Closed Strings in the Misner Universe

aka the Lorentzian orbifold

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based on hep-th/0307280 w/ M. Berkooz

and work in progress w/ M. Berkooz, B. Durin, D. Reichmann, M. Rozali

slides available from

`http://www.lpthe.jussieu.fr/~pioline/seminars.html`

Motivational string cosmology

- Observational Cosmology is now challenging string theory with high-precision data:

$$\Omega_{baryon} = 0.047 \; , \quad \Omega_{darkm} = 0.243 \; , \quad \Omega_\Lambda = 0.71 \; , \quad w = -0.98 \pm .12 \; , \ldots$$

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- With LHC still far in the future, understanding StringY Cosmology may be the only way to make contact with reality...

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- Perturbative string theory requires an Euclidean worldsheet, hence Euclidean target space. The analytic continuation may be ambiguous or ill-defined, Lorentzian observables may be very different from their Euclidean counterparts.
- String theory is not content on a finite time interval, and one is frequently forced into Big Bang / Big Crunch singularities, CTC in the process of maximally extending the geometry.

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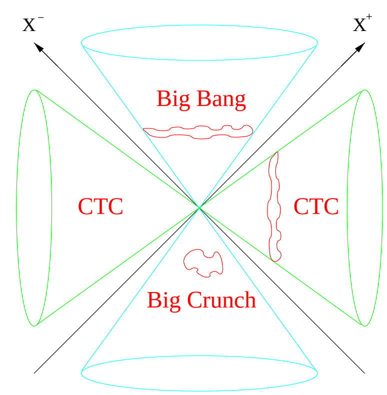

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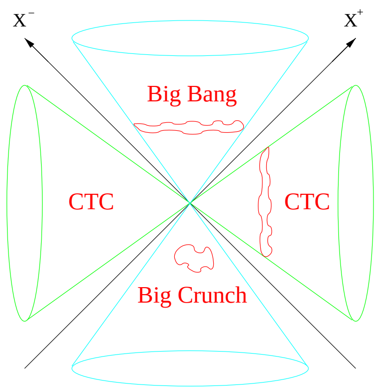


- Our main focus will be on the topological excitations which wind around the collapsing dimension: can the production of winding states resolve the singularity ?

Outline of the talk

1. Introduction

2. The Lorentzian orbifold and its avatars

   Misner, Taub-NUT, Grant...

3. Closed strings in Misner space: first pass

   Nekrasov

3. A detour: Open strings in electric fields

   Berkooz BP

4. Closed strings in Misner space: second pass

   Berkooz BP; Berkooz Durin BP Reichmann Rozali

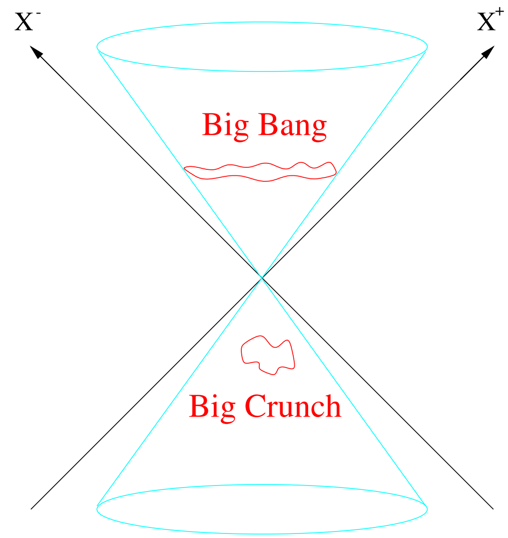
5. Conclusions, speculations

The Lorentzian orbifold

- One of the simplest examples of space-like singularities is the quotient of flat Minkowski space by a discrete boost, also known as Misner space (1967):

$$ds^2 = -2dX^+dX^- + (dX^i)^2$$

$$X^{\pm} \sim e^{\pm 2\pi\beta} X^{\pm}$$



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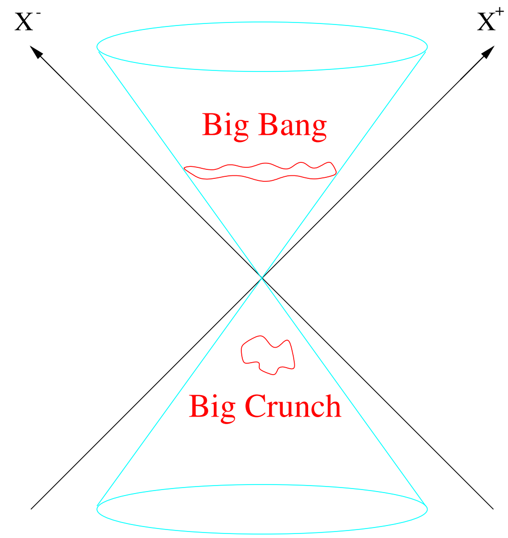

- The future (past) regions $X^+X^- > 0$ describes a cosmological universe often known as the Milne universe (1932), linearly expanding away from a Big Bang singularity (or contracting into a Big Crunch singularity):

$$ds^2 = -dT^2 + \beta^2 T^2 d\theta^2 + (dX^i)^2 \,, \quad \theta \equiv \theta + 2\pi \,, \quad X^\pm = Te^{\pm\beta\theta}/\sqrt{2}$$

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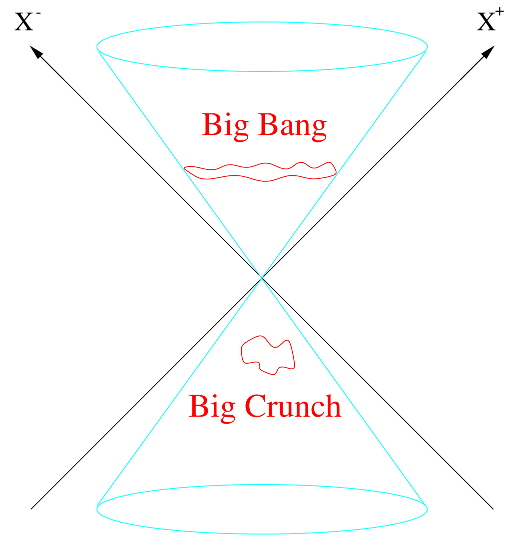

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This is a (degenerate) Kasner singularity, everywhere flat, except for a delta-function curvature at $T = 0$.

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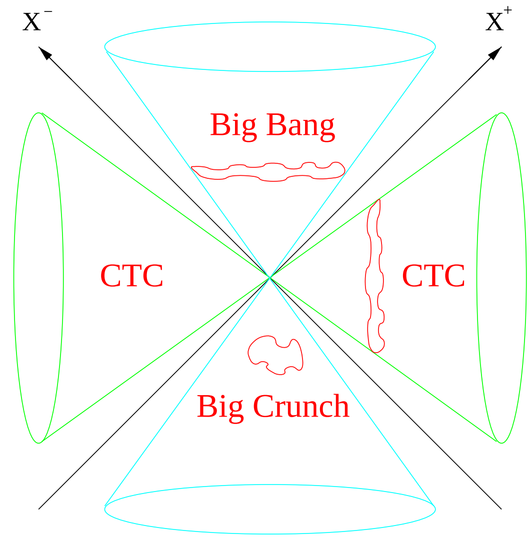

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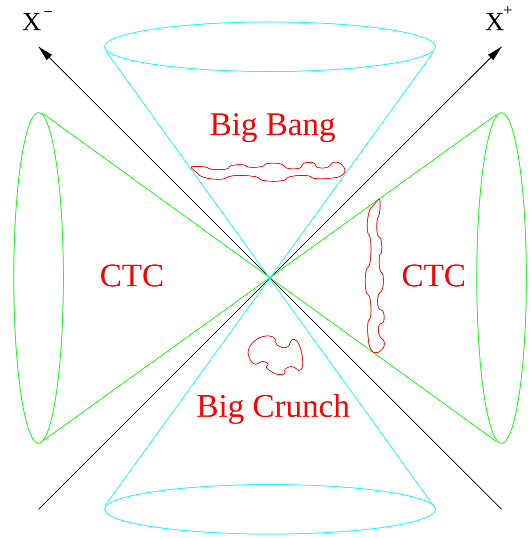

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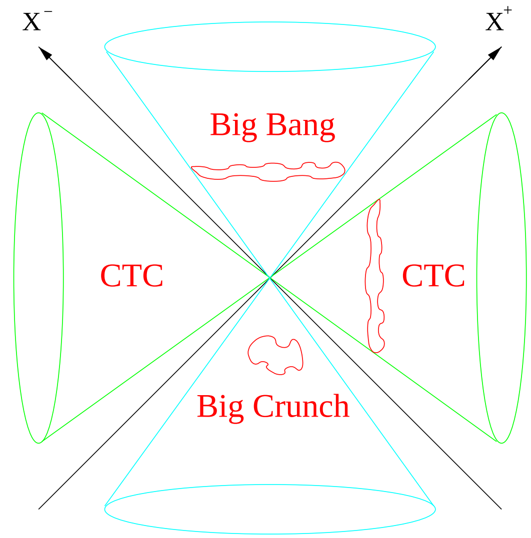

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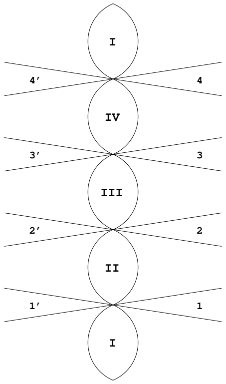
- Finally, the lightcone $X^+X^- = 0$ gives rise to a null, non-Hausdorff locus attached to the singularity.

Close relatives of the Misner Universe

- Misner space was first introduced as a local model of Lorentzian Taub-NUT space:

$$ds^2 = 4l^2 U(t)\sigma_3^2 + 4l\sigma_3 dt + (t^2+l^2)(\sigma_1^2+\sigma_2^2)\,, \quad U(t) = -1 + \frac{2mt + l^2}{t^2+l^2}$$



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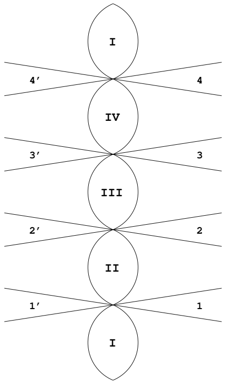

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This describes the space away from two moving cosmic strings. The cosmological singularity is smoothed out, but regions with CTC remain.

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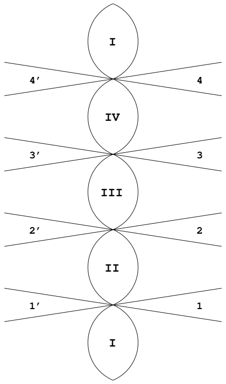

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- The Misner geometry arose again more recently as the M-theory lift of a simple (ekpyrotic) cosmological solution of Einstein-dilaton gravity with no potential.

Khoury Ovrut Seiberg Steinhard Turok

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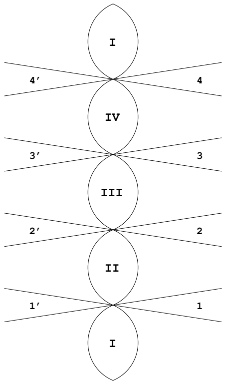

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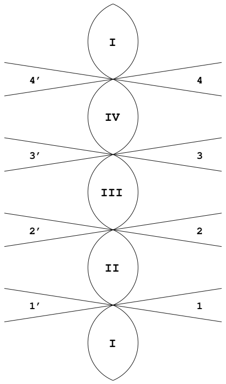

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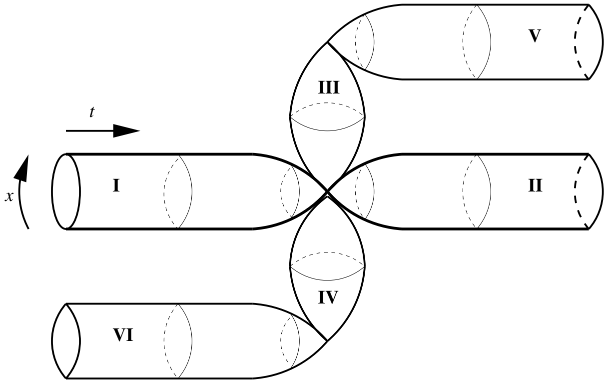

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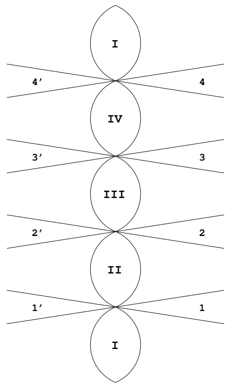

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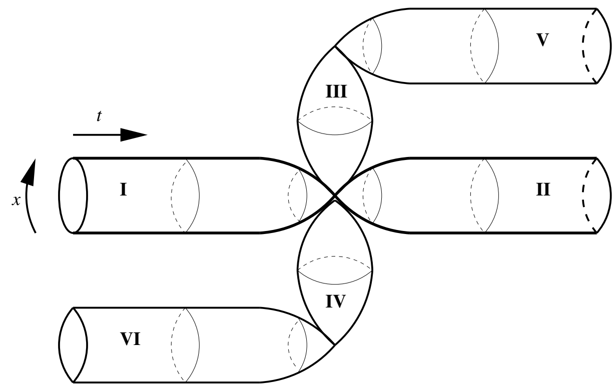
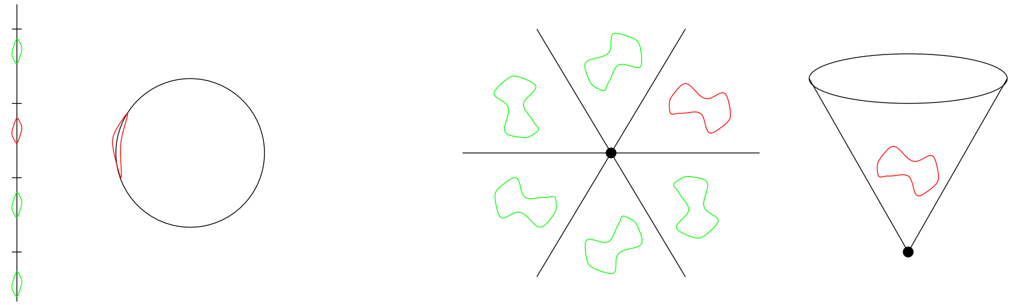


- The Lorentzian orientifold $IIB/[(-)^F boost]/[\Omega(-)^{F_L}]$ was also recently argued to describe orientifolds of non-supersymmetric strings with non-vanishing Neveu-Schwarz tadpoles.

Dudas Mourad Timirgaziu

Strings on Euclidean orbifolds - untwisted states

- Well-known examples of orbifolds are the circle, R/Z, and the rotation orbifold R^2/Z_k.
- The spectrum of the quotient theory contains closed string states of the parent theory which are invariant under G: untwisted states.



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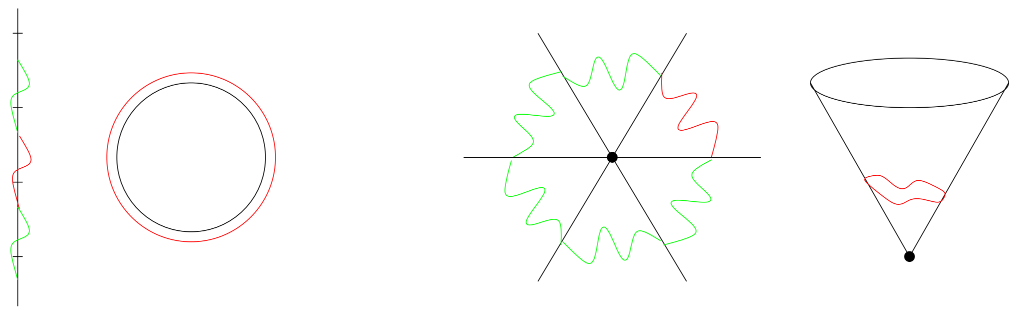
- Well-known examples of orbifolds are the circle, R/Z, and the rotation orbifold R^2/Z_k.
- Modular invariance requires that the spectrum should also include closed strings in the quotient theory which close up to the action of G in the parent theory: twisted states.



- When G acts non-freely, the twisted sector states are localized at the fixed points. They yield new localized degrees of freedom, which ensure the consistency of the background: anomaly free, divergence free...

Strings on Euclidean orbifolds - twisted sectors (cont.)

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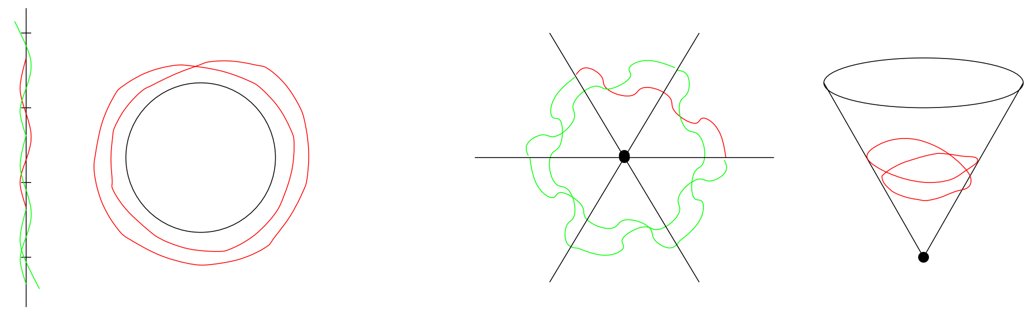
- Twisted sectors are labelled by conjugacy classes of G. Higher twisted sectors correspond to multiply wound states.



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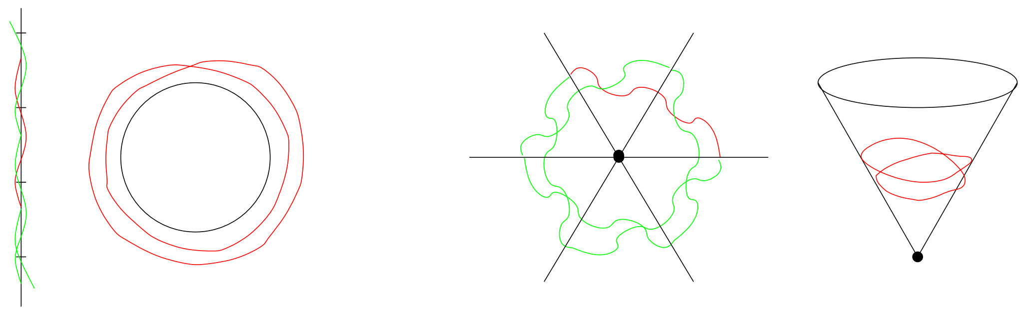

- Additionally, each twisted sector admits excited levels. The ground state can be thought as a Gaussian wave function centered at the origin.

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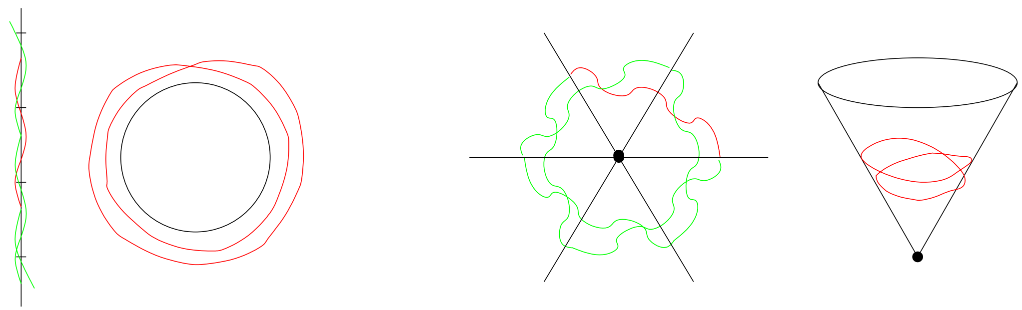

- Additionally, each twisted sector admits excited levels. The ground state can be thought as a Gaussian wave function centered at the origin.
- The condensation of these twisted states changes the vacuum, and effectively resolves the singularity: $R^2/Z_k \longrightarrow R^2/Z_{k-1} \longrightarrow \ldots$ (tachyon), $R^4/Z_k \longrightarrow$ multi-centered Eguchi-Hanson (massless mode).

Strings on Euclidean orbifolds - twisted sectors (cont.)

- Well-known examples of orbifolds are the circle, R/Z, and the rotation orbifold R^2/Z_k.
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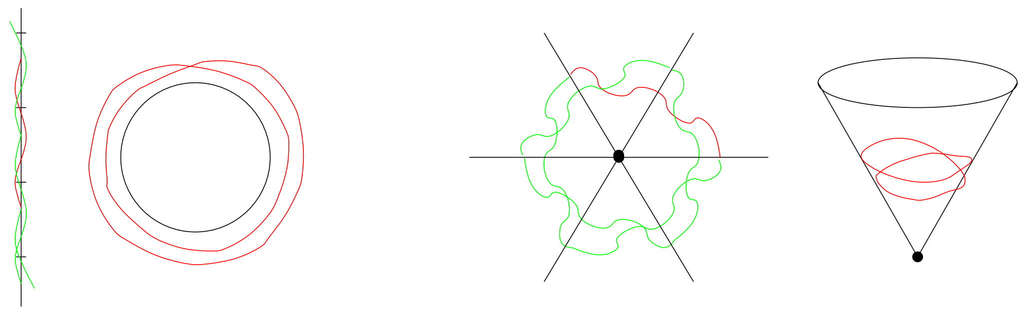

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- The Lorentzian orbifold share features with both examples: an infinite number of winding sectors, and a, non compact, fixed locus.

Closed strings in Misner space - untwisted states

- As usual in standard orbifolds, part of the spectrum involves closed strings on Minkowski covering space, which are invariant under the orbifold projection. In conformal gauge,

$$X^{\pm}(\sigma + 2\pi, \tau) = X^{\pm}(\sigma, \tau) \ , \quad (\partial_\tau^2 - \partial_\sigma^2) X^{\pm} = 0$$

satisfying the Virasoro (physical state) condition $(\dot{X}^\mu \pm X'^\mu)^2 = 0$.

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$$\sum_{n=-\infty}^{\infty}\partial X^+\bar{\partial}X^-\exp\left(ik^+X^-e^{-2\pi\beta n}+ik^-X^+e^{2\pi\beta n}+ik_ix^i\right)$$

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- Equivalently, after Poisson resummation over n, this is a superposition of states with integer boost momentum $j=x^+\partial_+-x^-\partial_-$,

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- The resulting eigenfunctions describe closed strings traveling around the Milne circle with integer momentum j.

Quantum fluctuations in field theory

- In the Minkowski vacuum (inherited from the covering space), the renormalized propagator can be obtained as a sum over images,

$$
\begin{aligned}
G(x;x') \quad = \quad & \sum_{n=-\infty,n\neq 0}^{\infty} \int_0^{\infty} d\tau \int dp^{\mu} \\
& \exp\left(-ip^-(x^+ - e^{2\pi\beta n}x^{+\prime}) - ip^+(x^- - e^{2\pi\beta n}x^{-\prime}) - ip^i(x^i - x^{i\prime})\right)
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- The one-loop stress-energy tensor follows from $G(x,x)$, e.g for a conformally coupled scalar,

$$\langle T_{ab}\rangle = \lim_{x\to x'}\left[(1-2\xi)\nabla_a\nabla'_b - 2\xi\nabla_a\nabla_b + (2\xi-\frac{1}{2})g_{ab}\nabla_c\nabla'^c\right] G(x,x')$$

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leading to a divergent quantum backreaction:

$$\langle T_\mu^\nu\rangle = \frac{K}{12\pi^2} T^{-4}\text{diag}(1,-3,1,1)\ , \quad K = \sum_{n=1}^{\infty} \frac{2+\cosh 2\pi n\beta}{[\cosh 2\pi n\beta - 1]^2}$$

Hiscock Konkowski 82

One-loop vacuum amplitude in field and string theory

- On the other hand, in string theory $\langle T_\mu^\nu \rangle (x)$ is an off-shell quantity, and only its integral over space-time is well defined:

$$\int dx^+ dx^- G(x,x) = \sum_{l=-\infty}^{+\infty} \int_0^\infty \frac{d\rho}{\rho^{D/2}} \frac{e^{-m^2 \rho}}{\sinh^2 (\pi \beta l)}$$

- This reproduces the zero-mode contribution to the string one-loop vacuum amplitude in the untwisted sector:

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- The local divergence in $\langle T_\mu^\nu \rangle(x)$ is integrable and yields a finite free energy.
- The existence of Regge trajectories with arbitrary high spin implies new (log) divergences in the bulk of the moduli space, not unlike long string poles in AdS_3.

Scattering of untwisted states

- Tree-level scattering amplitudes of untwisted sector states can be computed from those in flat space by the inheritance principle,

$$\langle V(j_1,k_1)\dots \quad V(j_n,k_n)\rangle_{Misner} = \int dv_1\dots dv_n\; e^{i(j_1 v_1+\dots+j_n v_n)}$$
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Berkooz Craps Rajesh Kutasov

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- It could be that eikonal resummation of ladder diagrams may lead to a finite result, e.g.

$$\mathcal{A} \sim -G\frac{s^2}{t} \quad \rightarrow \quad -G\frac{s^2}{t + (2\pi G s)^2} \qquad \text{(3D gravity)}$$

Deser McCarthy Steif; Cornalba Costa

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- In addition, there is an infinite set of twisted sectors, corresponding to strings on the covering space that close up to the action of the orbifold group:

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$$\begin{aligned} X_R^{\pm}(\tau - \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \pm i\nu)^{-1} \alpha_n^{\pm} \, e^{-i(n \pm i\nu)(\tau - \sigma)} \\ X_L^{\pm}(\tau + \sigma) &= \frac{i}{2} \sum_{n=-\infty}^{\infty} (n \mp i\nu)^{-1} \tilde{\alpha}_n^{\pm} \, e^{-i(n \mp i\nu)(\tau + \sigma)} \end{aligned}$$

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$$[\alpha_m^+, \alpha_n^-] = -(m + i\nu)\delta_{m+n} \quad , \quad [\tilde{\alpha}_m^+, \tilde{\alpha}_n^-] = -(m - i\nu)\delta_{m+n}$$
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- There are no translational zero-modes, instead two pairs of quasi zero-modes which are canonically conjugate real operators:

$$[\alpha_0^+,\alpha_0^-]=-i\nu\ ,\quad [\tilde{\alpha}_0^+,\tilde{\alpha}_0^-]=i\nu$$

Physical states (absence thereof)

- A natural way to quantize the system is to represent the oscillators on a Fock space with vacuum $|0\rangle$ annihilated by half of them, e.g.

$$\alpha^{\pm}_{n>0}\,, \quad \tilde{\alpha}^{\pm}_{n>0}\,, \qquad \alpha^{-}_{0}\,, \quad \tilde{\alpha}^{+}_{0}$$

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- This is the familiar result for the vacuum energy $\frac{1}{2}\theta(1-\theta)$ in the Euclidean rotation orbifold, after analytic continuation $\theta \longrightarrow i\nu$...
- Due to the $i\nu/2$ term in the ground state energy, all states obtained by acting on $|0\rangle$ by creation operators $\alpha^{\pm}_{n<0}$ and by α_0^+ will have imaginary energy, hence the physical state condition $L_0 = 0$ has no solutions.

Nekrasov

One-loop amplitude, twisted sector

- Independently of this fact, one may compute the one-loop path integral on an Euclidean worldsheet and Minkowskian target-space:

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- In the twisted sector, the left-moving zero-modes contribute

$$\frac{1}{2\sinh(\beta w\rho)]} = \sum_{n=1}^{\infty} q^{i(n+\frac{1}{2})\beta w}$$

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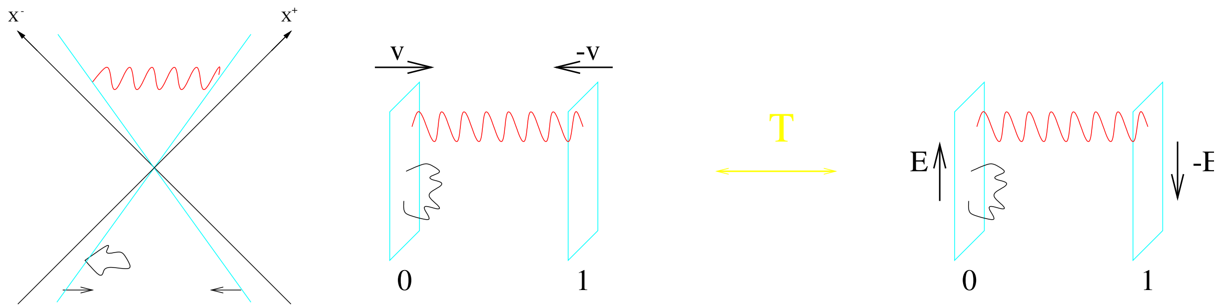
- The absence of physical twisted states crushes our hopes for resolving the singularity... yet does not sound very sensible. An important point: α_0^+ and α_0^- are not hermitian conjugate to each other, but rather self-hermitian...

A detour via Open strings in electric field

- A very similar puzzle is faced in the case of colliding D-branes, or in the T-dual process of charged open strings in a constant electric field:



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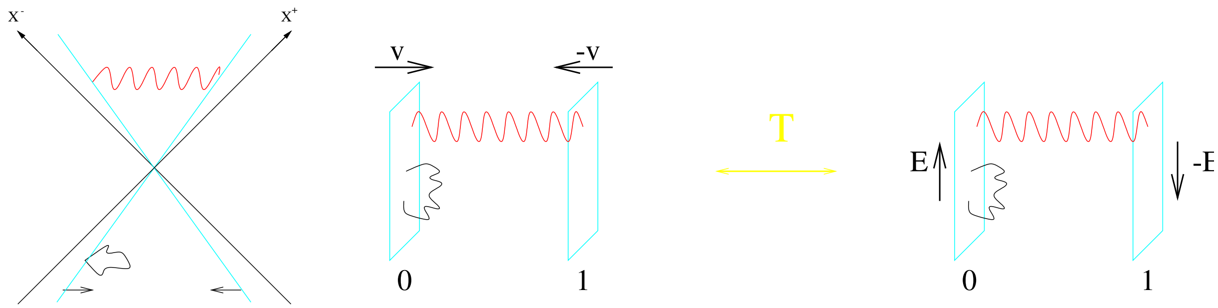

- Recall that for open strings stretched between two D-branes with electromagnetic fields F_0 and F_1, proper frequencies satisfy

$$e^{-2\pi i \omega_n} = \frac{1+F_0}{1-F_0} \cdot \frac{1-F_1}{1+F_1}$$

For $F_0 \neq F_1$, the open string carries a net electric charge, and the motion of its center of motion is that of a charged particle.

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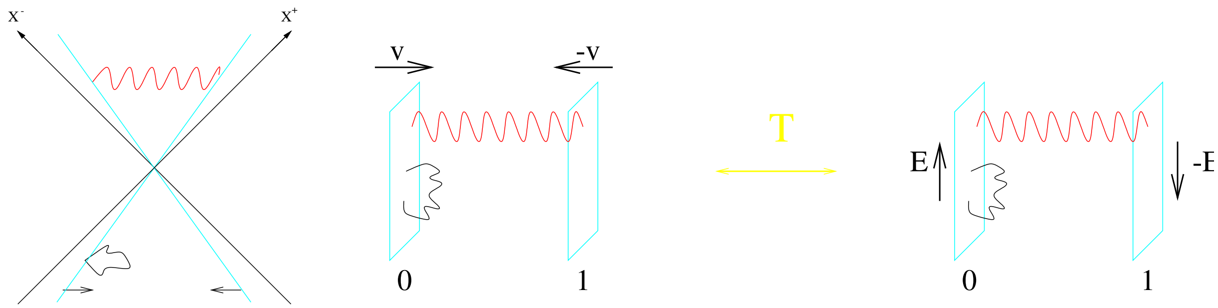

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For $F_0 \neq F_1$, the open string carries a net electric charge, and the motion of its center of motion is that of a charged particle.

- In the case of an electric field $F_1 = E dx^+ \wedge dx^-$, $F_1 = 0$, the resulting spectrum is

$$\omega_n = n + i\nu \ , \qquad \nu := \text{Arctanh} E = w\beta$$

just as in the Lorentzian orbifold case. The large winding limit amounts to a near critical electric field.

Open string mode expansion

- The light-cone embedding coordinates have the normal mode expansion

$$X^{\pm} = x_0^{\pm} + i \sum_{n=-\infty}^{+\infty} (-)^n (n \pm i\nu)^{-1} a_n^{\pm} e^{-i(n \pm i\nu)\tau} \cos[(n \pm i\nu)\sigma]$$

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- The world-sheet Hamiltonian, normal ordered with respect to the vacuum annihilated by $a_{n>0}^+, a_{n>0}^-$ and a_0^+, takes the form

$$L_0^{l.c.} = -\sum_{m=0}^{\infty} a_{-m}^+ a_m^- - \sum_{m=1}^{\infty} a_{-m}^- a_m^+ + \frac{i\nu}{2}(1 - i\nu) - \frac{1}{12}$$

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- By the same token, charged open strings should have no physical states... yet electrons and positrons do exist.

Charged particle and open string zero-modes

- Let us recall the quantization of a charged particle in an electric field:

$$L = \frac{1}{2}m\left(-2\partial_\tau X^+\partial_\tau X^- + (\partial_\tau X^i)^2\right) + \frac{e}{2}\left(X^+\partial_\tau X^- - X^-\partial_\tau X^+\right)$$

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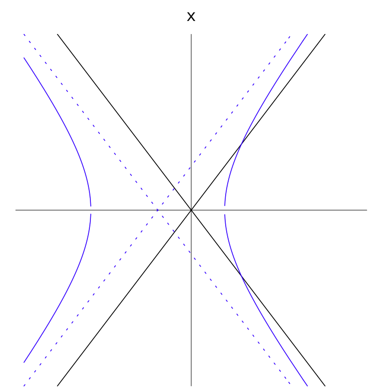
$\pm e x_0^\pm$ is the conserved linear momentum, and $a_0^\pm$ the velocity.



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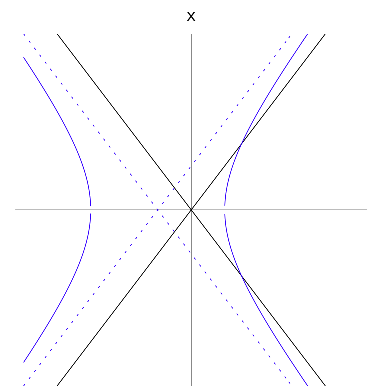

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- Starting from the canonical equal-time commutation rules

$$[\pi^+, x^-] = [\pi^-, x^+] = i\ , \quad [\pi^i, x^j] = i\delta_{ij}$$

one recovers the open string zero-mode commutation relations ($\nu = e$),

$$[a_0^+, a_0^-] = -i\nu\ , \quad [x_0^+, x_0^-] = -\frac{i}{\nu}$$

Charged particle and ppen string zero-modes

- Quantum mechanically, one may represent $\pi^{\pm} = i\partial/\partial x^{\mp}$ hence obtain $a_0^{\pm}, x_0^{\pm}$ as covariant derivatives

$$a_0^{\pm} = i\partial_{\mp} \pm \frac{\nu}{2} x^{\pm} \, , \quad x_0^{\pm} = \mp \frac{1}{\nu} \left(i\partial_{\mp} \mp \frac{\nu}{2} x^{\pm} \right)$$

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- The zero-mode piece of L_0, including the evil $\frac{i\nu}{2}$,

$$L_0^{(0)} = -a_0^+ a_0^- + \frac{i\nu}{2} = -\frac{1}{2}(\nabla_0^+\nabla_0^- + \nabla_0^-\nabla_0^+)$$

is just the Klein-Gordon operator of a particle of charge ν.

Klein-Gordon and the inverted harmonic oscillator

- Defining $\alpha_0^{\pm} = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator can be rewritten as an inverted harmonic oscillator:

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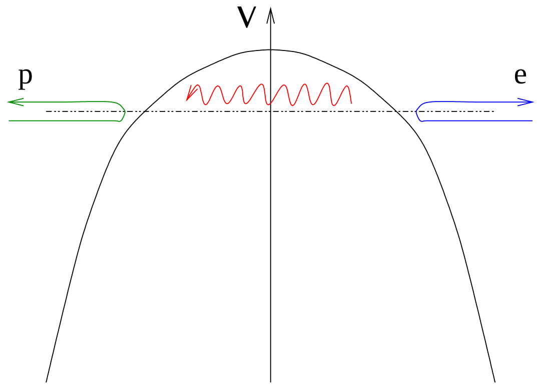
- The latter admits a respectable delta-normalizable spectrum of scattering states, in terms of parabolic cylinder functions, e.g:

$$\phi_{in}^+ = D_{-\frac{1}{2}+i\frac{M^2}{2\nu}}(e^{-\frac{3i\pi}{4}}u)e^{-i\tilde{p}t}e^{i\nu x t/2}$$



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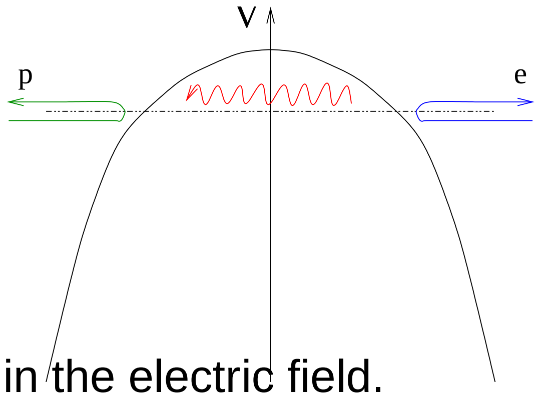

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- These correspond to non-compact trajectories of charged particles in the electric field. Tunnelling is just (stimulated) Schwinger pair creation,

$$e^- \longrightarrow (1+\eta)\, e^- + \eta\, e^+ \,, \quad \eta \sim e^{-\pi M^2/\nu}$$

Brezin Itzykson; Brout Massar Parentani Spindel

Lorentzian vs Euclidean states

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$$\frac{1}{2i\sin(\nu t/2)} = \sum_{n=1}^{\infty} e^{-i(n+\frac{1}{2})\nu t} = \int dM^2 \rho(M^2) e^{-M^2 t/2}$$

The density of states is obtained from the reflection phase shift,

$$\rho(M^2) = \frac{1}{\nu}\log\Lambda - \frac{1}{2\pi i}\frac{d}{dM^2}\log\frac{\Gamma\left(\frac{1}{2}+i\frac{M^2}{2\nu}\right)}{\Gamma\left(\frac{1}{2}-i\frac{M^2}{2\nu}\right)}$$

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- The physical spectrum can be explicitely worked out at low levels, and is free of ghosts: a tachyon at level 0, a transverse gauge boson at level 1, . . .

Charged particle in Rindler space

- For applications to the Milne universe, one should diagonalize the boost momentum J, ie consider an accelerated observer.

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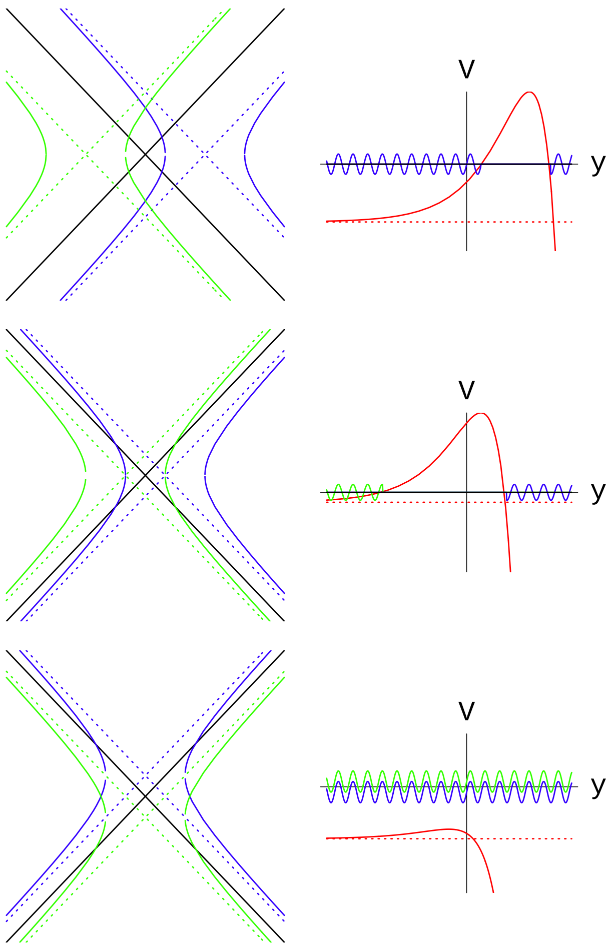

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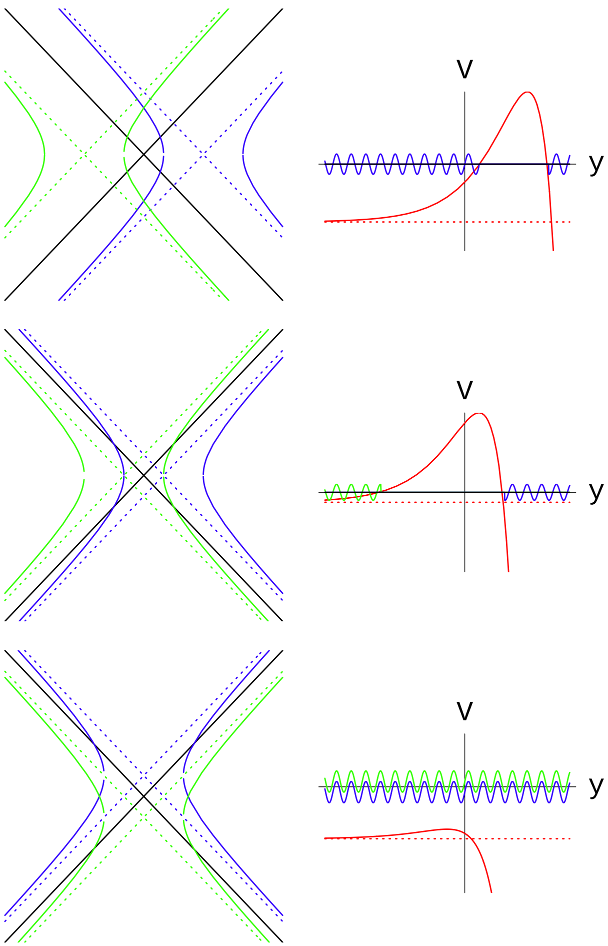

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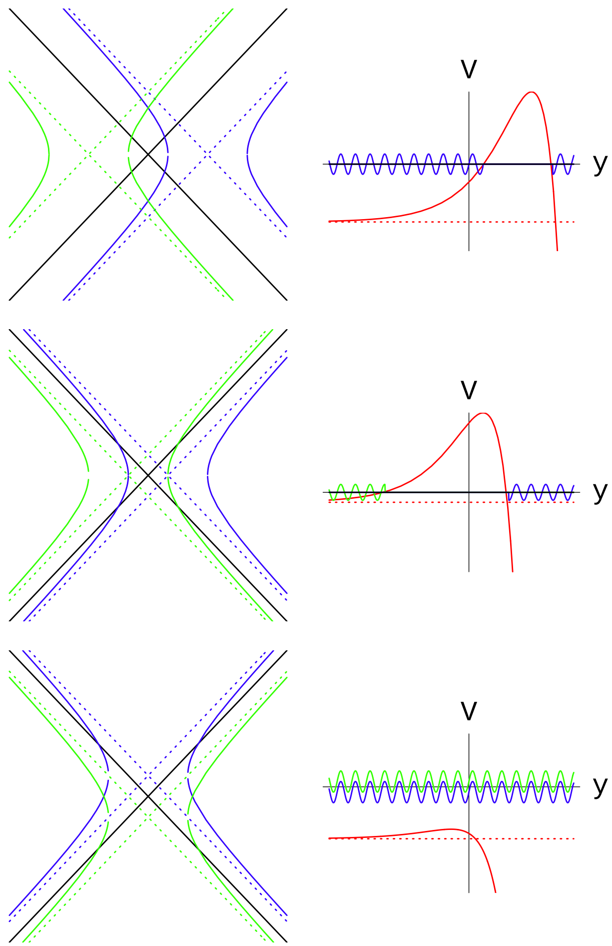

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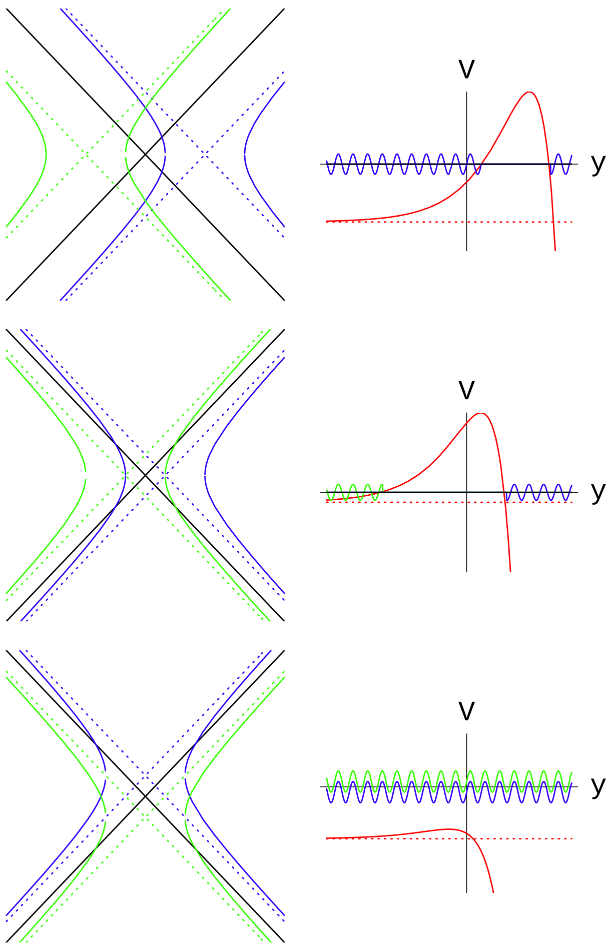

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- If $j > M^2/(2\nu)$, the electron branches extend in the Milne regions. There is no tunneling, but partial reflection amounts to a combination of Schwinger and Hawking emission.

Rindler modes

- Incoming modes from Rindler infinity I_R^- read, in terms of parabolic cylinder functions:

$$\mathcal{V}^j_{in,R} = e^{-ij\eta} r^{-1} M_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),-\frac{ij}{2}}(i\nu r^2/2)$$

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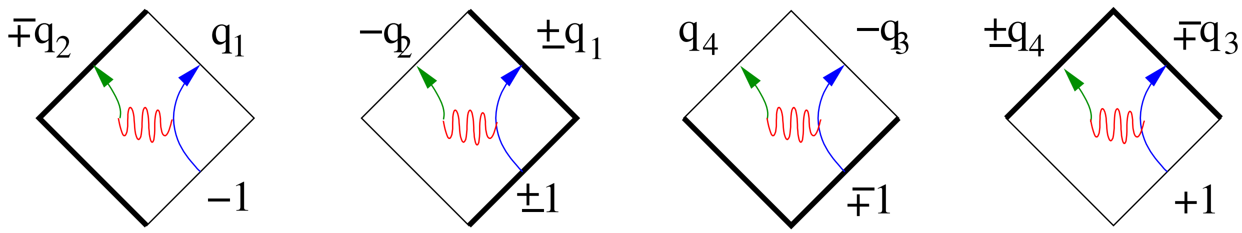

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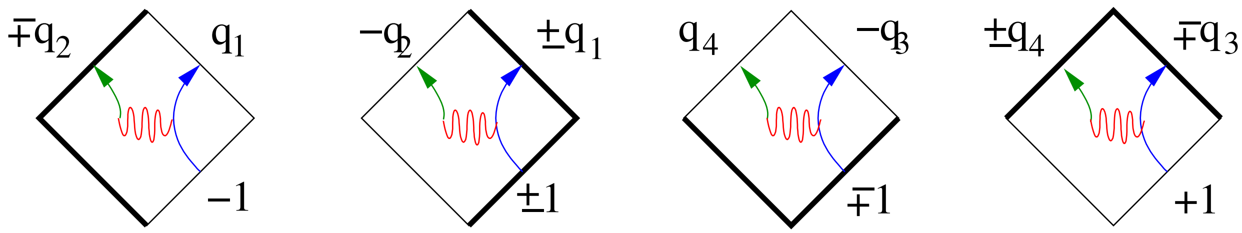

- The reflection coefficients can be computed:

$$q_1 = e^{-\pi j} \frac{\cosh\left[\pi \frac{M^2}{2\nu}\right]}{\cosh\left[\pi\left(j - \frac{M^2}{2\nu}\right)\right]}, \quad q_3 = e^{\pi\left(j-\frac{M^2}{2\nu}\right)} \frac{\cosh\left[\pi \frac{M^2}{2\nu}\right]}{|\sinh \pi j|}$$

and $q_2 = 1 - q_1$, $q_4 = q_3 - 1$, by charge conservation.

Global Charged Unruh Modes

- Global modes may be defined by patching together Rindler modes, ie by analytic continuation across the horizons. Unruh modes are those which are superposition of positive energy Minkowski modes,

$$\begin{aligned}\Omega^j_{in,+} &= \mathcal{V}^j_{in,P} = (-i\nu X^+X^-)[X^+/X^-]^{-ij/2} W_{-i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}} \\ \omega^j_{in,-} &= \mathcal{U}^j_{in,P} = (i\nu X^+X^-)[X^+/X^-]^{-ij/2} M_{i(\frac{j}{2}-\frac{m^2}{2\nu}),\frac{ij}{2}}\end{aligned}$$

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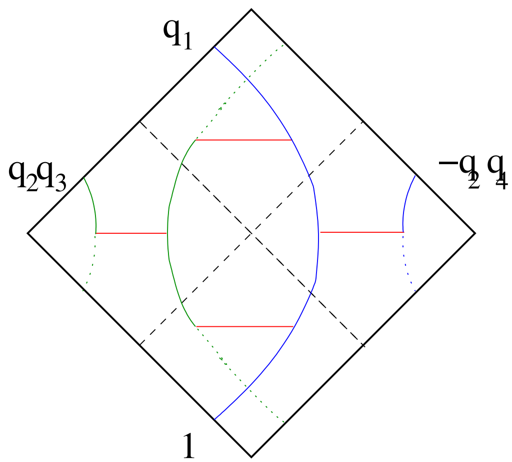
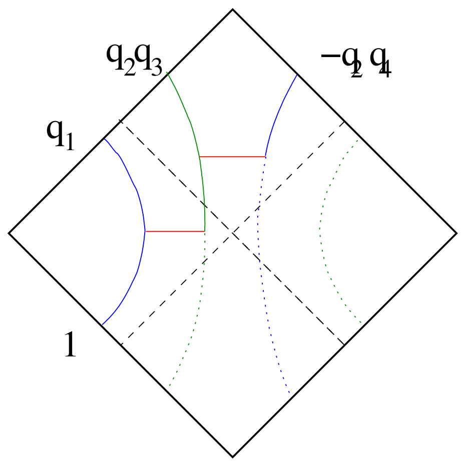
- There are two types of Unruh modes, involving 2 or 4 tunelling events:





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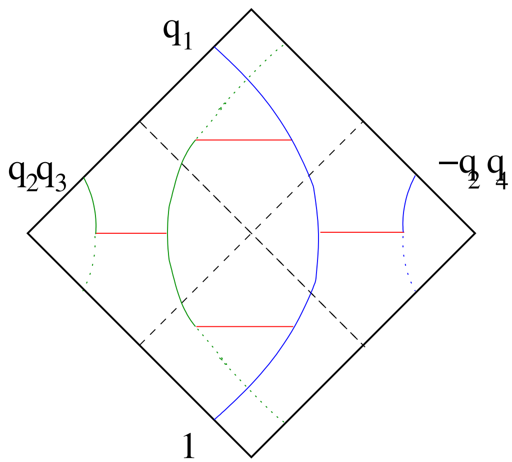

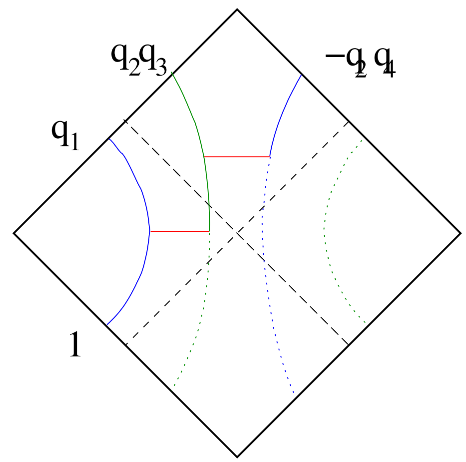

- Any state in Minkowski space can be represented as a state in the tensor product of the Hilbert spaces of the left and right Rindler patches. In contrast to neutral fields in Rindler space, Boulware-Fulling modes that vanish in L or R have positive Minkowski energy.

Closed string zero-modes

- Let us reanalyze the classical solutions for the closed string zero modes

$$X^{\pm}(\tau,\sigma) = e^{\mp\nu\sigma}\left[\pm\frac{1}{2\nu}\alpha_0^{\pm}e^{\pm\nu\tau} \mp \frac{1}{2\nu}\tilde{\alpha}_0^{\pm}e^{\mp\nu\tau}\right] \ , \quad \alpha_0^{\pm}, \tilde{\alpha}_0^{\pm} \in R$$

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- The Milne time, or Rindler radius, is independent of σ:

$$4\nu^2 X^+ X^- = \alpha_0^+\tilde{\alpha}_0^- e^{2\nu\tau} + \alpha_0^-\tilde{\alpha}_0^+ e^{-2\nu\tau} - \alpha_0^+\alpha_0^- - \tilde{\alpha}_0^+\tilde{\alpha}_0^-$$

We may thus follow the motion of a single point $\sigma = \sigma_0$ and obtain the rest of the worldsheet by smearing under the action of the boost.

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$$X^{\pm}(\tau,\sigma) = e^{\mp\nu\sigma}\left[\pm\frac{1}{2\nu}\alpha_0^{\pm}e^{\pm\nu\tau} \mp \frac{1}{2\nu}\tilde{\alpha}_0^{\pm}e^{\mp\nu\tau}\right] , \quad \alpha_0^{\pm}, \tilde{\alpha}_0^{\pm} \in R$$

- The Milne time, or Rindler radius, is independent of σ:

$$4\nu^2 X^+ X^- = \alpha_0^+\tilde{\alpha}_0^- e^{2\nu\tau} + \alpha_0^-\tilde{\alpha}_0^+ e^{-2\nu\tau} - \alpha_0^+\alpha_0^- - \tilde{\alpha}_0^+\tilde{\alpha}_0^-$$

  We may thus follow the motion of a single point $\sigma = \sigma_0$ and obtain the rest of the worldsheet by smearing under the action of the boost.

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- The behavior at early/late proper time now depends on $\epsilon\tilde{\epsilon}$: For $\epsilon\tilde{\epsilon} = 1$, the string begin/ends in the Milne regions. For $\epsilon\tilde{\epsilon} = -1$, the string begin/ends in the Rindler regions.

Short and long strings ($j = 0$)

Choosing $j = 0$ for simplicity, we have two very different types of solutions:

- $\epsilon = 1$, $\tilde{\epsilon} = 1$:

$$X^{\pm}(\sigma, \tau) = \frac{M}{\nu\sqrt{2}} \sinh(\nu\tau) e^{\pm\nu\sigma} \,, \quad T = \frac{M}{\nu} \sinh(\nu\tau) \,, \quad \theta = \nu\sigma$$

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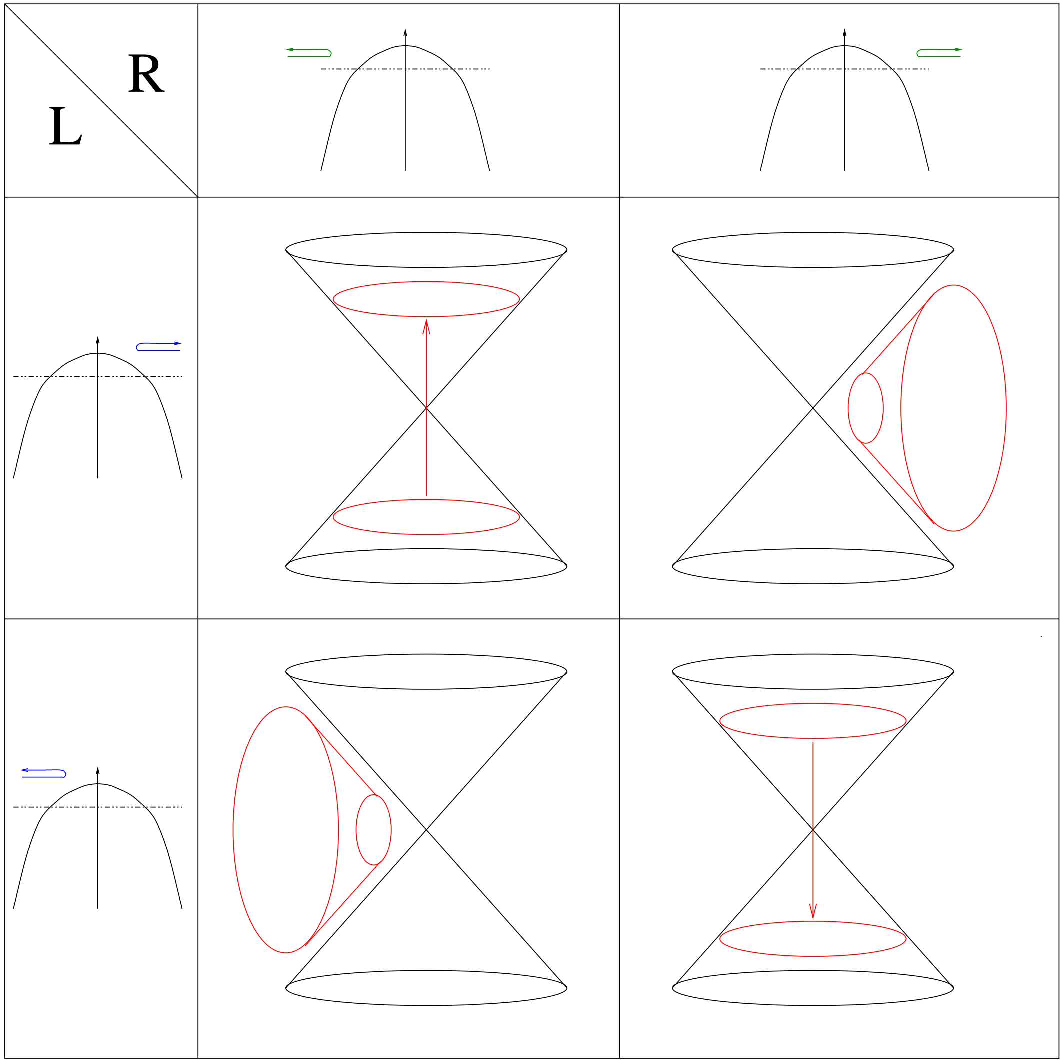
$\epsilon = -1$, $\tilde{\epsilon} = 1$ is the analogue in the left Rindler patch.

Short and long strings

Closed string trajectories are thus generated by the motion of two decoupled particles in inverted harmonic oscillators:



Relation to open string modes

- Instead of following the motion of a point at fixed σ, one may consider instead a point at fixed $\sigma + \tau$: this is precisely the trajectory of the open string zero-mode.
- Using the covariant derivative representation

$$\alpha_0^\pm = i\partial_\mp \pm \frac{\nu}{2} x^\pm \,, \quad \tilde{\alpha}_0^\pm = i\partial_\mp \mp \frac{\nu}{2} x^\pm$$

we observe that $x^\pm$ is the Heisenberg operator corresponding to the location of the closed string (at $\sigma = 0$):

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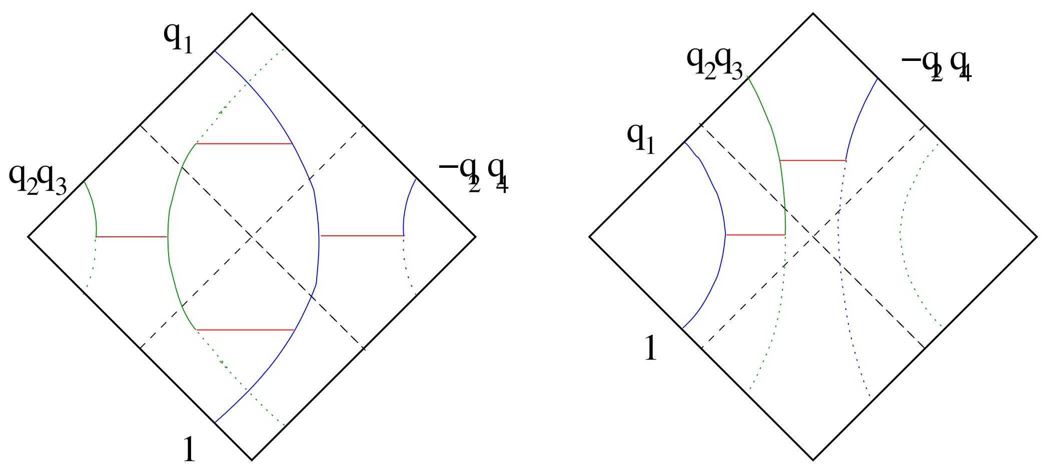

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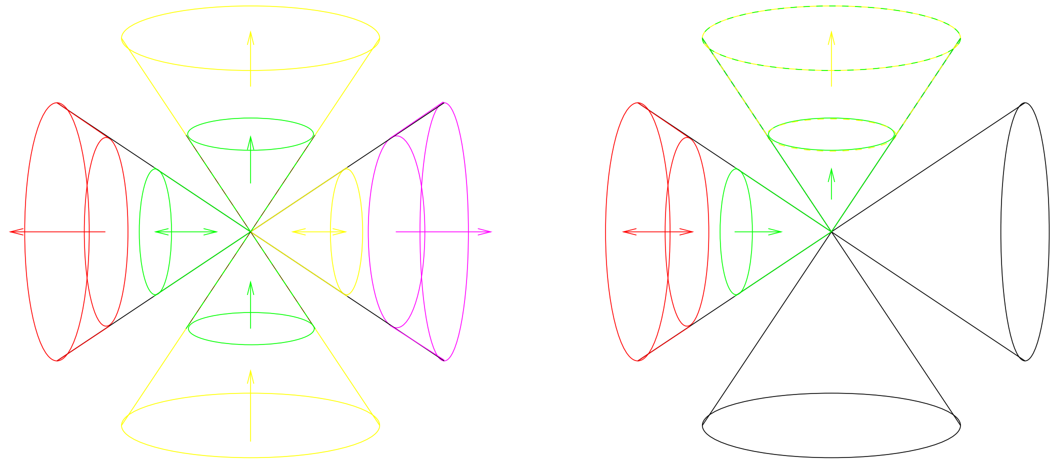
- The open string global wave functions are also the closed string wave functions...



Comments on winding string production

- The production rate of winding strings can be evaluated by WKB methods: for the Misner Universe,

$$R \sim \exp\left(-\pi \frac{M^2}{\beta w}\right) \quad \to 1 \text{ as } w \to \infty$$

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$$p_r^2 + M^2 - \left[wb(r) - \frac{j}{b(r)}\right]^2 = 0$$

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- One may instead deform the profile $a(T) \to \sqrt{\beta^2 T^2 + \epsilon^2}$, such that $a(T)$ does not vanish anymore. The twisted wave equation is well defined without continuation to Rindler space. What happens to the Rindler patches ?

Effective gravity analysis

- Consider a general Kasner ansatz

$$ds^2 = -dt^2 + \sum_{i=1}^{D} a_i^2(t) dx_i^2 \, , \qquad T_{\mu\nu} = \text{diag}(\rho, a_i^2 p_i \delta_{ij})$$

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- A bounce in dimension i requires $H_i' > 0$ when $H_i = 0$, hence

$$(D-2)p_i + \rho \geq \sum_{j \neq i} p_j$$

- In the presence of strings wrapped around dimension i,

$$\rho = \frac{T}{V} \,, \quad p_i = -\rho \,, \quad p_{j \neq i} = 0 \,, \quad V = \prod_{j \neq i} a_j \quad \Rightarrow D \leq 3$$

Effective gravity analysis (cont.)

- Modelling the dilaton as the radius of the $\sharp$ th direction, the strings become membranes wrapped around $(i, \sharp)$:

$$\rho = \frac{T}{V}\,, \quad p_i = p_\sharp = -\rho\,, \quad p_{j\neq i} = 0\,, \quad V = \prod_{j\neq(i,\sharp)} a_j$$

The bounce now takes place when $D \leq 4$. At the same time, the dilaton runs to infinity.

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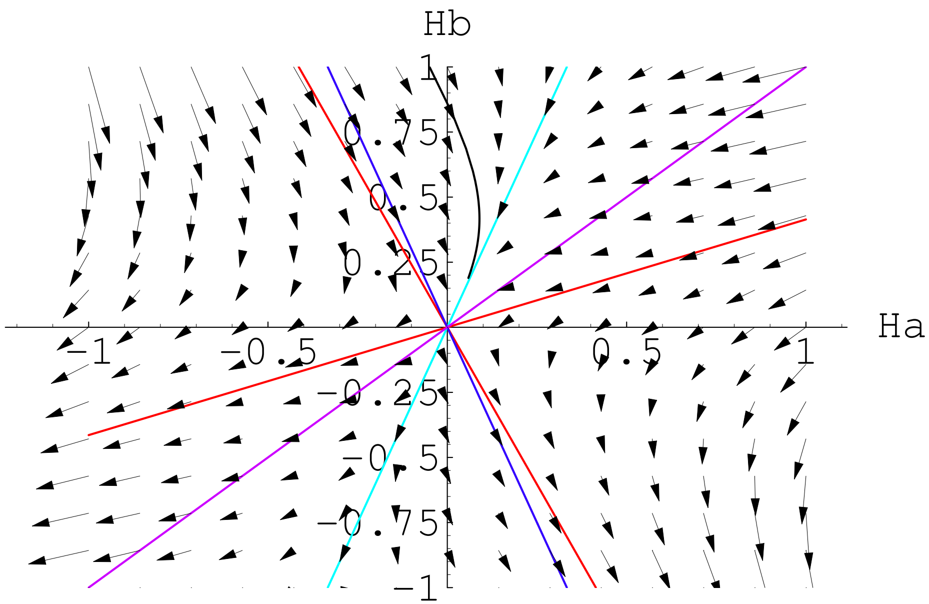

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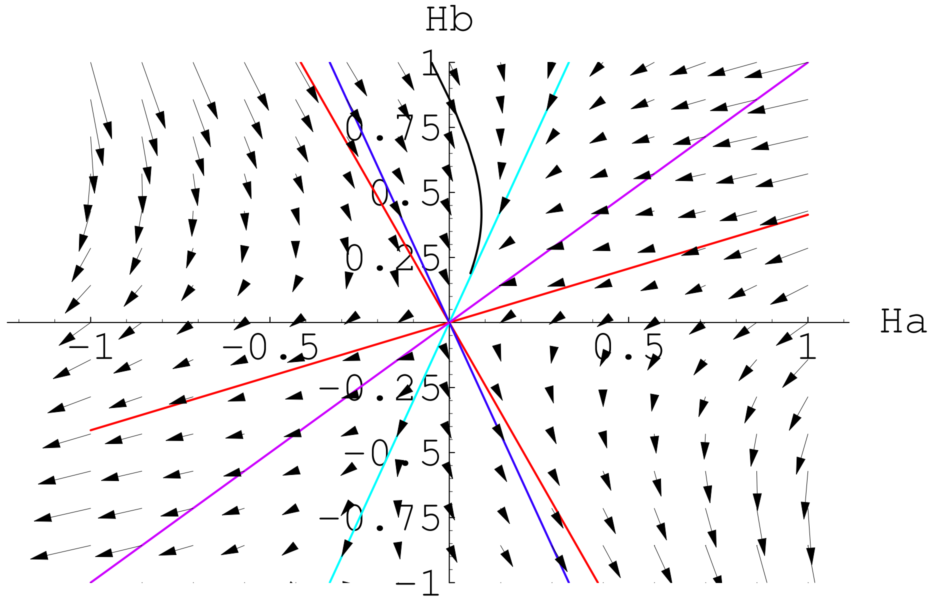

- This discussion assumed a constant number of wound strings: one should incorporate the dependence on the contraction rate.

Quantization in the Rindler patch

- For long strings in conformal gauge, the worldsheet time τ is in fact a spacelike coordinate wrt to the induced metric. For short strings, the induced metric undergoes a signature flip as it wanders in the Rindler patch.
- If so we should quantize the string with respect to the "time" coordinate σ rather than τ. The canonical generator of time translations

$$E = -\int_{-\infty}^{\infty} d\tau \left(X^+ \partial_\sigma X^- - X^- \partial_\sigma X^+ \right) = \int_{-\infty}^{\infty} d\tau \, r^2 \partial_\sigma \eta$$

is infinite: long strings carry an infinite Rindler energy.

- Introducing a cut-off $-T \leq \tau < T$, the Rindler energy

$$E_T \sim -\frac{e^{2\nu T}}{4\nu^2} \left(\tilde{\alpha}_0^+ \alpha_0^- + \tilde{\alpha}_0^- \alpha_0^+ \right)$$

can be understood as the tensive energy of the static stretched string.

- The Rindler energy spectrum is unbounded both above and below:

$$E_{short} < -e^{2\nu T} \frac{M \tilde{M}}{4\nu^2} < e^{2\nu T} \frac{M \tilde{M}}{4\nu^2} < E_{long}$$

How can one prevent the decay into short strings ?

Conclusions - speculations

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- In particular, since winding strings are spontaneously produced near the singularity, they contribute an energy proportional to the radius, hence akin to a two-dimensional positive cosmological constant: it seems plausible that the resulting transcient inflation may smooth out the singularity.
- If one manages to make sense of the winding production rate, and if the singularity gets resolved, what happens to the whiskers ? Can they provide some time-independent dual description of the cosmological evolution ?

Conclusions - speculations (cont.)

- To demonstrate that the singularity is resolved, one should in principle take into account the production of (an infinite number) of twisted sector states in correlated pairs, i.e. squeezed states: non-local deformations of the worldsheet ? closed string field theory ?

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- More generally, we still lack a framework to compute the production of closed strings in cosmological backgrounds. Those however are likely to lead to large departures from FRW cosmology, and possibly spectacular effects: cosmological bounce, Hagedorn phase transition...

  Lawrence Martinec, Gubser

Appendices (not shown during talk)

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$$H^{(1)}_{-ij}(mT)e^{-ij\theta} \sim e^{-ij\theta - imT}/\sqrt{T}$$

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Closed string one-loop vacuum amplitude

- Independently of this fact, one may compute the one-loop (Euclidean ws, Minkowskian target) free energy using path integral methods:

$$A_{bos} = \int_{\mathcal{F}} \sum_{l,w=-\infty}^{\infty} \frac{d\rho d\bar{\rho}}{(2\pi^2\rho_2)^{13}} \frac{e^{-2\pi\beta^2 w^2 \rho_2}}{|\eta^{21}(\rho)x\ \theta_1(i\beta(l+w\rho);\rho)|^2}$$

where θ_1 is the Jacobi theta function,

$$\theta_1(v;\rho) = 2q^{1/8}\sin\pi v \prod_{n=1}^{\infty}(1-e^{2\pi i v}q^n)(1-q^n)(1-e^{-2\pi i v}q^n)\ , \quad q = e^{2\pi i \rho}$$

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- In the untwisted sector, this reproduces the integrated vacuum free energy found by the method of images:

$$\int dx^+ dx^- G(x,x) = \sum_{l=-\infty}^{+\infty} \int_0^{\infty} \frac{d\rho}{\rho^{D/2}} \frac{e^{-m^2\rho}}{\sinh^2(\pi\beta l)}$$

One-loop amplitude and Schwinger pair production

- Using this quantization scheme, the one-loop (Euclidean worldsheet, Minkowskian target) vacuum free energy reads

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- This seems to support the quantization scheme based on a vacuum, hence the absence of physical states. But physical states do exist classically, how could quantization make them disappear altogether ?

Wick rotation to a rotation orbifold

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- Rotating $\beta = i\mu$, the Rindler region becomes get indeed an Euclidean metric,

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$$Z = X^+ = re^{i\mu\eta}\,, \quad \bar{Z} = -X^- = re^{-i\mu\eta} \qquad (r > 0)$$

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- By the same token, the left Rindler wedge rotates to another copy of the Euclidean plane with the origin removed: the complete analytic continuation of Misner space is therefore

$$\widetilde{R^2\backslash\{0\}}_L / e^{i\mu} \backslash \widetilde{R^2\backslash\{0\}}_R$$

and states of interest are non-normalizable !

D'Appolonio Kiritsis

The one-loop amplitude again

- Recall the (Euclidean ws, Minkowskian target) one-loop amplitude:

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$$i\beta(l+w\rho) = m + n\rho\ , \quad (l,w,m,n) \in Z$$

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- In contrast to the open string case, these poles do not yield an imaginary part: the overall cosmological particle production seems to vanish. *This is not to say that there is no particle production at intermediate stages !*

Physical spectrum at low level

- The ground state tachyon

$$|T\rangle = \phi(x^+, x^-)|0_{ex}, k\rangle$$

should satisfy the Virasoro constraint

$$L_0|T\rangle = \left[-\frac{1}{2}\left(a_0^+ a_0^- + a_0^- a_0^+\right) + \frac{1}{2}\nu^2 - 1 + \frac{1}{2}k_i^2\right]|T\rangle$$

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- The L_1 Virasoro constraint eliminates one polarization. Despite the non-vanishing two-dimensional mass $k_i^2 - \nu^2$, the spurious state $L_{-1}\phi|0\rangle$ is still physical, eliminating an extra polarization.
- One thus has $D-2$ transverse degrees of freedom, ie a massless gauge boson in D dimensions.

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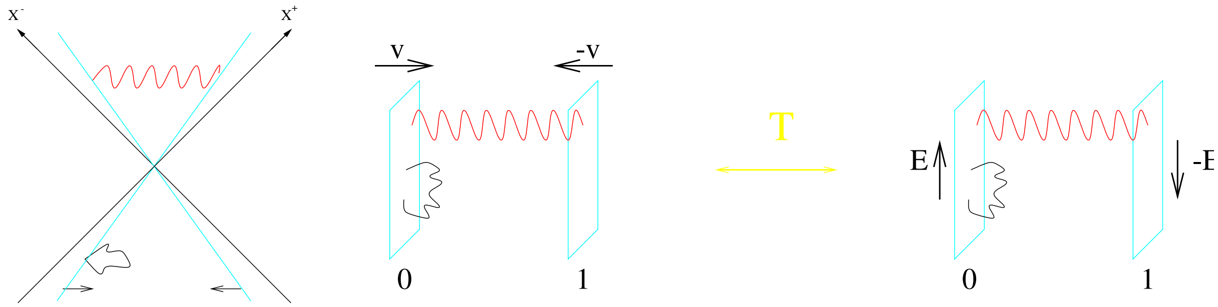

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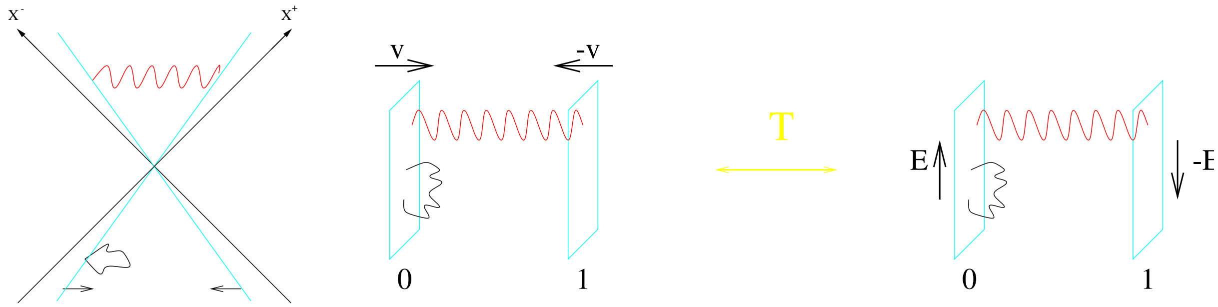

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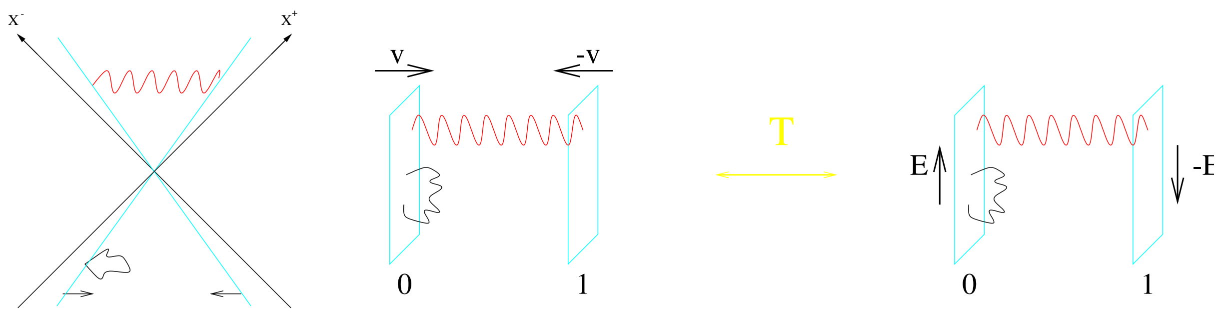

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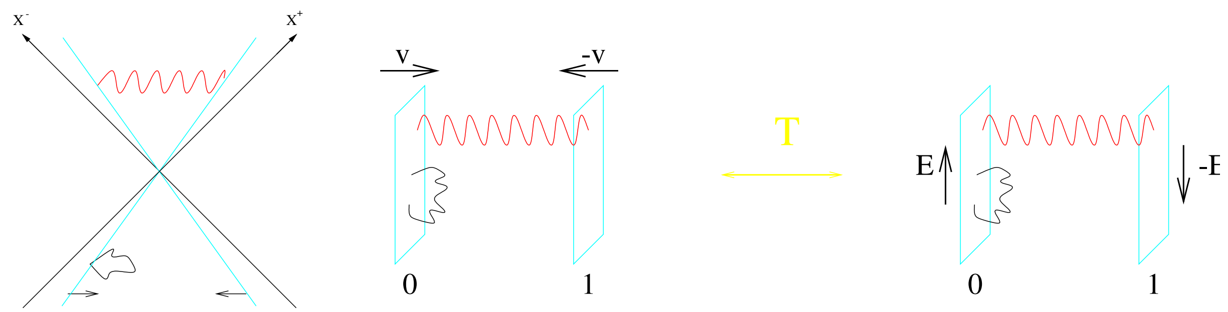

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- Can Schwinger production of twisted closed strings resolve the cosmological singularity of the Lorentzian orbifold ?

The Grant space

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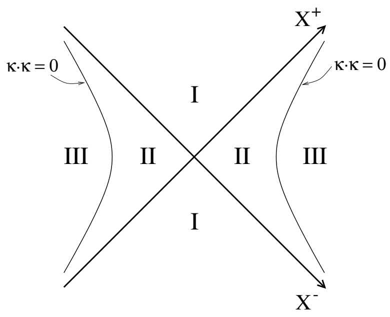
- Defining $Z^{\pm} = X^{\pm} e^{\mp \beta X/R}$, the metric can be written in the Kaluza-Klein form

$$ds^2 = R^2 (dX + A)^2 - 2dZ^+ dZ^- - \frac{E^2}{2R^2}(Z^+ dZ^- - Z^- dZ^+)^2\,, \quad X \equiv X + 2\pi$$

with radius R and KK electric field

$$R^2 = 1 + 2EZ^+ Z^-\,, \quad dA = \frac{E}{R^4} dZ^+ dZ^-\,, \quad E = \beta/R$$



Cornalba Costa

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- All CTC have to pass into $X^+X^- < -1/(2E)$, hence may be suppressed by excising this region: *orientifold boundary conditions ?*

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