DYNAMICS OF GLASSY SYSTEMS

1- SET THE STAGE. 1h

CLOSED / OPEN
CLASSICAL / QUANTUM
NON-INTEGRABLE / INTEGRABLE

2 - PHASE ORDERING KINETICS
DYN. ACROSS PHASE TRANS. 45'
(KIBBLE - ZUREK)
THE MEAN-FIELD VERSION $p=2$ 1h 15'

3- GLASSY DYN
DYNAMIC MEAN-FIELD THEORY 1h 45'
p-spin's FRAMEWORK
(cfr G. BIROLI'S)
OPEN VS. CLOSED — INTEGRABLE VS. NOT

AT ALL STEPS SOME COMMENT ON QUANTUM

1. SET THE STAGE

FIRST OF ALL, WE WILL BE INTERESTED IN LARGE MACROSCOPIC SYST

$N \to \infty$ THERMODYN LIMIT

TAKEN BEFORE ANY $t \to \infty$ LIMIT
OR TOGETHER WITH $t(N) \to \infty$

EQUILIBRIUM STAT. PHYS.

DISTINGUISH CLOSED - OPEN SYSTEMS

CLOSED TYPICALLY JUST N, V, E CONSTANT

WE USE MICROCANONICAL DESCRIPTION
(OFTEN HARD)

OPEN SOME CHOICES, SAY ΔE WITH SURROUNDINGS, T FIXED

CANONICAL
$$\mathcal{Z} = \underbrace{\sum_{\{c\}}}\, e^{-\beta H}$$

CLASSICALLY, A SUM OVER CONF. OR AN INTEGRAL OVER PHASE SPACE AND INCLUDES THE REMAINING CONSTRAINTS $(N, V) = \text{cts.}$

IF THERE ARE A FEW OTHER CONSTANTS OF MOTION, ONE IMPOSES THEM WITHIN $\sum_{\{c\}}$

IF THERE ARE OTHER MACROSC. QUANTITIES EXCHANGED WITH ENVIRONMENT $e^{-\gamma\theta}$

$\mathcal{Z} = \mathrm{Tr}\ e^{-\beta \hat{H}}$ IN QUANTUM.

DYNAMICS

CLOSED VS. OPEN

CLASSICAL VS. QUANTUM

CLASSICAL CLOSED

NEWTON / HAMILTON FOR PARTICLES

$$m\ddot{\vec{x}}_i = \vec{F}_i \qquad i = 1, \dots, N$$

BLOCH EQ. FOR MAGNETS

$$\dot{\vec{S}}_i = \mu \vec{S}_i \times \vec{B}_i$$

QUANTUM CLOSED

HEISENBERG / SCHRÖDINGER

$\hat{x}(t)$ HEISENBERG PICTURE

OPERATORS EVOLVE IN TIME

$$\frac{d\hat{x}(t)}{dt} = \frac{i}{\hbar}\left[\hat{H}, \hat{x}(t) \right] \qquad \text{etc.}$$

CLASSICAL OPEN

NEED A MODEL FOR THE ENVIRONMENT & COUPLING TO IT $\Longrightarrow$ NEW EQS

ENVIRONMENT $\gg$ SYSTEM ; NOT MUCH MODIF.

PROPOSE e.g. HARMONIC OSCILLATORS & COUPLING OR ENSEMBLE OF SPINS & COUPLING AND INTEGRATE OUT THE d.o.f. OF BATH TO DERIVE AN EQ. FOR SYST.

FEYNMAN-VERNON, ZWANZIG, KAWASAKI

GENERAL LANGEVIN EQS. MEMORY

$$m\ddot{\vec{x}}_i + \int_0^t dt' \, \Gamma(t-t') \, \dot{\vec{x}}_i(t') = \vec{F}_i + \vec{\xi}_i$$

$\vec{F}_i = -\vec{\nabla}_i V$ POTENTIAL FORCES

$$\langle \vec{\xi}_i(t) \rangle = 0 \quad \forall t$$

$$\langle \xi_i^a(t)\, \xi_j^b(t') \rangle = \underbrace{k_B T}_{\sim} \quad \Gamma(t-t') \quad \delta^{ab}\, \delta_{ij}$$

FROM EQUILIBRIUM ASSUMPTION ON BATH.

MORE USED TO WHITE NOISE LIMIT

$$\Gamma(t-t') \longrightarrow 2\gamma\, \delta(t-t')$$

$$\int_0^t dt' \; 2\gamma\, \delta(t-t') = \gamma$$

LANDAU - LIFSHITZ - GILBERT - BROWN EQ.

$$\dot{\vec{S}}_i = \mu\, \vec{S}_i \times \left[\vec{B}_i - \gamma \dot{\vec{S}}_i + \vec{\xi}_i \right]$$

ALREADY IN WHITE NOISE LIMIT

ENERGY $\quad K + V = E \quad$ IS NOT CONSERVED

$$\sum_{i=1}^{N} \vec{S}_i \cdot \vec{B}_i = E$$

ONE CAN MODEL THE QUANTUM BATH
(OPERATORS AND HEISENBERG DYNAMICS)
AND TRY TO TRACE OUT THE BATH.
EASIER TO DO IN A PATH INTEGRAL FORM.
$\Longrightarrow$ INFLUENCE FUNCTIONAL

SCHEMATIC REPRESENTATION   SCHWINGER-KELDYSH

$$Z_{dyn}[\eta] = \int \mathcal{D}x_\pm \, \mathcal{F}_\Gamma[x_\pm] \, e^{S_2[x_\pm]}$$

SOURCES

EFFECT OF HAVING INTEGRATED AWAY THE BATH
MOVE IT UP $\Longrightarrow$ MODIFIES S_η
TYPICALLY NON-LOCAL IN TIME
(MORE LATER)

BUT COULD ALSO THINK IN OPERATOR TERMS

e.g. $$\hat{H} = \sum_{a=1}^{\infty} \hat{S}^z \hat{b}_a^+ \hat{b}_a + \hat{H}[\hat{\vec{S}}]$$

SPIN BOSON - MODEL (OR OTHER COUPLINGS)

QUESTIONS

WHAT DOES EQUILIBRIUM MEAN?

UNDER WHICH CONDITIONS DO THESE SYSTEMS REACH EQUILIBRIUM?

IS IT ALWAYS THE GIBBS-BOLTZMANN OR OTHER?

EQUILIBRIUM MEANING

$$\lim_{t\to\infty} \lim_{N\to\infty} \overline{A_t} = \lim_{N\to\infty} \langle A \rangle$$

ERGODICITY

TIME AVERAGES COINCIDE WITH STATISTICAL AVERAGES $\forall$ $\underbrace{\text{NON-PATHOLOGICAL}}$ OBSERV. A

MEANS, e.g., "SOME GLOBALITY"

EX. CLASSICAL PARTICLES SYST

$$\overline{A}_t \equiv \lim_{\tau \gg t_0} \frac{1}{\tau} \int_t^{t+\tau} dt' \, A\left(\{\vec{x}_i(t'), \vec{p}_i(t')\}\right)$$

$$\langle A \rangle \equiv \underbrace{\int \prod_{i=1}^{N} d\vec{x}_i \int \prod_{i=1}^{N} d\vec{p}_i}_{\text{PHASE-SPACE AV.}} \; \underbrace{\rho(\vec{x}_i, \vec{p}_i)}_{\substack{\text{EQUIL MEASURE} \\ \text{MICROCAN. OR CANON.}}} \; A(\{x_i p_i\})$$

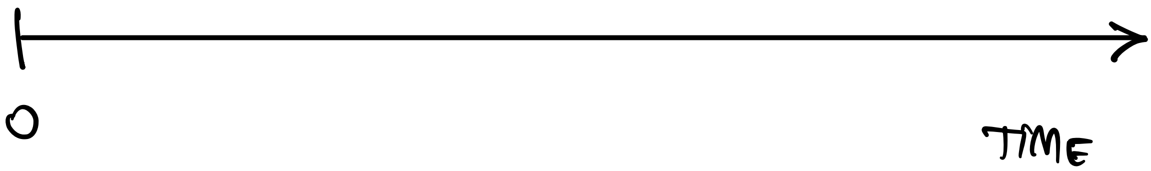
BUT IS THE ERGODIC HYPOTHESIS ALWAYS VALID ?

EQUILIBRIUM DYNAMICS

SOME NECESSARY CONDITIONS

EVOLUTION NEWTON OR LANGEVIN
WITH $\vec{F}_i = -\vec{\nabla}_i V(\{\vec{x}_i, \vec{p}_i\})$

BUT ALSO <u>BATH</u> $(\vec{\xi}_i)$ SHOULD BE EQUIL
ONE (SAME Γ IN FRICTION & NOISE CORR.)



↑ INITIAL TIME   SAMPLING I.C.

$P(\{\vec{x}_i(0), \vec{p}_i(0)\})$ COULD BE FROM $\mathcal{S}_{eq}$

WHICH KNOWS ABOUT $\mathcal{H}(\{\vec{x}_i, \vec{p}_i\}) = K + V$

SAMPLING OF SUCH INITIAL COND W/
NEWTON OR LANGEVIN DYN. REMAIN IN EQUIL

FINITE RELAXATION TIME

$\rho(\{\vec{x}_i(0), \vec{p}_i(0)\})$ DIFF FROM ρ_{eq}.

eg. PREPARE AT T_0 EVOLVE AT T w/ LANGEVIN OR EVOLVE WITH NEWTON

THERE CAN BE A FINITE RELAXATION TIME t_{eq} SUCH THAT

FOR $t \gg t_{eq}$

$$\rho(\{\vec{x}_i(t), \vec{p}_i(t)\}; t) \longrightarrow \rho_{eq}(\{\vec{x}_i, \vec{p}_i\})$$

AND ERGODICITY APPLIES

THANKS TO BATH OR INTERNAL INTERACTIONS

OTHER IMPORTANT PROPS OF EQUIL DYN

WE AVERAGED OVER TIME IN $\bar{A}_t$
(WE CAN ALSO AVERAGE OVER i.c.
IF WE SAMPLE OVER IC. BUT NOT NECESSARY)

ONE-TIME OBSERV APPROACH CONST.

$$\langle \bar{A}_t \rangle_{ic} \longrightarrow \text{CONST}$$

MANY-TIME OBS. BECOME TTI

$$\langle A(t)\, A(t') \rangle_{ic} \longrightarrow f(t-t')$$

$$\langle A(t_1) \dots A(t_n) \rangle_{ic} \longrightarrow f(t_1 - t_2, \dots, t_{n-1} - t_n)$$

OR STATIONARITY

FDT - WILL COME LATER

DIVERGING RELAX. TIMES

- THE SIMPLEST EXAMPLE · OVERDAMPED

NO POTENTIAL $\longrightarrow$ FLAT

$$\gamma \dot{x} = \xi$$

WHITE NOISE

$$\langle x^2(t) \rangle_\xi \longrightarrow 2Dt$$

DIFFUSION

NOT A CONST.

- CO-EXIST OF EQUIL VARIABLES AND OUT OF EQUIL. ONES

EXERCISE

$$m\ddot{x} + \gamma \dot{x} = \xi$$

$$\langle v^2(t) \rangle \longrightarrow \frac{k_B T}{m}$$

EQUIL (v)

$$\langle x^2(t) \rangle \longrightarrow 2Dt$$

DIFF (x)

MACROSCOPIC SYSTEMS $\longrightarrow$ QUENCHES

- MACROSC. SYST
  QUENCH TO A CRITICAL POINT
  CRITICAL DYN.

  SLOW QUENCH ACROSS A CRIT POINT
  KIBBLE-ZUREK

  FAST QUENCH INTO A SYMM BROKEN PHASE COARSENING
  (2nd PART OF LECTURE)
  RAMGOPAL AGRAWAL

- IN GLASSY SYSTEMS $t_{eq}(N) \xrightarrow[N\to\infty]{} \infty$
  AND ERGODICITY BROKEN.

  TOO LONG RELAXATION TIMES

  RELATED TO THE COMPLEX LANDSCAPES THAT BIROLI DISCUSSED

OTHER OPTIONS TO VIOLATE ERGODICITY

- EXTERNAL DRIVES
  NON POTENTIAL $\vec{F}_i \neq -\vec{\nabla}_i V$
  NON-RECIPROCAL FORCES $\vec{F}_{i\to j} \neq \vec{F}_{j\to i}$
  t-DEP FORCES $\vec{F}_i(t)$ LORENZO CORREALE
  e.g. CURRENTS
  (I won't discuss them, cfr GIAMARCHI's)

- TOO MANY CONST OF MOTION $\{I_\mu\}$
  INTEGRABLE SYSTEMS
  in NEWTON'S DYN. I'll GIVE AN EX.
  cfr MBL

ALL THESE CASES ARE FOUND / DESCRIBED WORKING WITH MODELS OF p-SPIN KIND

DYNAMIC MEAN-FIELD THEORY

cfr THE KOTLIAR-GEORGES ONE

V POTENTIAL ENERGY

$$V(\{x_i\}) = \sum_{i_1 \neq \dots \neq i_p} J_{i_1 \dots i_p} x_{i_1} \cdots x_{i_p}$$

$$\sum_{i=1}^{N} x_i^2 = N \qquad x_i \in \mathbb{R} \qquad p \text{ INTEGER}$$

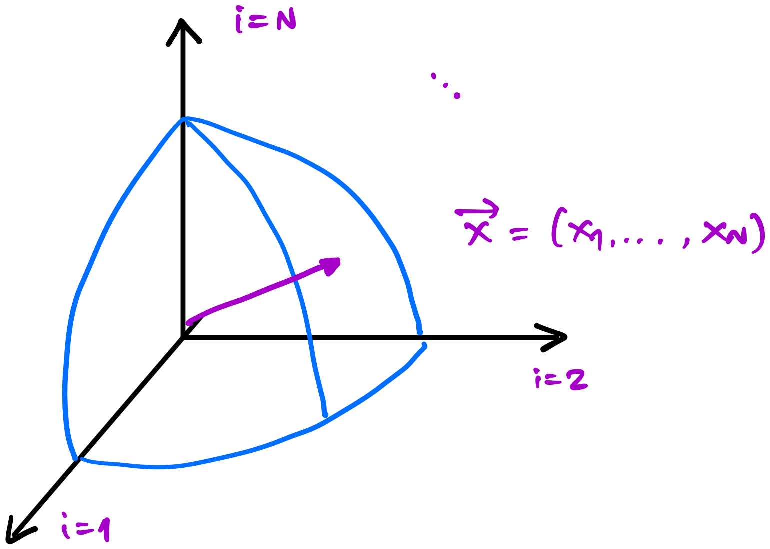
$J_{i_1 \dots i_p}$ iid GAUSSIAN DIST. $[J_{i_1 \dots i_p}] = 0$

$[J^2_{i_1 \dots i_p}] \propto \frac{1}{N^{p-1}}$ GOOD THERM. LIMIT

YOU SAW THESE MODELS IN G. BIROLI'S LECTURES

- ONE CAN USE NEWTON OR LANGEVIN DYN.
- ONE CAN QUANTIZE IT INTRODUCING $\{\hat{p}_i\}$
- FOR THOSE WHO KNOW, VERY SIMILAR TO SYK

THEY ARE MEAN-FIELD IN THE SENSE THAT THEY ARE FULLY CONNECTED (ALL i INTERACT WITH ALL OTHER j, ETC.)



COMING FROM SPIN-GLASS THEORY

NEXT GLASS REPORT

NOW MUCH MORE

- CAN BE THOUGHT OF AS THE MOTION OF A PARTICLE ON A SPHERE UNDER A VERY WEIRD POTENTIAL ENERGY

e.g. $p=2$: $-\frac{1}{2} \sum_{i \neq j} J_{ij} x_i x_j =$

$$= -\frac{1}{2} \sum_{\mu=1}^{N} \lambda_\mu x_\mu^2$$

HARM. POTENTIALS

$x_\mu \equiv \vec{x} \cdot \vec{v}_\mu$

$\lambda_\mu , \vec{v}_\mu$ EIGENVALUES & EIGENVECTORS OF J_{ij}

- WITH NEWTON DYN: NEUMANN'S INTEG. MODEL
- WITH LANGEVIN DYN: DESCRIBES COARSENING OR DOMAIN GROWTH.

- OR AS THE p-SPIN SPHERICAL MODEL. $p \geq 3$ REALIZING THE RFOT (BIROLI'S LECT.)

END INTRODUCTORY PART (1h)

PHASE ORDERING KINETICS

DYN ACROSS 2nd ORDER PHASE TRANS.

REFS BRAY, PURI, KRAPIVSKY - REDNER, ONUKI, LFC

MODELS (OPEN, COUPLED TO A BATH)

TAKE A CLASSICAL ISING MODEL W/ STOCH DYN

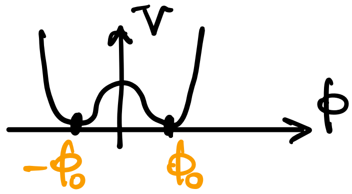
$$H_J[\{s_i\}] = -J \sum_{\langle ij \rangle} s_i s_j$$

OR IT'S t-DEP. GINZBURG LANDAU VERSION

$$\frac{\partial \phi(\vec{x},t)}{\partial t} = \nabla^2 \phi(\vec{x},t) - \frac{\delta V}{\delta \phi(\vec{x},t)} + \xi(\vec{x},t)$$

$$\langle \xi(\vec{x},t) \rangle = 0 \qquad \langle \xi(\vec{x},t)\, \xi(\vec{y},t') \rangle = 2\gamma k_B T\, \delta(t-t')$$

$$V(\phi) = \frac{r}{2}\phi^2 + \frac{\lambda \phi^4}{4}$$



INSTANTANEOUS QUENCH

RANDOM I.C. $s_i(0) = \pm 1$ prob $= 1/2$ OR

$\phi(\vec{x},t)$ GAUSSIAN pdf ZERO MEAN

AT TIME $t=0$ A PARAMETER IS SUDDENLY CHANGED

$T_0 \to \infty \longrightarrow T$ BATH

$J_0 = 0 \longrightarrow J$ IN POTENTIAL

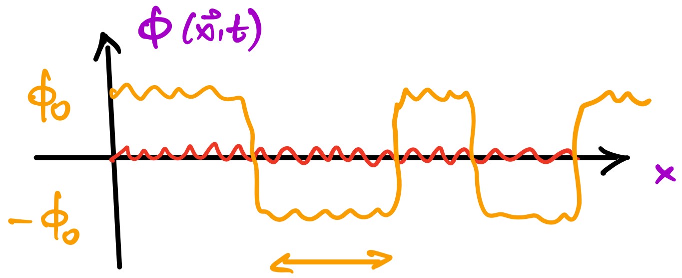
$r_0 > 0 \longrightarrow r < 0$ ENERGY

COULD SET THE EVOLUTION EXACTLY AT CRITICALITY OR IN THE SYMMETRY BROKEN PHASE

FOCUS ON 2nd CASE. BOTH J AND $\nabla^2\phi$ PENALIZE CHANGES IN $\{s_i\}$ OR ϕ NEARBY

MECHANISM $\longrightarrow$ ERASE WALLS $\Rightarrow$ DECREASE ENERGY

$\Rightarrow$ FM ORDERING BELOW A CRIT. POINT, PHASE WITH SPONT SYMM BREAK.



R(t) GROW IN TIME

AND CRIT. FLUCT AT CRIT. POINT $\Rightarrow$

- GROWTH OF CRIT EQUIL REGIONS AT CRIT POINT
- " " ORDERED DOMAINS IN ORD. PHASE

SIZE OF FLUCT DEPEND ON TEMP. T

THE EXCESS ENERGY WRT EQUIL ONE IS CONCENTRATED ON THE INTERFACES

FROM t-DEP RG, SIMU:

HOHENBERG - HALPERIN

$$R_c(t) \sim t^{1/z_c} \qquad z_c \gtrsim 2 \quad \text{CRITICALITY}$$

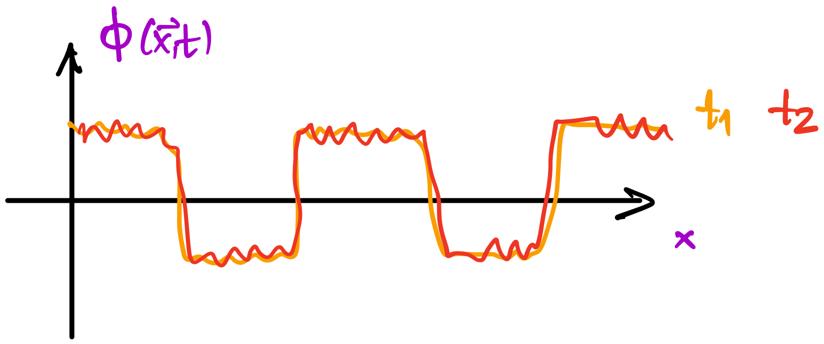
FROM SCALING ARGUMENTS, SIMU, APPROX:

$R(t) \sim t^{1/2}$ $\quad z = 2$ $\quad$ ALLEN CAHN (70's)

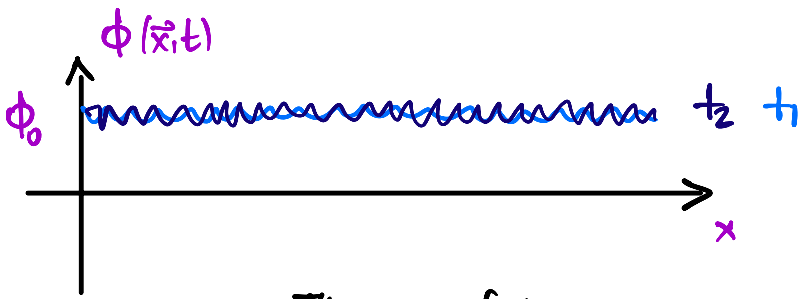
BRAY (90's)

PHENOMENON

IF ONE COMPARES THE SYST CONFS AT TWO TIMES SUCH THAT THE WALLS HAVE NOT MOVED $\Rightarrow$ ONLY THERMAL FLUCT WITHIN DOMAINS



AS IN EQUIL DYN THE WALLS ARE "UNSEEN"



THERMAL FLUCT. IN AN ORDERED SYST

QUANTITATIVE MEASUMENTS

ORDER PARAMETER

$$m(t) = \frac{1}{N} \sum_{i=1}^{N} \langle s_i(t) \rangle = 0$$

$$\tilde{\phi}(t) = \frac{1}{V} \int d^d r \, \langle \phi(\vec{x},t) \rangle = 0$$

TWO REASONS: THERMAL & SPATIAL AVER.

USELESS!

SELF - CORRELATION FUNCTION

$$C(t_1,t_2) \equiv \frac{1}{N} \sum_{i=1}^{N} \left\langle s_i(t_1)\, s_i(t_2) \right\rangle$$

AVERAGE OVER THERMAL NOISE AND/OR i.c.

$$C(t_1,t_2) = \frac{1}{V} \int d^d r \left\langle \phi(\vec{x},t_1)\, \phi(\vec{x},t_2) \right\rangle$$

LOCAL IN SPACE, NON-LOCAL IN TIME

INTERESTING

DISTINGUISH TIMES t_1, t_2

t_2 SUFFICIENTLY LARGE TO DEVELOP DOMAINS

$$\underbrace{\lim_{t_2 \to \infty}}\; C(t_1, t_2)$$

IN PRACTICE MEANS "LARGE"

$t_1 - t_2$ "FINITE" WRT t_2

SO THAT

DOMAINS DON'T MOVE

THEN,
$$\lim_{t_2 \to \infty} C(t_1, t_2) = C_{eq}(t_1 - t_2)$$

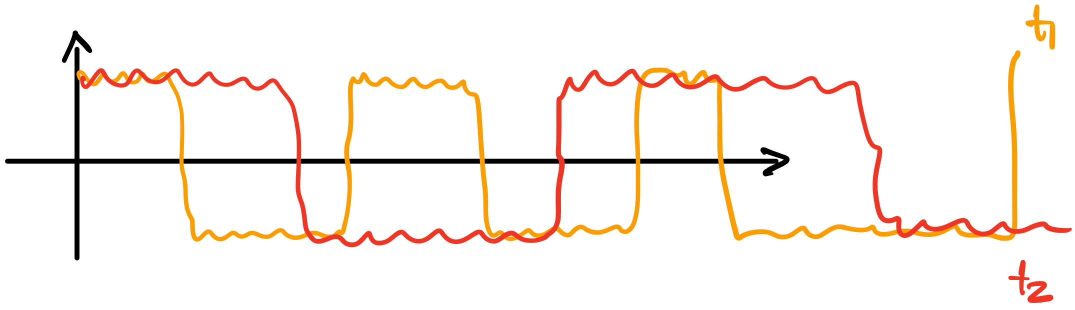
NB. IT'S STATIONARY

IF, ONE FURTHER TAKES $t_1 - t_2 \to \infty$

$$\lim_{t_1 - t_2 \to \infty} \lim_{t_2 \to \infty} C(t_1, t_2) = \phi_0^2$$

$$\lim_{t_1 - t_2 \to \infty} C_{eq}(t_1 - t_2) = \phi_0^2$$

HOWEVER, IF ONE LET'S THE WALLS MOVE
THERE IS A FURTHER DECORRELATION



<u>DYNAMIC SCALING</u> <u>IN THIS</u> $t_1 \simeq t_2$ <u>LIMIT</u>

SIMILAR TO EQUIL SCALING

$\exists$ A SINGLE GROWING LENGTH $R(t)$ WHICH
FOR $a \ll R(t) \ll L$ SCALES ALL CORR. fct

$$\lim_{t_1, t_2 \to \infty} C(t_1, t_2) = C_{ag}(t_1, t_2) = f_c\left(\frac{R(t_1)}{R(t_2)}\right)$$

$R(t_1) / R(t_2)$
FINITE

IT'S NOT STATIONARY

AGING

$$\partial_{t_1} C_{ag}(t_1,t_2) = f_c'\left(\frac{R(t_1)}{R(t_2)}\right) \frac{\dot{R}(t_1)}{R(t_2)}$$

$$= \underbrace{f_c'\left(\frac{R(t_1)}{R(t_2)}\right) \frac{R(t_1)}{R(t_2)}}_{g\left(\frac{R(t_1)}{R(t_2)}\right)} \frac{\dot{R}(t_1)}{R(t_1)}$$

$$g\left(\frac{R(t_1)}{R(t_2)}\right) \frac{d \ln R(t_1)}{dt_1}$$

if $R(t_1) \sim t_1^{1/2} \Longrightarrow \frac{1}{2}\frac{1}{t_1} \xrightarrow[t_1 \to \infty]{} 0$

- GIVEN t_2 FIXED, THE DERIVATIVE OF $C(t_1,t_2)$ WRT t_1 GETS SMALLER AND SMALLER

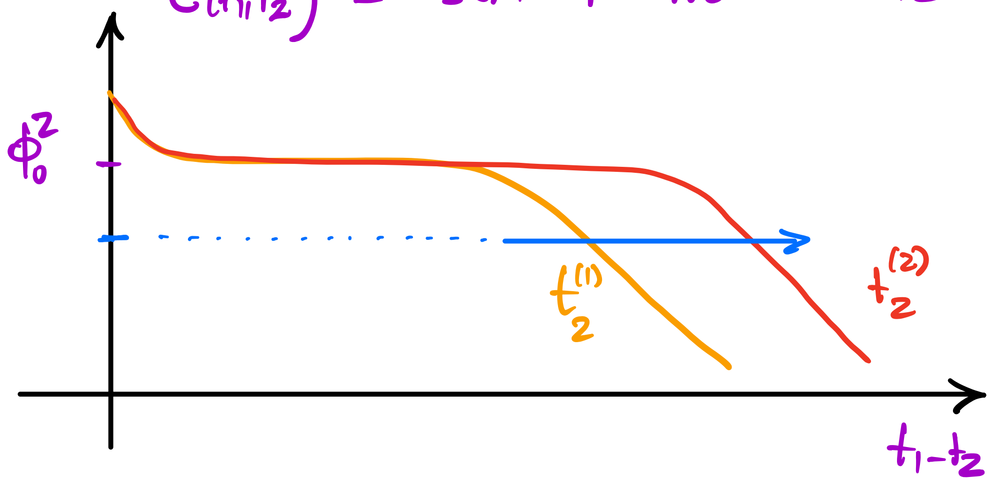
$$\partial_{t_2} C_{ag}(t_1,t_2) = f_c'\left(\frac{R(t_1)}{R(t_2)}\right) \frac{R(t_1)}{R^2(t_2)} \dot{R}(t_2)$$

$$= g\left(\frac{R(t_1)}{R(t_2)}\right) \frac{d \ln R(t_2)}{dt_2}$$

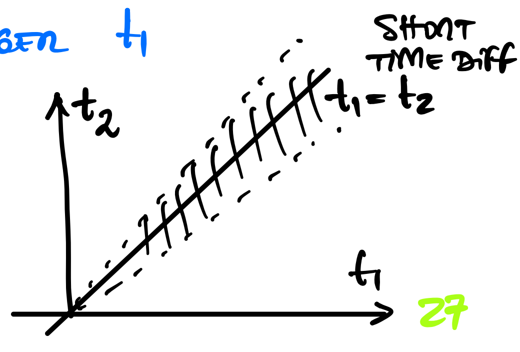
if $R(t_2) \sim t_2^{1/2} \implies \frac{1}{2}\frac{1}{t_2} \xrightarrow[t_2 \to \infty]{} 0$

- GIVEN t_1 FIXED, THE DERIVATIVE OF $C(t_1,t_2)$ WRT t_2 GETS SMALLER AND SMALLER

$C(t_1,t_2)$ = SUM OF TWO CONTRIB.



TO REACH THE SAME C VALUE BELOW ϕ_0^2 ONE NEEDS TO GO TO A LONGER t_1 FOR t_2 LONGER.



A WAY TO WRITE IT

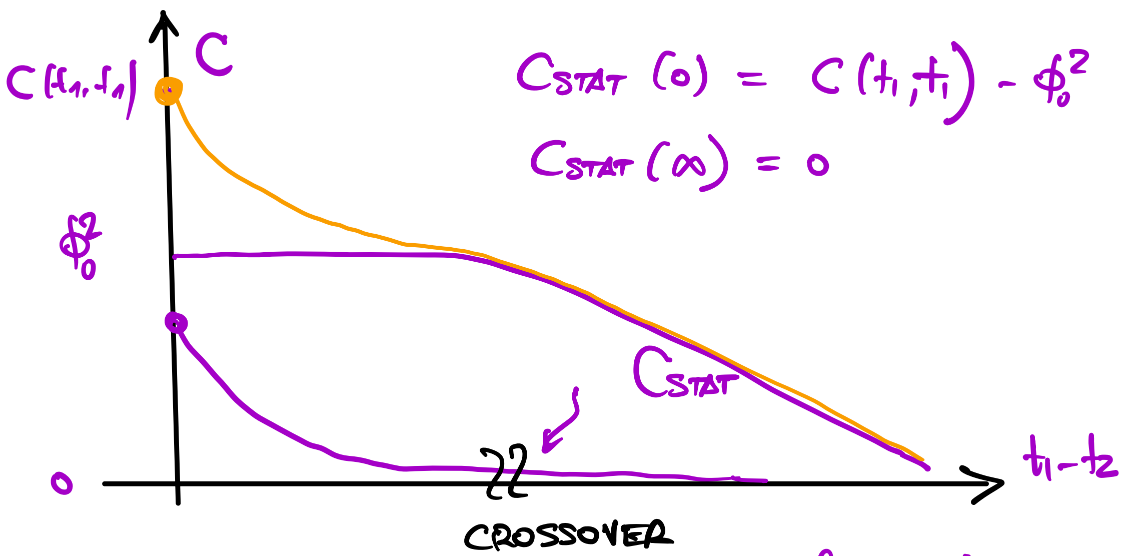
$$C(t_1, t_2) = C_{STAT}(t_1 - t_2) + C_{ag}(t_1, t_2)$$

$$\lim_{t_1 - t_2 \to \infty} C_{STAT}(t_1 - t_2) = 0$$

TWO POSITIVE CONTRIB IN FULLY RELAX. SYST.

$$\lim_{t_2 \to t_1^-} C_{ag}(t_1, t_2) = \phi_0^2$$

$$f_c(1) = \phi_0^2$$

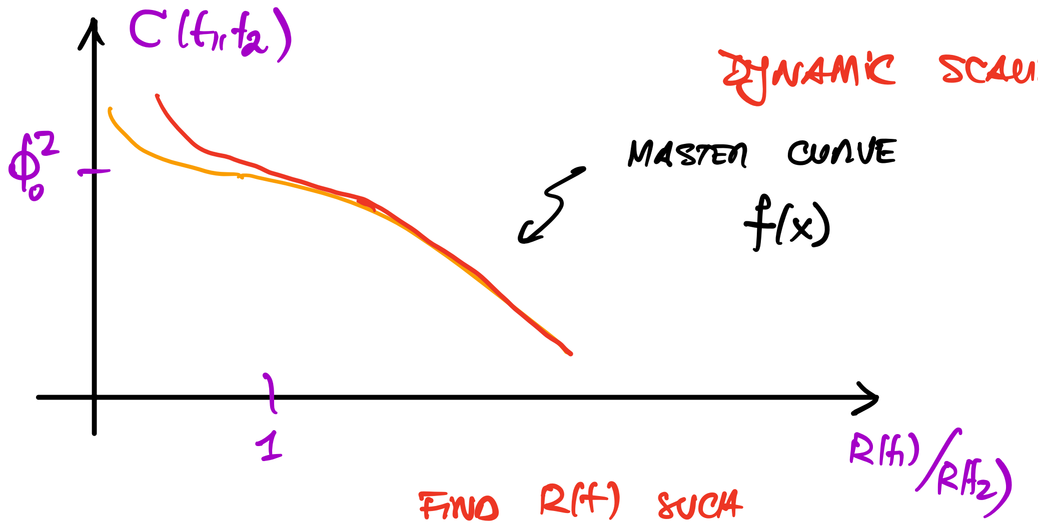


$$C_{STAT}(0) = C(t_1, t_1) - \phi_0^2$$

$$C_{STAT}(\infty) = 0$$

$$f_c(x \to \infty) = 0$$

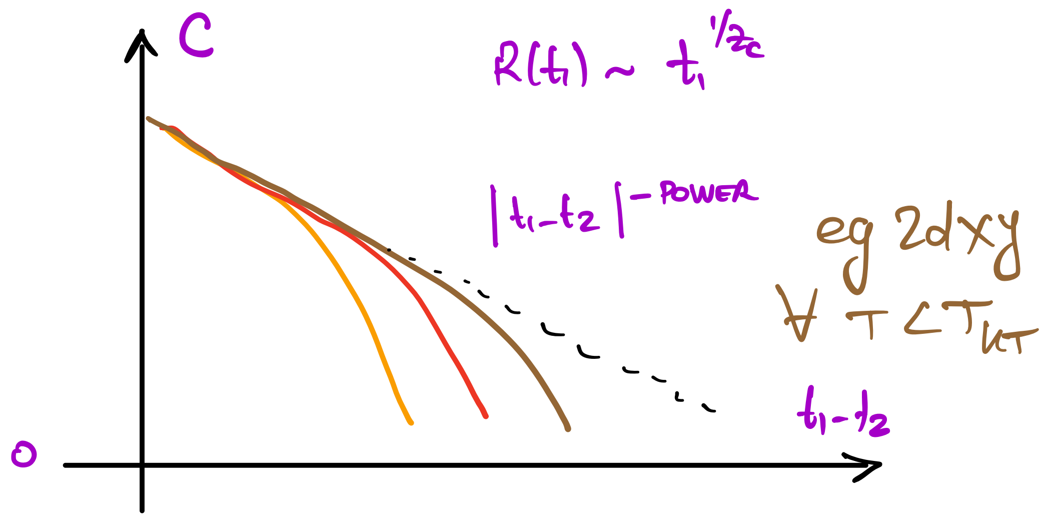
DYNAMIC SCALING



FIND $R(t)$ SUCH THAT SCALING WORKS

QUENCHES AT CRITICAL POINT. WHAT IS DIFF.?

SINCE $\phi_0 = 0$ THE APPROACH TO THE PLATEAU IS REPLACED BY A POWER LAW DECAY TOWARDS ZERO.



CONCEPTS

- CO- EXISTENCE OF FAST (VELOCITIES) AND SLOW (POSITIONS) VARIABLES

- SEPARATION OF TIME SCALES FOR SLOW VARIABLES IN MACROSC. SYST.

- AGING OF SLOW VARIABLES

- DYN. SCALING: (BEYOND EQUIL FLUCT.)
  A SINGLE GROWING LENGTH $R(t)$
  SCALING ALL CORR FUNCT.
  ALSO OBTAINED FROM $C(r,t)$

VERY NICE TESTS OF SCALING IN RECENT EXPS
DE ALMEIDA & TAKEUCHI

ALL THESE QUALITATIVE FEATURES ARE SHARED BY GLASSY SYST.

APART FROM

- $R(t)$ MAYBE NOT UNIQUE $\Longrightarrow$
- MANY REGIMES &
  NOT CLEAR TO "SEE" $R(t)$

AND, OF COURSE, THE QUANTITATIVE ONES

e.g. $R(t)$ DOESN'T NEED TO BE $t^{1/2}$

BACK TO QUENCHES BELOW CRIT POINT.

THINK IN TERMS OF WAVE-VECTORS

UNIFORM ORDERING $\Rightarrow$ $\phi(\vec{x},t) = \phi_0$ (APART FLUCT.)

IN FOURIER MEANS $\tilde{\phi}(\vec{k},t) = \phi_0 \, \delta_{\vec{k},0}$

(ALGEBRAIC) GROWTH OF ORDER MEANS

(ALGEBRAIC) GROWTH OF $\vec{k}=0$ MODE $\langle \tilde{\phi}(\vec{k}=0,t) \rangle$

MEAN-FIELD MODEL

SPHERICAL $p=2$

RECALL BIROLI'S p-SPIN

$$H = -\frac{1}{2} \sum_{i \neq j} J_{ij} s_i s_j = -\frac{1}{2} \sum_{\mu} \lambda_\mu x_\mu^2$$

+ CONSTRAINT

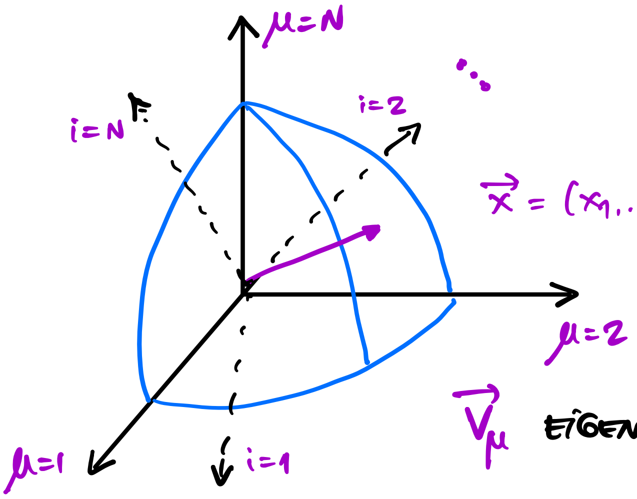
MEAN-FIELD IN THE SENSE THAT FULLY CONNECTED

(ALL i INTERACT WITH ALL OTHER j, ETC.)

COMING FROM SPIN-GLASS THEORY $s_i \in \mathbb{R}$

NEXT GLASS REPORT

NOW MUCH MORE



$\vec{x} = (x_1, \dots, x_N)$

$\vec{V}_\mu$ EIGENVECTORS OF J_{ij} (GOE)

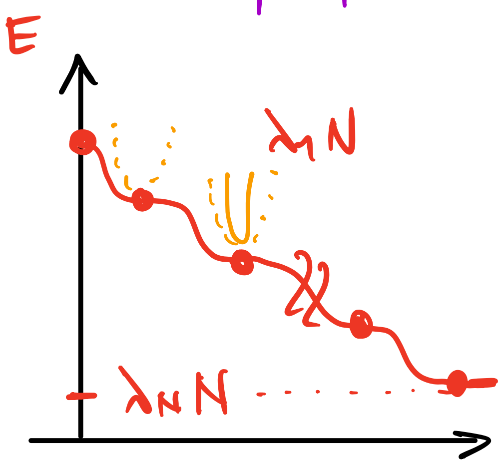
$$\rho(\lambda_\mu) = \frac{1}{\sqrt{2\pi J^2}} \sqrt{4J^2 - \lambda_\mu^2}$$

EIGENVALUES

$x_\mu = \vec{x} \cdot \vec{V}_\mu$ $\qquad \langle x_\mu \rangle_{eq} = 0 \quad \forall \mu \quad T \geq T_c$

ENERGY LANDSCAPE $\qquad \langle x_N \rangle_{eq} = \sqrt{(1 - T/J) N} \quad T < T_c$

$\vec{x} = \vec{V}_\mu \quad \mu = 1 \dots N$ $\qquad$ STAT. STATES $\qquad$ (BEC)



$\vec{V}_{\mu = N = \max}$ $\qquad$ ABS. MINIMUM

$\vec{V}_{\mu = N-1}$ $\qquad$ 1st EXC. W/ ONE UNST DIR.

$\vdots$

$\vec{V}_1$ $\qquad$ MAX.

LANGEVIN DYN

ALMOST INDEP HARMONIC OSCILLATORS

$$\dot{x}_\mu = -\lambda_\mu x_\mu + z_t x_\mu + \xi_\mu$$

z_t FIXED FROM $\frac{1}{N}\sum_\mu \langle x_\mu^2(t)\rangle = 1$

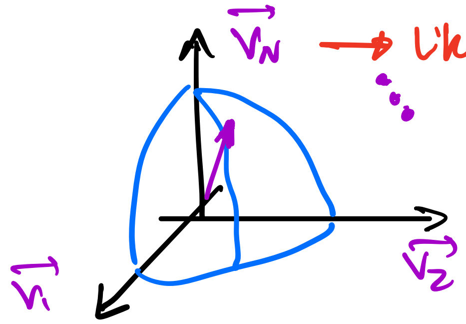
SOLVABLE ANALYTICALLY $x_\mu(t; z_t)$

$\langle x_{\mu_{MAX}} \rangle$ GROWS AS A POWER LAW t^{POWER}

WHILE ALL OTHER MODES TEND TO VANISH EXP.

$$\langle x_\mu \rangle \sim t^{POWER} e^{-(\lambda_N - \lambda_\mu)t}$$

PROJECTION ON MAX EIGENVECTOR GROWS IN t
(& OTHERS DECREASE)
BUT DOESN'T REACH
$\langle x_N \rangle_{eq}$ IN FINITE TIMES



$\vec{V}_N \longrightarrow$ LIKE $k=0$ MODE

$C(t_1,t_2) \longrightarrow$ SUM OF STAT / EQUIL CONT + AGING TERM

$$C_{STAT}(t_1-t_2) + \underbrace{C_{ag}(t_1,t_2)}_{f_c(t_1/t_2)}$$

$R(t) \sim t^{1/2}$ ALL THIS FOUND ANALYTICALLY EVEN SCALING

NOTE THERE IS NO SPACE IN THIS PROBLEM
CAN'T REALLY FIX THE POWER THIS WAY
BUT IT DOESN'T MATTER.

LFC & DEAN '95

A BETTER MF $N\phi^4$ IN LARGE N LIMIT
BRAY '94
SOME SPATIAL INFO
$\phi(\vec{r},t)$

WHAT CHANGES QUANTUM MECHANICALLY?

THE FUNCTIONAL FORM (TIME DEP.)
OF THE $\Gamma(t-t')$ IS IN GRAL DIFF.
ALBERTO ROSSO'S SEMINAR

- COULD HAVE LONG-TIME TAILS DEPENDING ON BATH MODEL
- COULD ALSO MODIFY THE CRITICAL PARAM.

BUT, IN LIMIT OF $\simeq \Gamma_{class}$:
SEPARATION OF TIME SCALES ✓

THE EQUIL DYN FEELS QUANTUM FLUCT $\Rightarrow$

- OSCILLATIONS — LFC + MÜLLER '23

- AGING CONTRIB. AS CLASSICAL
  $\sim$ DE-COHERENCE OF LARGE OBJECTS

MORE ON RESPONSES LATER

WHAT CHANGES IF DE-COUPLING FROM BATH?

D. BARBIER, LFC, G. LOZANO, N. NESSI & A. TARTAGLIA 15y)

QUITE A LOT CAN CHANGE. ENERGY CONSTANT BUT NOT ONLY LOOK AT $p=2$ VERSION

$$H = K + V$$

$$= \sum_{\mu=1}^{N} \frac{p_\mu^2}{2m} + \frac{1}{2} \sum_{\mu} \lambda_\mu x_\mu^2$$

& SPH. CONSTRAINT

N CONST OF MOTION $\{I_\mu\}_{\mu=1,\dots,N}$

INTEGRABLE !

$$I_\mu = x_\mu^2 + \frac{1}{mN} \sum_{\nu(\neq\mu)} \frac{(p_\mu x_\nu - p_\nu x_\mu)^2}{\lambda_\nu - \lambda_\mu}$$

ONE CAN DO QUENCHES BY CHANGING $\{\lambda_\mu\}$ FOR EX

$\mathcal{S}_{eq}(\{x_\mu(0)\}, T_0)$ EQUIL UNDER BATH

CUT BATH AT $t=0$

EVOLVE WITH NEWTON / HAMILTON $\neq H$ FROM H_0

$\{\lambda_\mu^{(0)}\} \to \{\lambda_\mu\}$

WHAT HAPPENS ? $\lim_{t\to\infty} \lim_{N\to\infty}$?

EQUIL. TO GGE $\qquad Z_{GGE} = \sum_{conf} e^{-\sum_\mu \gamma_\mu I_\mu}$

WITH γ_μ FIXED FROM

$I_\mu(0^+) = I_\mu(t) = \langle I_\mu \rangle_{GGE}$

ALL CALCULABLE ANALYTICALLY !

CHECKED BY STUDYING

$$\lim_{t\to\infty} \lim_{\tau\to\infty} \lim_{N\to\infty} \overline{\langle x_\mu^2(t) \rangle_{ic}} =$$

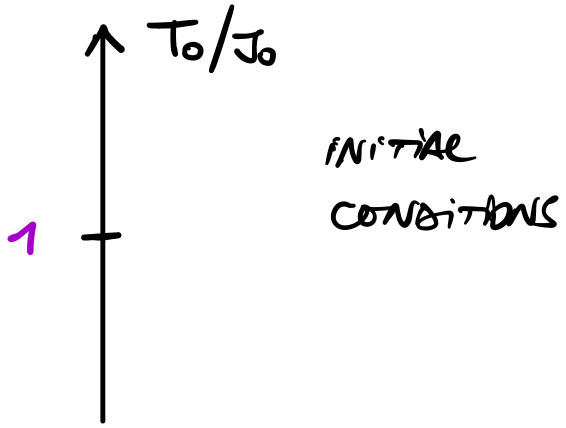
$$= \lim_{N\to\infty} \langle x_\mu^2 \rangle_{GGE}$$

ETC.

PHASE TRANSITION IN INITIAL CONDITIONS

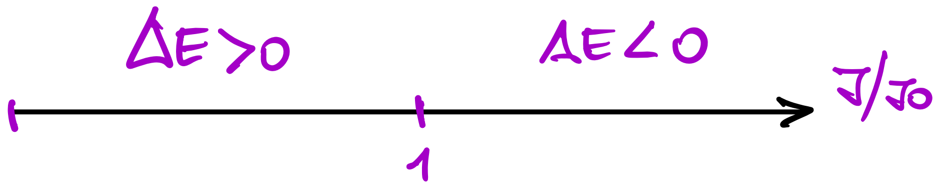
$$\rho_{eq}(\{p_\mu(0), x_\mu(0)\}) = e^{-\beta_0 H_0}$$

$\{\lambda_\mu^{(0)}\}$



EVOLVE WITH H WITH $\{\lambda_\mu\}$

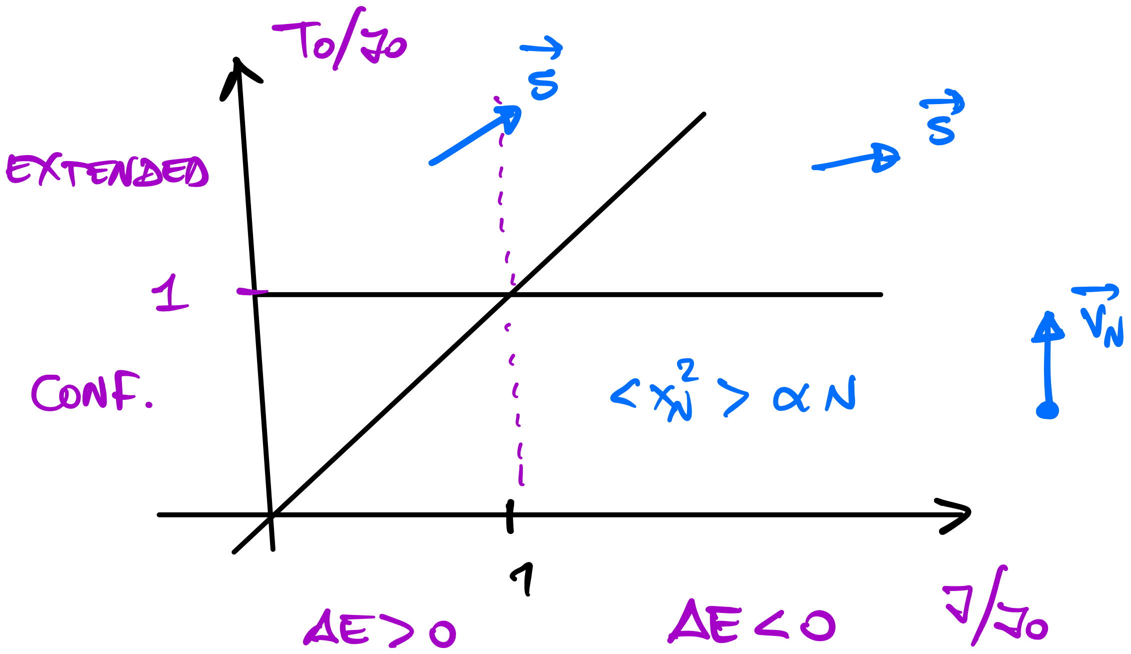
$$\lambda_\mu = J/J_0 \, \lambda_\mu \quad \text{(SIMPLEST QUENCH)}$$



NO QUENCH

ENERGY CHANGE

SCHWINGER-DYSON DMFT EQS.



$$\lim_{N\to\infty} \overline{\langle x_\mu^2 \rangle_{ic}} = \lim_{N\to\infty} \langle x_\mu^2 \rangle_{GBE}$$

NUMERICS FINITE BUT LARGE N

SOME ANALYTICS TOO

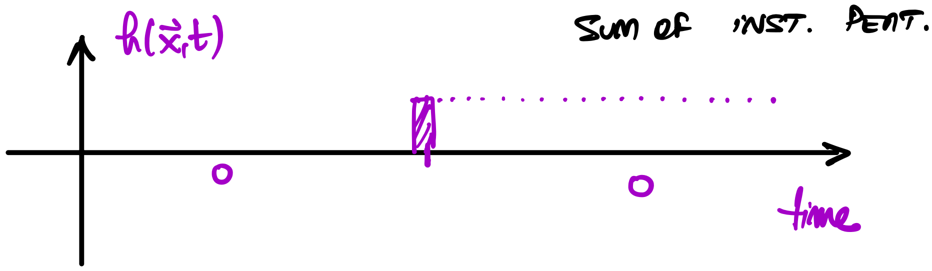
TRICKS LIKE HUBBARD-STRATO & RANDOM MATRICES

30 min

ANOTHER WAY TO PROBE A SYSTEM

THROUGH AN EXTERNAL PERTURBATION

APPLY A KICK / FIELD



$$\frac{\partial\phi(\vec{x},t)}{\partial t} = \nabla^2\phi(\vec{x},t) - \frac{\delta V_h}{\delta\phi(\vec{x},t)} + \xi(\vec{x},t)$$

$$V_h = V - \int d^d r \; h(\vec{r},t)\,\phi(\vec{r},t)$$

SIMPLEST LINEAR COUPLING.

$$R(t_1,t_2) = \int \left.\frac{\delta\langle\phi(\vec{x},t_1)\rangle}{\delta h(\vec{x},t_2)}\right|_{h=0} d^d r \; \frac{1}{Vol}$$

$$\chi(t_1,t_2) = \int_{t_2}^{t_1} dt' \; R(t_1,t')$$ INTEGRATED RESP.

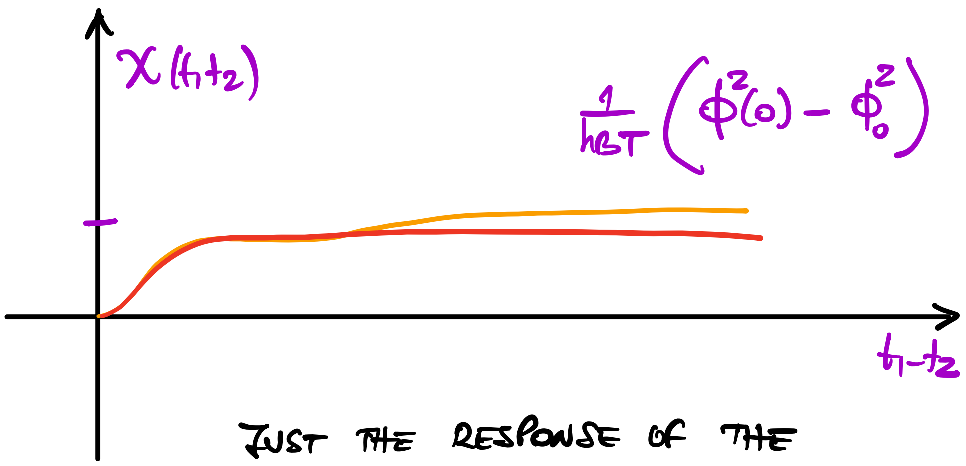
IN EQUIL. $R(t_1,t_2) = \frac{1}{k_BT} \partial_{t_2} C(t_1,t_2) \qquad t_1 > t_2$

$$\chi(t_1,t_2) = \frac{1}{k_BT}\left[C(t_1,t_1) - C(t_1,t_2)\right] \qquad \underline{\text{FDT}}$$

LINEAR RELATION

OUT OF EQUIL NO REASON TO HOLD !

IN COARSENING MODELS



JUST THE RESPONSE OF THE INTERIOR OF DOMAINS, AS IN EQUIL.

EQUIL. FDT SATISFIED FOR $t_1 - t_2$ NOT TOO LONG

VIOLATED LATER SINCE NO OUT OF EQ CONTRIB. TO χ WHILE THERE'S ONE TO C

HOW DOES THE RESPONSE BEHAVE
IN $p=2$ MODEL W/ LANGEVIN DYN ?

$$\gamma\, \dot{x}_\mu = + \lambda_\mu x_\mu - z(t)\, x_\mu + h_\mu + \xi_\mu$$

$$R(t,t') \equiv \frac{1}{N} \sum_\mu \frac{\partial \langle x_\mu(t) \rangle^{(h)}}{\partial h_\mu(t')} \Bigg|_{h \to 0}$$

$$R(t,t') = R_{STAT}(t-t') + t^{-3/2} f_R(t/t')$$

$\downarrow$ RATHER FAST !

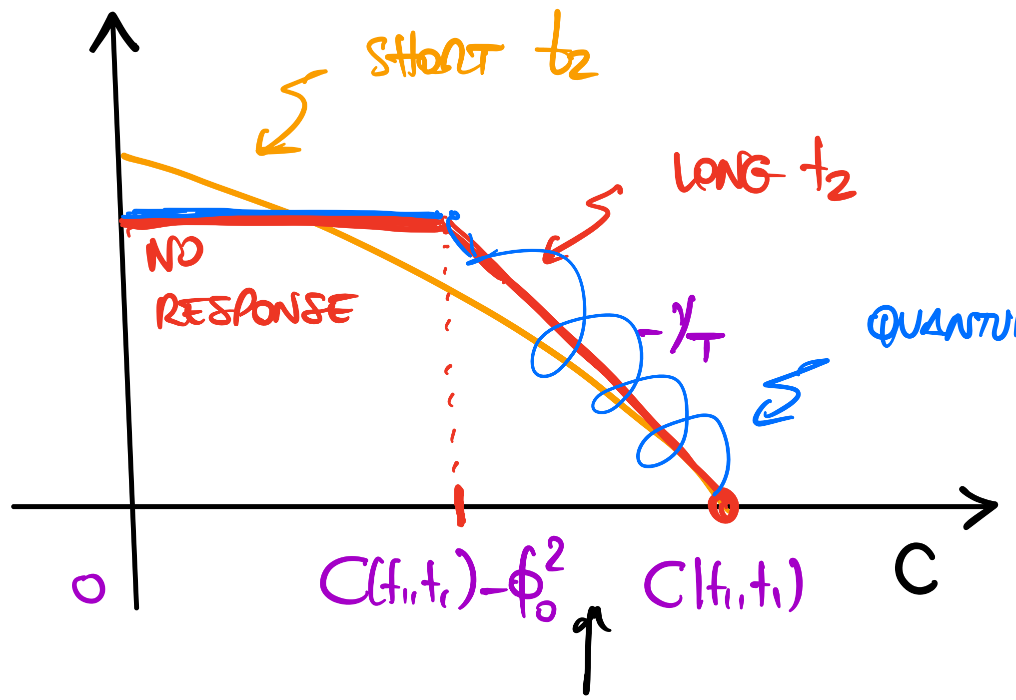
$$\chi(t,t') = \chi_{STAT}(t-t') + t^{-1/2} \chi_c(t/t')$$

DIFFERENT IN GLASSY PROBLEMS

PARAMETRIC CONSTRUCTION

$$\lim_{t_2\to\infty} \chi(t_1,t_2) = f_\chi(C)$$

$C(t_1,t_2)$ FINITE



RESPONSE & CORR } WITHIN DOMAINS

QUANTUM FDT OK IN EQ. REGIME

ADD TO CONCEPTS

- RESPONSE & CORR NOT OBVIOUSLY RELATED
- FDT BROKEN IN AGING REGIME
- A LOT OF KNOWLEDGE TO BE GATHERED FROM THE X BEHAVIOUR AND RELATION WITH C

(NO TIME TO DEVELOP HERE)

DIFFERENT RESPONSE OF GLASSES & COARS.

30 min

p-SPIN MODELS IN GENERAL

BASICALLY TWO METHODS TO STUDY THEM ON AVER OVER QUENCHED DISORDER

- SCHWINGER-DYSON EQS ON

$$C(t_1,t_2) \qquad R(t_1,t_2)$$

BOTH $\left\{ \begin{array}{l} \text{CLASSICAL OR QUANTUM} \\ \text{OPEN OR CLOSED} \\ \text{WHATEVER } p \end{array} \right.$

- SINGLE VARIABLE EFFECT.
  DIFF. EQ.

  ALSO GENERIC

REFS. LES HOUCHES '02
ANN. REV. COND. MATT '23

SCHWINGER - DYSON EQS. $(t_1 \geq t_2)$

OHMIC FRICTION

$$G_0^{-1}(t_1) \equiv m\,\partial_{t_1}^2 + \gamma\,\partial_{t_1} - z(t_1)$$

$$G_0^{-1}(t_1)\,C(t_1,t_2) = \int^{t_1} dt'\,\Sigma(t_1,t')\,C(t',t_2)$$

$$+ \int_0^{t_2} dt'\,D(t_1,t')\,R(t_2,t')$$

$$+ \beta_0\,D(t_1,0)\,C(t_2,0)$$

INITIAL COND S_{eq}

$$+ \gamma\,k_B T\,R(t_2,t_1)$$

$$G_0^{-1}(t_1)\,R(t_1,t_2) = \int_{t_2}^{t_1} dt'\,\Sigma(t_1,t')\,R(t',t_2)$$

$$+ \delta(t_1-t_2)$$

INTERACTIONS

$$\Sigma(t_1,t_2) = f''(C(t_1,t_2))\;R(t_1,t_2)$$

$$D(t_1,t_2) = f'(C(t_1,t_2))$$

IF QUANTUM MODIFIED & BATH COMES HERE TOO + $(\Sigma_B,\,D_B)$

SINGLE VARIABLE EQ.

(CLOSE TO DMFT IN COND-MATTER)

$$G_0^{-1}(t_1)\, s(t_1) = \int_0^{t_1} dt'\, \Sigma(t_1, t')\, s(t') + \xi(t_1) + \eta(t_1) + \beta_0\, D(t_1, 0)\, s(0)$$

$$\langle \xi(t_1) \rangle = 0 \qquad \langle \xi(t_1)\, \xi(t_2) \rangle = 2\gamma\, k_B T\, \delta(t_1 - t_2)$$

$$\langle \eta(t_1) \rangle = 0 \qquad \langle \eta(t_1)\, \eta(t_2) \rangle = D(t_1, t_2)$$

WITH

$$D(t_1, t_2) = f'(C(t_1, t_2))$$

$$\Sigma(t_1, t_2) = f''(C(t_1, t_2))\, R(t_1, t_2)$$

AS IN SCHWINGER-DYSON EQS.

WHAT IS $f(x)$ HERE? THE RANDOM POTENTIAL CORRELATION

$$\left[V(\{s_i\})\; V(\{s_i'\})\right] = f\left(\frac{\vec{s}\cdot\vec{s}\,'}{N}\right) \propto \left(\frac{\vec{s}\cdot\vec{s}\,'}{N}\right)^p$$

NOW IT'S A PROBLEM IN ANALYSIS OF INTEGRO-DIFF EQS. (OR STOCHASTIC ONES IF 2nd CHOICE TAKEN)

SURPRISINGLY, THE SCHWINGER-DYSON EQS. CAN BE SOLVED

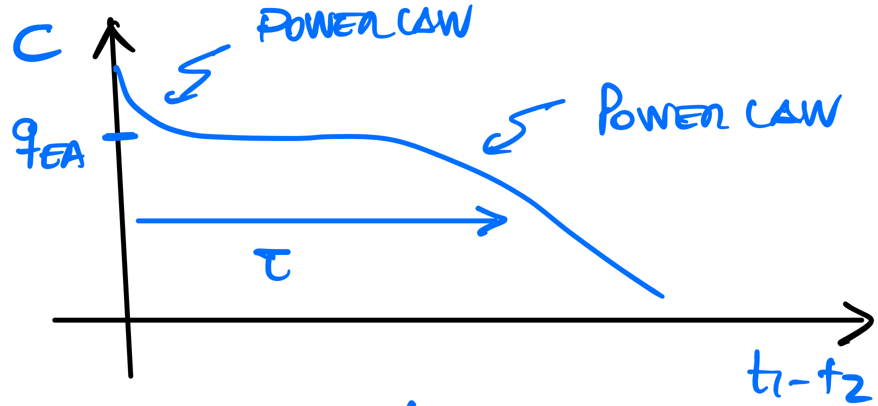
- NUMERICALLY
- ANALYTICALLY FOR $t_2 \to \infty$

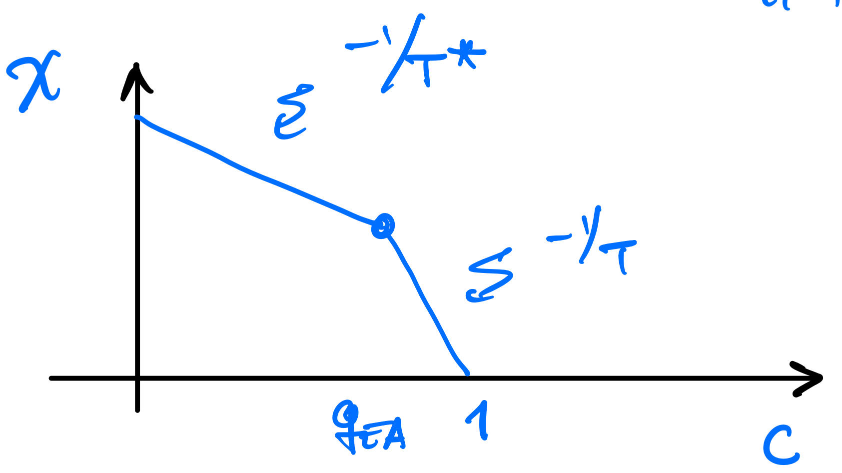
LOT'S OF INFORMATION ABOUT DYN FROM

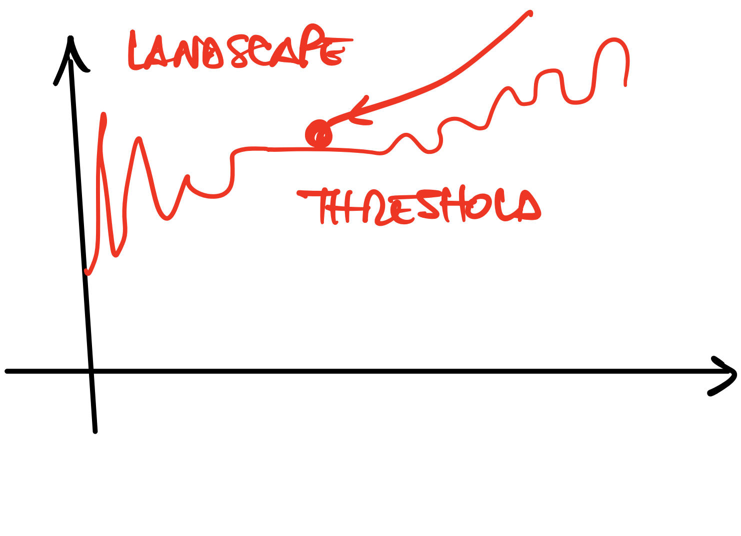
$T_0 > T_d$ EQUIL ABOVE TEMP AT WHICH MINIMA IN LANDSC. START DOMINATING

$T_0 < T_d$ BELOW IT $\Rightarrow$ WITHIN ONE OF THOSE MINIMA

• $T_0 > T_d$

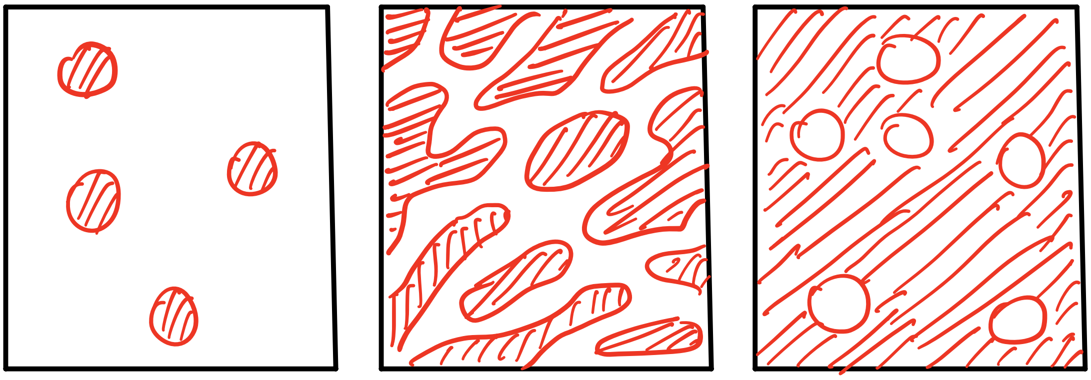






LFC &
KURCHAN 93

TOP VIEW OF LANDSCAPE



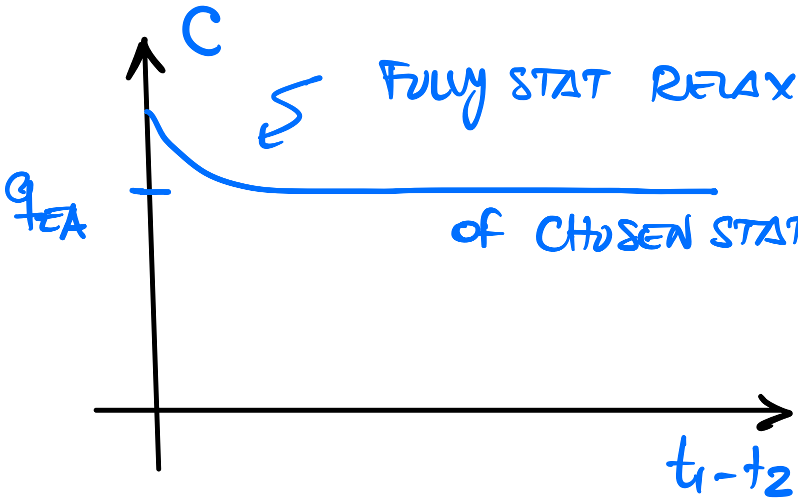
MOUNTAINS | THRESHOLD MESA PLATEAU | ISLANDS

EASY RELAX AT VERY SHORT TIMES | APPROACH TO PLATEAU | ONLY ACCESSIBLE AT $t(N)$ OR SPECIAL i.c.

$$\lim_{t\to\infty}\lim_{N\to\infty}\frac{\langle H(t)\rangle}{N} = e_{th}$$

- $T_0 < T_d$

SPECIAL i.c. IN EQUIL WITHIN A MINIMUM



FULLY STAT RELAX & FDT AT TEMP of CHOSEN STATE T HOLDS

EXTENSIONS

QUANTUM   LFC & LOZANO (LATE 90's)

- THE EQ PART OF RELAX. MODIFIED,
  THE REST NOT.

- PHASE DIAGRAM CHANGED
  → 1st ORDER QUANTUM
  → INCREASE OF ORDER BY
  QUANTUM

LFC & GREMPEL, DA SILVA SANTOS (00's)

QUANTUM W/ STRONG BATH

- ORDERED PHASE IS ENHANCED
  eg. $T_c(T=0, \alpha) \uparrow \quad \alpha \uparrow$
  γ MEASURING COUPLING TO BATH
  LFC, GRAEMPEL, LOZANO, LOZZA, DA SILVA SANTOS (02-04)
  cfr A. ROSSO'S TALK.

NO BATH

LFC, LOZANO & NESSI (17-18)

$p \geq 3$ NOT INTEGRABLE

$\Longrightarrow$ IT CAN EQUIL. AT $e^{-\beta H}$
FOR CERTAIN IC & QUENCHES

BUT CAN ALSO AGE IF SET
AT THRESHOLD