Closed Strings in the Misner Universe

Boris Pioline
LPTHE and LPTENS, Paris

Copenhagen, Dec 2, 2003

based on hep-th/0307280 w/ M. Berkooz
and work in progress w/ M. Berkooz, B. Durin, D. Reichmann, M. Rozali

slides available from
`http://www.lpthe.jussieu.fr/~pioline/seminars.html`

Motivational string cosmology

- With LHC still far in the future, Observational Cosmology is now challenging string theory with high-precision data:

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- Time dependence is presumably incompatible with target space supersymmetry.
- Perturbative string theory requires an Euclidean worldsheet, hence Euclidean target space. The analytic continuation may be ambiguous or ill-defined, Lorentzian observables may be very different from their Euclidean counterparts.
- String theory is not content on a finite time interval, and one is frequently forced into Big Bang / Big Crunch singularities, CTC in the process of maximally extending the geometry.

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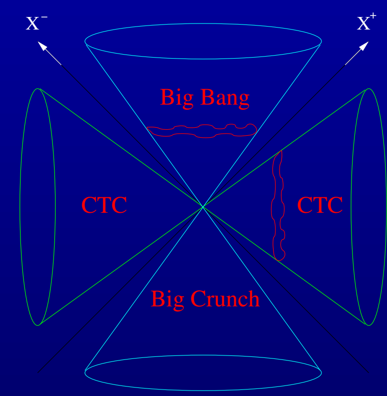

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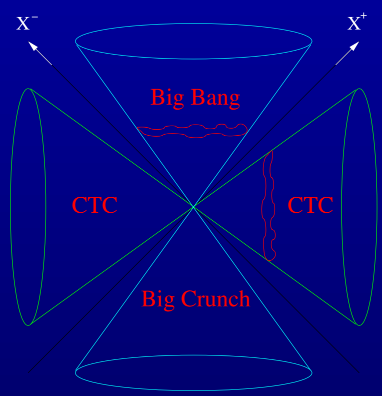

- We shall focus in particular on the extended perturbative string states which wind around the collapsing dimension.

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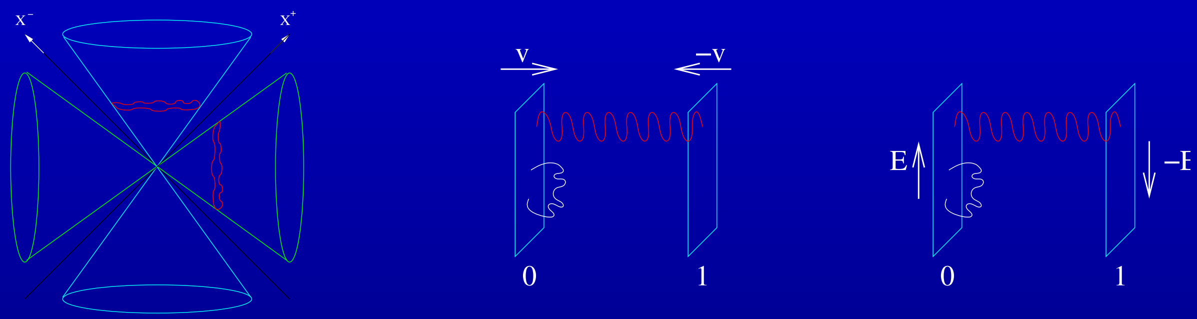

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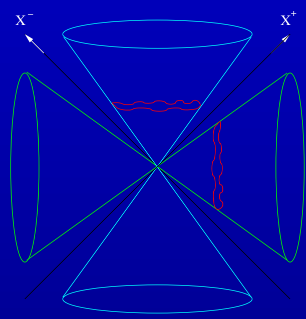

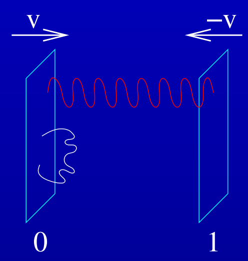

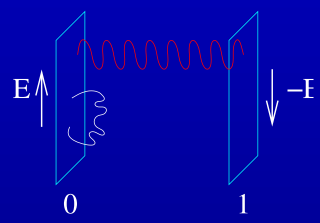

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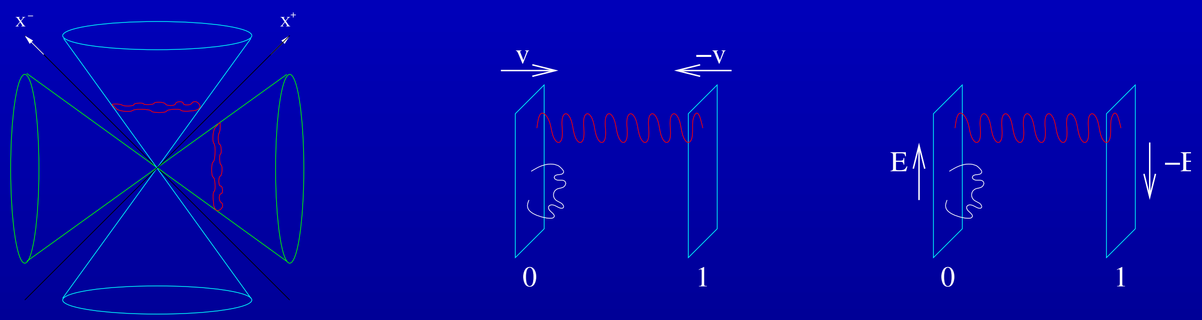

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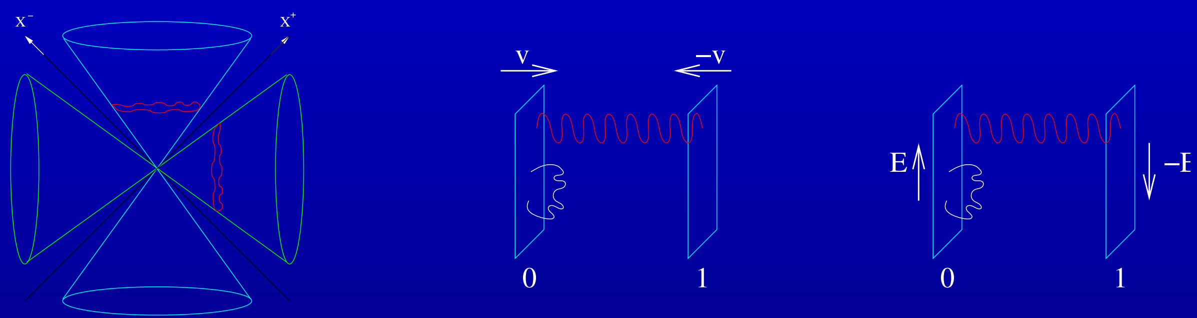

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- Can Schwinger production of twisted closed strings resolve the cosmological singularity of the Lorentzian orbifold ?

Outline of the talk

1. Introduction

2. The Lorentzian orbifold and its avatars

Misner, Taub-NUT, Grant...

3. Closed strings in Misner space: first pass

Nekrasov

3. Open strings in electric fields, revisited

Berkooz BP

4. Closed strings in Misner space: second pass

Berkooz BP; Berkooz Durin BP Reichmann Rozali

5. Conclusions, speculations

The Lorentzian orbifold

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$$ds^2 = dr^2 - \beta^2 r^2 d\eta^2 + (dX^i)^2 \quad \eta \equiv \eta + 2\pi \,, \quad X^\pm = \pm r e^{\pm\beta\eta}/\sqrt{2}$$

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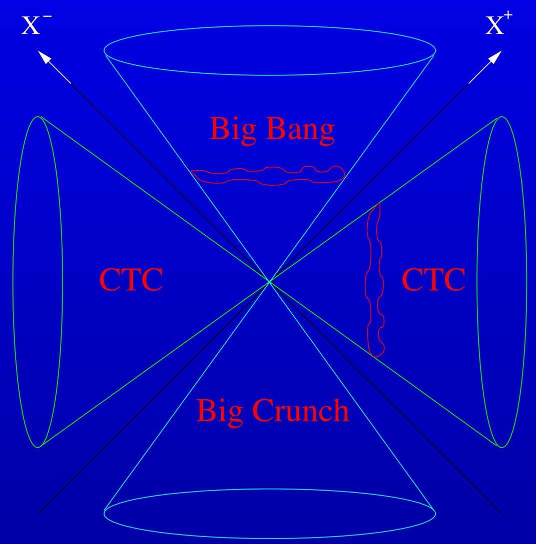
- Finally, the lightcone $X^+X^- = 0$ gives rise to non-Hausdorff null lines, attached to the singularities.

The Misner/Milne Universe



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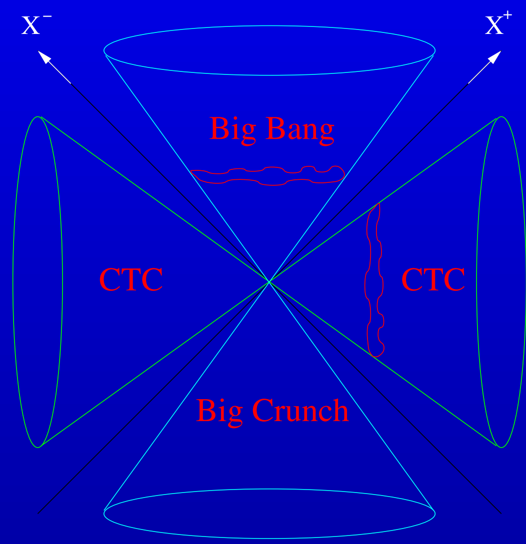

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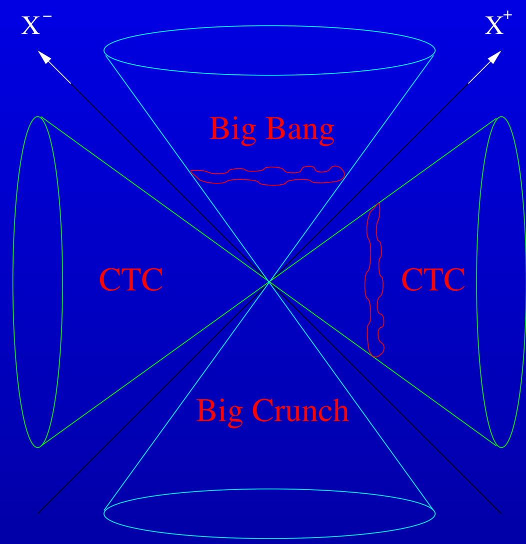

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- Misner space was first introduced as a local model of Lorentzian Taub-NUT space:

$$ds^2 = 4l^2 U(t)\sigma_3^2 + 4l\sigma_3 dt + (t^2 + l^2)(\sigma_1^2 + \sigma_2^2) \ , \quad U(t) = -1 + \frac{2mt + l^2}{t^2 + l^2}$$

A bouncing universe, isomorphic to $R^{1,1}/boost \times S^2$ around each singularity.

Stringy Misner-type Universes

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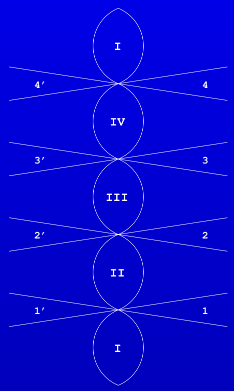

Nappi Witten; Elitzur Giveon Kutasov Rabinovici

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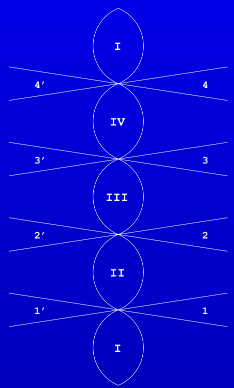

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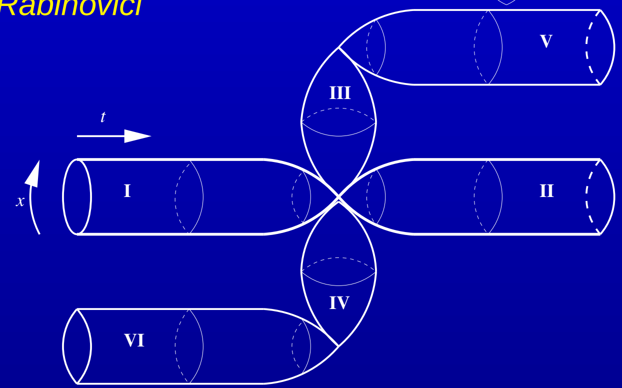

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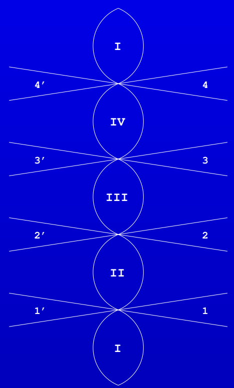

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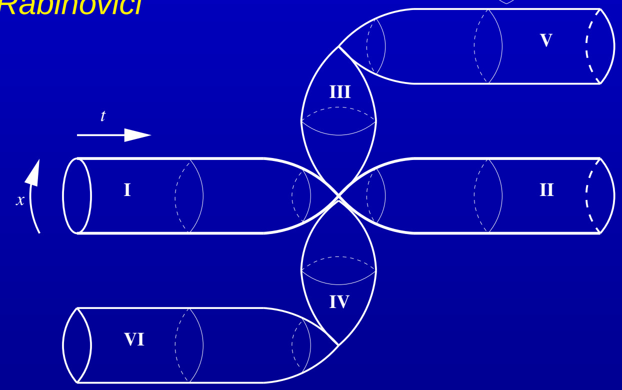

- The Lorentzian orientifold $IIB/[(-)^F boost]/[\Omega(-)^{F_L}]$ was also recently argued to describe orientifolds of non-supersymmetric strings with non-vanishing Neveu-Schwarz tadpoles.

Dudas Mourad Timirgaziu

The Grant space

- A close variant of Misner space is the quotient of flat space by the combination of a discrete boost and a translation on an extra direction, often known as the Grant space:

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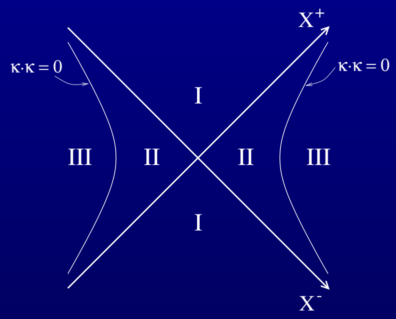
Gott 91, Grant 93

- Defining $Z^\pm = X^\pm e^{\mp \beta X/R}$, the metric can be written in the Kaluza-Klein form

$$ds^2 = R^2(dX + A)^2 - 2dZ^+dZ^- - \frac{E^2}{2R^2}(Z^+dZ^- - Z^-dZ^+)^2 \,, \quad X \equiv X + 2\pi$$

with radius R and KK electric field

$$R^2 = 1 + 2EZ^+Z^- \,, \quad dA = \frac{E}{R^4} dZ^+ dZ^- \,, \quad E = \beta/R$$



Cornalba Costa

The Grant space (cont)

- In particular, the compact direction X becomes time-like in the region $X^+X^- < -1/(2E)$: there are still CTC !

The Grant space (cont)

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- All CTC have to pass into $X^+X^- < -1/(2E)$, hence may be suppressed by excising this region: *orientifold boundary conditions ?*

Cornalba Costa

Closed strings in Misner space - untwisted states

- As usual in orbifold constructions, part of the spectrum involves string which close on the (flat) covering space, and are invariant under the orbifold projection:

$$X^{\pm}(\sigma + 2\pi, \tau) = X^{\pm}(\sigma, \tau) \,, \quad (\partial_\tau^2 - \partial_\sigma^2) X^{\pm} = 0$$

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- The resulting eigenfunctions describe strings with momentum j along the Milne circle. For $k^+>0, k^->0$, they are exponentially decreasing in the Rindler wedges. j is now the (quantized) Rindler energy.

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Quantum fluctuations and backreaction

- In the Minkowski vacuum (inherited from the covering space), the renormalized propagator can be obtained as a sum over images, e.g in D=4

$$G(x;x') = \sum_{n=-\infty, n\neq 0}^{\infty} [-2(X^+ - e^{2\pi\beta n} X^{+'})(X^- - e^{2\pi\beta n} X^{-'}) + (X^i - X^{i'})^2]^{-1}$$

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- The one-loop stress-energy tensor follows from $G(x,x)$, e.g for a conformally coupled scalar,

$$\langle T_{ab} \rangle = \lim_{x\to x'} \left[(1-2\xi)\nabla_a \nabla'_b - 2\xi \nabla_a \nabla_b + (2\xi - \frac{1}{2}) g_{ab} \nabla_c \nabla'^c \right] G(x,x')$$

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leading to a divergent quantum backreaction:

$$\langle T_\mu^\nu\rangle = \frac{K}{12\pi^2}T^{-4}\text{diag}(1,-3,1,1)\ , \quad K = \sum_{n=1}^{\infty}\frac{2+\cosh 2\pi n\beta}{[\cosh 2\pi n\beta - 1]^2}$$

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- In the case of the Grant space, the one-loop energy momentum tensor diverges as $1/(R^2T^2)$ on the chronological horizon, and $1/(T-T_n)^3$ on the polarized hypersurfaces. This is at the basis of Hawking's chronology protection conjecture. The divergence seems to have more to do with the CTC than with the singularity...

Scattering of untwisted states

- Scattering amplitudes of untwisted sector states can be computed from those in flat space by the inheritance principle,

$$\langle V(j_1,k_1) \quad \ldots \quad V(j_n,k_n)\rangle_{Misner} = \int dv_1 \ldots dv_n e^{ij_1v_1+\cdots+j_nv_n}$$
$$\langle V(e^{\beta v_1}k_1^+, e^{-\beta v_1}k_1^-, k_1^i) \ldots V(e^{\beta v_n}k_n^+, e^{-\beta v_n}k_n^-, k_n^i)\rangle_{Minkowski}$$

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- The integral diverges due to Regge behavior in the large momentum, fixed angle regime. E.g, the four-tachyon scattering amplitude in bosonic string leads to

$$\int dv \ v^{-\frac{1}{2}(k_1^i - k_3^i)^2 + i(j_2 - j_4)}$$

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- The divergence mostly disappears for the Grant space, except for a localized contribution at $k_1^i = k_3^i$. The amplitude is also finite for transverse gravitons in type II superstring on Misner space, but reappears for longitudinal gravitons.

  Berkooz Durin Pioline Reichmann, unpublished

Closed string in Misner space - twisted sectors

- In addition, there are closed string modes on the orbifold that descend from strings on the covering space that close up to the action of the orbifold group:

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- They have a normal mode expansion:

$$\begin{aligned} X_R^{\pm}(\tau - \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha_n^{\pm} e^{-i(n \pm i\nu)(\tau-\sigma)} \\ X_L^{\pm}(\tau + \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \mp i\nu)^{-1} \tilde{\alpha}_n^{\pm} e^{-i(n \mp i\nu)(\tau+\sigma)} \end{aligned}$$

with canonical commutation relations

$$[\alpha_m^+, \alpha_n^-] = -(m + i\nu)\delta_{m+n} \quad , \quad [\tilde{\alpha}_m^+, \tilde{\alpha}_n^-] = -(m - i\nu)\delta_{m+n}$$

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- There are no translational zero-modes, instead two pairs of quasi zero-modes which are canonically conjugate hermitian operators:

$$[\alpha_0^+, \alpha_0^-] = -i\nu \ , \quad [\tilde{\alpha}_0^+, \tilde{\alpha}_0^-] = i\nu$$

Physical states (absence thereof)

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated, e.g., by

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- This is the analytic continuation of the familiar result for a rotation orbifold $\frac{1}{2}\theta(1-\theta)$ under $\theta \to i\nu$...
- Due to the $i\nu/2$ term in the ground state energy, all states obtained by acting on $|0\rangle$ by creation operators $\alpha^{\pm}_{n<0}$ and by α^{+}_{0} will have imaginary energy, hence the physical state condition $L_0 = 0$ has no solutions.

Nekrasov

One-loop amplitude

- Independently of this fact, one may compute the one-loop (Euclidean ws, Minkowskian target) free energy using path integral methods:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=0}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho) x\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

where θ_1 is the Jacobi theta function,

$$\theta_1(v;\rho) = 2q^{1/8} \sin \pi v \prod_{n=1}^{\infty} (1-e^{2\pi i v} q^n)(1-q^n)(1-e^{-2\pi i v} q^n)\ , \quad q = e^{2\pi i \rho}$$

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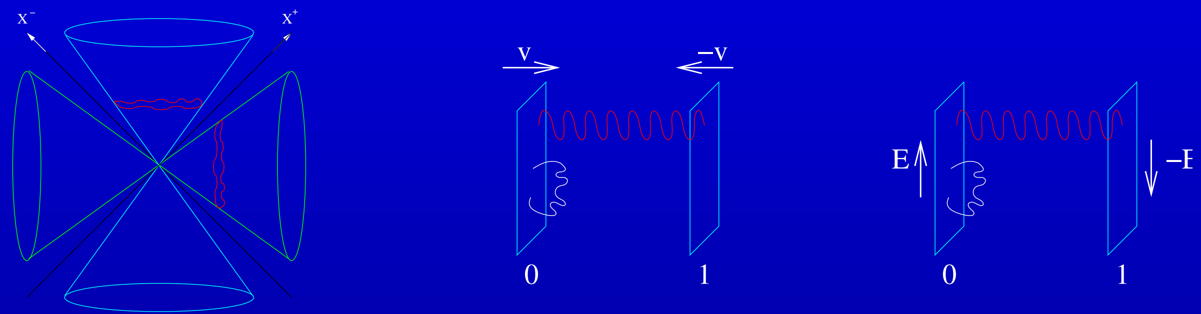
- The absence of physical twisted states crushes our hopes for resolving the singularity... but is also rather hard to swallow. For one, α_0^+ and α_0^- are not hermitian conjugate to each other, but rather self-hermitian...

Open strings in electric field vs Lorentzian orbifold

- A very similar puzzle is faced in the case of moving D-branes, or charged open strings in a constant electric field:



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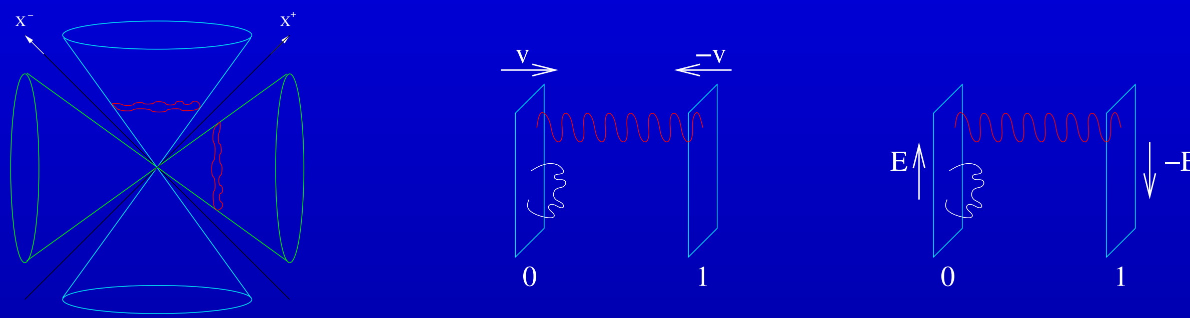

- Recall that for open strings stretched between two D-branes with electromagnetic fields F_0 and F_0(resp. twisted closed strings on an orbifold $X^\mu \equiv R^\mu_\nu X^\nu$, proper frequencies satisfy

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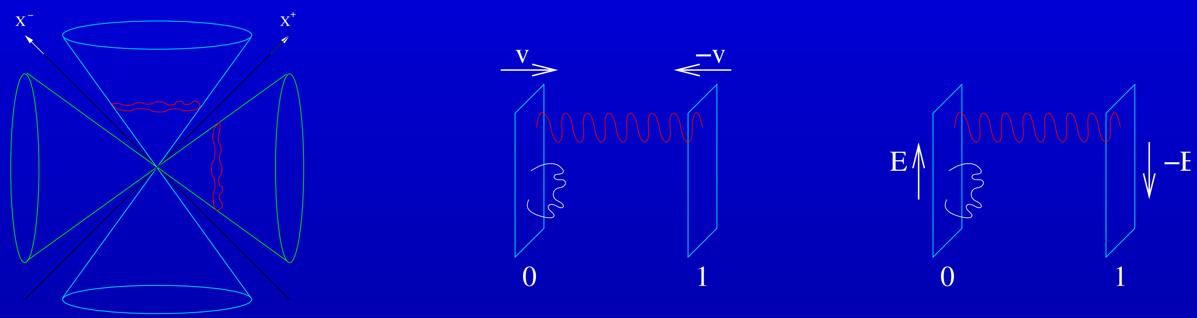

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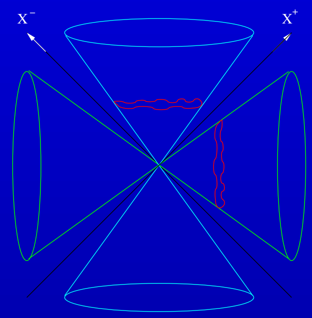

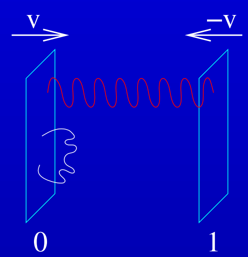

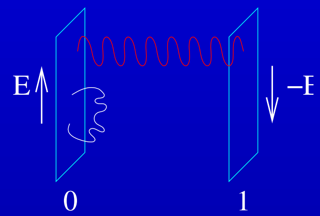

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- In particular, twisted closed strings on the Lorentzian orbifold have the same eigenfrequencies as charged open strings in an electric field $\nu :=$ Arctanh$E = w\beta$.
- More precisely, the charged open string has half the excited modes of the twisted closed strings, and isomorphic quasi-zero modes.

Open string mode expansion

- The light-cone embedding coordinates have the normal mode expansion

$$X^{\pm} = x_0^{\pm} + i \sum_{n=-\infty}^{+\infty} (-)^n (n \pm i\nu)^{-1} a_n^{\pm} e^{-i(n \pm i\nu)\tau} \cos[(n \pm i\nu)\sigma]$$

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$$[a_m^+, a_n^-] = -(m + i\nu)\delta_{m+n} \ , \quad [x_0^+, x_0^-] = -\frac{i}{E}$$

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- The world-sheet Hamiltonian, normal ordered with respect to the vacuum annihilated by $a_{n>0}^+, a_{n>0}^-$ and a_0^+, takes the form

$$L_0^{l.c.} = -\sum_{m=0}^{\infty} a_{-m}^+ a_m^- - \sum_{m=1}^{\infty} a_{-m}^- a_m^+ + \frac{i\nu}{2}(1 - i\nu) - \frac{1}{12}$$

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- By the same token, charged open strings should have no physical states...

One-loop amplitude and Schwinger pair production

- Using this quantization scheme, the one-loop (Euclidean worldsheet, Minkowskian target) vacuum free energy reads

$$A_{bos} = \frac{i\pi V_{26}(e_0+e_1)}{2}\int_0^\infty \frac{dt}{(4\pi^2 t)^{13}}\frac{e^{-\pi\nu^2 t/2}}{\eta^{21}(it/2)\,\theta_1(t\nu/2;it/2)}$$

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- Each of the poles at $t = 2k/\nu$ contributes to the imaginary part, yielding the rate for charged string pair production,

$$\mathcal{W} = \frac{1}{2(2\pi)^{25}}\frac{(e_0+e_1)}{\nu}\sum_{k=1}^{\infty}(-)^{k+1}\left(\frac{|\nu|}{k}\right)^{13}\sum_{N=-1}^{\infty}c_b(N)\exp\left(-2\pi k\frac{N}{|\nu|}-2\pi k|\nu|\right)$$

Bachas Porrati

where $\eta^{-24}(q) = \sum_{N=-1}^{\infty} c_b(N)q^N$. This can be viewed as the sum of the Schwinger production rates for each state in the spectrum, of mass $m^2 = 2N+\nu^2$.

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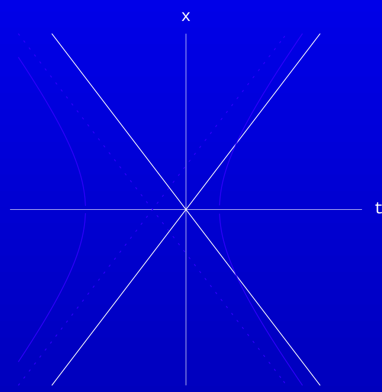
- This seems to support the quantization scheme based on a vacuum, hence the absence of physical states. But physical states do exist classically, how could quantization make them disappear altogether ?

Open string zero-modes



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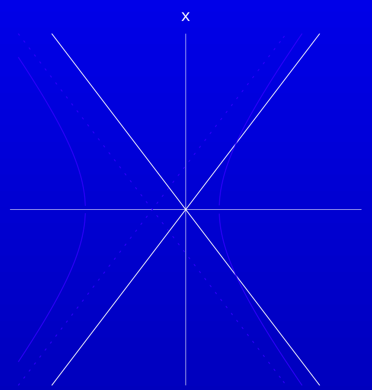

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$$L = \frac{1}{2} m \left(-2 \partial_\tau X^+ \partial_\tau X^- + (\partial_\tau X^i)^2 \right) + \frac{e}{2} \left(X^+ \partial_\tau X^- - X^- \partial_\tau X^+ \right)$$

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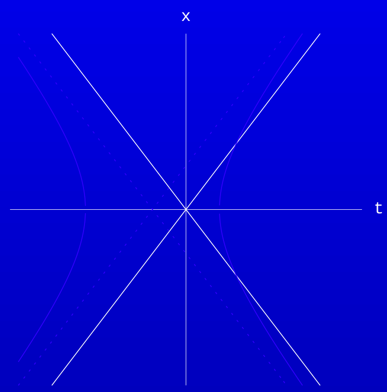

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- The canonical momenta

$$\pi^{\pm} = m\, \partial_\tau X^{\pm} \mp \frac{e}{2} X^{\pm} = \mp \frac{e}{2} x_0^{\pm} + \frac{1}{2} a_0^{\pm} e^{\pm e\tau/m} \,, \quad \pi^i = m\, \partial_\tau X^i = p^i$$

satisfy the usual equal-time commutation rules

$$[\pi^+, x^-] = [\pi^-, x^+] = i \,, \quad [\pi^i, x^j] = i \delta_{ij}$$

Quantizing the open string zero-modes

- At $\tau = 0$, one can thus express

$$a_0^{\pm} = \pi^{\pm} \pm \frac{\nu}{2} x^{\pm} \,, \quad x_0^{\pm} = \mp \frac{1}{\nu} \left(\pi^{\pm} \mp \frac{\nu}{2} x^{\pm} \right)$$

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- Quantum mechanically, one may represent $\pi^\pm = i\partial/\partial x^\mp$ so that $a_0^\pm$ become covariant derivatives in the electric field ν.
- The zero-mode piece of L_0, including the evil $\frac{i\nu}{2}$,

$$L_0^{(0)} = -a_0^+ a_0^- + \frac{i\nu}{2} = -\frac{1}{2}(a_0^+ a_0^- + a_0^- a_0^+)$$

is just the Klein-Gordon operator of a particle of 2D mass $M^2 = -2L_0^{(0)}$ and charge ν.

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^{\pm} = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator just becomes an inverted harmonic oscillator:

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- More explicitely, in terms of $u = (\tilde{p} + \nu x)\sqrt{2/\nu}$,

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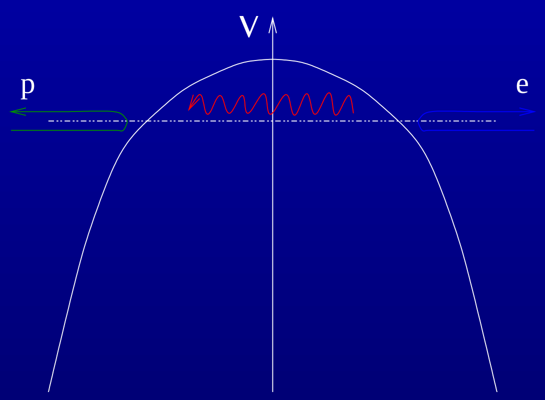
- The latter admits a respectable delta-normalizable spectrum of scattering states, in terms of parabolic cylinder functions, e.g:

$$\phi_{in}^+ = D_{-\frac{1}{2}+i\frac{M^2}{2\nu}}(e^{-\frac{3i\pi}{4}} u) e^{-i\tilde{p}t} e^{i\nu x t/2}$$



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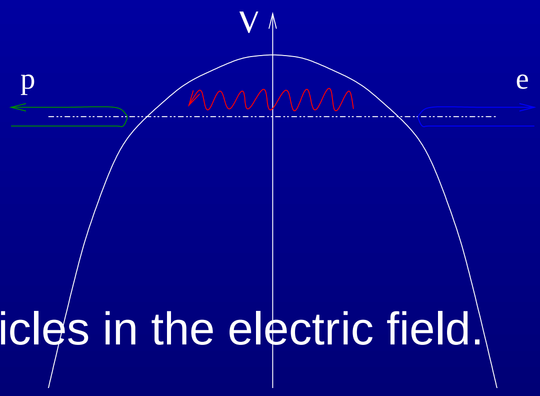

- These correspond to non-compact trajectories of charged particles in the electric field. Tunnelling is just (stimulated) Schwinger pair creation,

$$e^- \longrightarrow (1+\eta)\, e^- + \eta\, e^+ \,, \quad \eta \sim e^{-\pi M^2/\nu}$$

Brezin Itzykson; Brout Massar Parentani Spindel

Lorentzian vs Euclidean states

- Analytic continuation $X^0 \to -iX^0$, $\nu \to i\nu$ takes us from an electric field in $R^{1,1}$ to a magnetic field in R^2. At the same time, one should Wick rotate the worldsheet time.

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- The contribution of the zero-modes to the one-loop amplitude can be interpreted either way,

$$\frac{1}{2i\sin(\nu t/2)} = \sum_{n=1}^{\infty} e^{-i(n+\frac{1}{2})\nu t} = \int dM^2 \rho(M^2) e^{-M^2 t/2}$$

where the density of states is obtained from the reflection phase shift,

$$\rho(M^2) = \frac{1}{\nu}\log\Lambda - \frac{1}{2\pi i}\frac{d}{dM^2}\log\frac{\Gamma\left(\frac{1}{2}+i\frac{M^2}{2\nu}\right)}{\Gamma\left(\frac{1}{2}-i\frac{M^2}{2\nu}\right)}$$

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- The physical spectrum can be explicitely worked out at low levels, and is free of ghosts: a tachyon at level 0, a transverse gauge boson at level 1, . . .

Physical spectrum at low level

- The ground state tachyon

$$|T\rangle = \phi(x^+, x^-)|0_{ex}, k\rangle$$

should satisfy the Virasoro constraint

$$L_0|T\rangle = \left[-\frac{1}{2}\left(\alpha_0^+\alpha_0^- + \alpha_0^-\alpha_0^+\right) + \frac{1}{2}\nu^2 - 1 + \frac{1}{2}k_i^2\right]|T\rangle$$

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- The L_1 Virasoro constraint eliminates one polarization. Despite the non-vanishing two-dimensional mass $k_i^2 - \nu^2$, the spurious state $L_{-1}\phi|0\rangle$ is still physical, eliminating an extra polarization.
- One thus has $D-2$ transverse degrees of freedom, ie a massless gauge boson in D dimensions.

Charged particle in Rindler space

- For applications to the Milne universe, one should diagonalize the boost momentum J, ie consider an accelerated observer.

Gabriel Spindel; Mottola Cooper

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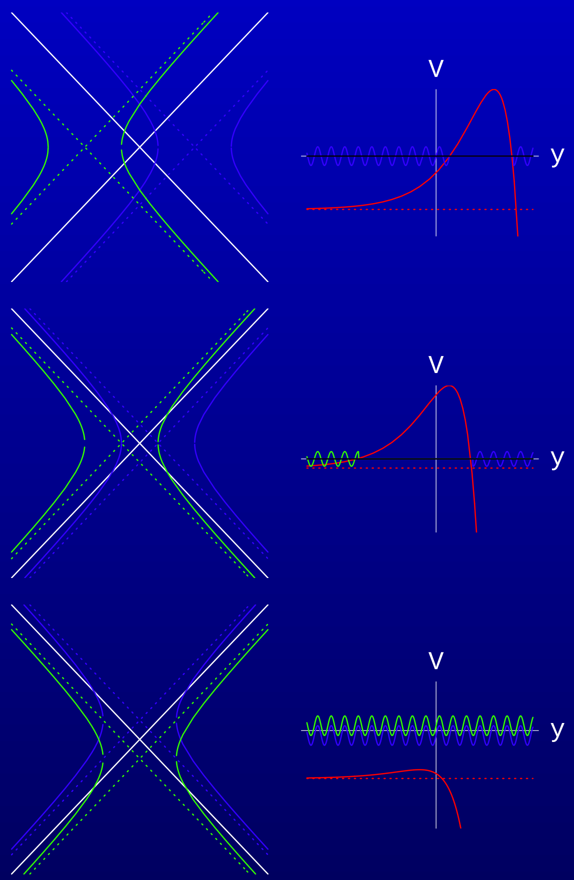

- In the Rindler patch R, letting $f(r,\eta) = e^{-iJ\eta} f_J(r)$ and $r = e^y$, one gets a Schrodinger equation for a particle in a potential

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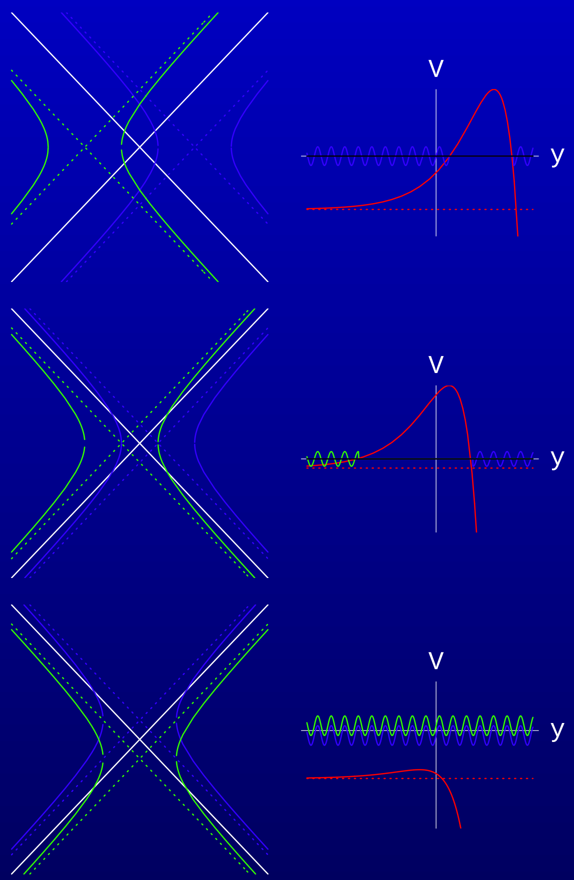

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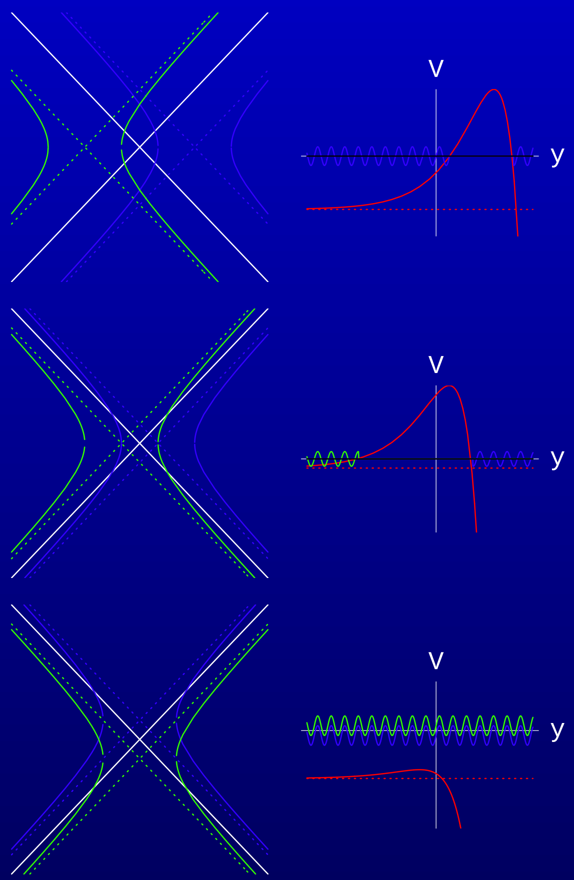

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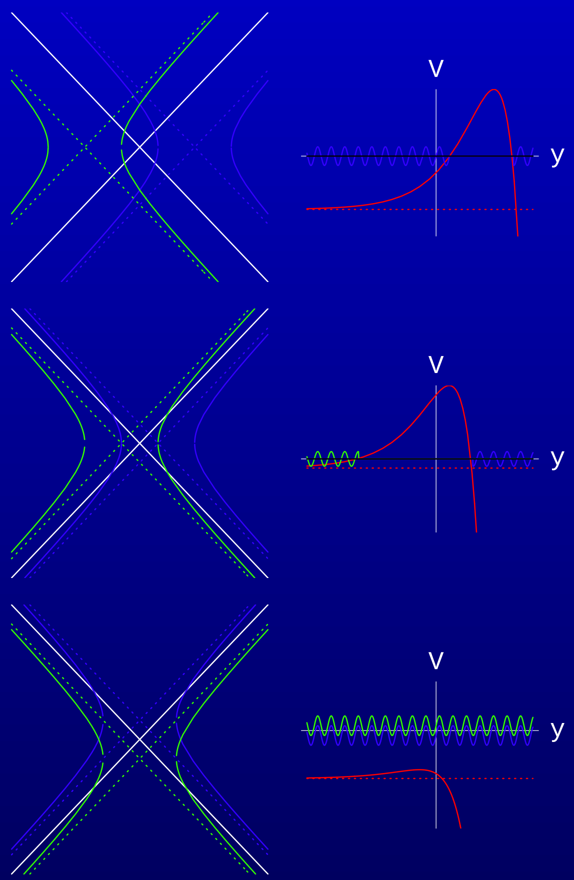

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- If $j > M^2/(2\nu)$, the electron branches extend in the Milne regions. There is no tunneling, but partial reflection amounts to a combination of Schwinger and Hawking emission.

Rindler modes

- Solutions are expressable in terms of parabolic cylinder functions:
  Incoming modes from Rindler infinity I_R^- read

$$\mathcal{V}^j_{in,R} = e^{-ij\eta} r^{-1} M_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),-\frac{ij}{2}}(i\nu r^2/2)$$

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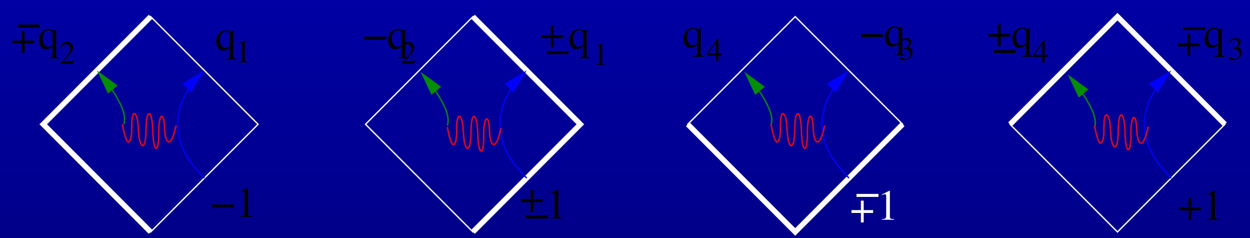
$$\mathcal{U}^j_{in,R} = e^{-ij\eta} r^{-1} W_{i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}(-i\nu r^2/2)$$



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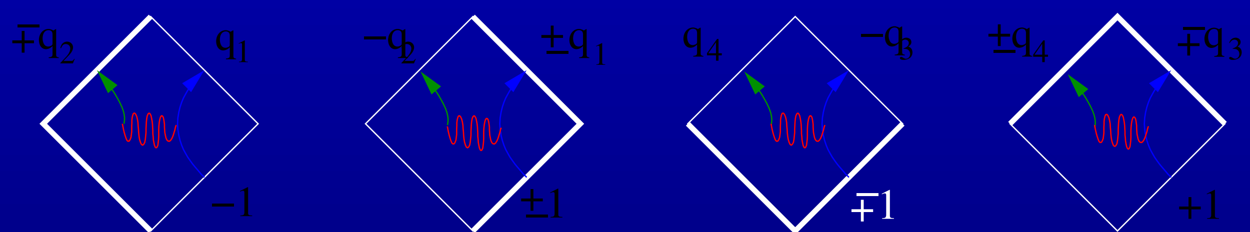

- The reflection coefficients can be computed:

$$q_1 = e^{-\pi j} \frac{\cosh\left[\pi \frac{M^2}{2\nu}\right]}{\cosh\left[\pi\left(j - \frac{M^2}{2\nu}\right)\right]}\,, \quad q_3 = e^{\pi\left(j-\frac{M^2}{2\nu}\right)} \frac{\cosh\left[\pi \frac{M^2}{2\nu}\right]}{|\sinh \pi j|}$$

  and $q_2 = 1 - q_1$, $q_4 = q_3 - 1$, by charge conservation.

Global Charged Unruh Modes

- Global Unruh modes may be defined by patching together Rindler modes, ie by analytic continuation across the horizons:

$$\Omega^j_{in,+} = \mathcal{V}^j_{in,P} = W_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}(-i\nu X^+X^-)[X^+/X^-]^{-ij/2}$$

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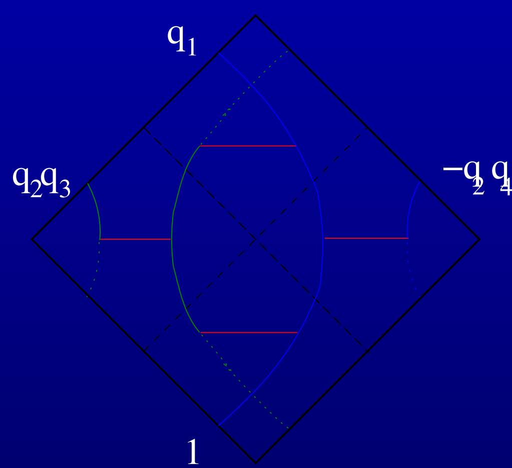
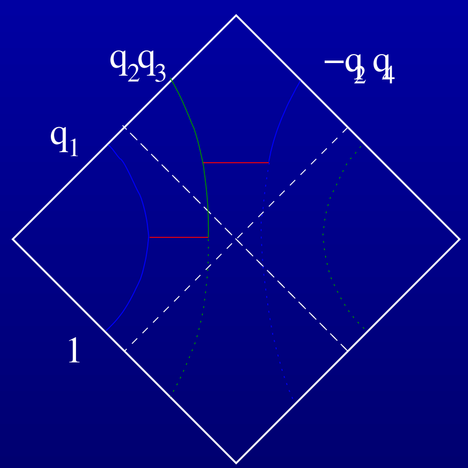
- There are two types of modes, involving 2 or 4 tunelling events:





Closed string zero-modes

- Let us analyze the classical solutions for the closed string zero modes

$$X^{\pm}(\tau,\sigma) = \pm\frac{1}{2\nu}\alpha_0^{\pm}e^{\pm\nu(\tau-\sigma)} \mp \frac{1}{2\nu}\tilde{\alpha}_0^{\pm}e^{\mp\nu(\tau+\sigma)}\ , \quad \alpha_0^{\pm}, \tilde{\alpha}_0^{\pm} \in R$$

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$$4\nu^2 X^+ X^- = \alpha_0^+\tilde{\alpha}_0^- e^{2\nu\tau} + \alpha_0^-\tilde{\alpha}_0^+ e^{-2\nu\tau} - \alpha_0^+\alpha_0^- - \tilde{\alpha}_0^+\tilde{\alpha}_0^-$$

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- The behavior at early/late proper time now depends on $\epsilon\tilde\epsilon$: For $\epsilon\tilde\epsilon = 1$, the string begin/ends in the Milne regions. For $\epsilon\tilde\epsilon = -1$, the string begin/ends in the Rindler regions.

Short and long strings ($j = 0$)

- $\epsilon = 1$, $\tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma, \tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma} \; , \quad T = \frac{M}{\nu} \sinh(\nu\tau) \; , \quad \theta = \nu\sigma$$

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- For $|j| > 0$, the short string in the Milne region attaches to a short string in the Rindler region stretching from $r = 0$ to $r_0 = |j|/(M + \tilde{M})$ and back. The induced worldsheet metric is of Misner type at the light-cone:

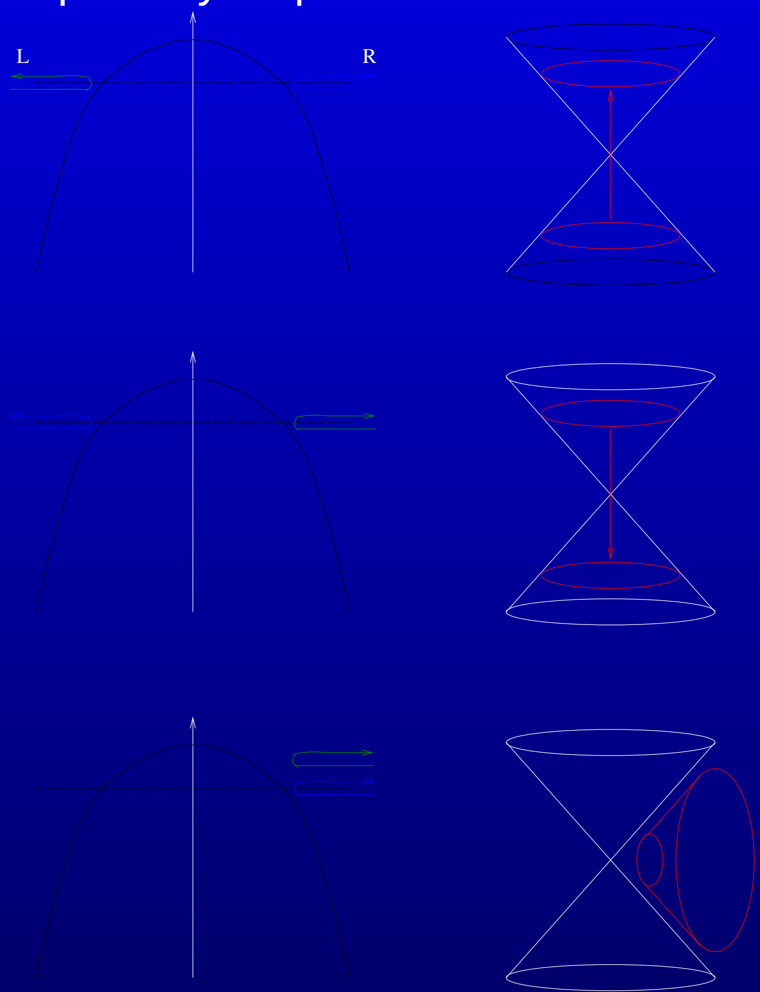
$$-2dX^+dX^- = -\nu j d\tau d\sigma + \nu|j|(\tau - \tau_0)d\sigma^2 - \frac{1}{2}(M^2 + \tilde{M}^2)d\tau^2$$

much like long strings or supertubes in Gödel Universe.

Drukker Fiol Simon; Israel

Short and long strings (static modes)

Just as in the open string case, we may now quantize the left and right-moving zero-modes separately as particles in inverted harmonic oscillator:



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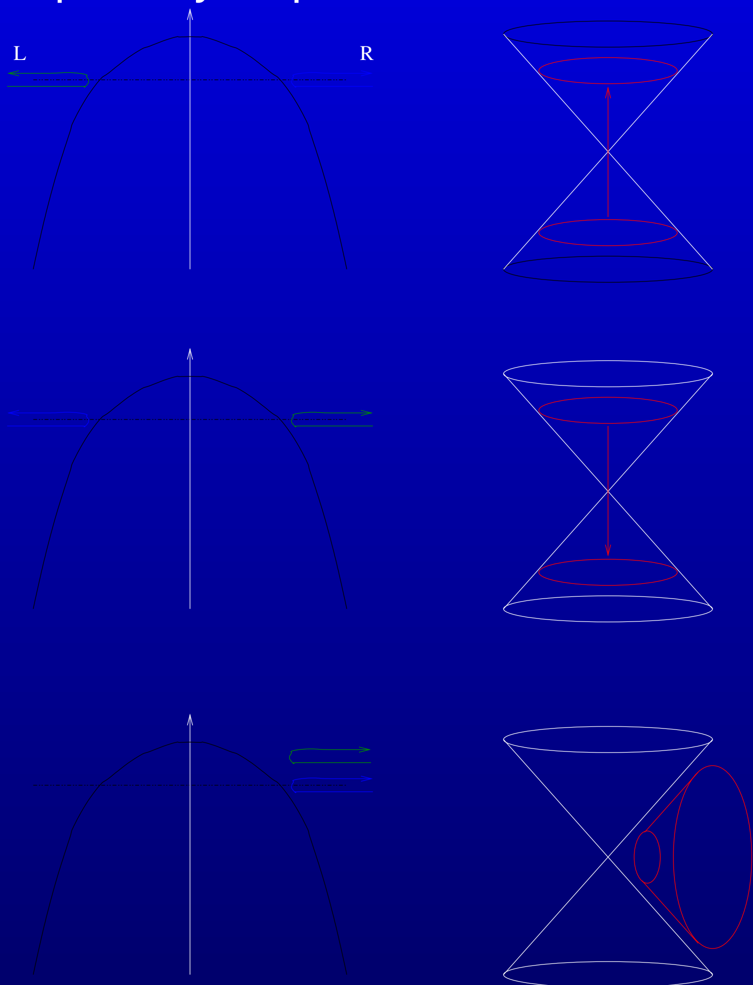

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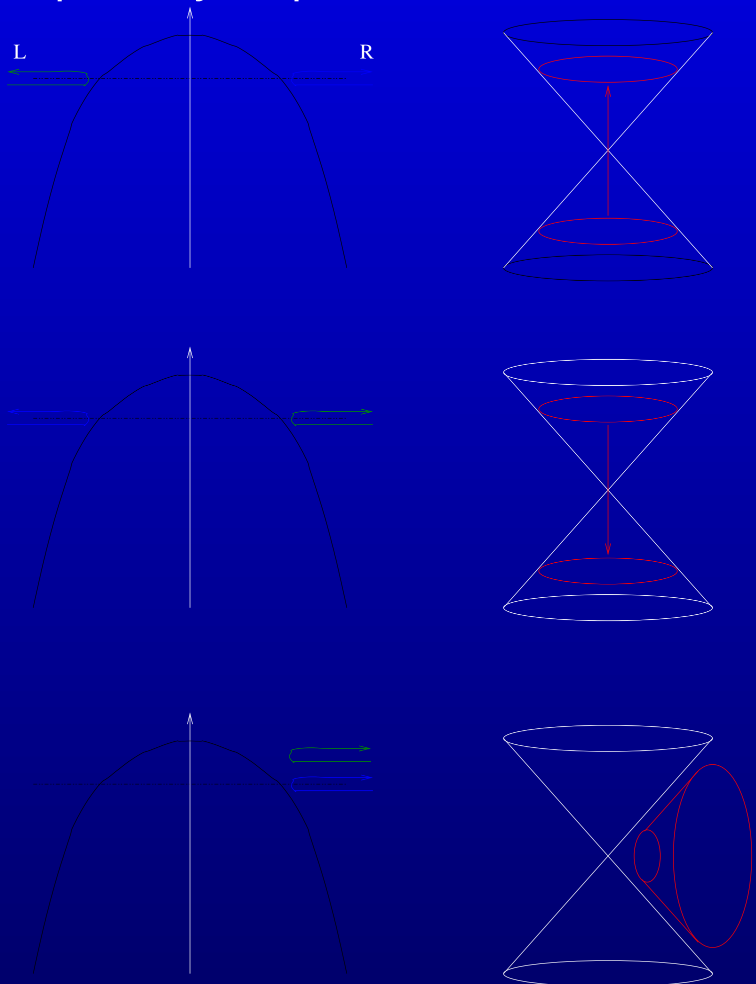

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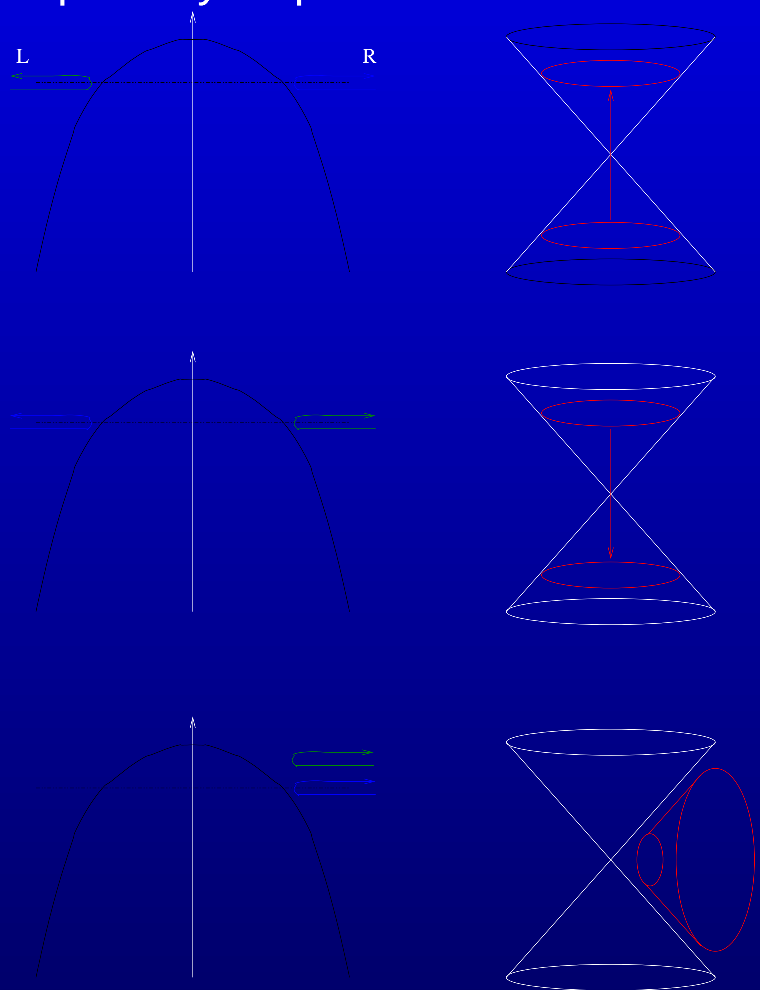

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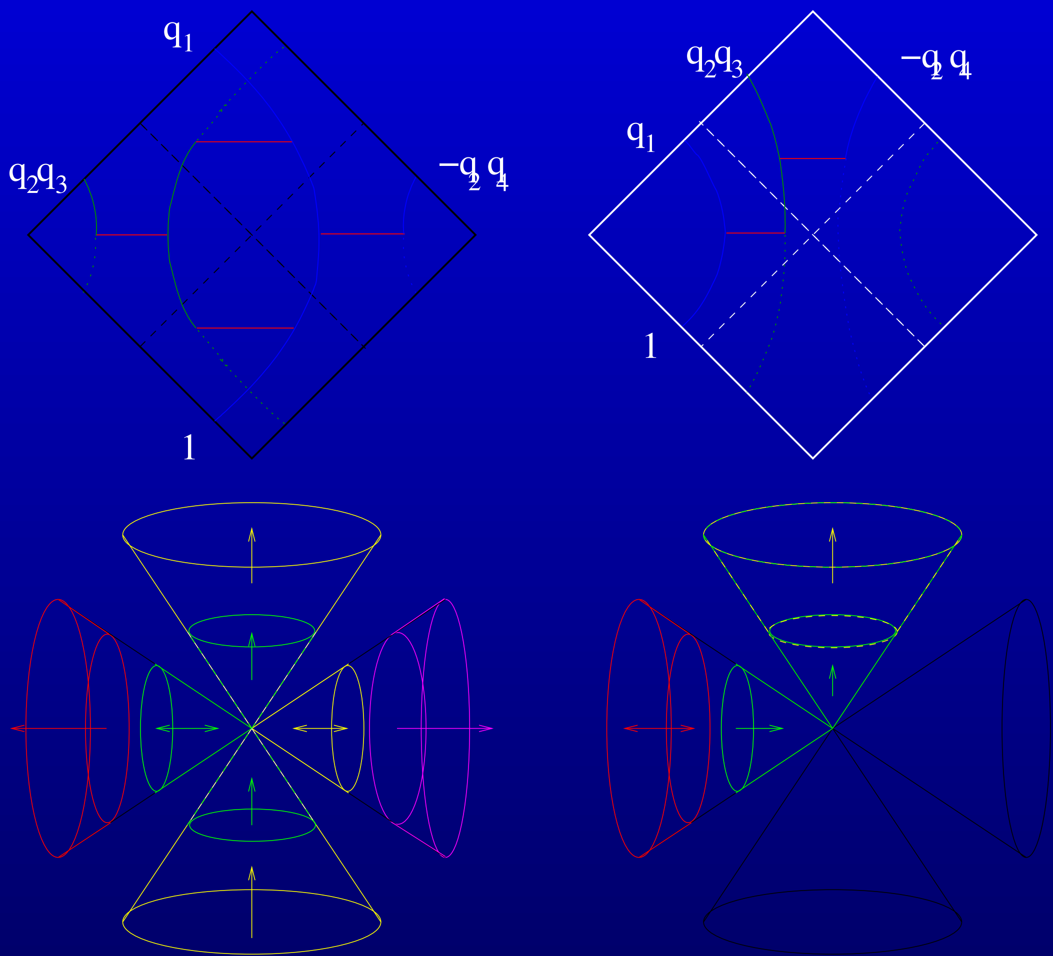
- $\epsilon = -\tilde{\epsilon} = 1$: $\phi = \phi_{p,in}\tilde{\phi}_{p,in}$ is a string-anti-string state, in the left whisker, but its seems awkward to take it as conjugate to the previous one...

Short and long strings, Unruh modes

- Instead of following the motion of a point at fixed σ, one may consider instead fixed $\sigma + \tau$: these are the trajectories of the open string zero-mode, in Rindler coordinates.



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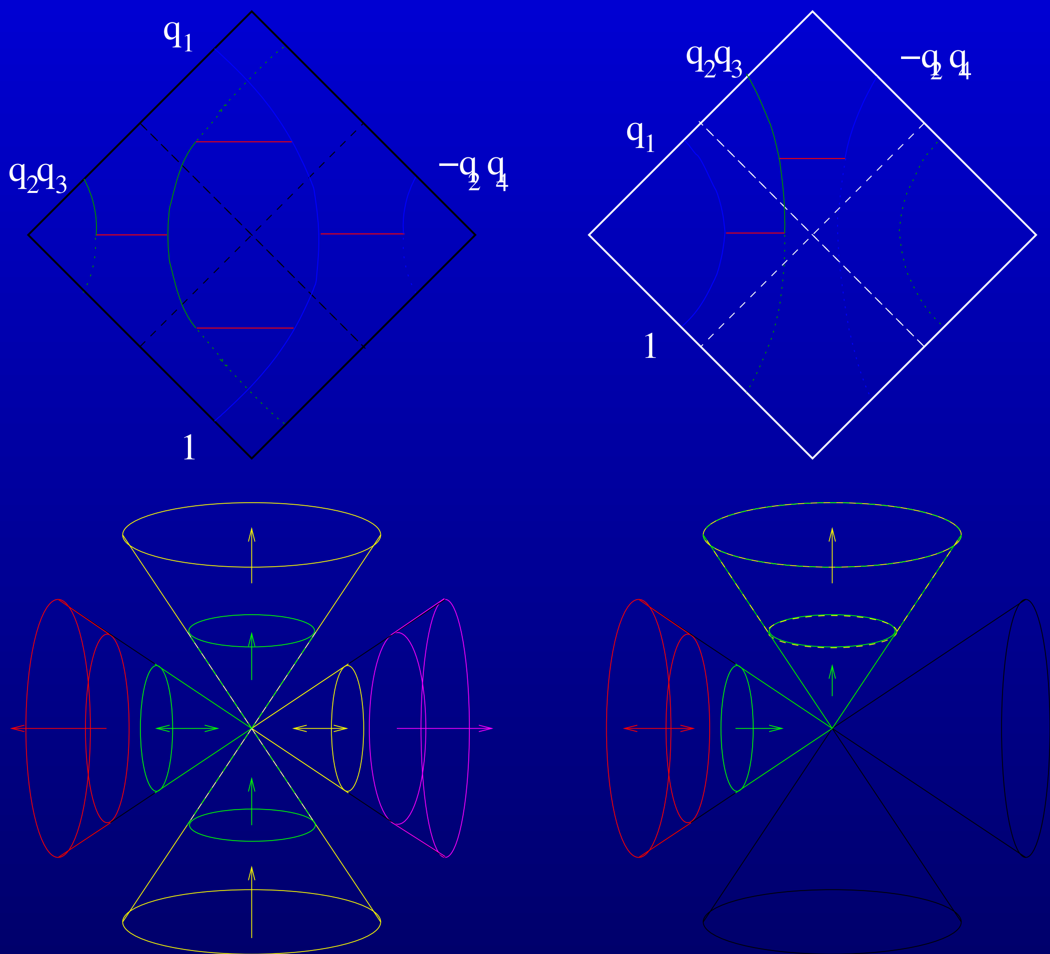

- The probability amplitude of winding strings at $T=+\infty$, *assuming that there are no stretched pairs in the whiskers*, is q_1 times the incoming amplitude at $T=-\infty$.

$$q_1 = e^{-\pi j} \frac{\cosh\left[\pi \frac{M^2}{2\nu}\right]}{\cosh\left[\pi\left(j-\frac{M^2}{2\nu}\right)\right]},$$

hence $q_1 = 0$ if $j=0$.

The one-loop amplitude again

- Recall the (Euclidean ws, Minkowskian target) one-loop amplitude:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=0}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho)\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

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leading to logarithmic divergences $\int d\rho d\bar{\rho}/|\rho-\rho_0|^2 \sim log\epsilon$, analogous to the long strings in AdS_3.

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- These can be traced to the existence of infinite families of periodic orbits, localized on the light-cone (currently under investigation)

Wick rotation to a rotation orbifold

- Note first that the (future) Milne region $ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + dx_i^2$ cannot be directly Wick-rotated to Euclidean.

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- Rotating $\beta = i\mu$, the Rindler region becomes get indeed an Euclidean metric,

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$$Z = X^+ = re^{i\mu\eta}\,, \quad \bar{Z} = -X^- = re^{-i\mu\eta} \qquad (r > 0)$$

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- By the same token, the left Rindler wedge rotates to another copy of the Euclidean plane with the origin removed: the complete analytic continuation of Misner space is therefore

$$\widetilde{R^2 \backslash \{0\}}_L / e^{i\mu} \backslash \widetilde{R^2 \backslash \{0\}}_R$$

and states of interest are non-normalizable !

D'Appolonio Kiritsis

Conclusions - speculations

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Conclusions - speculations

- As a less ambitious goal, can one compute scattering amplitudes of twisted states, and check if they are better behaved than untwisted states. For this, the relation with negative level $Sl(2)/U(1)$ and double analytic continuation of the Nappi-Witten plane wave may be useful.

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- The closed string orbifold we have discussed are highly non-generic trajectories on the cosmological billiard: Do whiskers feature also for more general Kasner-like singularities ?