In[165]:=
```
J = 1.
```
Out[165]= 1.

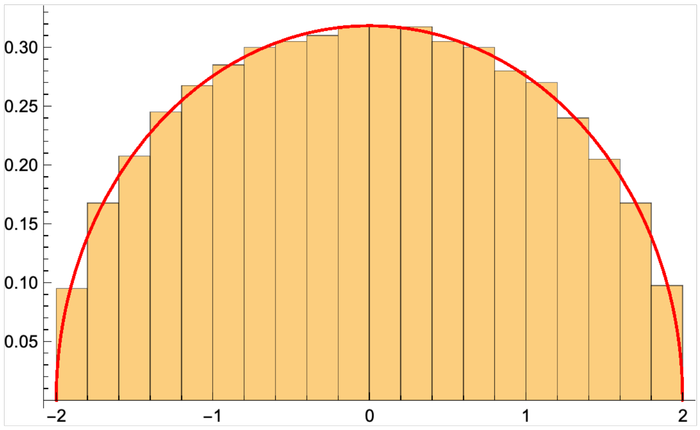
Wigner Semicircle Law

n = 2000

In[∘]:=
```
n = 2000
```
Out[∘]= 2000

In[∘]:=
```
sigmaJ = Sqrt[ 2 J^2 / n]
```
Out[∘]= 0.0316228

In[∘]:=
```
M2000 = Array[a2000, {n, n}]
```
In[∘]:=
```
Do[m[i, j] = RandomVariate[NormalDistribution[0, sigmaJ]], {i, 1, n}, {j, 1, n}]
```
In[∘]:=
```
Do[a2000[i, j] = (m[i, j] + m[j, i]) / 2, {i, 1, n}, {j, 1, n}]
```
In[∘]:=
```
Show[Histogram[Eigenvalues[M2000], Automatic, "PDF"],
 Plot[1 / (2 Pi J^2) Sqrt[(2 J)^2 - x^2], {x, -2 J, 2 J}, PlotStyle → Red]]
```
Out[∘]=



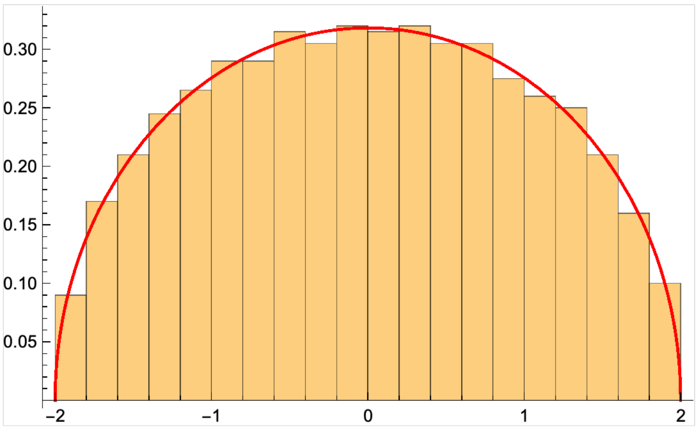
n = 1000

In[8]:=
```
n = 1000
```
Out[∘]= 1000

In[9]:=
```
sigmaJ = Sqrt[ 2 J^2 / n]
```
Out[∘]= 0.0447214

In[10]:=
```
M1000 = Array[a1000, {n, n}]
```
In[11]:=
```
Do[m[i, j] = RandomVariate[NormalDistribution[0, sigmaJ]], {i, 1, n}, {j, 1, n}]
```
In[12]:=
```
Do[a1000[i, j] = (m[i, j] + m[j, i]) / 2, {i, 1, n}, {j, 1, n}]
```

In[13]:=
```
Show[Histogram[Eigenvalues[M1000], Automatic, "PDF"],
 Plot[1 / (2 Pi J^2) Sqrt[(2 J)^2 - x^2], {x, -2 J, 2 J}, PlotStyle → Red]]
```

Out[•]=


n = 100

In[14]:=
```
n = 100
```
Out[•]= 100

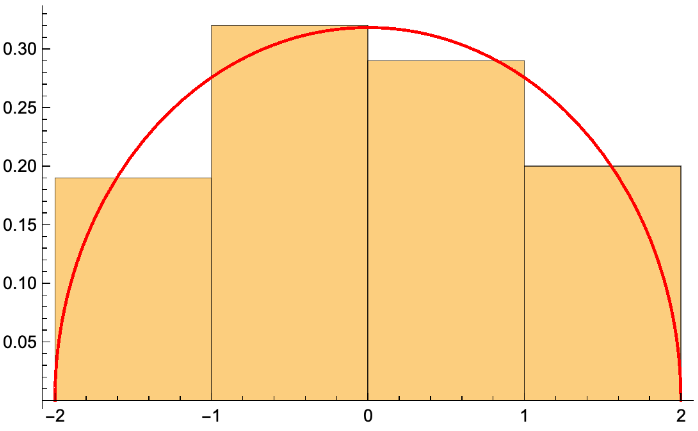
In[15]:=
```
sigmaJ = Sqrt[ 2 J^2 / n]
```
Out[•]= 0.141421

In[16]:=
```
M100 = Array[a100, {n, n}]
```

In[17]:=
```
Do[m[i, j] = RandomVariate[NormalDistribution[0, sigmaJ]], {i, 1, n}, {j, 1, n}]
```

In[18]:=
```
Do[a100[i, j] = (m[i, j] + m[j, i]) / 2, {i, 1, n}, {j, 1, n}]
```

In[19]:=
```
Show[Histogram[Eigenvalues[M100], Automatic, "PDF"],
 Plot[1 / (2 Pi J^2) Sqrt[(2 J)^2 - x^2], {x, -2 J, 2 J}, PlotStyle → Red]]
```

Out[•]=


n = 10

In[20]:=
```
n = 10
```
Out[•]=
```
10
```

In[21]:=
```
sigmaJ = Sqrt[ 2 J^2 / n]
```
Out[•]=
```
0.447214
```

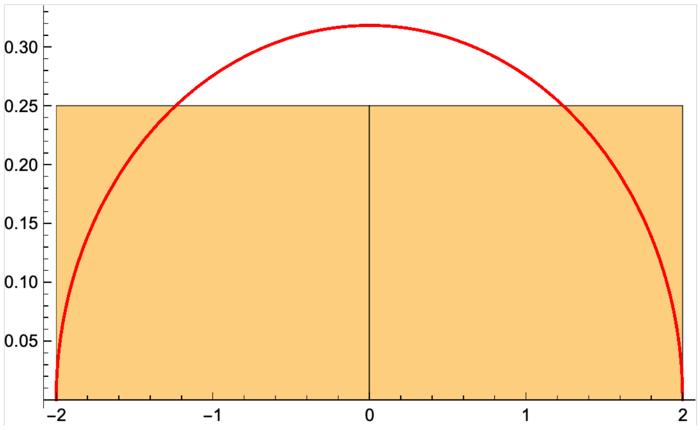
In[22]:=
```
M10 = Array[a10, {n, n}]
```

In[23]:=
```
Do[m[i, j] = RandomVariate[NormalDistribution[0, sigmaJ]], {i, 1, n}, {j, 1, n}]
```

In[24]:=
```
Do[a10[i, j] = (m[i, j] + m[j, i]) / 2, {i, 1, n}, {j, 1, n}]
```

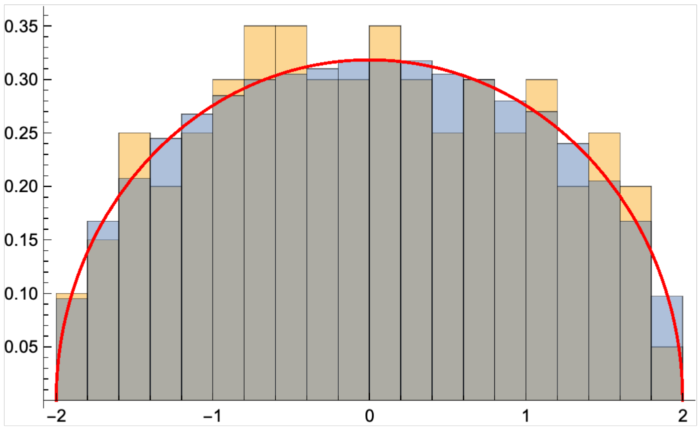
In[25]:=
```
Show[Histogram[Eigenvalues[M10], Automatic, "PDF"],
 Plot[1 / (2 Pi J^2) Sqrt[(2 J)^2 - x^2], {x, -2 J, 2 J}, PlotStyle → Red]]
```

Out[•]=



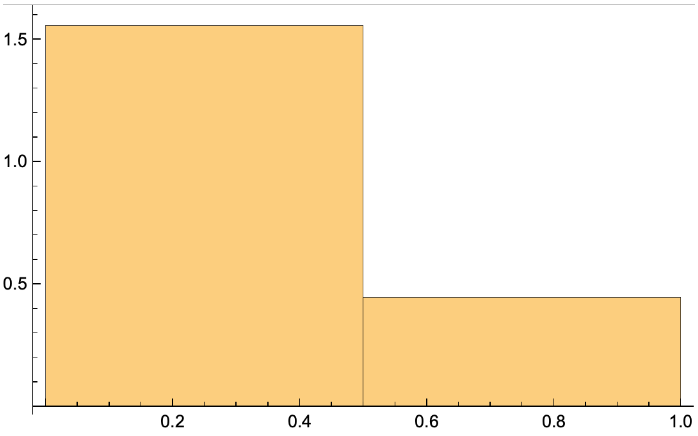
In[•]:=
```
Show[Histogram[{Eigenvalues[M100], Eigenvalues[M2000]}, Automatic, "PDF"],
 Plot[1 / (2 Pi J^2) Sqrt[(2 J)^2 - x^2], {x, -2 J, 2 J}, PlotStyle → Red]]
```

Out[•]=

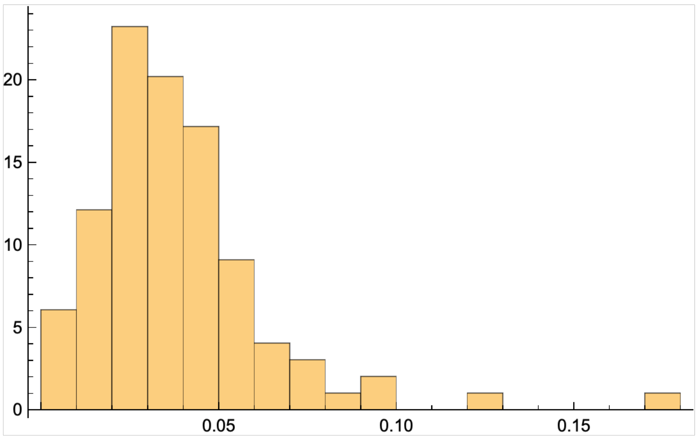


Level repulsion

```
In[∘]:= v10 = NumericalSort[Eigenvalues[M10]]
Out[∘]= {-1.67036, -0.874687, -0.625707, -0.254182,
  -0.11284, 0.230875, 0.444219, 0.64214, 1.41129, 1.52521}

In[∘]:= s10 = Table[v10[[i+1]] - v10[[i]], {i, 1, 9}]
Out[∘]= {0.795673, 0.24898, 0.371525, 0.141342,
  0.343715, 0.213345, 0.19792, 0.769148, 0.113927}

In[∘]:= Histogram[s10, Automatic, "PDF"]
Out[∘]=
```

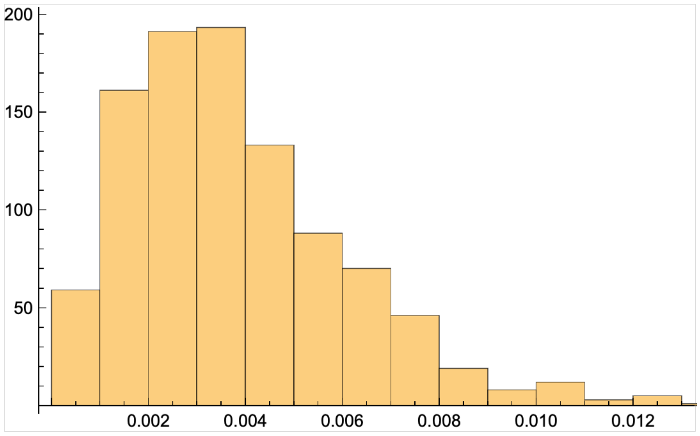



```
In[∘]:= v100 = NumericalSort[Eigenvalues[M100]]

In[∘]:= s100 = Table[v100[[i+1]] - v100[[i]], {i, 1, 99}]

In[∘]:= Histogram[s100, Automatic, "PDF"]
Out[∘]=
```




```
In[∘]:= v1000 = NumericalSort[Eigenvalues[M1000]]

In[∘]:= s1000 = Table[v1000[[i+1]] - v1000[[i]], {i, 1, 999}]
```

In[∘]:= Histogram[s1000, Automatic, "PDF"]

Out[∘]=


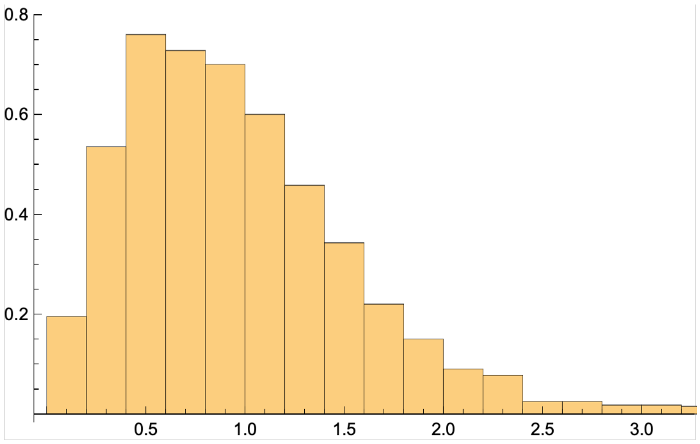
In[∘]:= v2000 = NumericalSort[Eigenvalues[M2000]]

In[∘]:= s2000 = Table[v2000[[i + 1]] - v2000[[i]], {i, 1, 1999}]

In[∘]:= ave2000 = Mean[s2000]

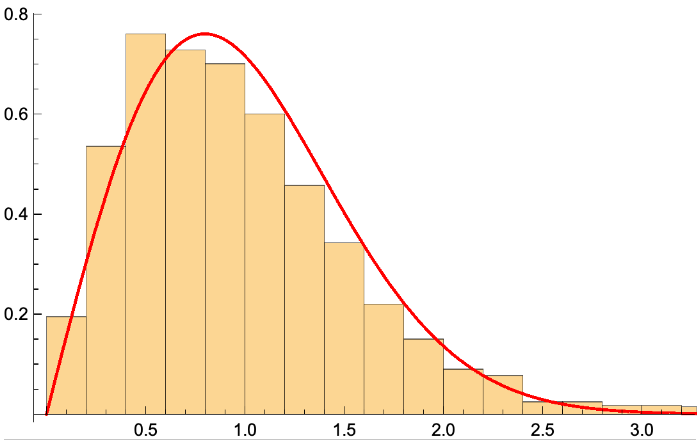
Out[∘]= 0.00199159

In[∘]:= Histogram[s2000 / ave2000, Automatic, "PDF"]

Out[∘]=


Comparison to Wigner's form

In[∘]:= Show[Histogram[{s2000 / ave2000}, Automatic, "PDF"],
 Plot[Pi / 2 x E^(-Pi x^2 / 4), {x, 0, 4}, PlotStyle → Red]]

Out[∘]=


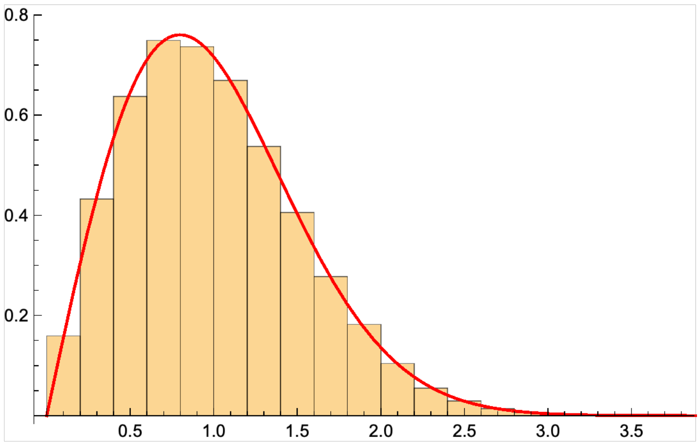
n = 2 and sampling over many matrices

In[237]:=
```
n = 2; sigmaJ = Sqrt[ 2 J^2 / n]
```
Out[237]=
```
1.
```

In[238]:=
```
s2 = {}; lambdasampling2 = {};
Do[M2 = Array[a2, {n, n}];   Do[
   m[i, j] = RandomVariate[NormalDistribution[0, sigmaJ]], {i, 1, n}, {j, 1, n}];
  Do[a2[i, j] = (m[i, j] + m[j, i]) / 2, {i, 1, n}, {j, 1, n}];
  lambdan2 = NumericalSort[Eigenvalues[M2]];
  lambdasampling2 = Append[lambdasampling2, lambdan2[[2]]];
  lambdasampling2 = Append[lambdasampling2, lambdan2[[1]]];
  s2 = Append[s2, lambdan2[[2]] - lambdan2[[1]]], {a, 1, 100000}]
```

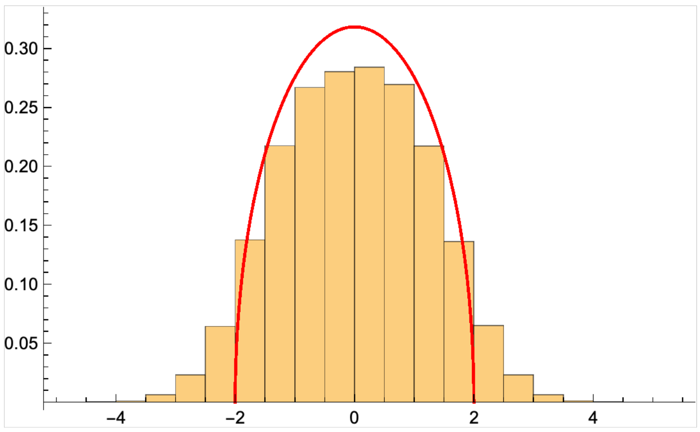
In[239]:=
```
Show[Histogram[{s2 / Mean[s2]}, Automatic, "PDF"],
 Plot[Pi / 2 x E^(-Pi x^2 / 4), {x, 0, 4}, PlotStyle → Red]]
```

Out[239]=



In[240]:=
```
Show[Histogram[lambdasampling2, Automatic, "PDF"],
 Plot[1 / (2 Pi J^2) Sqrt[(2 J)^2 - x^2], {x, -2 J, 2 J}, PlotStyle → Red]]
```

Out[240]=

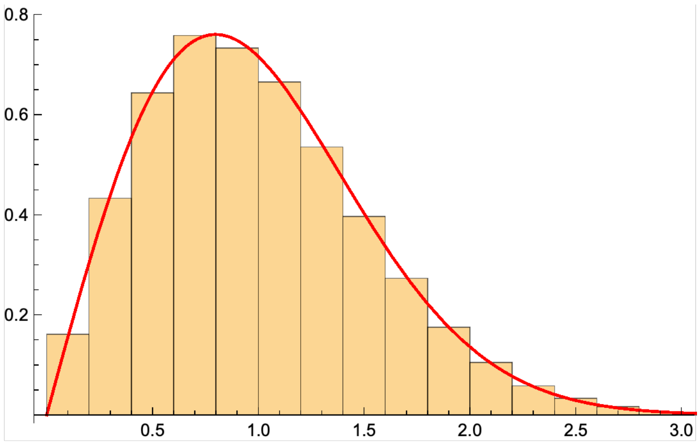


n = 3 and sampling over many matrices

In[252]:=
```
n = 3;
sigmaJ = Sqrt[ 2 J^2 / n];
s3 = {};
lambdasampling3 = {};
Do[M3 = Array[a3, {n, n}];   Do[
  m[i, j] = RandomVariate[NormalDistribution[0, sigmaJ]], {i, 1, n}, {j, 1, n}];
 Do[a3[i, j] = (m[i, j] + m[j, i]) / 2, {i, 1, n}, {j, 1, n}];
 lambdan3 = NumericalSort[Eigenvalues[M3]];
 lambdasampling3 = Append[lambdasampling3, lambdan3[[3]]];
 lambdasampling3 = Append[lambdasampling3, lambdan3[[2]]];
 lambdasampling3 = Append[lambdasampling3, lambdan3[[1]]];
 s3 = Append[s3, lambdan3[[2]] - lambdan3[[1]]];
 s3 = Append[s3, lambdan3[[3]] - lambdan3[[2]]], {a, 1, 50 000}]
```

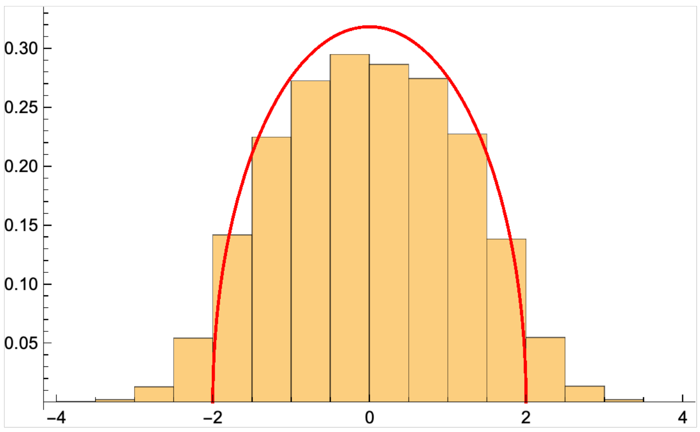
In[253]:=
```
Show[Histogram[{s3 / Mean[s3]}, Automatic, "PDF"],
 Plot[Pi / 2 x E^(-Pi x^2 / 4), {x, 0, 4}, PlotStyle → Red]]
```

Out[253]=



In[254]:=
```
Show[Histogram[lambdasampling3, Automatic, "PDF"],
 Plot[1 / (2 Pi J^2) Sqrt[(2 J)^2 - x^2], {x, -2 J, 2 J}, PlotStyle → Red]]
```

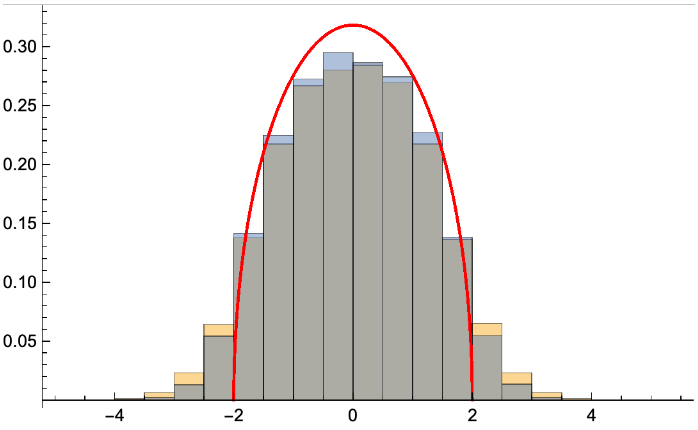
Out[254]=



Comparing the n = 2, 3 cases
(semi – circle law and tails)

In[255]:=
```
Show[Histogram[{lambdasampling2, lambdasampling3}, Automatic, "PDF"],
 Plot[1 / (2 Pi J^2) Sqrt[(2 J)^2 - x^2], {x, -2 J, 2 J}, PlotStyle → Red]]
```

Out[255]=



n = 4 and sampling over many matrices

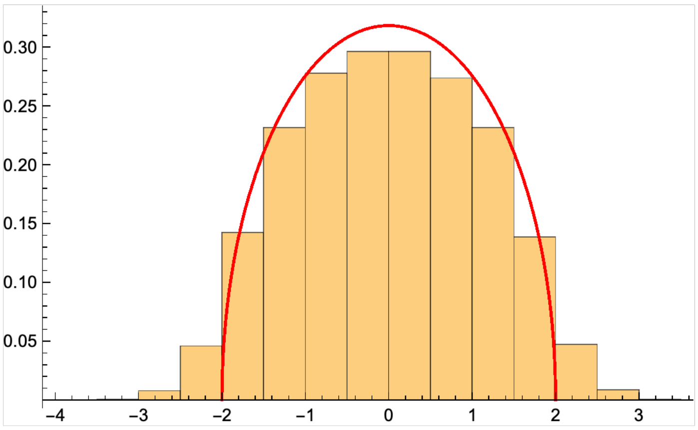
In[232]:=
```
n = 4; sigmaJ = Sqrt[ 2 J^2 / n]
```

Out[232]=
```
0.707107
```

In[233]:=
```
s4 = {};
lambdasampling4 = {};
Do[M4 = Array[a4, {n, n}];   Do[
  m[i, j] = RandomVariate[NormalDistribution[0, sigmaJ]], {i, 1, n}, {j, 1, n}];
 Do[a4[i, j] = (m[i, j] + m[j, i]) / 2, {i, 1, n}, {j, 1, n}];
 lambdan4 = NumericalSort[Eigenvalues[M4]];
 lambdasampling4 = Append[lambdasampling4, lambdan4[[4]]];
 lambdasampling4 = Append[lambdasampling4, lambdan4[[3]]];
 lambdasampling4 = Append[lambdasampling4, lambdan4[[2]]];
 lambdasampling4 = Append[lambdasampling4, lambdan4[[1]]];
 s4 = Append[s4, lambdan4[[2]] - lambdan4[[1]]];
 s4 = Append[s4, lambdan4[[3]] - lambdan4[[2]]];
 s4 = Append[s4, lambdan4[[4]] - lambdan4[[3]]], {a, 1, 10 000}]
```

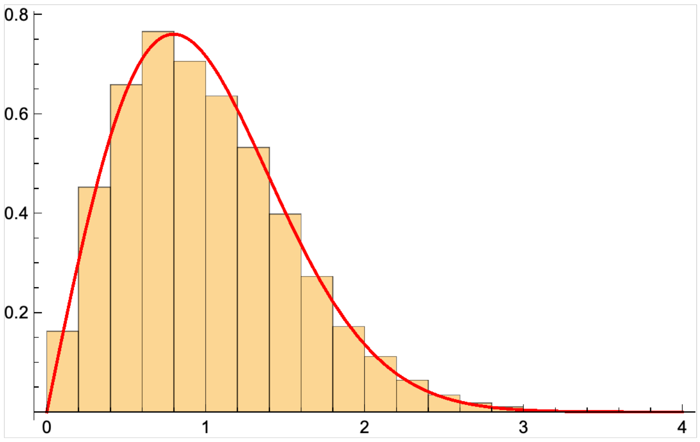
In[234]:=
```
Show[Histogram[lambdasampling4, Automatic, "PDF"],
 Plot[1 / (2 Pi J^2) Sqrt[(2 J)^2 - x^2], {x, -2 J, 2 J}, PlotStyle → Red]]
```

Out[234]=



In[235]:=
```
Show[Histogram[{s4 / Mean[s4]}, Automatic, "PDF"],
 Plot[Pi / 2 x E^(-Pi x^2 / 4), {x, 0, 4}, PlotStyle → Red]]
```

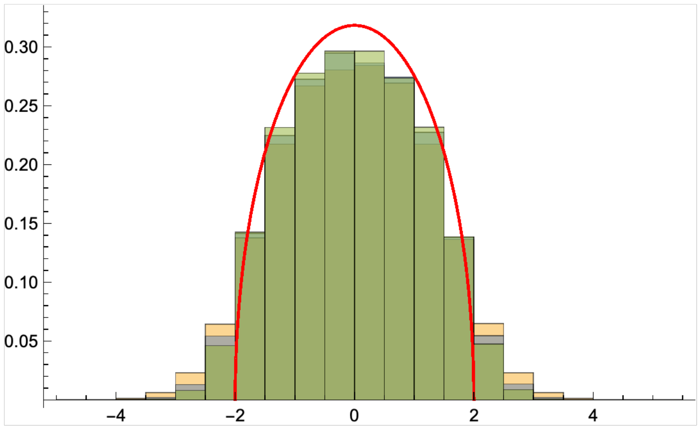
Out[235]=



Comparing the n = 2, 3, 4 cases (semi – circle law and tails)

In[256]:=
```
Show[Histogram[{lambdasampling2, lambdasampling3, lambdasampling4}, Automatic,
  "PDF"], Plot[1 / (2 Pi J^2) Sqrt[(2 J)^2 - x^2], {x, -2 J, 2 J}, PlotStyle → Red]]
```

Out[256]=



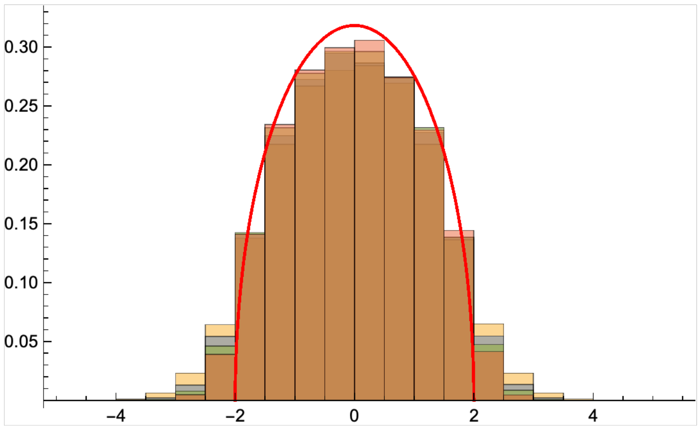
n = 5 and sampling over many matrices

In[243]:=
```
n = 5;
sigmaJ = Sqrt[ 2 J^2 / n];
s5 = {};
lambdasampling5 = {};
Do[M5 = Array[a5, {n, n}];   Do[
  m[i, j] = RandomVariate[NormalDistribution[0, sigmaJ]], {i, 1, n}, {j, 1, n}];
 Do[a5[i, j] = (m[i, j] + m[j, i]) / 2, {i, 1, n}, {j, 1, n}];
 lambdan5 = NumericalSort[Eigenvalues[M5]];
 lambdasampling5 = Append[lambdasampling5, lambdan5[[5]]];
 lambdasampling5 = Append[lambdasampling5, lambdan5[[4]]];
 lambdasampling5 = Append[lambdasampling5, lambdan5[[3]]];
 lambdasampling5 = Append[lambdasampling5, lambdan5[[2]]];
 lambdasampling5 = Append[lambdasampling5, lambdan5[[1]]];
 s5 = Append[s5, lambdan5[[2]] - lambdan5[[1]]];
 s5 = Append[s5, lambdan5[[3]] - lambdan5[[2]]];
 s5 = Append[s5, lambdan5[[4]] - lambdan5[[3]]];
 s5 = Append[s5, lambdan5[[5]] - lambdan5[[4]]], {a, 1, 10 000}]
```

Comparing the n = 2, 3, 4, 5 cases (semi – circle law and tails)

In[257]:=
```
Show[Histogram[{lambdasampling2, lambdasampling3,
   lambdasampling4, lambdasampling5}, Automatic, "PDF"],
 Plot[1 / (2 Pi J^2) Sqrt[(2 J)^2 - x^2], {x, -2 J, 2 J}, PlotStyle → Red]]
```

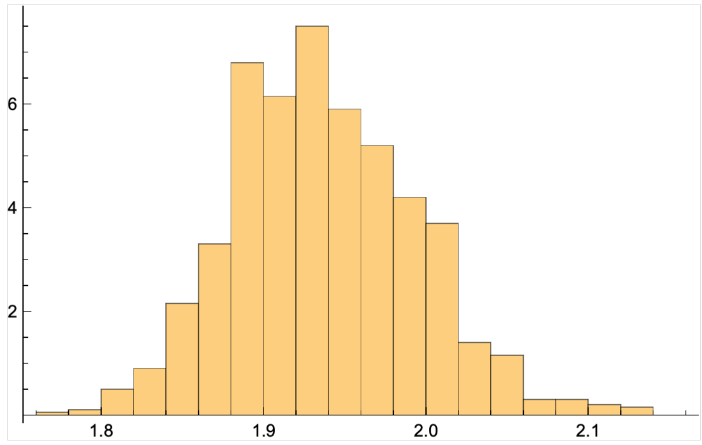
Out[257]=



Sampling over many matrices the maximum eigenvalue

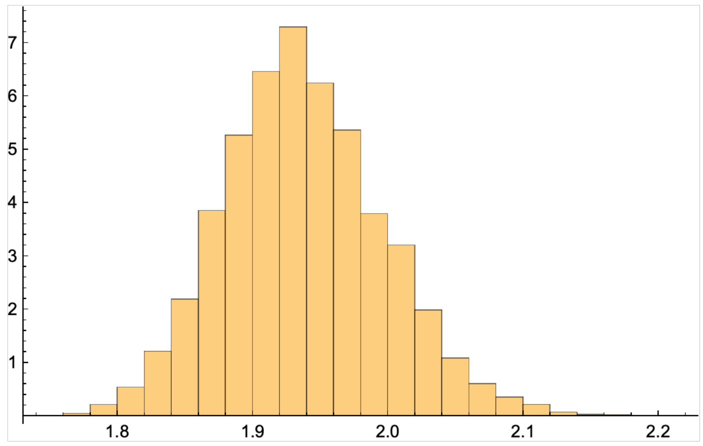
In[617]:=
```
J = 1;
n = 100;
sigmaJ = Sqrt[ 2 J^2 / n];
lambdamax = {};
Do[M = Array[a, {n, n}];   Do[
   m[i, j] = RandomVariate[NormalDistribution[0, sigmaJ]], {i, 1, n}, {j, 1, n}];
 Do[a[i, j] = (m[i, j] + m[j, i]) / 2, {i, 1, n}, {j, 1, n}];
 lambda = NumericalSort[Eigenvalues[M]];
 lambdamax = Append[lambdamax, lambda[[n]]], {index, 1, 1000}];
Show[Histogram[lambdamax, Automatic, "PDF"]]
```

Out[618]=



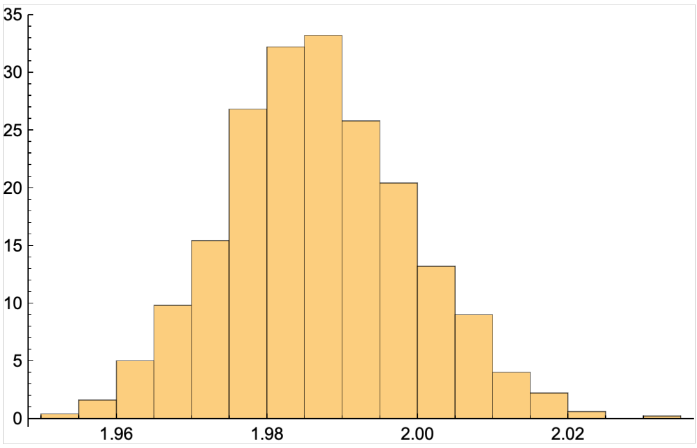
In[619]:=
```
J = 1;
n = 100;
sigmaJ = Sqrt[ 2 J^2 / n];
lambdamax = {};
Do[M = Array[a, {n, n}];   Do[
   m[i, j] = RandomVariate[NormalDistribution[0, sigmaJ]], {i, 1, n}, {j, 1, n}];
 Do[a[i, j] = (m[i, j] + m[j, i]) / 2, {i, 1, n}, {j, 1, n}];
 lambda = NumericalSort[Eigenvalues[M]];
 lambdamax = Append[lambdamax, lambda[[n]]], {index, 1, 10 000}];
Show[Histogram[lambdamax, Automatic, "PDF"]]
```

Out[620]=



In[621]:=
```
J = 1;
n = 1000;
sigmaJ = Sqrt[ 2 J^2 / n];
lambdamax = {};
Do[M = Array[a, {n, n}];   Do[
   m[i, j] = RandomVariate[NormalDistribution[0, sigmaJ]], {i, 1, n}, {j, 1, n}];
 Do[a[i, j] = (m[i, j] + m[j, i]) / 2, {i, 1, n}, {j, 1, n}];
 lambda = NumericalSort[Eigenvalues[M]];
 lambdamax = Append[lambdamax, lambda[[n]]], {index, 1, 1000}];
Show[Histogram[lambdamax, Automatic, "PDF"]]
```

Out[622]=



In[623]:=
```
Show[Out[620], Out[622]]
```

Out[623]=