Cécile de Witt - Morette

Le chemin integré

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Cécile de Witt-Morette 100th Birth Anniversary, 2022

Quotations

In “Path integrals, Les Houches, and other adventures of Cécile DeWitt-Morette",

T. Feder, Physics Today, 2008

– *She wished she was better known for her science*

– *In the 1980s, she had a grant proposal turned down with the comment that she was “busy raising children and doing administrative work”*

I will focus on **her science**

My thesis

Long ago and far away







Volume 253, number 1,2 PHYSICS LETTERS B 3 January 1991

Path-integral measure for Chern–Simons theory within the stochastic quantization approach

L.F. Cugliandolo, G. Lozano
Departamento de Física, Universidad Nacional de La Plata, C.C.67, 1900 La Plata, Argentina

and

F.A. Schaposnik [1,2]
International Centre for Theoretical Physics, I-34100 Trieste, Italy

Les Houches & path integrals

Path integrals

A bit of history

N. Wiener 20s

P. A. M. Dirac 30s

R. P. Feyman 40s

C. de Witt-Morette 50s

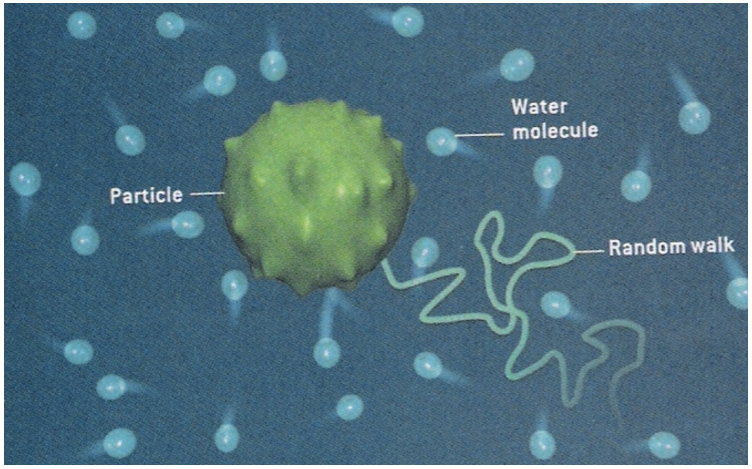
I'm no historian though !

Path integrals

A bit of history

N. Wiener 20s on Brownian motion,

the erratic motion of a massive particle immersed in a fluid



N. Wiener, Acta Math. 55, 117 (1930)

Path integrals

A bit of history

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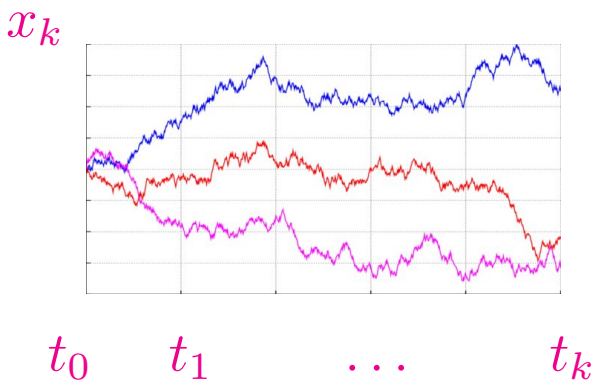


$$\Delta x_k = x_{k+1} - x_k = \Delta t\, \xi_k$$

with ξ_k a random Gaussian force with zero mean and no memory

Transition probability $\mathbb{P}(x_\mathrm{f}, t_\mathrm{f} | x_0, t_0) = \frac{1}{A} \int \prod_{k=1}^{N-1} \frac{\mathrm{d}x_k}{A}\, \mathrm{e}^{-\frac{\Delta t}{2}\left(\frac{\Delta x_k}{\Delta t}\right)^2}$

a sum over paths joining x_0 at t_0 and x_f at $t_\mathrm{f} = N\Delta t = N(t_{k+1} - t_k)$

each one weighted with its own probability

A is a normalization constant, $A = (2\pi\Delta t)^{1/2}$

Path integrals

A bit of history

N. Wiener 20s on Brownian motion

P. A. M. Dirac 30s on Lagrangian representation of quantum mechanics

The transition amplitude between states $\langle x(t_\mathrm{f})|x(t_0)\rangle$ is *analogous* to a sum over paths, weighted with

$$A\ \mathrm{e}^{\mathrm{i}S[x]/\hbar}$$

with $S[x] = \sum_{k=1}^{N-1} \mathcal{L}(x_k, \frac{(x_{k+1}-x_k)}{\Delta t})$ the classical action

and $\mathcal{L}(x_k, \frac{(x_{k+1}-x_k)}{\Delta t})$ the classical Lagrangian

prefactor A ? measure $\prod_k \mathrm{d}x_k$?

P. A. M. Dirac, *Lagrangian in Quantum Mechanics*, Physikalische Zeitschrift der Sowjetunion, Band 3, Heft 1 (1933)

Path integrals

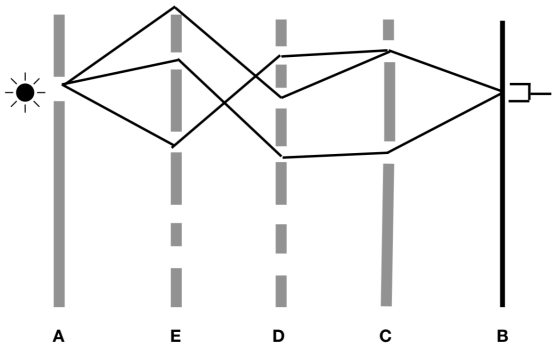
A bit of history

N. Wiener 20s on Brownian motion

P. A. M. Dirac 30s on Lagrangian representation of quantum mechanics

R. P. Feynman 40s following H. Jehle's suggestion to read Dirac's, formalisation of the idea, practical calculational setting and use

PhD thesis *The Principle of Least Action in Quantum Mechanics* (1942)



As in **Feynman & Hibbs 65**

A and $\prod_k \mathrm{d}x_k$ identified to make things work for $\Delta t \to 0$ and $N \to \infty$

Path integrals

A bit of history

PHYSICAL REVIEW VOLUME 81, NUMBER 5 MARCH 1, 1951

On the Definition and Approximation of Feynman's Path Integrals

CÉCILE MORETTE*†
Institute for Advanced Study, Princeton, New Jersey
(Received July 10, 1950)

A general and compact expression for Feynman's path integral has been obtained. A classical method is given for the computation of such expressions. The example of a Dirac particle in a constant external electromagnetic field is treated by this method.

In Feynmans' work[1] the definition of the path integrals involves an infinite product of "normalization factors." For his purposes Feynman determined these normalization factors in the cases in which the potential is velocity independent and gave their expressions in rectangular coordinates; and he indicated also the existence of a relationship between these factors and the action, S. We shall give here the general formula for the normalization factors valid for all actions and all frames of reference; moreover, we shall give a compact expression for the infinite product of the normalization factors. We shall give first the general formula for $K(x^{k+1}, x^k)$ for two points corresponding to an interval $\tau^{k+1}-\tau^k=\epsilon$ infinitesimally small; then we shall obtain $K(x^B, x^A)$ by iteration. (The essential formulas are given before their proofs.)

$$\text{①}\quad K(x^{k+1}, x^k)=\exp[(i/\hbar)\bar{S}(x^{k+1}, x^k)] \times(2\pi\hbar i)^{-\frac{1}{2}l}(\det_{\mu\nu}a^{\mu\nu}{}_{k+1,k})^{\frac{1}{2}}. \quad (2)$$

Here l is the number of degrees of freedom ($l=4$ in the actual case),

$$a^{\mu\nu}{}_{k+1,k}\equiv\partial^2\bar{S}/\partial x_\mu{}^{k+1}\partial x_\nu{}^k$$

$\det_{\mu\nu}$ means the determinant with respect to the indices μ and ν.

Post-doc at Princeton

invited by J. R. Oppenheimer

$$A=\frac{1}{(2\pi i\hbar)^{N/2}}\det{}^{1/2}\frac{\delta^2 S[x]}{\delta x_{k+1}\delta x_k}$$

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"the work of Morette-DeWitt starting with her early paper [76] gave rise to a beautiful theory of the semiclassical expansion in powers of $\hbar$"

C. Grosche, *An introduction into the Feynman path integral*

ICTP Trieste 1992

Path integrals

A bit of history

However, fundamental questions needed answers:

– How does one compute the N-tuple integral?

– Does the N-tuple integral have a unique limit for $N \to \infty$?

Difference between ∞ dimensional and $N \to \infty$ integral

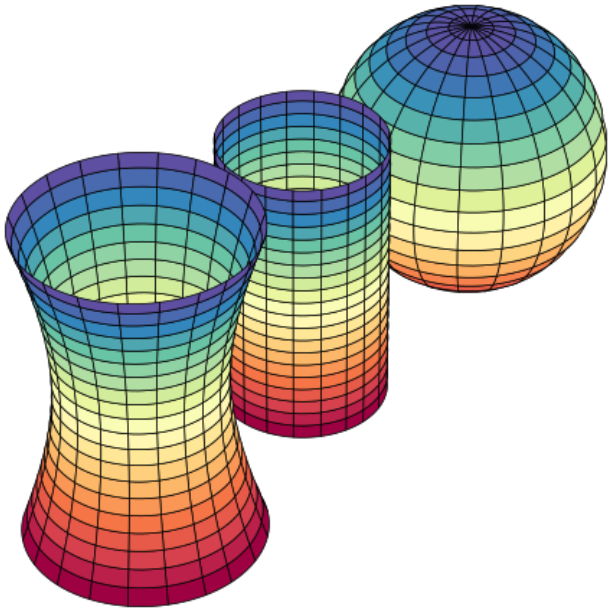
Do away with the N-tuple integrals and identify the function spaces which serve as domain of integration for functional integrals

Axiomatic framework which ensures the uniqueness of the measure sought

C. Morette-DeWitt *Feynman's Path Integral: Definition Without Limiting Procedure*, Comm. Math. Phys. 28, 47 (1972). **P. Cartier & C. DeWitt-Morette**, *Functional Integration: Action and Symmetries*, (Cambridge University Press, Cambridge, 2006).

Mathematics

Analysis, Manifolds & Physics



BOOKS

HELP IN KEEPING UP WITH THEORETICAL PHYSICS

Analysis, Manifolds, and Physics.

Yvonne Choquet-Bruhat and Cecile DeWitt-Morette

Part I: Basics
North Holland, New York, 1989 [1982]. Revised edition. 449 pp. $40.00 hc ISBN 0-444-87071-7
Part II: 92 Applications
North Holland, New York, 1989. 630 pp. $36.00 hc ISBN 0-444-86017-7

Reviewed by Arthur S. Wightman

You go to the local theoretical physics seminar in 1989 or 1990 and hear some young sprout talking about what is presumably elementary particle theory. The speaker takes for granted that every literate theoretical physicist knows the definition and main applications of the following notions: principal fiber bundle, connection, Chern class, Stieffel-Whitney class, Pontrjagin class, Sobolev space, spin bundle, almost complex structure, Kähler manifold, Calabi–Yau manifold and holonomy.

What do you do if you do not have a clue? You could take one-year courses in algebraic topology and differential geometry, or you could try to catch up with the help of a good book. There are really no other alternatives if you want to follow current theoretical particle physics. For better or for worse, it is being expressed in the same geometrical language that one hears spoken in current mathematics seminars.

In the reviewer's opinion, the two volumes under review constitute precisely what is needed to catch up. The authors' clear intention is to make the books useful to those who want to pursue what they call physical mathematics. They have provided excellent indexes. (You will find entries for all the above listed notions.) The names of new concepts are printed in boldface when introduced and are also printed in the margin. Crucial developments are elaborated in the context of significant applications. There are worked problems. Pains have evidently been taken to make the exposition clear. The book is the kind of mixed reference work and textbook that is most useful to the independent reader.

For what audience is the book intended? Although I have emphasized above the book's usefulness for ignorant old folks, I think that advanced graduate students with some preparation could also handle it. The authors have provided in the first 111 pages of the first volume a review (mostly without proofs) of the fundamentals of analysis (set theory, algebraic structures, topology, integration and functional analysis) and differential calculus in Banach spaces (foundations, calculus of variations, implicit function theorem and differential equations). If the volumes were used as the basis of a one year course in physical mathematics, this review could be presented with class participation to get things started. Used in this way, the books could be the basis for a lovely course in physical mathematics.

Here is a brief indication of the remaining contents: In the first volume there are three chapters on various aspects of the geometry of finite-dimensional differentiable manifolds, including one chapter on integration on manifolds and another on Riemannian manifolds, Kähler manifolds and connections on a principal fiber bundle. In the last two chapters of volume one, the emphasis is on functional analysis: distribution theory; Sobolev spaces and partial differential equations; differentiable manifolds of infinite dimension; and the Wiener integral.

The chapter headings in the second volume are the same as those in the first, but the contents of these chapters are applications or further elaborations of the corresponding chapters of the first volume. An exception is the additional material on infinite dimensional manifolds that is incorporated in the six chapters of volume two.

The authors' long experience in research on the general theory of relativity and quantum theory shows throughout in their sure judgment on the selection of examples. I will not attempt to classify the 92 applications of the second volume, not to speak of listing them. Suffice it to say that there are many choice plums in this pudding. Of course, the books cannot include every mathematical idea that currently is being explored in the world of theoretical physics. For example, these volumes contain no discussion of Teichmüller space, conformal equivalence classes of Riemann surfaces and related matters.

I think these books will, in the course of the 1990s, come to be regarded as classics.

Arthur Wightman has been teaching and doing research in mathematical physics at Princeton University for 41 years.

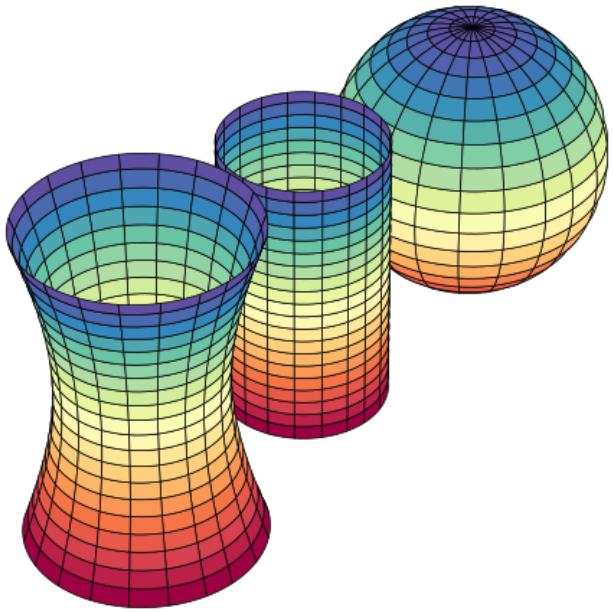
Atomic Collisions: Electron and Photon Projectiles

Earl W. McDaniel
Wiley, New York, 1989. 699 pp. $74.95 hc ISBN 0-471-85307

The field of atomic collisions touches and influences other fields of science and engineering to an extent perhaps unrivaled among the areas of modern physics. As a field that has been studied for more than 80 years, its venerability doesn't bespeak its vitality: technological advances, continued discoveries of new phenomena and demands from evolving related fields insure change and growth. Such a wide-ranging field needs to be "scoped out" periodically, to put new ideas

Mathematics

Analysis, Manifolds & Physics



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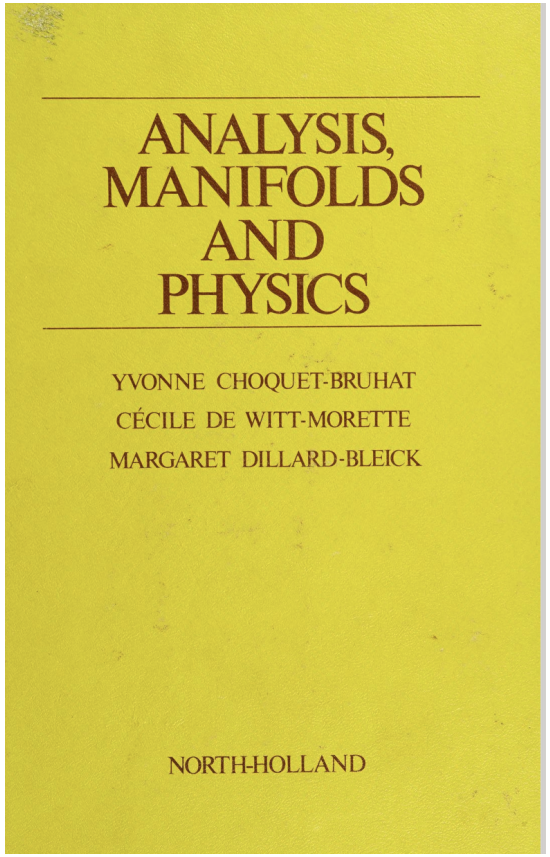
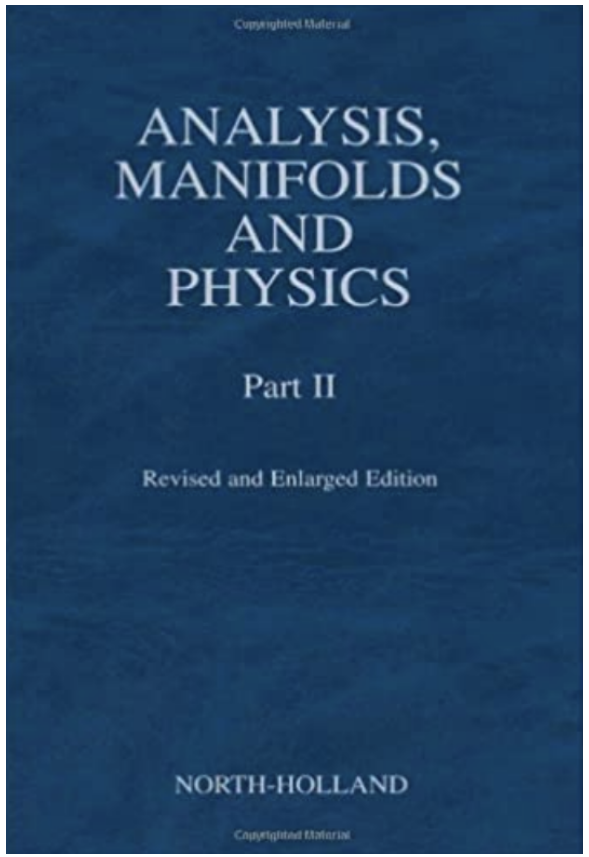
Reference books

Y. Choquet-Bruhat, C. DeWitt-Morette & M. Dillard-Bleick





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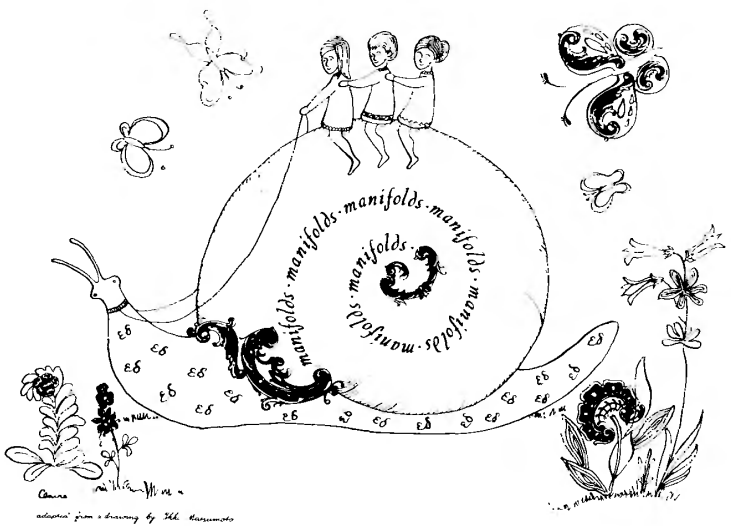

Esta es una sucinta historia de tres mujeres en torno a un libro de un gran calado en la época, y en el que con sus casi 600 páginas cubría prácticamente todo el instrumental matemático que un físico necesitaba para investigar. El libro conoció varias ediciones más, revisadas y aumentadas, aunque ya sin Margaret, quién se dedicó posteriormente a la medicina, falleciendo en 2008.

https://www.madrimasd.org/blogs/matematicas/2017/02/09/143225

Cécile

Visit to Austin 2017



Photo credit Chris DeWitt