issues such as choice of vacua and production of open strings are retained.
- More powerful techniques are available for open strings: boundary states, string field theory, gauge theory . . .
- Moving D-branes and electric fields are often easier to visualize...

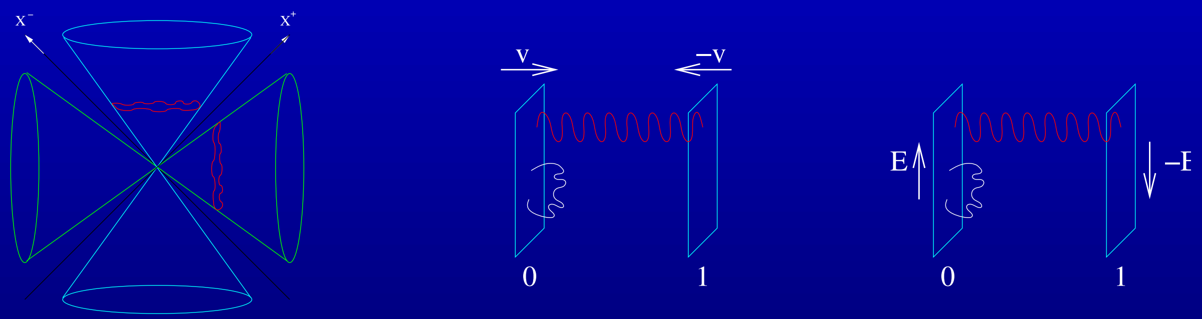
Open strings in time dep. backgrounds

In order to disentangle gravitational instabilities from time-dependence, it may be simpler to consider time-dependent open-string configurations: moving D-branes or electric fields:

- Backreaction in the closed string sector may be neglected as $g_s \longrightarrow 0$. Yet general issues such as choice of vacua and production of open strings are retained.
- More powerful techniques are available for open strings: boundary states, string field theory, gauge theory . . .
- Moving D-branes and electric fields are often easier to visualize...
- Time dependent open string backgrounds may be holographically dual to time-dependent closed string backgrounds...

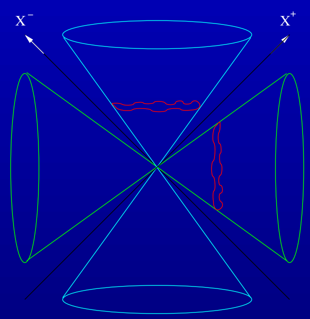
closed strings on orbifolds vs open strings in electric fields

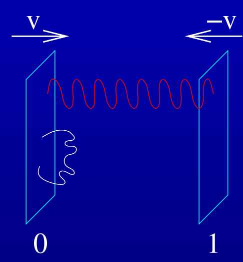
- An analogue of the Lorentzian orbifold is a D-brane head-on collision, or open strings in a constant electric field:

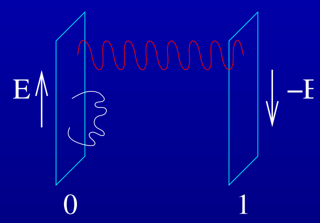


closed strings on orbifolds vs open strings in electric fields

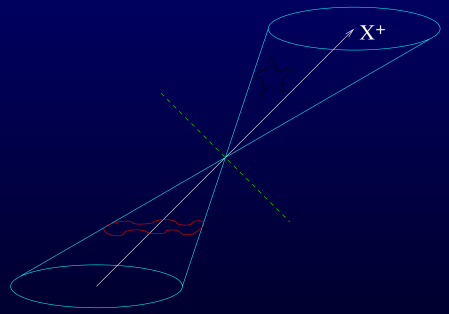
- An analogue of the Lorentzian orbifold is a D-brane head-on collision, or open strings in a constant electric field:

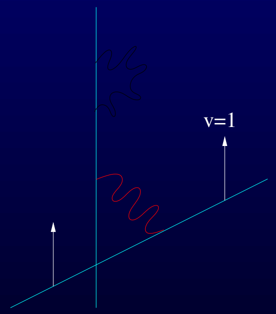


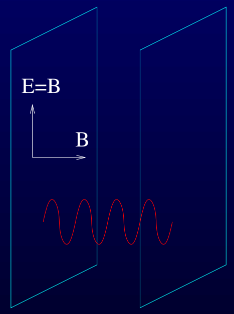




- Analogues of the parabolic orbifold are null scissor configurations, or open strings in a constant null field $F = f dx^2 \wedge dx^+$:



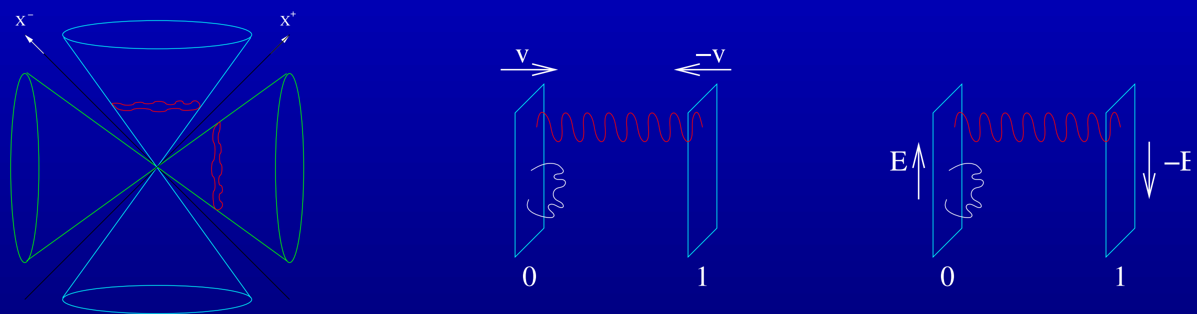




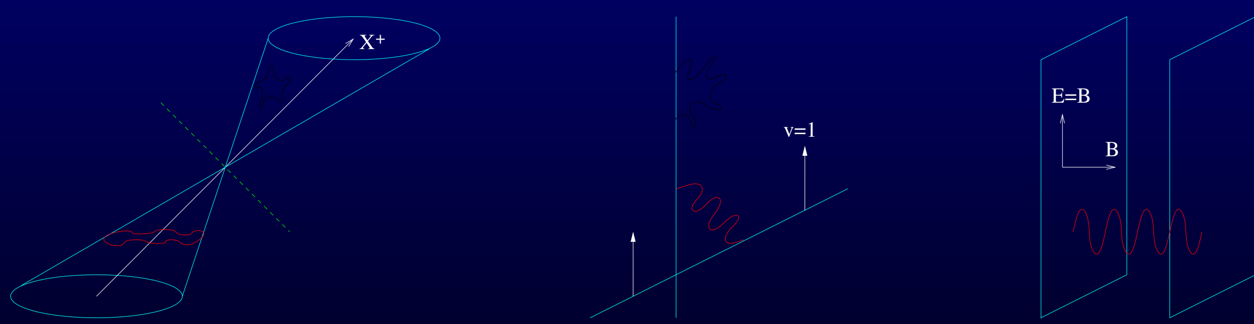
Bachas Hull

closed strings on orbifolds vs open strings in electric fields

- An analogue of the Lorentzian orbifold is a D-brane head-on collision, or open strings in a constant electric field:



- Analogues of the parabolic orbifold are null scissor configurations, or open strings in a constant null field $F = f\,dx^2 \wedge dx^+$:



Bachas Hull

- Charged open strings have (half) the same mode structure as twisted closed strings.

Open strings in constant electromag. field vs orbifolds

- Open strings couple to an electromagnetic field through their boundary only. The embedding coordinates are therefore free bosons in the bulk of the Minkowskian strip $0 < \sigma < \pi, \quad \tau \in R$,

$$X^\mu(\tau, \sigma) = f^\mu(\tau + \sigma) + g^\mu(\tau - \sigma)$$

Open strings in constant electromag. field vs orbifolds

- Open strings couple to an electromagnetic field through their boundary only. The embedding coordinates are therefore free bosons in the bulk of the Minkowskian strip $0 < \sigma < \pi, \quad \tau \in R$,

$$X^\mu(\tau, \sigma) = f^\mu(\tau + \sigma) + g^\mu(\tau - \sigma)$$

- The electric field may be different on each of the two D-branes. The boundary conditions at $\sigma = \sigma_a \in \{0, \pi\}$

$$\partial_\sigma X^\mu + (2\pi\alpha')F^\mu{}_{\nu;a}(X)\partial_\tau X^\nu = 0$$

Open strings in constant electromag. field vs orbifolds

- Open strings couple to an electromagnetic field through their boundary only. The embedding coordinates are therefore free bosons in the bulk of the Minkowskian strip $0 < \sigma < \pi, \quad \tau \in R$,

$$X^\mu(\tau, \sigma) = f^\mu(\tau + \sigma) + g^\mu(\tau - \sigma)$$

- The electric field may be different on each of the two D-branes. The boundary conditions at $\sigma = \sigma_a \in \{0, \pi\}$

$$\partial_\sigma X^\mu + (2\pi\alpha')F^\mu{}_{\nu;a}(X)\partial_\tau X^\nu = 0$$

- For a constant F, this is a linear system of non-local ODEs, which can be solved in Fourier space. Assuming $[F_0, F_1] = 0$,

$$e^{-2\pi i\omega_n} = \frac{1 + F_0}{1 - F_0} \cdot \frac{1 - F_1}{1 + F_1}$$

Open strings in constant electromag. field vs orbifolds

- Open strings couple to an electromagnetic field through their boundary only. The embedding coordinates are therefore free bosons in the bulk of the Minkowskian strip $0 < \sigma < \pi, \quad \tau \in R$,

$$X^\mu(\tau, \sigma) = f^\mu(\tau + \sigma) + g^\mu(\tau - \sigma)$$

- The electric field may be different on each of the two D-branes. The boundary conditions at $\sigma = \sigma_a \in \{0, \pi\}$

$$\partial_\sigma X^\mu + (2\pi\alpha')F^\mu{}_{\nu;a}(X)\partial_\tau X^\nu = 0$$

- For a constant F, this is a linear system of non-local ODEs, which can be solved in Fourier space. Assuming $[F_0, F_1] = 0$,

$$e^{-2\pi i\omega_n} = \frac{1+F_0}{1-F_0} \cdot \frac{1-F_1}{1+F_1}$$

- Twisted closed strings in orbifolds satisfy

$$X^\mu(\sigma + 2\pi, \tau) = R^\mu_\nu \, X^\nu(\sigma, \tau) \quad \Rightarrow \quad e^{-2\pi i\omega_n} = R^\mu_\nu$$

- Twisted closed strings and charged open strings behave analogously when $R = (1+F)/(1-F)$.

Open strings in a constant electromag field

The dispersion relation again:

$$e^{-2\pi i\omega_n} = \frac{1+F_0}{1-F_0} \cdot \frac{1-F_1}{1+F_1}$$

Open strings in a constant electromag field

The dispersion relation again:

$$e^{-2\pi i\omega_n} = \frac{1+F_0}{1-F_0} \cdot \frac{1-F_1}{1+F_1}$$

- Magnetic field: $F = b \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \to \{ib, -ib\}$ hence $|T| = 1$ and frequencies are real:

$$\omega_n = n \pm \nu \ , \quad \pi\nu = \text{ArcTan}\ b_1 - \text{ArcTan}\ b_0$$

where ν is the stringy Larmor frequency. The string c.o.m. follows stable Landau orbits.

Open strings in a constant electromag field

The dispersion relation again:

$$e^{-2\pi i\omega_n} = \frac{1+F_0}{1-F_0}\cdot\frac{1-F_1}{1+F_1}$$

- Magnetic field: $F = b\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \to \{ib, -ib\}$ hence $|T| = 1$ and frequencies are real:

$$\omega_n = n \pm \nu\ , \quad \pi\nu = \text{ArcTan}\ b_1 - \text{ArcTan}\ b_0$$

  where ν is the stringy Larmor frequency. The string c.o.m. follows stable Landau orbits.

- Electric field: $F = e\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \to \{e, -e\}$ hence $|T| \neq 1$ and frequencies have an imaginary part:

$$\omega_n = n \pm i\nu\ , \quad \pi\nu = \text{ArcTanh}\ e_1 - \text{ArcTanh}\ e_0$$

Open strings in a constant electromag field

The dispersion relation again:

$$e^{-2\pi i\omega_n} = \frac{1+F_0}{1-F_0}\cdot\frac{1-F_1}{1+F_1}$$

- Magnetic field: $F = b\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix} \rightarrow \{ib, -ib\}$ hence $|T| = 1$ and frequencies are real:

$$\omega_n = n \pm \nu\ , \quad \pi\nu = \text{ArcTan } b_1 - \text{ArcTan } b_0$$

  where ν is the stringy Larmor frequency. The string c.o.m. follows stable Landau orbits.

- Electric field: $F = e\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \rightarrow \{e, -e\}$ hence $|T| \neq 1$ and frequencies have an imaginary part:

$$\omega_n = n \pm i\nu\ , \quad \pi\nu = \text{ArcTanh } e_1 - \text{ArcTanh } e_0$$

  This instability is due to Schwinger production of charged pairs. The rate production rate follows from the one-loop amplitude,

$$\Im(A) = \sum_{\text{states}} \sum_{k=1}^{\infty} (-1)^{F(k+1)} \left(\frac{\nu}{k}\right)^{D/2} \exp\left(-\frac{\pi k M^2}{\nu}\right)$$

Bachas Porrati

Open strings in a null electric field

- A generic $F_{\mu\nu}$ can always be brought to electric or magnetic form depending on sgn $F_{\mu\nu}F^{\mu\nu}$. However there is a third non-generic possibility,

$$F = f dx^2 \wedge dx^+ = f \begin{pmatrix} 0 & 1 & \\ & 0 & 1 \\ & & 0 \end{pmatrix} \quad , \qquad x^{\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$$

  which satisfies $F_{\mu\nu}F^{\mu\nu} = 0$. In 4D, it amounts to a configuration with crossed fields $\vec{E} \perp \vec{B}$ of equal magnitude $|\vec{E}| = |\vec{B}|$.

Open strings in a null electric field

- A generic $F_{\mu\nu}$ can always be brought to electric or magnetic form depending on sgn $F_{\mu\nu}F^{\mu\nu}$. However there is a third non-generic possibility,

$$F = f dx^2 \wedge dx^+ = f \begin{pmatrix} 0 & 1 & \\ & 0 & 1 \\ & & 0 \end{pmatrix} \quad , \qquad x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$$

  which satisfies $F_{\mu\nu}F^{\mu\nu} = 0$. In 4D, it amounts to a configuration with crossed fields $\vec{E} \perp \vec{B}$ of equal magnitude $|\vec{E}| = |\vec{B}|$.

- The eigenvalues of F are 0, so there is no vacuum polarization nor particle production, as expected in an electromagnetic wave.

Open strings in a null electric field

- A generic $F_{\mu\nu}$ can always be brought to electric or magnetic form depending on sgn $F_{\mu\nu}F^{\mu\nu}$. However there is a third non-generic possibility,

$$F = f dx^2 \wedge dx^+ = f \begin{pmatrix} 0 & 1 & \\ & 0 & 1 \\ & & 0 \end{pmatrix} \quad , \qquad x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$$

  which satisfies $F_{\mu\nu}F^{\mu\nu} = 0$. In 4D, it amounts to a configuration with crossed fields $\vec{E} \perp \vec{B}$ of equal magnitude $|\vec{E}| = |\vec{B}|$.

- The eigenvalues of F are 0, so there is no vacuum polarization nor particle production, as expected in an electromagnetic wave.

- After T-duality on x, this describes a null scissor configuration, i.e. two intersecting straight D-branes whose intersection point moves with the speed of light.

Bachas Hull

Open strings in a null electric field

- A generic $F_{\mu\nu}$ can always be brought to electric or magnetic form depending on sgn $F_{\mu\nu}F^{\mu\nu}$. However there is a third non-generic possibility,

$$F = f dx^2 \wedge dx^+ = f \begin{pmatrix} 0 & 1 & \\ & 0 & 1 \\ & & 0 \end{pmatrix} , \qquad x^{\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$$

  which satisfies $F_{\mu\nu}F^{\mu\nu} = 0$. In 4D, it amounts to a configuration with crossed fields $\vec{E} \perp \vec{B}$ of equal magnitude $|\vec{E}| = |\vec{B}|$.

- The eigenvalues of F are 0, so there is no vacuum polarization nor particle production, as expected in an electromagnetic wave.

- After T-duality on x, this describes a null scissor configuration, i.e. two intersecting straight D-branes whose intersection point moves with the speed of light.

  Bachas Hull

- This open string background bears similarities with the closed string parabolic orbifold.

Relativistic string in a pulse

- More generally, one may allow an arbitrary dependence in the light-cone time x^+, and an harmonic profile in transverse coordinates:

$$A = \Phi(x^+, x^i) dx^+ \, , \qquad \partial_i^2 \Phi = 0$$

This is an exact SUSY solution of the open strings eom to all orders in α'.

Relativistic string in a pulse

- More generally, one may allow an arbitrary dependence in the light-cone time x^+, and an harmonic profile in transverse coordinates:

$$A = \Phi(x^+, x^i) dx^+ \,, \qquad \partial_i^2 \Phi = 0$$

This is an exact SUSY solution of the open strings eom to all orders in α'.

- Strings can be quantized in the light-cone gauge $X^+ = x_0^+ + p^+ \tau$.

Relativistic string in a pulse

- More generally, one may allow an arbitrary dependence in the light-cone time x^+, and an harmonic profile in transverse coordinates:

$$A = \Phi(x^+, x^i) dx^+ \,, \qquad \partial_i^2 \Phi = 0$$

This is an exact SUSY solution of the open strings eom to all orders in α'.

- Strings can be quantized in the light-cone gauge $X^+ = x_0^+ + p^+ \tau$.

- For $\Phi = f_i(x^+) x^i$, the boundary conditions receive a time-dependent source term,

$$\partial_\sigma X^i + (2\pi\alpha') p^+ f_i(X^+) = 0 \,, \quad \sigma \in \{0, \pi\}$$

while X^i remains free in the bulk. Classical solutions can be computed by linear response.

Relativistic string in a pulse

- More generally, one may allow an arbitrary dependence in the light-cone time x^+, and an harmonic profile in transverse coordinates:

$$A = \Phi(x^+, x^i) dx^+ \, , \qquad \partial_i^2 \Phi = 0$$

This is an exact SUSY solution of the open strings eom to all orders in α'.

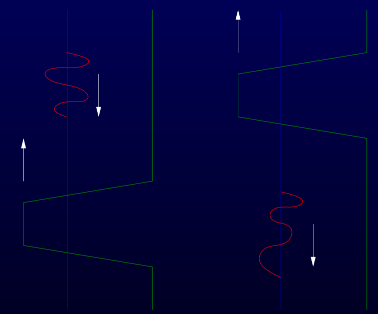
- Strings can be quantized in the light-cone gauge $X^+ = x_0^+ + p^+ \tau$.

- For $\Phi = f_i(x^+) x^i$, the boundary conditions receive a time-dependent source term,

$$\partial_\sigma X^i + (2\pi\alpha') p^+ f_i(X^+) = 0 \, , \quad \sigma \in \{0, \pi\}$$

while X^i remains free in the bulk. Classical solutions can be computed by linear response.

- After T-duality along x, on obtains "null scissors" configurations, with arbitrary shape. An incoming string in its ground state will in general emerge in an excited state, depending on the profile $\Phi(x^+)$: the S-matrix can be computed.



Bachas

Relativistic string in a pulse

- More generally, one may allow an arbitrary dependence in the light-cone time x^+, and an harmonic profile in transverse coordinates:

$$A = \Phi(x^+, x^i) dx^+ \ , \qquad \partial_i^2 \Phi = 0$$

This is an exact SUSY solution of the open strings eom to all orders in α'.

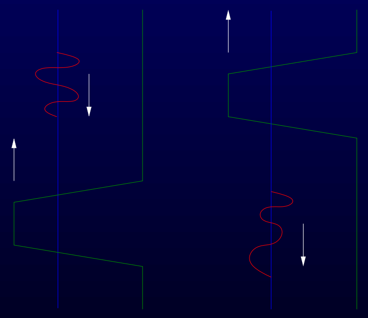
- Strings can be quantized in the light-cone gauge $X^+ = x_0^+ + p^+ \tau$.

- For $\Phi = f_i(x^+) x^i$, the boundary conditions receive a time-dependent source term,

$$\partial_\sigma X^i + (2\pi\alpha') p^+ f_i(X^+) = 0 \ , \quad \sigma \in \{0, \pi\}$$

while X^i remains free in the bulk. Classical solutions can be computed by linear response.

- After T-duality along x, on obtains "null scissors" configurations, with arbitrary shape. An incoming string in its ground state will in general emerge in an excited state, depending on the profile $\Phi(x^+)$: the S-matrix can be computed.



Bachas

Relativistic string in a quadrupolar potential

- The next simplest harmonic function is a quadratic potential

$$\Phi(x^+, x^i) = h_{ij}(x^+)x^i x^j/2$$

Relativistic string in a quadrupolar potential

- The next simplest harmonic function is a quadratic potential

$$\Phi(x^+, x^i) = h_{ij}(x^+) x^i x^j / 2$$

In the light-cone gauge, this leads to a massive boundary deformation:

$$\partial_\sigma X^i + p^+ (h_a)_{ij}(x^+) X^j = 0 \ , \quad \sigma = \sigma^a$$

hence conformal invariance is broken on the boundary.

Durin, BP

Relativistic string in a quadrupolar potential

- The next simplest harmonic function is a quadratic potential

$$\Phi(x^+, x^i) = h_{ij}(x^+) x^i x^j / 2$$

In the light-cone gauge, this leads to a massive boundary deformation:

$$\partial_\sigma X^i + p^+ (h_a)_{ij}(x^+) X^j = 0 \ , \quad \sigma = \sigma^a$$

hence conformal invariance is broken on the boundary.

Durin, BP

- This is an open string analogue of the gravitational wave, indeed the open string metric reads

$$ds^2 = 2dx^+ dx^- + (dx^i)^2 + (2\pi\alpha')^2 (h_{ij} x^j)^2 (dx^+)^2$$

Relativistic string in a quadrupolar potential

- The next simplest harmonic function is a quadratic potential

$$\Phi(x^+, x^i) = h_{ij}(x^+) x^i x^j / 2$$

In the light-cone gauge, this leads to a massive boundary deformation:

$$\partial_\sigma X^i + p^+ (h_a)_{ij}(x^+) X^j = 0 \ , \quad \sigma = \sigma^a$$

hence conformal invariance is broken on the boundary.

Durin, BP

- This is an open string analogue of the gravitational wave, indeed the open string metric reads

$$ds^2 = 2dx^+ dx^- + (dx^i)^2 + (2\pi\alpha')^2 (h_{ij} x^j)^2 (dx^+)^2$$

- Such deformations are also reminiscent of open string tachyon condensation in BSFT, however (i) h_{ij} is traceless, and (ii) the worldsheet is Lorentzian.

Non-relativistic dipole and critical gradient

- On the light-cone, a relativistic string behaves like a non-relativistic dipole. This implies that its tensive energy is proportional to the square of its length:

$$V_t = \frac{1}{\alpha' p^+}(x_L - x_R)^2$$

Non-relativistic dipole and critical gradient

- On the light-cone, a relativistic string behaves like a non-relativistic dipole. This implies that its tensive energy is proportional to the square of its length:

$$V_t = \frac{1}{\alpha' p^+}(x_L - x_R)^2$$

- For a quadrupolar wave, the electrostatic energy scales also like the square of the distance,

$$V_e = p^+ \left(h^0_{ij} x^i_L x^j_L - h^1_{ij} x^i_R x^j_R \right)$$

Non-relativistic dipole and critical gradient

- On the light-cone, a relativistic string behaves like a non-relativistic dipole. This implies that its tensive energy is proportional to the square of its length:

$$V_t = \frac{1}{\alpha' p^+}(x_L - x_R)^2$$

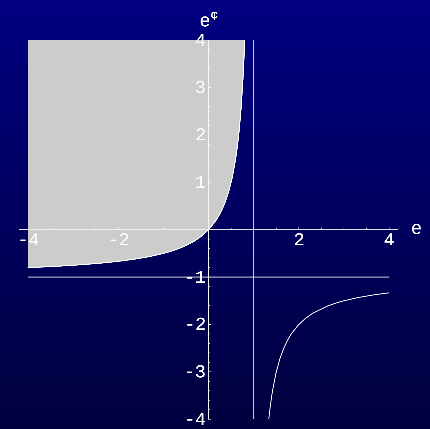
- For a quadrupolar wave, the electrostatic energy scales also like the square of the distance,

$$V_e = p^+ \left(h^0_{ij} x^i_L x^j_L - h^1_{ij} x^i_R x^j_R \right)$$

- There is a critical line

$$h_1 - h_0 - \pi (p^+)^2 \alpha' h_0 h_1 = 0$$

where the two forces balance against each other, leading to stretched macroscopic strings: A non-relativistic analogue of the Born-Infeld critical field.



Non-relativistic dipole and critical gradient

- On the light-cone, a relativistic string behaves like a non-relativistic dipole. This implies that its tensive energy is proportional to the square of its length:

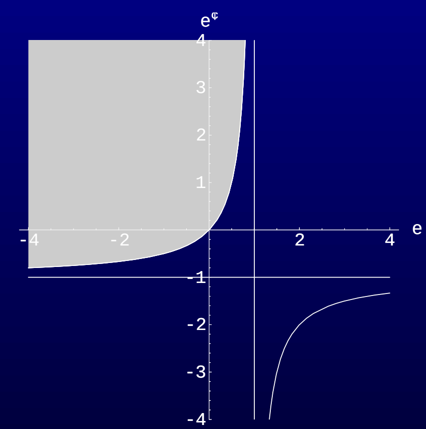
$$V_t = \frac{1}{\alpha' p^+}(x_L - x_R)^2$$

- For a quadrupolar wave, the electrostatic energy scales also like the square of the distance,

$$V_e = p^+ \left(h^0_{ij} x^i_L x^j_L - h^1_{ij} x^i_R x^j_R \right)$$

- There is a critical line

$$h_1 - h_0 - \pi (p^+)^2 \alpha' h_0 h_1 = 0$$



where the two forces balance against each other, leading to stretched macroscopic strings: A non-relativistic analogue of the Born-Infeld critical field.

What is the analogue of the open string metric for stretched strings ? is there a decoupled theory analogous to NCOS ?

Dynamical instability

- For a traceless quadratic potential h_{ij}, the motion is always unstable, due to the convexity of the stability domain. However, this is a kinematical instability of the string probe, much like the divergence of geodesics in purely gravitational plane waves.

Marolf Zayas; Brecher Gregory Saffin

Dynamical instability

- For a traceless quadratic potential h_{ij}, the motion is always unstable, due to the convexity of the stability domain. However, this is a kinematical instability of the string probe, much like the divergence of geodesics in purely gravitational plane waves.

Marolf Zayas; Brecher Gregory Saffin

- A well-known way to cure these instabilities is to introduce a magnetic field, as in the Penning trap:

$$V(x) = \frac{e}{2}(2z^2 - x^2 - y^2)\ , \quad B = b dx \wedge dy$$

stable if $b^2 > e$ and $e > 0$

Dynamical instability

- For a traceless quadratic potential h_{ij}, the motion is always unstable, due to the convexity of the stability domain. However, this is a kinematical instability of the string probe, much like the divergence of geodesics in purely gravitational plane waves.

Marolf Zayas; Brecher Gregory Saffin

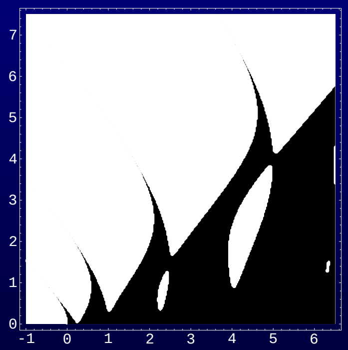
- A well-known way to cure these instabilities is to introduce a magnetic field, as in the Penning trap:

$$V(x) = \frac{e}{2}(2z^2 - x^2 - y^2)\ , \quad B = b\,dx \wedge dy$$

stable if $b^2 > e$ and $e > 0$

- One may also use parametric resonance as in the RF trap:

$$V = (\omega^2 + \alpha^2 \cos t)(x^2 - y^2)$$



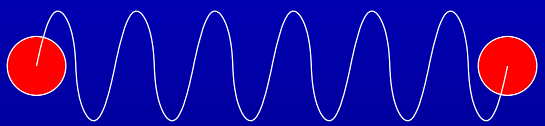
These mechanisms carry over to open strings in electromagnetic waves, or closed strings in gravitational waves.

Some more comments

- In terms of the T-dual coordinate $\tilde{X}^i = f^i(\tau + \sigma) - g^i(\tau - \sigma)$ the bc becomes, after differentiating once,

$$\partial_\tau^2 \tilde{X}^i + p^+ (h_a)_{ij} \partial_\sigma \tilde{X}^j = 0,$$

This is an open string with two beads of mass h_a^{-1}/p^+ at its ends. When $h = 0$, it reduces to a Dirichlet bc.

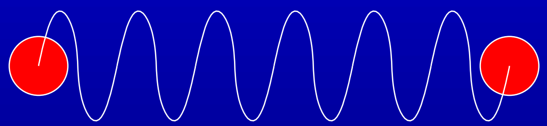


Some more comments

- In terms of the T-dual coordinate $\tilde{X}^i = f^i(\tau+\sigma) - g^i(\tau-\sigma)$ the bc becomes, after differentiating once,

$$\partial_\tau^2 \tilde{X}^i + p^+ (h_a)_{ij} \partial_\sigma \tilde{X}^j = 0,$$

This is an open string with two beads of mass h_a^{-1}/p^+ at its ends. When $h=0$, it reduces to a Dirichlet bc.



- A T-dual-like but inequivalent bc would be to take

$$\partial_\tau X^i + p^+ \partial_i \Phi(X) = 0$$

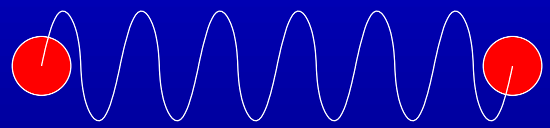
The ends of the open string follow the gradient lines of Φ: null scissors of arbitrary shape.

Some more comments

- In terms of the T-dual coordinate $\tilde{X}^i = f^i(\tau+\sigma) - g^i(\tau-\sigma)$ the bc becomes, after differentiating once,

$$\partial_\tau^2 \tilde{X}^i + p^+ (h_a)_{ij} \partial_\sigma \tilde{X}^j = 0,$$

  This is an open string with two beads of mass h_a^{-1}/p^+ at its ends. When $h=0$, it reduces to a Dirichlet bc.



- A T-dual-like but inequivalent bc would be to take

$$\partial_\tau X^i + p^+ \partial_i \Phi(X) = 0$$

  The ends of the open string follow the gradient lines of Φ: null scissors of arbitrary shape.

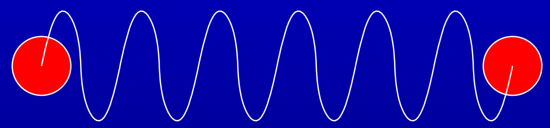
- For time-dependent quadrupolar fields, the Bogolioubov transformation between one-string states can be computed in the in the Born and adiabatic approximation.

Some more comments

- In terms of the T-dual coordinate $\tilde{X}^i = f^i(\tau+\sigma) - g^i(\tau-\sigma)$ the bc becomes, after differentiating once,

$$\partial_\tau^2 \tilde{X}^i + p^+ (h_a)_{ij} \partial_\sigma \tilde{X}^j = 0,$$

  This is an open string with two beads of mass h_a^{-1}/p^+ at its ends. When $h=0$, it reduces to a Dirichlet bc.



- A T-dual-like but inequivalent bc would be to take

$$\partial_\tau X^i + p^+ \partial_i \Phi(X) = 0$$

  The ends of the open string follow the gradient lines of Φ: null scissors of arbitrary shape.

- For time-dependent quadrupolar fields, the Bogolioubov transformation between one-string states can be computed in the in the Born and adiabatic approximation.

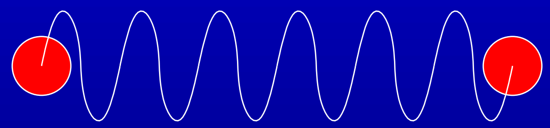
- The open string has a hidden half degree of freedom in in the splitting of the zero-mode between left- and right-movers. It undergoes a non-trivial backreaction.

Some more comments

- In terms of the T-dual coordinate $\tilde{X}^i = f^i(\tau+\sigma) - g^i(\tau-\sigma)$ the bc becomes, after differentiating once,

$$\partial_\tau^2 \tilde{X}^i + p^+ (h_a)_{ij} \partial_\sigma \tilde{X}^j = 0,$$

This is an open string with two beads of mass h_a^{-1}/p^+ at its ends. When $h=0$, it reduces to a Dirichlet bc.



- A T-dual-like but inequivalent bc would be to take

$$\partial_\tau X^i + p^+ \partial_i \Phi(X) = 0$$

The ends of the open string follow the gradient lines of Φ: null scissors of arbitrary shape.

- For time-dependent quadrupolar fields, the Bogolioubov transformation between one-string states can be computed in the in the Born and adiabatic approximation.

- The open string has a hidden half degree of freedom in in the splitting of the zero-mode between left- and right-movers. It undergoes a non-trivial backreaction.

- Open/closed duality can be checked (rather) straightforwardly.

Bergman Gaberdiel Green

Colliding plane waves

- A string probe with $p^+ \neq 0$ can be thought as a perturbation colliding with the background wave. Its state after the collision can be extracted simply from the Bogolioubov transformation in light-cone gauge.

Colliding plane waves

- A string probe with $p^+ \neq 0$ can be thought as a perturbation colliding with the background wave. Its state after the collision can be extracted simply from the Bogolioubov transformation in light-cone gauge.

- Quantum mechanically, part of the string will be scattered off the background wave, hence alter or back-react on the background wave through emission of $p^+ = 0$ states.

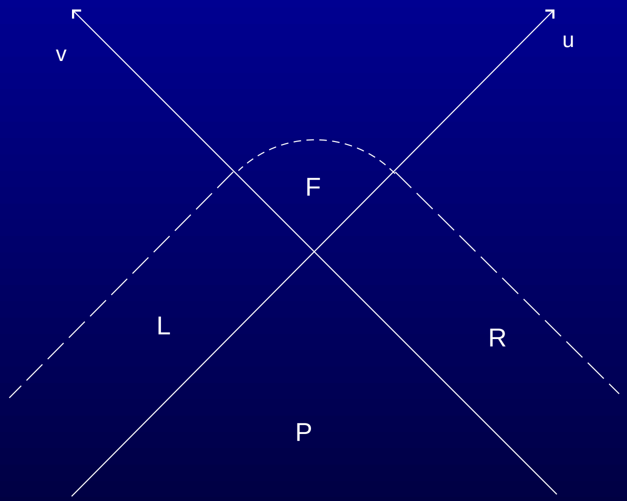
Colliding plane waves

- A string probe with $p^+ \neq 0$ can be thought as a perturbation colliding with the background wave. Its state after the collision can be extracted simply from the Bogolioubov transformation in light-cone gauge.

- Quantum mechanically, part of the string will be scattered off the background wave, hence alter or back-react on the background wave through emission of $p^+ = 0$ states.

- The same issue arises in the context of gravitational waves. For this one needs to go beyond the light-cone gauge.

Gutperle BP.



Electric field vs Milne Universe

- Eigenmodes of closed strings in the twisted sector of order w are free fields satisfying

$$X^{\pm}(\sigma + 2\pi, \tau) = e^{\pm\nu} X^{\pm}(\sigma, \tau) \ , \quad \nu = w\beta$$

hence the normal mode expansion:

$$\begin{aligned} X_R^{\pm}(\tau - \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1/2} \alpha_n^{\pm} e^{-i(n \pm i\nu)(\tau - \sigma)} \\ X_L^{\pm}(\tau + \sigma) &= -\frac{i}{2} \sum_{n=-\infty}^{\infty} (-n \mp i\nu)^{-1/2} \tilde{\alpha}_n^{\pm} e^{-i(-n \mp i\nu)(\tau + \sigma)} \end{aligned}$$

with canonical commutation relations

$$[\alpha_m^+, \alpha_n^-] = -(m + i\nu)\delta_{m+n} \quad , \quad [\tilde{\alpha}_m^+, \tilde{\alpha}_n^-] = (m + i\nu)\delta_{m+n}$$
$$(\alpha_m^{\pm})^* = \alpha_{-m}^{\pm} \quad , \quad (\tilde{\alpha}_m^{\pm})^* = \tilde{\alpha}_{-m}^{\pm}$$

Electric field vs Milne Universe

- Eigenmodes of closed strings in the twisted sector of order w are free fields satisfying

$$X^{\pm}(\sigma + 2\pi, \tau) = e^{\pm\nu} X^{\pm}(\sigma, \tau) \ , \quad \nu = w\beta$$

hence the normal mode expansion:

$$\begin{aligned} X_R^{\pm}(\tau - \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1/2} \alpha_n^{\pm} e^{-i(n \pm i\nu)(\tau-\sigma)} \\ X_L^{\pm}(\tau + \sigma) &= -\frac{i}{2} \sum_{n=-\infty}^{\infty} (-n \mp i\nu)^{-1/2} \tilde{\alpha}_n^{\pm} e^{-i(-n \mp i\nu)(\tau+\sigma)} \end{aligned}$$

with canonical commutation relations

$$[\alpha_m^+, \alpha_n^-] = -(m + i\nu)\delta_{m+n} \quad , \quad [\tilde{\alpha}_m^+, \tilde{\alpha}_n^-] = (m + i\nu)\delta_{m+n}$$
$$(\alpha_m^{\pm})^* = \alpha_{-m}^{\pm} \quad , \quad (\tilde{\alpha}_m^{\pm})^* = \tilde{\alpha}_{-m}^{\pm}$$

- Charged open strings are obtained by identifying

$$\alpha_n^{\pm} = \tilde{\alpha}_{-n}^{\pm} \ , \quad \nu = \frac{2}{\pi} \text{arctanh}(E/\pi) \ ,$$

and adding a canonical pair of constant zero modes $x_0^{\pm}$ with $[x_0^+, x_0^-] = i\pi/E$.

Zero-modes

- In either cases, zero-modes consist of two pairs of hermitian canonically conjugate variables,

$$[\alpha_0^+, \alpha_0^-] = -i\nu \ , \quad [\tilde{\alpha}_0^+, \tilde{\alpha}_0^-] = i\nu \ ,$$

where, in the open string case, $\tilde{\alpha}_0^\pm = \pm\sqrt{2\nu E} x_0^\pm$

Zero-modes

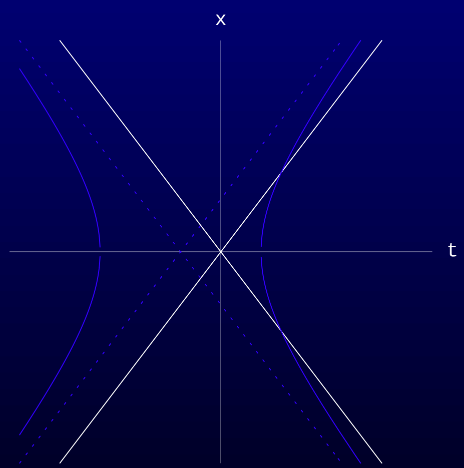
- In either cases, zero-modes consist of two pairs of hermitian canonically conjugate variables,

$$[\alpha_0^+, \alpha_0^-] = -i\nu \ , \quad [\tilde{\alpha}_0^+, \tilde{\alpha}_0^-] = i\nu \ ,$$

where, in the open string case, $\tilde{\alpha}_0^\pm = \pm\sqrt{2\nu E} x_0^\pm$

- In the open string case, $x_0^\pm$ is the linear momentum of a charged particle in an electric field, and $\alpha_0^\pm$ its velocity. The trajectory is an hyperbola centered at (x_0^+, x_0^-),

$$(X^+ - x_0^+)(X^- - x_0^-) + \frac{M^2}{2\nu^2} = 0$$



Are there twisted physical states ?

- Representing these oscillators on a Fock space with vacuum $|0\rangle$ annihilated by all $\alpha^{\pm}_{n>0}$ and by α^{-}_{0}, the normal ordered worldsheet Hamiltonian reads

$$L_0 = -\sum_{n=0}^{\infty} (\alpha_n^+)^* \alpha_n^- \; - \sum_{n=1}^{\infty} (\alpha_n^-)^* \alpha_n^+ + \frac{1}{2} i\nu(1 - i\nu) - 1 + L_{int}$$

with a similar answer for $\tilde{L}_0$.

Are there twisted physical states ?

- Representing these oscillators on a Fock space with vacuum $|0\rangle$ annihilated by all $\alpha^{\pm}_{n>0}$ and by α_0^-, the normal ordered worldsheet Hamiltonian reads

$$L_0 = -\sum_{n=0}^{\infty} (\alpha_n^+)^* \alpha_n^- \; - \sum_{n=1}^{\infty} (\alpha_n^-)^* \alpha_n^+ + \frac{1}{2} i\nu(1-i\nu) - 1 + L_{int}$$

  with a similar answer for $\tilde{L}_0$.
- Due to the $i\nu/2$ term in the ground state energy, all states obtained by acting on $|0\rangle$ by creation operators $\alpha^{\pm}_{n<0}$ and by α_0^+ will have imaginary energy, hence the physical state condition $L_0 = 0$ has no solutions.

Nekrasov

Space-time representation of zero-modes

- Quantum mechanically, zero-modes can be unitarily represented as covariant derivatives acting on wave functions $f(x^+, x^-)$ of the center of motion of the charged string,

$$\alpha_0^\pm = i\partial_\mp \mp \frac{\nu}{2} x^\pm \,, \quad \tilde{\alpha}_0^\pm = i\partial_\mp \pm \frac{\nu}{2} x^\pm$$

- The zero-mode piece of L_0, including the evil $\frac{i\nu}{2}$,

$$L_0^{(0)} = -\alpha_0^+ \alpha_0^- + \frac{i\nu}{2} = -\frac{1}{2}(\alpha_0^+ \alpha_0^- + \alpha_0^- \alpha_0^+)$$

is just the Klein-Gordon operator of a particle of 2D mass $M^2 = -2L_0^{(0)}$ and charge ν.

Space-time representation of zero-modes

- Quantum mechanically, zero-modes can be unitarily represented as covariant derivatives acting on wave functions $f(x^+, x^-)$ of the center of motion of the charged string,

$$\alpha_0^\pm = i\partial_\mp \mp \frac{\nu}{2} x^\pm \,, \quad \tilde{\alpha}_0^\pm = i\partial_\mp \pm \frac{\nu}{2} x^\pm$$

- The zero-mode piece of L_0, including the evil $\frac{i\nu}{2}$,

$$L_0^{(0)} = -\alpha_0^+ \alpha_0^- + \frac{i\nu}{2} = -\frac{1}{2}(\alpha_0^+ \alpha_0^- + \alpha_0^- \alpha_0^+)$$

is just the Klein-Gordon operator of a particle of 2D mass $M^2 = -2L_0^{(0)}$ and charge ν.

- For closed strings, the difference $L_0^{(0)} - \tilde{L}_0^{(0)}$ is the zero-mode boost momentum,

$$\mathcal{M}^2 - \tilde{\mathcal{M}}^2 = -i\nu \left(x^+ \partial_+ - x^- \partial_- \right) := J^{(0)}$$

subject to the matching condition $\beta w J = N_L - N_R$ In addition, the orbifold projection requires the total boost momentum J to be integer.

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^\pm = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator just becomes an inverted harmonic oscillator:

$$M^2 = \alpha_0^+ \alpha_0^- + \alpha_0^- \alpha_0^+ = \frac{1}{2}(P^2 - Q^2)$$

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^\pm = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator just becomes an inverted harmonic oscillator:

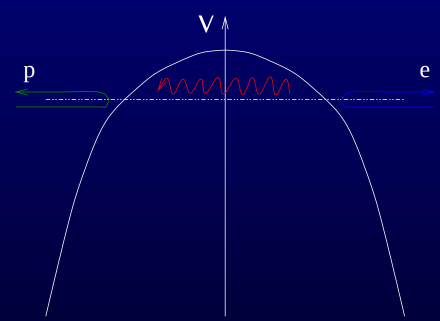
$$M^2 = \alpha_0^+ \alpha_0^- + \alpha_0^- \alpha_0^+ = \frac{1}{2}(P^2 - Q^2)$$

- More explicitely, in terms of $u = (\tilde{p} + \nu x)\sqrt{2/\nu}$,

$$\left(-\partial_u^2 - \frac{1}{4}u^2 + \frac{M^2}{2\nu} \right) \psi_{\tilde{p}}(u) = 0$$

- The latter admits a respectable delta-normalizable spectrum of scattering states, in terms of parabolic cylinder functions, e.g:

$$\phi_{in}^+ = D_{-\frac{1}{2}+i\frac{M^2}{2\nu}}(e^{-\frac{3i\pi}{4}}u)e^{-i\tilde{p}t}e^{i\nu x t/2}$$



Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^{\pm} = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator just becomes an inverted harmonic oscillator:

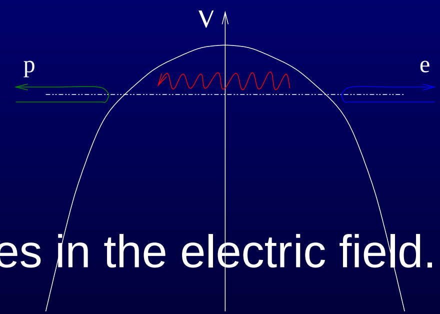
$$M^2 = \alpha_0^+ \alpha_0^- + \alpha_0^- \alpha_0^+ = \frac{1}{2}(P^2 - Q^2)$$

- More explicitely, in terms of $u = (\tilde{p} + \nu x)\sqrt{2/\nu}$,

$$\left(-\partial_u^2 - \frac{1}{4}u^2 + \frac{M^2}{2\nu} \right) \psi_{\tilde{p}}(u) = 0$$

- The latter admits a respectable delta-normalizable spectrum of scattering states, in terms of parabolic cylinder functions, e.g:

$$\phi_{in}^+ = D_{-\frac{1}{2}+i\frac{M^2}{2\nu}}(e^{-\frac{3i\pi}{4}} u) e^{-i\tilde{p}t} e^{i\nu x t/2}$$



- These correspond to non-compact trajectories of charged particles in the electric field. Tunnelling is just (stimulated) Schwinger pair creation,

$$e^- \longrightarrow (1+\eta)\, e^- + \eta\, e^+$$

Brezin Itzykson; Brout Massar Parentani Spindel

Lorentzian vs Euclidean states

- Analytic continuation $X^0 \longrightarrow e^{-i\pi/2} X^0$, $\nu \longrightarrow e^{i\pi/2}\nu$ takes us from an electric field in $R^{1,1}$ to a magnetic field in R^2. At the same time, one should Wick rotate the worldsheet time.

Lorentzian vs Euclidean states

- Analytic continuation $X^0 \longrightarrow e^{-i\pi/2} X^0$, $\nu \longrightarrow e^{i\pi/2}\nu$ takes us from an electric field in $R^{1,1}$ to a magnetic field in R^2. At the same time, one should Wick rotate the worldsheet time.
- The discrete spectrum with complex energy comes by analytic continuation of the normalizable (Landau) states of the (stable) harmonic oscillator.

Lorentzian vs Euclidean states

- Analytic continuation $X^0 \longrightarrow e^{-i\pi/2} X^0$, $\nu \longrightarrow e^{i\pi/2}\nu$ takes us from an electric field in $R^{1,1}$ to a magnetic field in R^2. At the same time, one should Wick rotate the worldsheet time.
- The discrete spectrum with complex energy comes by analytic continuation of the normalizable (Landau) states of the (stable) harmonic oscillator.
- Instead, the continuous scattering states of the inverted harmonic oscillator come by analytic continuation from non-normalizable states of the stable harmonic oscillator.

Lorentzian vs Euclidean states

- Analytic continuation $X^0 \to e^{-i\pi/2} X^0$, $\nu \to e^{i\pi/2}\nu$ takes us from an electric field in $R^{1,1}$ to a magnetic field in R^2. At the same time, one should Wick rotate the worldsheet time.
- The discrete spectrum with complex energy comes by analytic continuation of the normalizable (Landau) states of the (stable) harmonic oscillator.
- Instead, the continuous scattering states of the inverted harmonic oscillator come by analytic continuation from non-normalizable states of the stable harmonic oscillator.
- The contribution of the zero-modes to the one-loop amplitude can be interpreted either way,

$$\frac{1}{2i\sin(\nu t/2)} = \sum_{n=1}^{\infty} e^{-i(n+\frac{1}{2})\nu t} = \int dM^2 \rho(M^2) e^{-M^2 t/2}$$

where the density of states is obtained from the reflection phase shift,

$$\rho(M^2) = \frac{1}{\nu}\log\Lambda - \frac{1}{2\pi i}\frac{d}{dM^2}\log\frac{\Gamma\left(\frac{1}{2}+i\frac{M^2}{2\nu}\right)}{\Gamma\left(\frac{1}{2}-i\frac{M^2}{2\nu}\right)}$$

- The physical spectrum can be explicitely worked out at low levels, and is free of ghosts: e.g. for open strings at level 1, one finds a transverse gauge boson; for closed strings, a zero-momentum transverse graviton in each twisted sector.

Charged Klein-Gordon Eq. in Rindler space

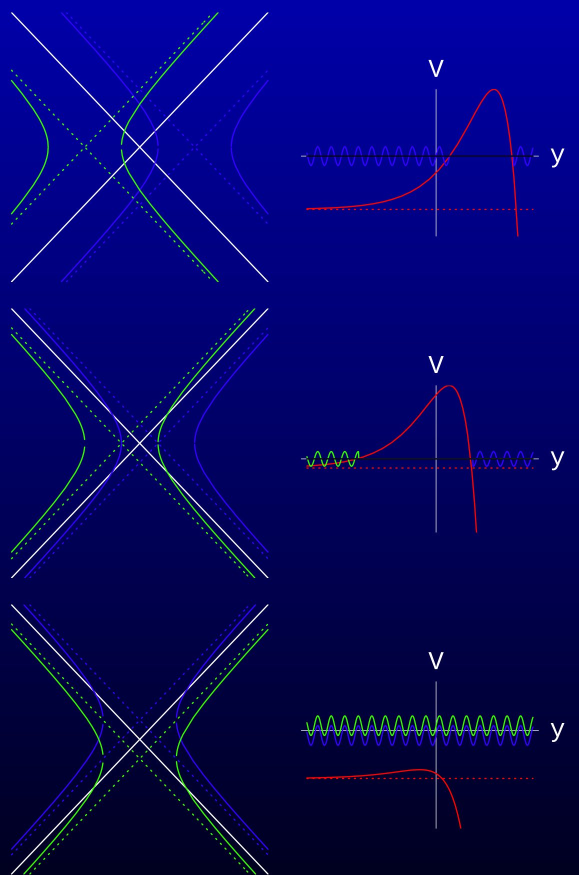
- For applications to the Milne universe, one should diagonalize the boost momentum J, ie consider an accelerated observer.

Gabriel Spindel; Mottola Cooper

Charged Klein-Gordon Eq. in Rindler space

- For applications to the Milne universe, one should diagonalize the boost momentum J, ie consider an accelerated observer.

Gabriel Spindel; Mottola Cooper



- In the Rindler patch R, letting $f(r,\eta) = e^{-iJ\eta} f_J(r)$ and $r = e^y$, one gets a Schrodinger equation for a particle in a potential

$$V(y) = M^2 e^{2y} - (J + \frac{1}{2}\nu\, e^{2y})^2$$

- Incoming and outgoing Rindler/Unruh modes can be defined as usual $\Rightarrow$ Schwinger pair creation in the bulk, thermal particle production from the horizon.

Twisted state production in Milne

- By analogy with the electric field, one expects Schwinger production of correlated pairs of twisted stateson the Milne orbifold. Due to orbifold projection, lightest states will be produced in $J = 0$ states, thereby breaking spatial homogeneity.

Twisted state production in Milne

- By analogy with the electric field, one expects Schwinger production of correlated pairs of twisted stateson the Milne orbifold. Due to orbifold projection, lightest states will be produced in $J = 0$ states, thereby breaking spatial homogeneity.
- For a given state of mass M, the local rate of pair creation can be evaluated semi-classically by computing the Jacobian

$$w(X^+, X^-) = \int dx_0^+ dx_0^- \delta[(X^+ - x_0^+)(X^- - x_0^-) + \frac{M^2}{2\nu^2}]$$
$$\times \delta[\nu x_0^+ x_0^- + \frac{M^2}{2\nu} - J] = \frac{1}{\sqrt{X^+X^-(\nu^2 + 2M^2X^+X^-)}}$$

hence diverges on the light-cone.

Twisted state production in Milne

- By analogy with the electric field, one expects Schwinger production of correlated pairs of twisted stateson the Milne orbifold. Due to orbifold projection, lightest states will be produced in $J = 0$ states, thereby breaking spatial homogeneity.
- For a given state of mass M, the local rate of pair creation can be evaluated semi-classically by computing the Jacobian

$$w(X^+, X^-) = \int dx_0^+ dx_0^- \delta[(X^+ - x_0^+)(X^- - x_0^-) + \frac{M^2}{2\nu^2}]$$
$$\times \delta[\nu x_0^+ x_0^- + \frac{M^2}{2\nu} - J] = \frac{1}{\sqrt{X^+ X^-(\nu^2 + 2M^2 X^+ X^-)}}$$

hence diverges on the light-cone.

- One may expect that pair creation will decrease the contraction rate so as to avoid the singularity. This is hard to demonstrate with present technology.

Twisted state production in Milne

- By analogy with the electric field, one expects Schwinger production of correlated pairs of twisted stateson the Milne orbifold. Due to orbifold projection, lightest states will be produced in $J = 0$ states, thereby breaking spatial homogeneity.
- For a given state of mass M, the local rate of pair creation can be evaluated semi-classically by computing the Jacobian

$$w(X^+, X^-) = \int dx_0^+ dx_0^- \delta[(X^+ - x_0^+)(X^- - x_0^-) + \frac{M^2}{2\nu^2}]$$

$$\times \, \delta[\nu x_0^+ x_0^- + \frac{M^2}{2\nu} - J] = \frac{1}{\sqrt{X^+X^-(\nu^2 + 2M^2X^+X^-)}}$$

  hence diverges on the light-cone.
- One may expect that pair creation will decrease the contraction rate so as to avoid the singularity. This is hard to demonstrate with present technology.
- A more tractable situation perhaps is to send in squeezed twisted states from $-\infty$, along with states one is interested to scatter.

Conclusions - speculations

- Open strings in electric fields, or D-brane collisions, are a simpler arena to address conceptual issues of Time and string theory. They often exhibit formal similarities with closed string orbifolds.

Conclusions - speculations

- Open strings in electric fields, or D-brane collisions, are a simpler arena to address conceptual issues of Time and string theory. They often exhibit formal similarities with closed string orbifolds.
- Using intuition from open strings, we have found that physical states do exist in the twisted sector of the Lorentzian orbifold, and can be pair produced. Can one compute scattering amplitudes, backreaction ?

D'Appollonio, Kiritsis

Conclusions - speculations

- Open strings in electric fields, or D-brane collisions, are a simpler arena to address conceptual issues of Time and string theory. They often exhibit formal similarities with closed string orbifolds.
- Using intuition from open strings, we have found that physical states do exist in the twisted sector of the Lorentzian orbifold, and can be pair produced. Can one compute scattering amplitudes, backreaction ?

D'Appollonio, Kiritsis

- Twisted sector states are produced in correlated pairs, i.e. squeezed states, whose condensation should involve non-local deformations of the worldsheet. Can the techniques developed in the context of strong electric fields be useful ?

Aharony Berkooz Silverstein

Cooper, Eisenberg, Kluger, Mottola and Svetitsky

Conclusions - speculations

- Open strings in electric fields, or D-brane collisions, are a simpler arena to address conceptual issues of Time and string theory. They often exhibit formal similarities with closed string orbifolds.
- Using intuition from open strings, we have found that physical states do exist in the twisted sector of the Lorentzian orbifold, and can be pair produced. Can one compute scattering amplitudes, backreaction ?

D'Appollonio, Kiritsis

- Twisted sector states are produced in correlated pairs, i.e. squeezed states, whose condensation should involve non-local deformations of the worldsheet. Can the techniques developed in the context of strong electric fields be useful ?

Aharony Berkooz Silverstein

Cooper, Eisenberg, Kluger, Mottola and Svetitsky

- The closed string orbifold we have discussed are highly non-generic trajectories on the cosmological billiard: how about more general Kasner-like singularities ?

Conclusions - speculations

- Open strings in electric fields, or D-brane collisions, are a simpler arena to address conceptual issues of Time and string theory. They often exhibit formal similarities with closed string orbifolds.
- Using intuition from open strings, we have found that physical states do exist in the twisted sector of the Lorentzian orbifold, and can be pair produced. Can one compute scattering amplitudes, backreaction ?

D'Appollonio, Kiritsis

- Twisted sector states are produced in correlated pairs, i.e. squeezed states, whose condensation should involve non-local deformations of the worldsheet. Can the techniques developed in the context of strong electric fields be useful ?

Aharony Berkooz Silverstein

Cooper, Eisenberg, Kluger, Mottola and Svetitsky

- The closed string orbifold we have discussed are highly non-generic trajectories on the cosmological billiard: how about more general Kasner-like singularities ?
- More generally, what is the fate of the classical chaos near a cosmological singularity, once quantum/string effects are taken into account ? Can exceptional theta series provide the wave function of the Universe ?

Damour Henneaux Julia . . . ; BP Waldron