

DYNAMICS

TIME EVOLUTION OF A SYST COUPLED TO
A BATH

LANGEN DYNAMICS	(CALC.)
MONTÉ CARLO	(SIM.)

CALCULATIONS

$$m\ddot{x} + \gamma\dot{x} = -V'(x) + \xi$$

ASSUMPTION WHITE NOISE

$$\langle \xi(t) \rangle = 0$$

$$\langle \xi(t) \xi(t') \rangle = 2\gamma k_B T \delta(t-t')$$

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BATH IN EQUIL.

NB. LANGEVIN EQ. CAN BE DERIVED FROM  
SYST + BATH OSC  
BUT FREQ. OF OSC & TYPE OF COUPLING

DECIDE FORM OF LANGEVIN EQ.

OVERDAMPED LIMIT DROP  $m\ddot{x}$

$$\gamma \dot{x} = -V'(x) + \xi$$

REDEFINE TIME  $\Rightarrow$  ELIMINATE  $\gamma$

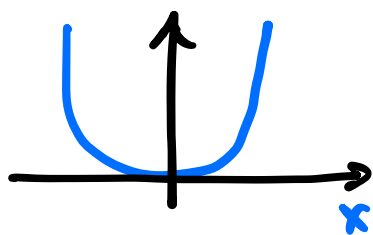
$$\dot{x} = -V'(x) + \xi$$

$$\langle \xi(t) \xi(t') \rangle = 2k_B T \delta(t-t')$$

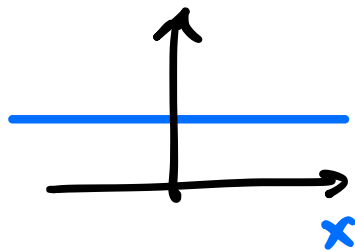
USUAL FORM FOR A SIMPLE VARIABLE  
EXTENSION TO VECTORS, FIELDS, ETC.

$V(x)$  POTENTIAL ENERGY

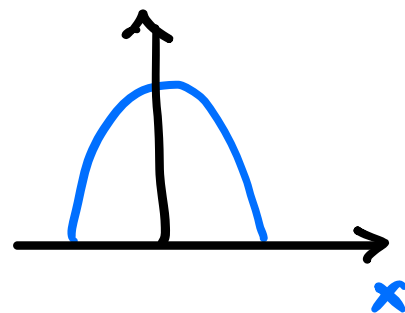
### THREE EXAMPLES



(a)



(b)



(c)

CONFINING



DECAY TO  
MIN

FLAT

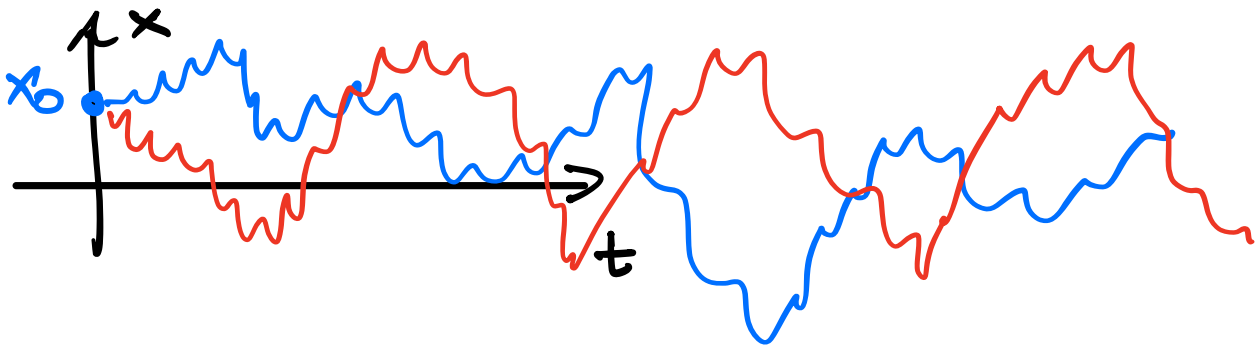
DIFF.

UNSTABLE

ESCAPE TO  $\infty$

## NOISY MOTION

IN CASE (a)



$$\langle X(t) \rangle \xrightarrow[t \rightarrow \infty]{} 0$$

EX. IN (b) & (c) TOO

PLAY A ROLE IN UNDERSTANDING THE DYN  
OF COMPLEX SYST AS THE EVOL. OF A  
REPRESENT. POINT IN THE COMPLEX  
FREE-ENERGY LANDSCAPE

BUT HIGH DIM SO VERY  
UNUSUAL FEATURES!

REAL SPACE

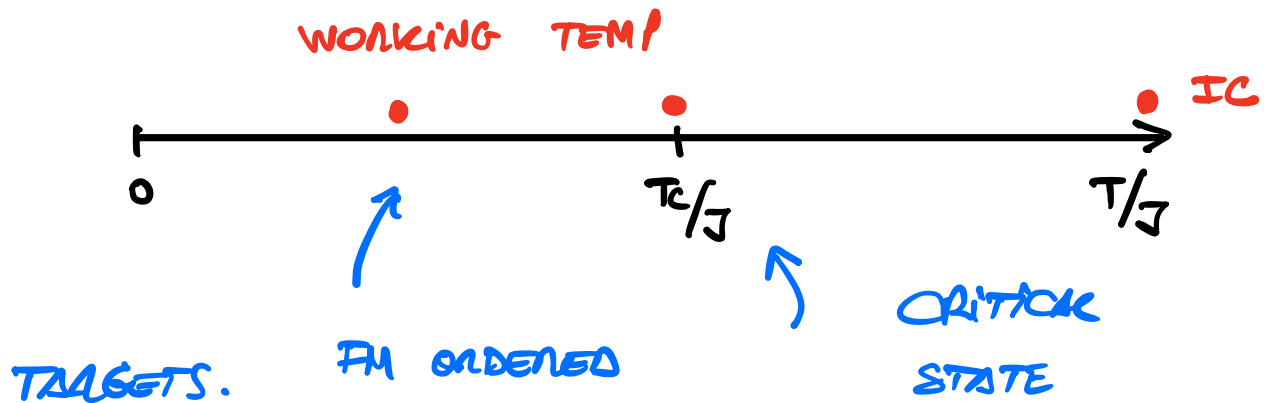
QUENCHING A SYST

DOMAIN GROWTH - 2d IM

$$\{S_i(t) = \pm 1\} \quad p = 1/2$$

EVOLVE WITH GLAUBER / MC  $T < T_c$

INITIAL COND UNST



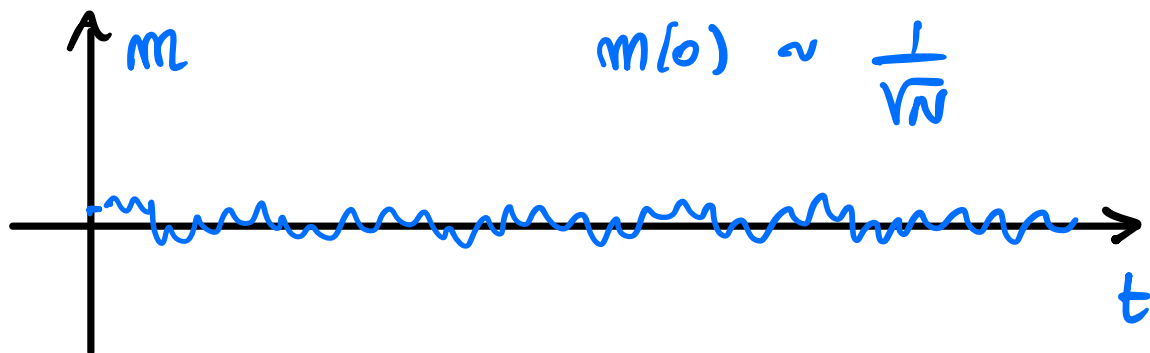
SIMPLEST OBSERVABLE

GLOBAL MAGN

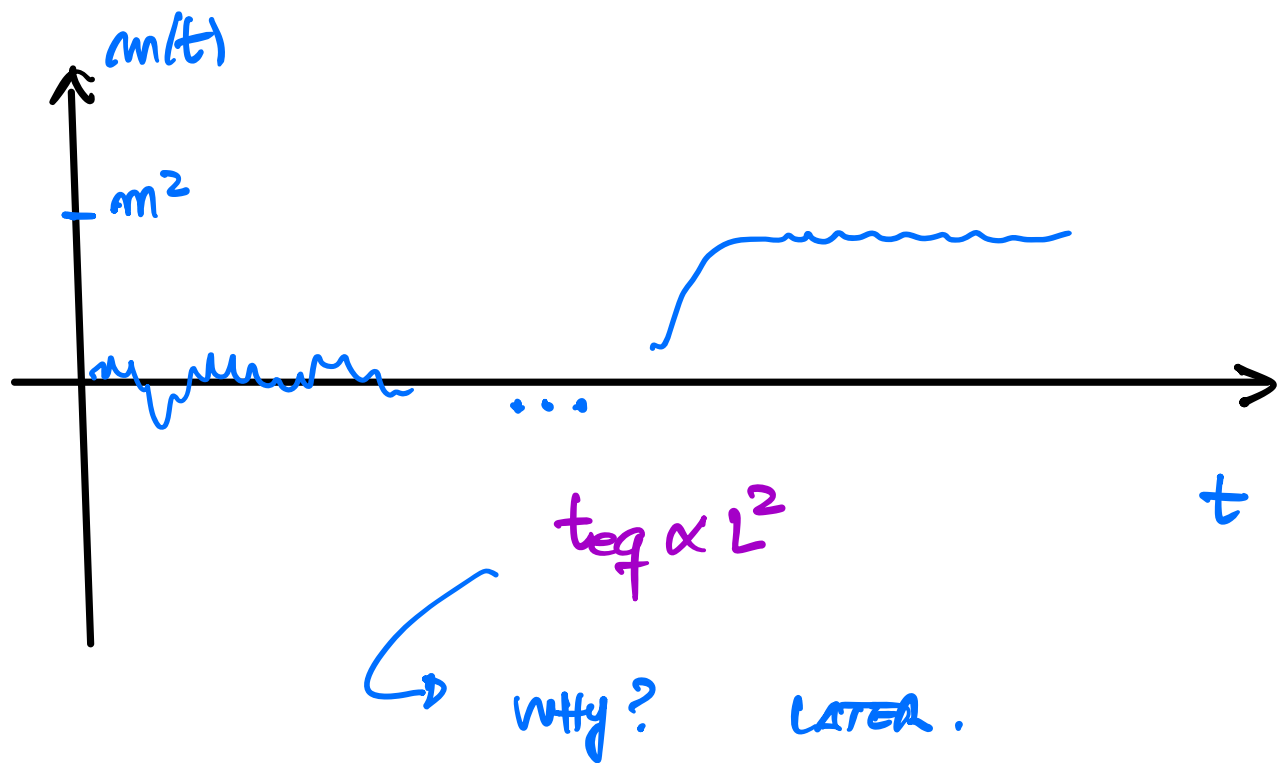
$$m(t) = \frac{1}{N} \sum_i s_i(t)$$

DEPENDS ON NOISE HISTORY

BUT NOT MUCH IF  $N \gg 1$



$m(t)$  TOO AS SUM OF  
ALMOST INDEP. VALUES



STONT. SYMM BREAKING

CHOOSE POSIT MAGN CASE  
w/ PROB  $1/2$

IF AVERAGE OVER NOISES  $\langle m(t) \rangle = 0 \quad \forall t$

WHAT HAPPENS DURING PRE-REGIME?

$$t < t_{eq} \quad ?$$

COARSENING .

PHASE ORDERING KINETICS

DOMAIN- GROWTH .

$$T < T_c$$

$$T = T_c$$

DOMAINS FORM

CRITICAL REGIONS FORM

$\Sigma$  THEY GROW IN TIME

TRANSIENT  $t_s \rightarrow t \gg t_s$

DYN SCALING REGIME

TYPICAL LENGTH  $R(t) \sim \lambda(t) t^{1/2}$

DYNAMIC GROWTH EXP  $\begin{matrix} \mathbb{Z} & T < T_c \\ \mathbb{Z}_c & T = T_c \end{matrix}$

QUENCH BELOW  $T_c$

LET SYST EVOLVE UP TO  $t_w$

LET IT EVOLVE FURTHER  $t > t_w$

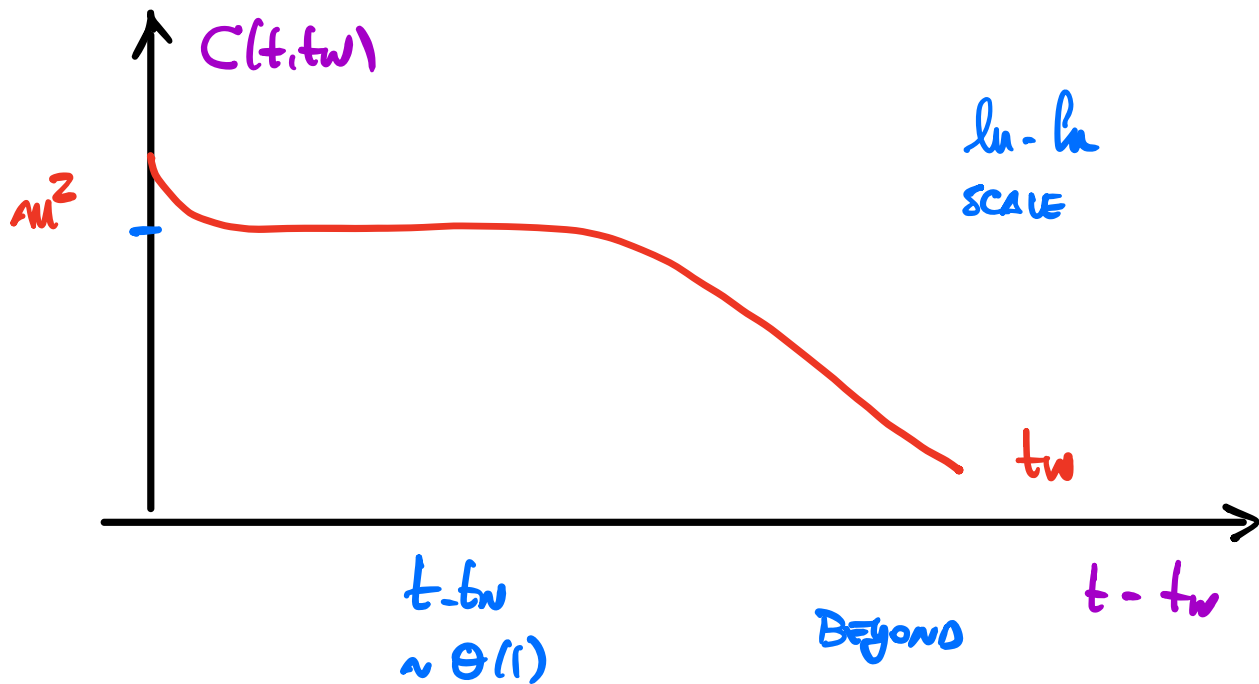
COMPARE  $t - t_w$  TO  $t_w$

$$\frac{t - t_w}{t_w} < 1$$

$$\frac{t - t_w}{t_w} > 1$$

ONLY THERMAL  
FUCT WITHIN  
DOMAINS

NOTION OF  
DOMAIN WALLS  
OPERATIONAL



DECOR WITHIN  
DOMAINS  
AS IN EQU'L.

DECOR DUE TO  
MOTION OF  
WALLS.  
TWO PICT LOOK  
VERY DIFF.

SHOW SKETCH AS A  
CHECKER BOARD

STAT RELAX

NON-STAT RELAX  
AGING

$$C_{st}(t, t_w) = f_{st}(t - t_w)$$

$$C_{ag}(t, t_w) = f_{ag}\left(\frac{R(t)}{R(t_w)}\right)$$

Full CORR

$$\begin{aligned} C(t, t_w) &= C_{st}(t - t_w) + C_{ag}(t, t_w) \\ &= f_{st}(t - t_w) + f_{ag}\left(\frac{R(t)}{R(t_w)}\right) \end{aligned}$$

MATCHING OF TIME- SCALES

$$C(t, t) = 1 = C_{st}(0) + C_{ag}(1)$$

PROPOSE

$$C_{st}(0) = 1 - m^2$$

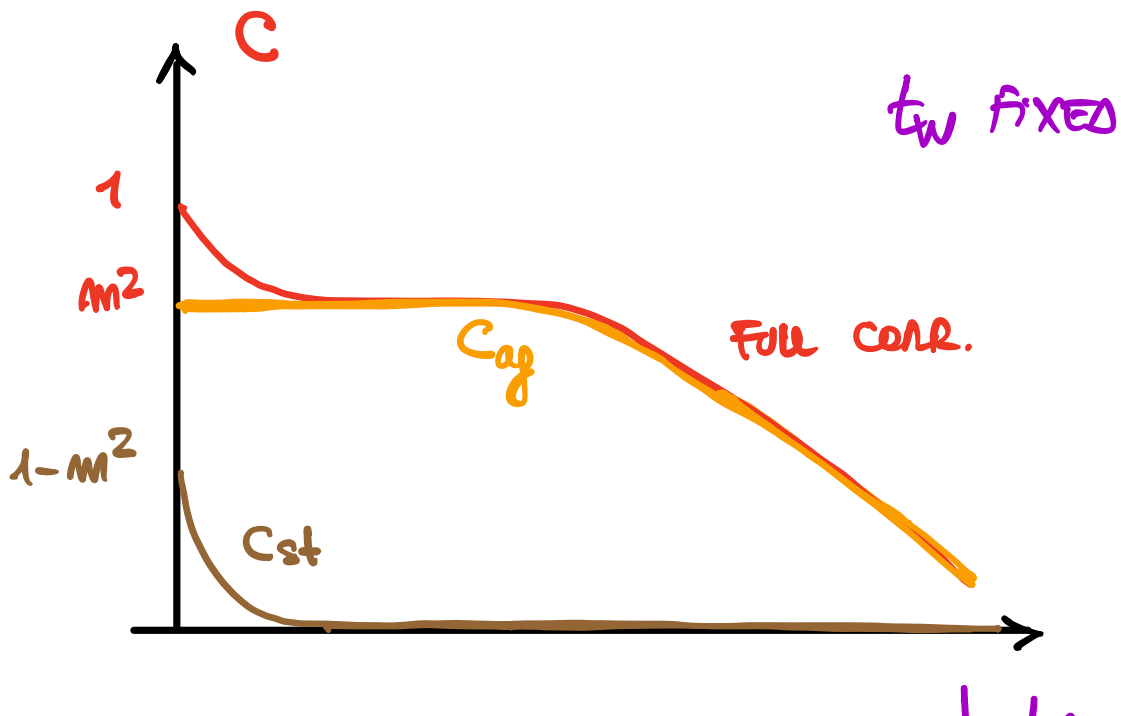
$$C_{ag}(1) = m^2$$

$$\lim_{t-t_w \rightarrow \infty} \lim_{t \rightarrow \infty} C(t, t_w) = m^2$$

$$\Rightarrow C_{st}(t-t_w) = 0 \quad C_{ag}(1) = m^2$$

THE CONC. DECAYS TO ZERO AT SUFF LONG TIME DELAYS

$$\lim_{\substack{t \gg t_w \\ \rightarrow \infty}} C(t, t_w) = 0 \quad \Rightarrow \quad \lim_{\substack{t \gg t_w \\ \rightarrow \infty}} C_{ag}\left(\frac{R(t)}{R(t_w)}\right) = 0$$



$\tau - t_w$

ADDITIVE BEHAVIOUR

SUM OF TWO CONTRIB

SINCE  $m^2 \neq 0$

QUENCH AT  $T_c$

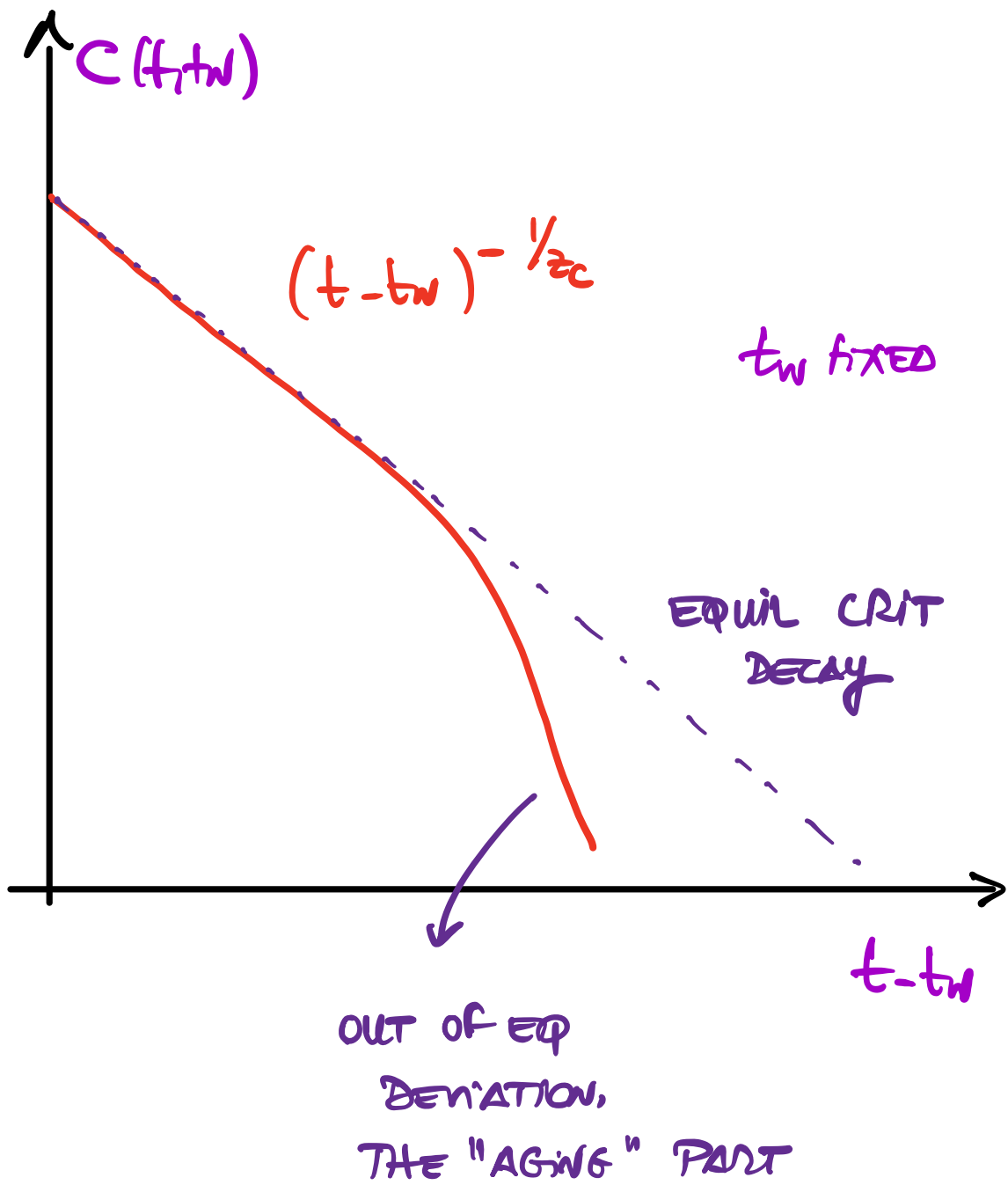
$m^2 = 0$

MULTIPLICATIVE BEHAVIOUR

$$C(t, t_w) = C_{st}(t - t_w) \text{Cap} \left( \frac{R(t)}{R(t_w)} \right)$$

$$C_{st}(t - t_w) \sim (t - t_w)^{-1/2c}$$

$$R(t) \sim t^{1/2c}$$



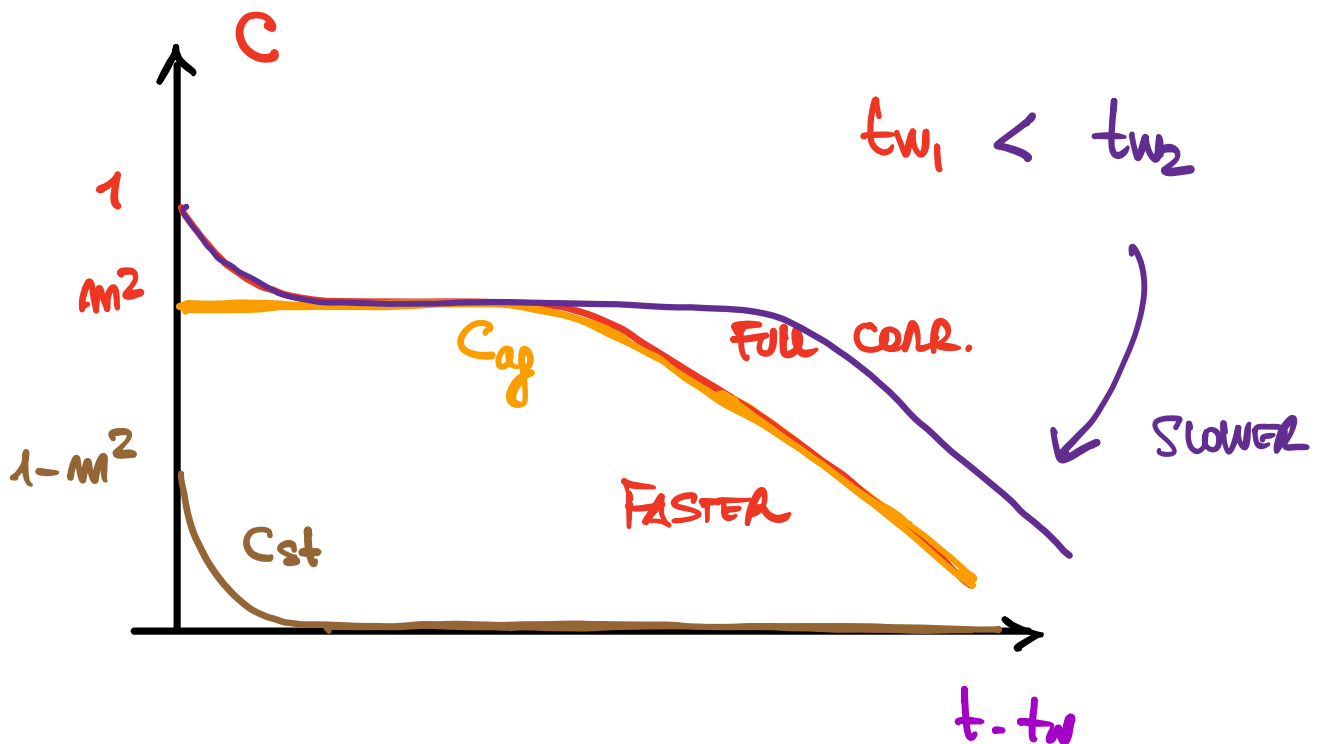
2d IM AT  $T_c$

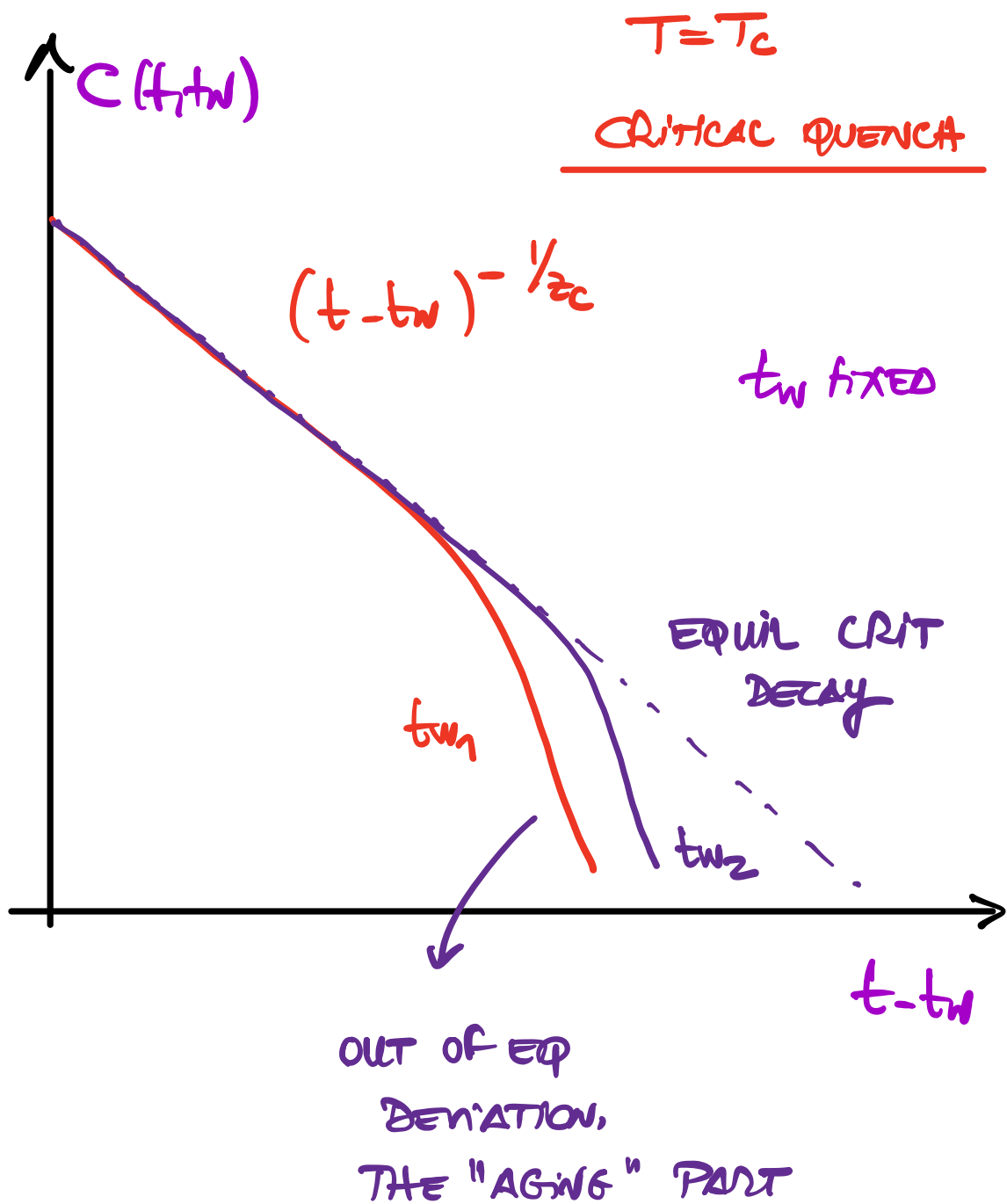
2d XY AT  $T < T_{KT}$

# AGING

THE SECOND RELAXATION IS NOT  
STATIONARY  
DEPENDS ON  $t_w$

$T < T_c$  SUB-CRITICAL QUENCH





IN BOTH CASES THE SYSTEM PROGRESSIVELY  
GOES CLOSER TO EQUIL. BUT IT DOESN'T  
REACH IT UNTIL

$$R(t) \sim L$$

$$\left. \begin{array}{l} t^{1/2} \sim L \\ t^{1/2c} \sim L \end{array} \right\} \Rightarrow \begin{array}{l} t \sim L^2 \\ t \sim L^{2c} \end{array}$$

DIVERGING WITH SYSTEM SIZE IN BOTH  
CASES

HOW DO WE KNOW THAT

$$R(t)$$

IS A LENGTH ?

## SPACE - TIME CORRELATIONS

$$C(r, t) = \langle S_i(t) S_j(t) \rangle$$

$$|\vec{r}_i - \vec{r}_j| = r$$

SAME TIME, DIFFERENT PLACES IN SPACE

AT  $T_c$

$$C(r, t) \sim C_{eq}(r) \cdot f_c(r/R(t))$$

$$C_{eq}(r) \sim r^{2-d-\eta}$$

BELOW  $T_c$

$$C(r, t) \sim C_{eq}(r) + f(r/R(t))$$

$$C_{eq}(r) \sim m^2 + \text{DECAY}$$

## THEORY

TIME-DEP. GINZBURG-LANDAU

$$F[\phi] = \int d^d x \left\{ \frac{1}{2} (\nabla \phi)^2 + V(\phi) \right\}$$

$$V(\phi) = \frac{\Gamma}{2} \phi^2 + \frac{\lambda}{4} \phi^4 \quad \Gamma < 0$$

OVER-DAMPED LANG. EQ FOR  $\phi$

$$\gamma \frac{\partial \phi(\vec{x}, t)}{\partial t} = - \frac{\delta F[\phi]}{\delta \phi(\vec{x}, t)} + S(\vec{x}, t)$$

NB. WE USE  $F[\phi]$  THE

FREE-ENERGY AS THE "FUNCTIONAL  
POTENTIAL"  $\Rightarrow$  DETERM. FORCE

$$\langle S(\vec{x}, t) \rangle = 0$$

$$\langle \xi(\vec{x}, t) \xi(\vec{y}, t') \rangle =$$

$$= 2\gamma k_B T \delta^d(\vec{x} - \vec{y}) \delta(t - t')$$

IT'S A NON-LINEAR EQ.  $\Rightarrow$   
HARD TO SOLVE !

NUMERICS

ANALYTIC

- ~~t-DEP~~ RG AT  $T_c$
- ARGUMENTS

$$R(t) \sim t^{1/z_c} \quad z_c \sim 2.1$$

$$R(t) \sim t^{1/2} \quad z=2$$

$\hat{=}$  DYNAMIC UNIVERSALITY CLASSES

A CLOSER MODEL TO THE ONES WE STUDIED

$p=2$  SPHERICAL KOSTENITZ-THOULESS-JONES  
LFC & DEAN

$$H = -\frac{1}{2} \sum_{i \neq j} J_{ij} s_i s_j$$

$$-\infty \leq s_i \leq \infty \quad \sum_{i=1}^N s_i^2 = N$$

$$\dot{s}_i = - \frac{\delta H}{\delta s_i} + \int s_i + s_i$$

↑  
TIME DEP LAGRANGE  
MULTIPLIER

$$= - J_{ij} s_j + \int s_i + s_i$$

$$\dot{s}_\mu = - \lambda_\mu s_\mu + \int s_\mu + s_\mu$$

Solve LINEAR EQ. FOR

$$s_\mu(t; s(t))$$

Fix  $\mathcal{I}(t)$  FROM CONSTRAINT

$$\frac{1}{N} \sum_{\mu} \langle s_{\mu}^2(t) \rangle_{ic} = 1$$

AVERAGE OVER NOISE AND INITIAL CONDITIONS

$\mathcal{I}(t)$  DEPENDS ON I.C. AND WORKING TEMP.

$C(t, t_N)$  WITH THE PROPERTIES EXPLAINED  
ABOVE EVEN THOUGH NO SPACE  
SINCE FULLY CONNECTED MODEL

HOW TO DEAL WITH MORE COMPLEX CASES, LIKE

$p$ -SPIN MODELS OR  
SK MODEL ?

SOFT SPINS  $\Rightarrow$  LANGEVIN EQS. FOR  
CONTINUOUS VARIABLES

BUT NON-LINEAR, SO CANNOT SOLVE THEM

NO PERTURBATION EITHER SINCE "STRONG COUPLING LIMIT"

$Z_{\text{dyn}}$  DYNAMIC GENERATING FUNCTIONAL

MARTIN-SIGGIA-ROSE-JANSSEN-DE DOMINICIS

IN A SENSE, SIMILAR TO WHAT WE DID WITH

KAC-RICE

BUT TIME PLAYS A ROLE TOO.

IMPOSE LANG. EQ.

$$Z_{\text{dyn}}[\text{SOURCES}] = \int \mathcal{D}\vec{\xi} \mathcal{P}[\vec{\xi}] \int \mathcal{D}\vec{S} \delta(\text{LANG. EQ.}) \det e^{\int \vec{\eta} \cdot \vec{S}}$$

$$\vec{S} = (S_1, \dots, S_N)$$

$$\mathcal{D}\vec{S} = \prod_{i=1}^N \prod_{h=1}^{\mathcal{N}} dS_i(t_h) \quad \text{PATH INTEGRAL}$$

$$\vec{\Sigma} = (\Sigma_1, \dots, \Sigma_N)$$

$$\mathcal{D}\vec{\Sigma} = \prod_{i=1}^N \prod_{k=1}^N d\Sigma_i(t_k)$$

FROM THIS GENERATING FUNCTIONAL ONE DERIVES

### SCHWINGER-DYSON EQS

INTEGRAL-DIFFERENTIAL EQS. WHICH COUPLE TWO-TIME CORR. & LINEAR RESP.

ONE CAN IMPOSE  $\neq$  KINDS OF INITIAL COND

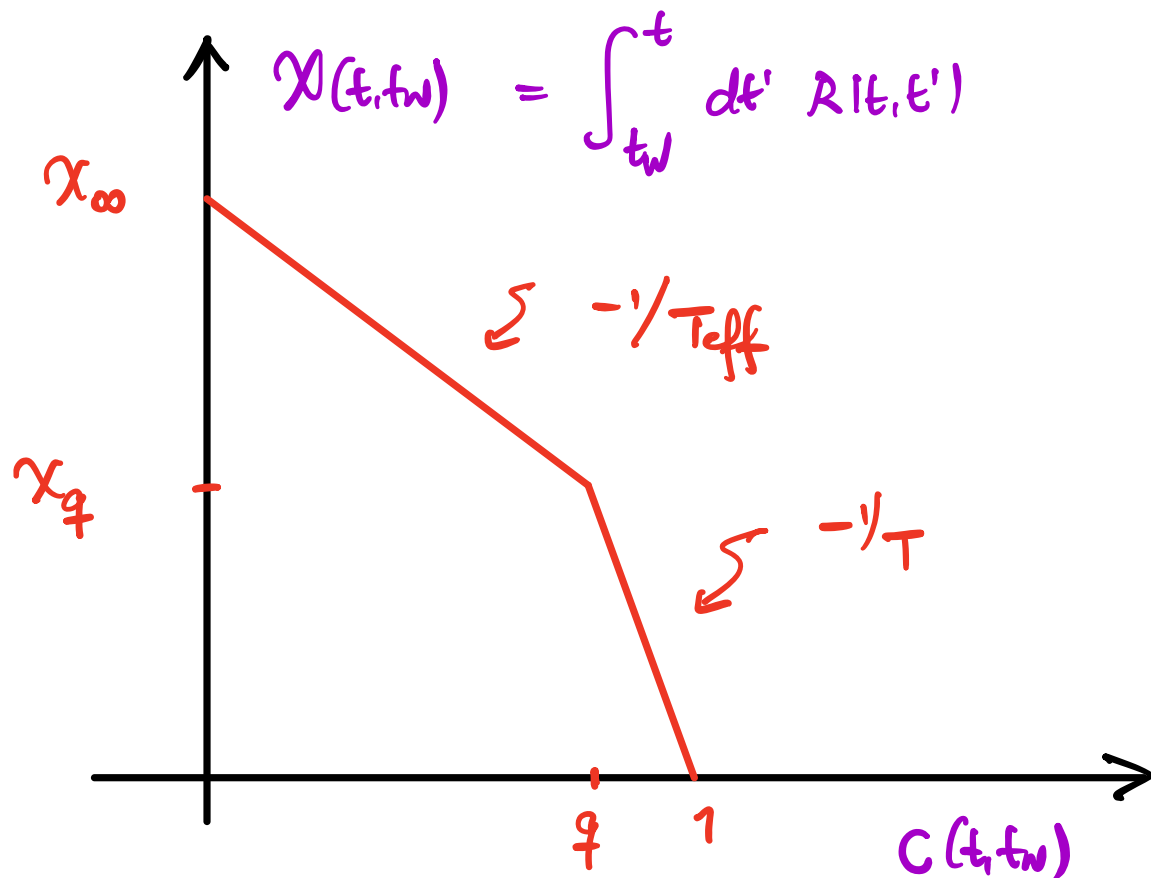
$$\frac{\partial C(H, t_w)}{\partial t} = \int dt' (\Sigma \cdot C + \mathcal{D} \cdot R)$$

$$\frac{\partial R(H, t_w)}{\partial t} = \int dt' \Sigma \cdot R + \delta$$

COMPACT NOTATION

$\Rightarrow$  ASYMPTOTIC  $t_w \rightarrow \infty$  SOLUTION

## NEW FEATURES



DYNAMICS APPROACHING THE THRESHOLD FOR  
p-Spin SPHERICAL MODELS

$$e(t) \xrightarrow{t \rightarrow \infty} e_h$$

EFFECTIVE TEMPERATURE

$$T_{eff}(C)$$

WITH TWO "SCALES"

$$T_{\text{eff}} = T \quad \text{BATH} \quad C > q_{\text{EA}}$$

$$T_{\text{eff}} > T \quad C < q_{\text{EA}}$$

FOR QUENCHES FROM  $T > T_c$