STATISTICAL PHYSICS of CLASSICAL MANY BODY SYSTEMS EVOLVING OUT OF EQUILIBRIUM

GENERALITIES AND AN EXAMPLE COVERING / CAPTURING MANY SALIENT FEATURES

www. lpthe. jussieu. fr /~ leticia

leticia @ lpthe. jussieu. fr

- MANY BODY $N \gg 1$

particles, rotors / spins, agents, birds ...

$N \gg 1 \Rightarrow$ Laws of large numbers $\Rightarrow$

Probability Methods $\Rightarrow$

Statistical Physics

- CLASSICAL ($\hbar = 0$, no operators)

ENERGY CONSERVATION / NO DISSIPATION

CLOSED

Newton - Hamilton
Block
Other

DISSIPATION / NOISE

OPEN

Langevin
Landau - Lifshitz / Gilbert
Brown
Vicsek, other

OUT OF EQUILIBRIUM

→ No usual thermodynamics

→ No usual Stat Mech
  eg. No standard Gibbs ensembles
  No Maxwell Botzmann pdfs

Attn. Systems could evolve in stationary conditions but out of usual equilibrium
NESS

Take a system made by N particles in interaction

MECHANICAL | STATISTICAL $N \gg 1$ | DESCRIPTIONS

Newton-Hamilton
$(\vec{x}, \vec{p})_i$

Liouville

$\rho(\vec{x}, \vec{p}; t) \xrightarrow[\text{STATIONARY ASSUMPTION}]{} \rho(\vec{x}, \vec{p})$

Particle density in phase space

If furthermore usual situation

$\rho_{GB}(\vec{x}, \vec{p})$

Gone from $\mathcal{O}(N)$ equations (Newton-Hamilton) to a field-equation

OBSERVABLES

$$\bar{A} = \lim_{\tau \to \infty} \frac{1}{\tau} \int_{t_s}^{t_s+\tau} dt' \, A(t')$$

if $\boxed{=}$ $\Rightarrow$ ERGODICITY

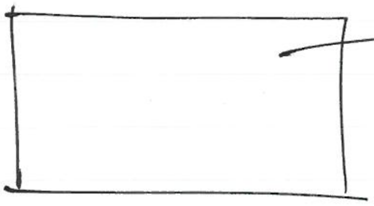
$$\langle A \rangle = \prod_i \int d\vec{x}_i \, d\vec{p}_i \quad A(\vec{p}_i, \vec{x}_i) \; \rho(\vec{p}_i, \vec{x}_i)$$

CAREFULLY : "ERGODICITY" COULD HOLD WITH $\rho \neq \rho_{GB}$;
ONLY FOR SOME A
ONLY FOR $t \gg t_s$

$\Rightarrow$ OUT OF EQUIL.

EQUILIBRIUM ENSEMBLES

N PART IN d-DIM SPACE
$\vec{P} = \left(\vec{P}^{(1)}, \ldots \vec{P}^{(N)}\right)$ ETC.



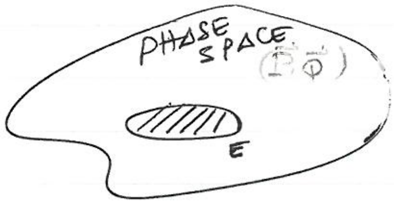
ISOLATED SYSTEM
W/ CONSERVED ENERGY
NEWTON DYN.

$$H(\vec{p}, \vec{\varphi}) = E$$

$\Longrightarrow$ MICRO CANONICAL ENSEMBLE

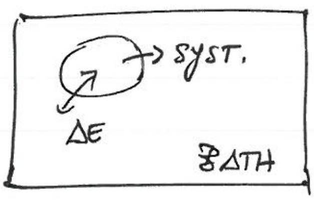
$$\rho_{MC}(\vec{p}, \vec{\varphi}) = \rho_{MC}(H(\vec{p}_i, \vec{\varphi}_i)) \propto \delta(H - E)$$

INITIAL VALUES



Uniform Distribution on const energy E subspace
Hard to compute w/it
Equivalence of ensembles Claimed $\Longrightarrow$

CANONICAL ENSEMBLE



ISOLATED TOTAL: SMALL SYST + BATH

$$\rho_C(\vec{p}, \vec{\varphi}) = \frac{1}{Z(\beta)} e^{-\beta H_{syst}(\vec{p}, \vec{\varphi})}$$

$$\beta = \frac{1}{k_B T} = \frac{1}{k_B} \left.\frac{\partial S(E)}{\partial E}\right|_{E}^{BATH}$$

ASSUMED CONSTANT & $\mathcal{O}(1)$

$$E = E_{syst} + E_{BATH} + \underbrace{\cancel{E_{INT}}}_{\ll E_{BATH}} \approx E_{BATH}$$

ADDITIVITY

LES HOUCHES 2008 DAUXOIS, RUFFO & LFC ON LONG-RANGE INT SYST.

STATISTICAL PHYSICS ⟹ PHASE TRANSITIONS

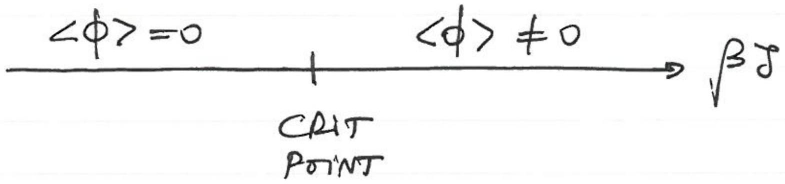
CLASSICAL STAT PHYS ⟹ USUALLY SYST COUPLED TO BATHS ⟹ CANONICAL SETTING*

SAY JUST ENERGY EXCHANGE

$$\frac{e^{-\beta H}}{Z(\beta)}$$

H: SOME COUPLING CONST / ENERGY SCALE, SAY J

βJ: CONTROL PARAMETER (ADIMENSIONAL)



- TYP. PHASE TRANS → 1 GLOBAL ORDER PARAM. $\langle\phi\rangle$

eg. ISING MODEL $\quad H = -\frac{J}{2}\sum_{\langle ij\rangle} s_i s_j$

$m = \frac{1}{N}\sum_i s_i \quad \begin{cases}\langle m\rangle = 0 \\ \langle m\rangle \neq 0\end{cases}$

$P(m) = \frac{1}{2}\delta_{m,|m_{eq}|} + \frac{1}{2}\delta_{m,-|m_{eq}|}$

Sampling done over measurements (numerical or experimental)

GINZBURG - LANDAU

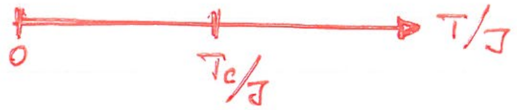
- MORE COMPLEX ONES → eg. LOCAL ORDER PARAM $\langle s_i\rangle$

$f(\langle s_i\rangle = m_i)$ ⟹ COMPLEX LANDSCAPE.

CONSTRUCT GLOBAL OBSERV FROM $\{m_i\}$ ⟶ CHARACT THEIR pdfs.

* THOUGH EQUIVALENCE W/ MICROCANONICAL ASSUMED.

- MANY BODY $N \gg 1$ $\Rightarrow$ IN EQUILIBRIUM ONE CAN HAVE PHASE TRANSITIONS



DYNAMICS ACROSS PHASE TRANSITIONS

OUT OF EQUILIBRIUM OVER VERY LONG TIMES

$t_s = \mathcal{O}(L^a) \xrightarrow[L \to \infty]{} \infty$

A KIND OF RATHER SIMPLE "COMPLEX" DYNAMICS.

e.g. $\left\{\begin{array}{l}\text{ANNEALING – KIBBLE-ZUREK} \\ \text{QUENCHES ACROSS } T_c \text{ IN 2d IM}\end{array}\right.$

$$H = -\frac{J}{2} \sum_{\langle ij \rangle} s_i s_j$$ COUPLED TO A BATH.

At $T_c \neq 0$ TRANS. BTW $\left\{\begin{array}{l}\text{DISORDERED (ENTROPY)} \\ \text{2 ORDERED (ENERGY)} \\ \quad |m| \neq 0 \text{ states}\end{array}\right.$

Locally spins tend to order $\uparrow$ or $\downarrow$ w/ = prob. to reduce the energy $\Rightarrow$ domain formation

NB $\phi(\vec{r},t) = \frac{1}{V_{\vec{r}}} \sum_{i \in V_{\vec{r}}} s_i$ tends to be constant within the domains

$\phi(\vec{r},t) \to \pm \bar{\phi}$ $\Rightarrow$ Fourier transform of a constant is $\delta(\vec{k})$

$\left\{\begin{array}{l}\text{COARSENING} \\ \text{DOMAIN GROWTH} \\ \text{PHASE ORDERING KINETICS}\end{array}\right.$

CONDENSATION PHENOM $\Rightarrow$ BACK TO IT LATER.

NON-STATIONARY.

- EXTERNAL DRIVES

  eg. ACTIVE MATTER (MICROSC.)
  GRANULAR MATTER (MACROSC.)

  Input of energy by drives have to be evacuated somehow - environment

- CONSTANTS OF MOTION ?

  Usually just E, maybe $\vec{P}, \vec{L}$ but what happens in cases with a macrosc. number of integrals of motion?

  INTEGRABLE SYSTEMS. $\{I_k\}$ $k = 1, \dots, Nd$

  As many as degrees of freedom.

  $\rho(\vec{x}, \vec{p})$?

AIM OF 3 LECTURES

FAMILIARIZE YOU w/ QUESTION & TECHNIQUES OF DISORDERED SYSTEMS

USING A/THE SIMPLE/ST MODEL

1- STUDY OF POT ENERGY LANDSCAPE

2- " " FREE ENERGY LANDSC (TAP)

3 - STATIC (CANONICAL) ANALYSIS

- DIRECT
- REPLICAS

4- STOCH DYN → OUT OF EQ RELAX.

→ EQ REL. FOR SPECIAL INITIAL COND.

DIRECT & MSR ⇔ SD

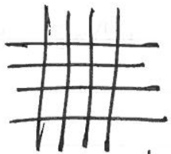
5_ CLOSE / NEWTON DYN & INTEGRABILITY

OGE.

DIRECT & MSR ⇔ SD.

FROM ISING TO BERLIN-KAC TO $\begin{cases} p=2 \text{ SPHERICAL} \\ \text{SPIN-GLASS} \\ \text{\& NEUMANN MODEL} \end{cases}$

$$(1) \quad H = -\frac{J}{2} \sum_{\langle ij \rangle} s_i s_j \qquad s_i = \pm 1$$



i LABELS THE SITE

d-dim LATTICE

$\langle ij \rangle$ nn.

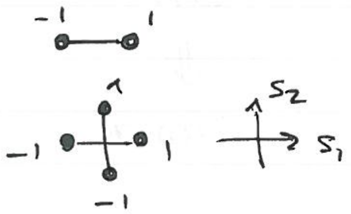
STATICS ONSAGER SOL IN $d=2$
BUT NO SOL FOR $d>2$

DYNAMICS STOCHASTIC GLAUBER IN $d=1$
MC SIMUL. IN $d>1$

SOLVABLE MODEL W/ SAME OR SIMILAR PHENOMENOLOGY ?
BERLIN-KAC
KAC-THOMPSON

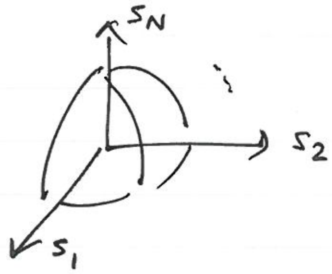
$$(s_1, \dots, s_N) = \vec{s}$$

IF ISING : ~~HYPERCUBE~~ $\square$ LATTICE, eg $N=1$
$N=2$



MAKE $-\infty \leq s_i \leq \infty$ BUT CONSTRAIN $\vec{s}$ TO BE ON THE N-DIM SPHERE $|\vec{s}| = \sqrt{N}$ W/ RADIUS $N^{1/2}$

$$\sum_i s_i^2 = N$$



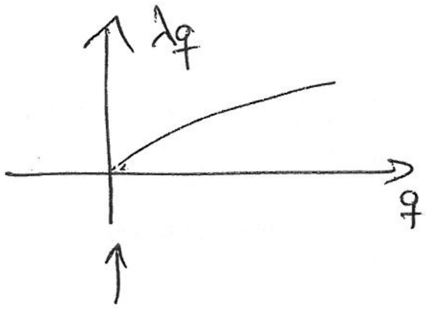
W/ SAME HAMILTONIAN (1).
ie, ON THE SAME LATTICE IN d-dim. $\Rightarrow$
FOURIER TRANSFORM

$$(2) \quad H = -\frac{J}{2} \sum_q s_{\vec{q}} \left(2 \sum_{a=1}^{d} (1 - \cos q_a) \right) s_{-\vec{q}}$$

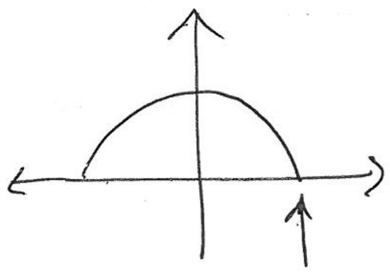
$$= -\frac{1}{2} \sum_q \lambda_q \, s_{\vec{q}} \, s_{-\vec{q}} \qquad \text{"DIAG QUADRATIC HAMILTONIAN"}.$$

$$\approx -\frac{1}{2} \sum_q q^2 s_{\vec{q}} s_{-\vec{q}} \qquad \text{SMALL } \vec{q} \text{ BEHAV.}$$

SPECTRA



EDGE $q=0$



EDGE λ_{MAX}

$\Downarrow$

ENERGY BOUNDED FROM BELOW

$q=0$
UNIFORM
HOMOGENEOUS
CONF
$\Downarrow$
ORDER.

SPHERICAL $p=2$ MODEL (MEAN)

$V_J[\{s_i\}] \Rightarrow$ DISTINCTION BETWEEN V WITHOUT CONST LAGRANGE MULT & WITH

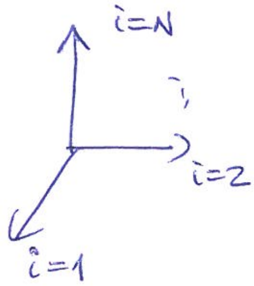
$$V_J^{(z)}[\{s_i\}] = -\frac{1}{2}\sum_{ij} J_{ij} s_i s_j + \frac{z}{2}\left(\sum_i s_i^2 - N\right)$$

$-\infty \leq s_i \leq \infty \qquad \sum_{ij} : i \neq j \qquad J_{ji} = J_{ij}$ i.i.d. SYMMETRIC

$[J_{ij}] = 0 \qquad [J_{ij}^2] = \frac{J^2}{N}$ GAUSSIAN $p(J_{ij})$

- JUSTIFICATION OF SCALING W/N: IT'LL LEAD TO INTERESTING RESULTS $\Rightarrow E = \Theta(N)$ AS IN $N \to \infty$ $\Theta(N)$ FIELD TH. SCALING $\frac{\lambda}{N}\phi^4$

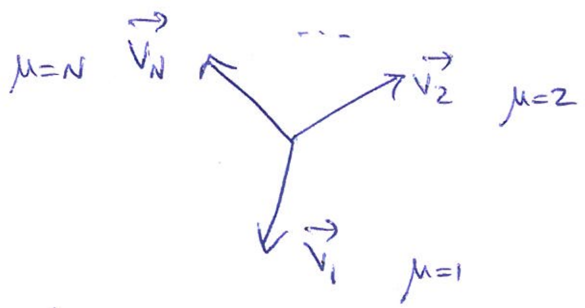
ORIGINAL BASIS



CONF. SPACE $\{s_i\}$ N-DIM

$\vec{s} = (s_1, \ldots, s_N)$

$\Rightarrow$ ROTATED BASIS



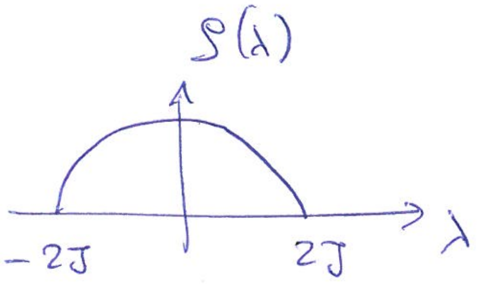
$J_{ij} v_j^{(\mu)} = \lambda_\mu v_j^{(\mu)}$

$\{\lambda_\mu, \vec{v}_\mu\} \quad \mu = 1, \ldots, N$ EIGENVALUES & EIGENVECTORS OF J MATRIX

- WE ORDER λ_μ's / $\lambda_1 < \lambda_2 < \ldots \lambda_N$

SCALING OF $[J_{ij}^2] = \frac{J^2}{N} \Rightarrow \lambda_\mu = \mathcal{O}(1)$

SYMMETRY & STATISTICS OF J_{ij} $\Rightarrow$ FOR $N \to \infty$
GOE



SEMICIRCLE LAW
WIGNER'S pdf
RECALL ALSO LEVEL REPULSION
$\lambda_\mu \neq \lambda_\nu$

OF COURSE, WE COULD HAVE TAKEN SOMETHING ELSE BUT THIS IS WELL-KNOWN IN SPIN-GLASS THEORY

KOSTERLITZ, THOULESS & JONES 70'S.

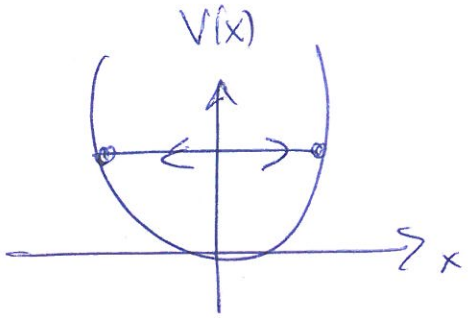
LOOKING FOR A "SIMPLE" SPIN-GLASS MODEL BUT ACTUALLY EVEN MUCH SIMPLER THAN THAT

STUDIES

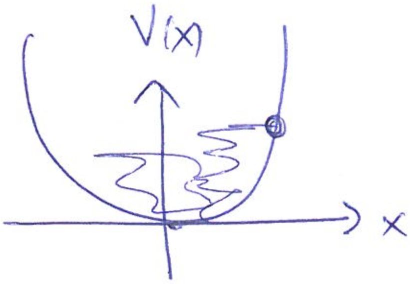
- POTENTIAL ENERGY / FREE-ENERGY LANDSCAPE
  METAST STATES
- EQUIL. PROPS, STUDY OF $Z(\beta) \to f(\beta)$
  PHASE DIAGRAM
- RELAX. DYNAMICS (LANGEVIN, COUPLING TO BATH)
  QUENCHES

POTENTIAL ENERGY LANDSCAPE

eg 1D HARM. POT



LANGEVIN RELAX.



KNOWING & LOOKING AT IT HELPS UNDERST. THE DYN OF THE PROB.

BUT IN HIGHER DIMENSIONS $\to$ COMPLEX LANDSCAPES

EXTREMA OF $V_J^{(z)}[\{s_i\}, z]$ WORKING AT FIXED REALIZATION OF J_{ij}.

$$0 = \left.\frac{\partial V_J^{(z)}}{\partial s_i}\right|_{\{s_i^*, z^*\}} = -J_{ij} s_j^* + z^* s_i^* = 0$$

THIS IS AN EIGENVALUE EQ
N-SOLS FOR
$\mu = 1, \dots, N$

$$\begin{cases} z^* = \lambda_\mu \\ \vec{s}^* = \vec{v}_\mu \end{cases}$$

EACH ONE OF THE EIGENVALUES OF $\mathbb{J}$

STABILITY OF $\{\vec{s}^* = \vec{v}_\mu, z^* = \lambda_\mu\}$:

$$\left.\frac{\partial V_J^{(z)}}{\partial s_i \partial s_j}\right|_{\{\vec{s}_i^*, z^*\}} = -J_{ij} + z^* \delta_{ij} = \mathcal{H}_{ij} \text{ HESSIAN}$$

$$\mathcal{H}_C = \begin{pmatrix} -\lambda_1 + z^* & & 0 \\ & \ddots & \\ 0 & & -\lambda_N + z^* \end{pmatrix}$$

IN DIAGONAL FORM

EIGENVALUES IN ORDER

$$\underbrace{+\lambda_1 \; - - - - \; \lambda_{\mu-1}} \quad \lambda_\mu \quad \underbrace{\lambda_{\mu+1} \; - - - \; \lambda_N}$$

$-\lambda_\nu + z^* > 0$

THE μth EXTREM IS A MINIMUM

$-\lambda_\mu + z^* = 0$

FLAT

$-\lambda_\eta + z^* < 0$

DIRECTIONS IN WHICH THE μth EXTREMUM IS A MAXIMUM

THE POTENTIAL ENERGY AT THE EXTREMA

$$V\left[\{\vec{S}^* \to \vec{V}_\mu\},\, z^* = \lambda_\mu\right] = -\frac{1}{2}\sum_\nu \lambda_\nu S_\nu^{*2} + \frac{z^*}{2}\left(\sum_\nu S_\nu^2 - N\right)$$

EVALUATED AT $\begin{cases} \vec{S}^* = \vec{V}_\mu \\ z^* = \lambda_\mu \end{cases}$

$S^2_{\nu(\neq\mu)} = 0 \quad S_\mu^2 = N$

$$= -\frac{\lambda_\mu}{2} N$$

$S_\mu = \vec{S} \cdot \vec{V}_\mu$

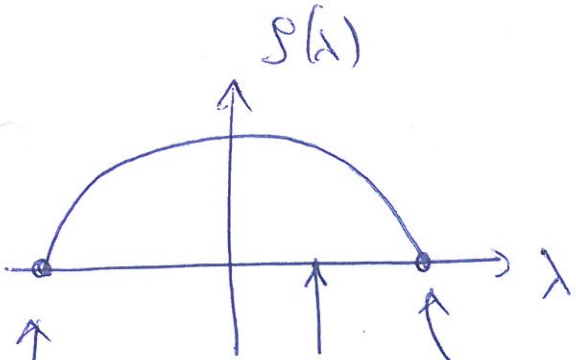
IF $\vec{S} = \vec{V}_\mu$

$S_\mu = V_\mu^2 = N$

$\Downarrow$

THE ABSOLUTE MINIMUM IS THE N-th EIGENVECTOR $\vec{V}_N$ FOR WHICH $\lambda_N = 2J$ (GOE)

AND $V_J^{(2)}[\vec{V}_N, \lambda_N] = -\frac{2JN}{2} = -JN$



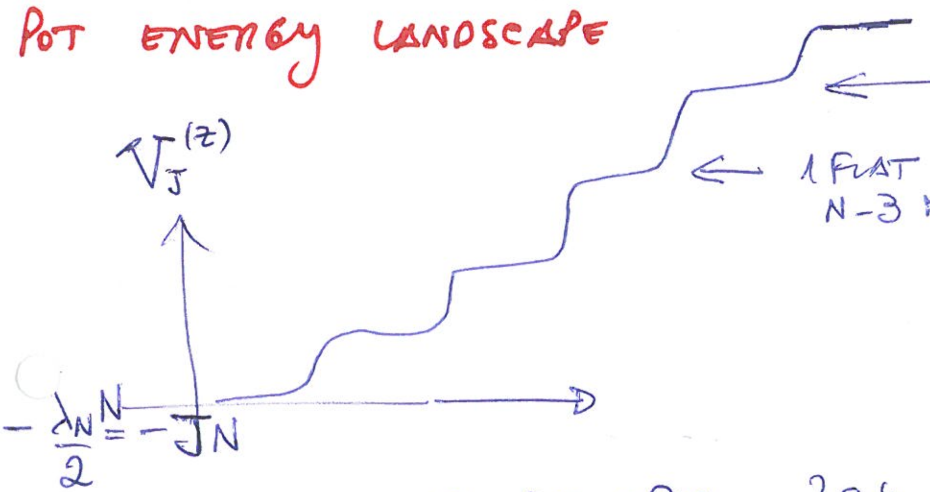
DENS EIGENVALUES OF J

ABSOLUTE MINIMUM (ONE FLAT DIRECTION)

ABSOLUTE MAXIMUM

INTERMEDIATE STATE W/
ONE FLAT DIRECTION
& "TO THE LEFT" DIRECT IN WHICH IT'S STABLE
"TO THE RIGHT" DIRECT IN WHICH IT'S UNSTABLE

SIMPLE "HIERARCHICAL" POT ENERGY LANDSCAPE



$-\frac{\lambda_1 N}{2} = JN$ MAXIMUM

1 FLAT $N-2$ MAX, 1 MIN.

1 FLAT $N-3$ MAX, 2 MIN

ETC.

CONFIG. SPACE. $\{S_i\}$

"UP-DOWN" SYMMETRY $S_i \Longleftrightarrow -S_i$ IS A SYMM OF HAMILT

$\Rightarrow$ TWO MINIMA, TWO MAX, ETC. ALL DOUBLED.

DEPENDENCE ON DISORDER J

IF $N \gg 1 \Rightarrow$ THE $\rho(\lambda)$ DOES NOT "FLUCTUATE"
BUT THE $\vec{V}_\mu$ WILL DO, MOST PROBABLY.

FREE-ENERGY LANDSCAPES

TEMP $\neq 0$ EFFECTS

WE NOW THAT AT $T=0 \Rightarrow$ GROUND STATE (OR EVEN METAST STATES) EXTREME OF POT. ENERGY.

NOW, IF $T \neq 0$ WHICH IS THE RELEVANT LANDSCAPE? THOULESS-ANDERSON & PALMER 70s.

RECALL AGAIN GINZBURG LANDAU:

$$\{s_i\} \longrightarrow \phi(\vec{r})$$

WITH A $\lambda\phi^4$ TH BELOW T_c
WHERE IS THE ϕ^4 TERM COMING FROM?
THE $\phi \simeq 0$ (2nd ORDER PH. TRANS.)

$$\mathcal{N}[\phi] \approx e^{S[\phi]}$$

EXPANSION OF $S[\phi]$ ENTROPY ($k_B = 1$)

$$\text{ex} \quad -\frac{J}{2}\sum_{i \neq j} s_i s_j = V$$

FULLY CONN. ISING MODEL

$$Z(\beta) = \sum_{\{s_i = \pm 1\}} e^{-\beta V} \longrightarrow \int dy \, e^{-\beta N \underbrace{f(y)}_{\text{FREE-EN. DENSITY}}}$$

$$y = \frac{1}{N}\sum_i s_i$$

I DON'T WRITE THE Z IN $e(y)$ BUT IT'S THERE, AS IN $f(y)$.

$f(y)$ DEPENDS ALSO ON β, J.

$$f(y) = e(y) - T\,s(y)$$

THE POT ENERGY CONTRIB.

THE ENTROPY DENSITY $s(y)$ COMES FROM # $\{s_i\}$ CONFS W/ SAME y. (W/ THE LAGRANGE MULT.)

EASY & STRAIGHTFORWARD FOR FM-ISING BUT FOR MODELS W/ DISORDER IT'S HARDER

y (STILL A "FLUCTUATION" OBJECT) $\Longrightarrow$ FIXED BY SADDLE-POINT

$$y_{SP} = \langle y \rangle = m.$$

WITH DISORDER $y \longrightarrow \tilde{m}_i$ LOCAL AND THE FLUCTUATING ONES

$f(\{\tilde{m}_i\})$ SHOULD HAVE
- AN ENERGY CONT
- AN ENTROPIC ONE
- A CORRECTION TERM (KILLING SELF-EFFECT)

SYSTEMATIC WAY OF DERIVING $f(\{\tilde{m}_i\})$
- YEDIDIA & GEORGES 80S
- BIROLI 0D (DYNAMICS LANGEVIN)
- BIROLI & ICF (QUANTUM)
- BARBIER ET AL (NEWTON)

$$\downarrow$$

S.P. $\dfrac{\partial f(\{\tilde{m}_i\})}{\partial \tilde{m}_i} = 0 \Longrightarrow$ EXTREME OF FREE-ENERGY LANDSCAPE.

WHAT HAPPENS WITH THESE "METASTABLE STATES" WHEN TEMP IS MODIFIED? DEPENDS ON THE MODEL.

- FOR TWO-BODY PBM w/ SPHERICAL CONST NOTHING WEIRD ⇒ STATE "FOLLOWING".
- FOR TWO BODY w/ ISING → SK MODEL
- " THREE BODY w/ ISING / SPH. MORE COMPLEX

POSSIBILITY OF $\begin{cases}\text{CROSSING / MERGING / BIRTH /}\\ \text{DEATH}\end{cases}$

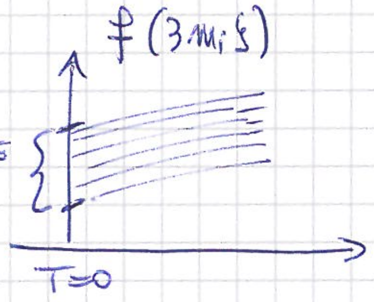
DEPENDING ON COMPLEXITY OF MODEL.

NOT FOR THE MODEL WE'LL STUDY HERE.

RECALL: $$f(\{m_i\}) = \frac{1}{N}\sum_i (1-m_i^2)$$

+ ONSAGER REACTION TERM

LITTLE TECHNICALITY



COLLAPSE TO THE SAME FOR p=2 MODEL

"LEVEL OF FREE-ENERGY" if THEY DON'T CROSS NOR DISAPPEAR / MERGE

$e_{th} = e_0$

$$q = \frac{1}{N}\sum_i m_i^2 = 1 - T/J$$ ∀ SOLS TO TAP EQS. (N-EQS.)

DUE TO ONSAGER TERM!

EQUILIBRIUM PROPERTIES $\longrightarrow Z(\beta)$

$$Z_J(\beta) = \underbrace{\sum_{\{conf\}}}\, e^{-\beta V_J[\{s_i\}]}$$

IMPOSE CONST w/ LAGRANGE MULT z
(ON AVERAGE)

$$\longrightarrow -\beta f_J(\beta) = \frac{1}{N} \ln Z(\beta)$$

WITH THE i's AND CONTOURS THAT SHOULD BE CHOSEN

$$\sum_{\{conf\}} \longrightarrow \int \prod_i ds_i \int dz\, e^{-\frac{\beta z}{2}\left(\sum_i s_i^2 - N\right)}$$

$\hookrightarrow$ CONVENIENT TO INTRO β

$$\sum_i \langle s_i^2 \rangle = \sum_\nu \langle s_\nu^2 \rangle = N$$

CONST ON AVERAGE HERE

QUADRATIC INTEGRALS $\Rightarrow$

$$Z_J(\beta) \propto \int dz \int \prod_\nu ds_\nu \exp\left\{ \frac{\beta}{2} \sum_\nu \lambda_\nu s_\nu^2 - \frac{\beta z}{2}\left(\sum_\nu s_\nu^2 - N\right)\right\}$$

IDEA FIX z BY IMPOSING THE SPH CONST

QUADRATIC INT OVER $\{s_\nu\}$ $\longrightarrow$

$$\langle s_\nu^2 \rangle = \frac{T}{z - \lambda_\nu}$$

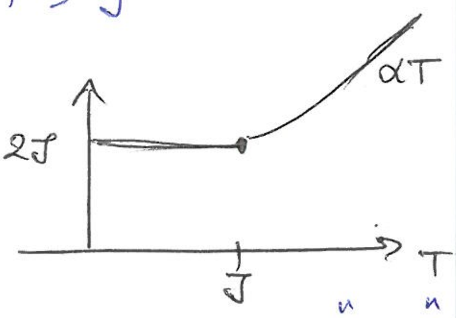
& THE EQ FIXING z IS:

$$\sum_\nu s_\nu^2 = N = \sum_\nu \frac{T}{z - \lambda_\nu} \overset{N \gg 1}{\longrightarrow} 1 = T \int d\lambda\, \frac{1}{z - \lambda}\, \rho(\lambda)$$

THIS EQ ADMITS A $z(T)$ SOL $\forall\ T > J$

$$\begin{cases} z = T + \dfrac{J^2}{T} & T > J \\ z = 2J & T < J \end{cases}$$

SINCE THERE'S NO MORE SOL TO EQ FOR z. z FIXED TO BEST POSSIBLE VALUE



BUT IF $Z = 2J$ FOR $T < J$ $\Rightarrow$

$$T \int d\lambda \, \frac{\rho(\lambda)}{Z - \lambda} < 1$$

THE MISSING PIECE IS ATTRIBUTED TO THE LARGEST MODE (SIMILAR BEC) **BEC**

$$\frac{1}{N} \langle S_N^2 \rangle + \underbrace{T \int^{1} d\lambda \, \frac{\rho(\lambda)}{Z - \lambda}}_{1/J} = 1$$

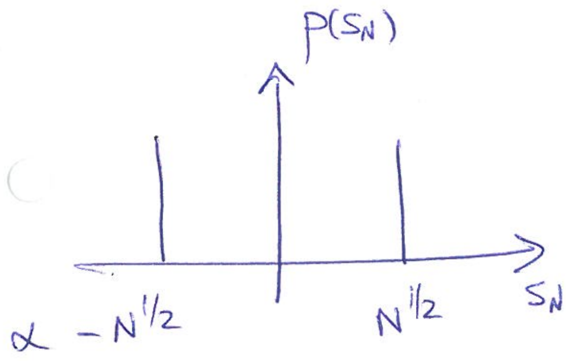
NO PROBLEM WITH DIVERG AT BORDER SINCE NUM $\to 0$ TOO & INTEGRAL CONVER.

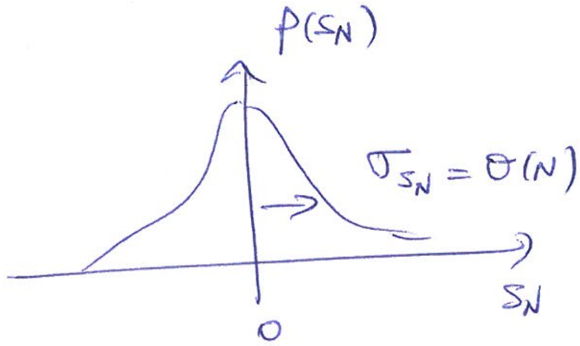
$$\| $$

$$\underbrace{\left(1 - \frac{T}{J}\right)}_{\text{PROPOSE IT}} + \frac{T}{J} = 1 \quad \checkmark$$

$\boxed{\text{PHASE TRANSITION}}$ DUE TO SPH CONST !

NOW $\langle S_N^2 \rangle \propto N$ CAN BE DUE TO:





BIMODAL DIST

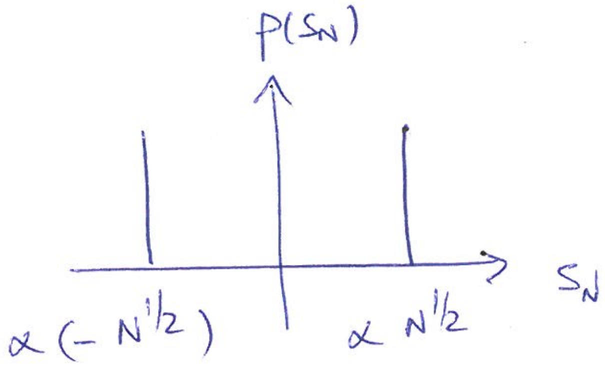
$\Rightarrow$ SYMM / ERG BREAKING
SPONTANEOUS
BROKEN SYMMETRY

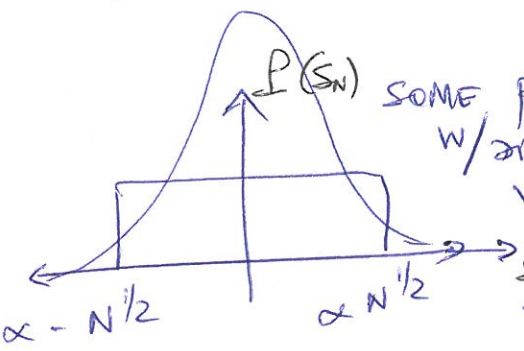
CONDENSATION À LA BEC	OR	CONDENSATION OF FLUCTUATIONS.
BEC CANONICAL		BEC GRAN-CANONICAL

BERLIN - KAC // KAC - THOMSON // M. ZANNETTI

TWO OPTIONS → $S_N = \pm\, a(T)\, N^{1/2}$

→ OR simply $\langle S_N^2 \rangle = \Theta(N)$





SOME pdf w/ DIVERGING VARIANCE & ZERO MEAN

CONDENSATION OF Nth MODE

TO DECIDE ⟹ HIGHER ORDER CORRELATIONS.

CONDENSATION of FLUCT of Nth MODE

$P(S_N)$ GAUSSIAN, WEIGHT

$$e^{+\left(\frac{\lambda_N}{2} S_N^2 - \frac{z}{2} S_N^2\right)\beta}$$

$\boxed{z^* \to \lambda_N + \Theta(N^{-1})}$ CORRECTIONS

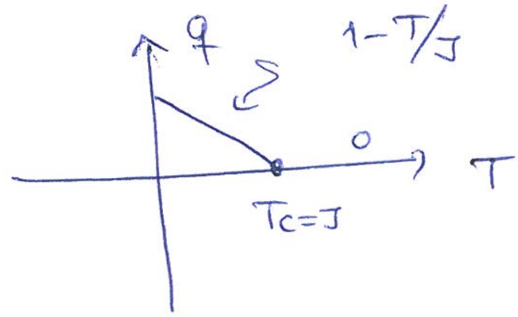
REPLICA CALCULATION

AVERAGE OVER DISORDER HERE

$$-\beta\, [f(\beta)] = \frac{1}{N} [\ln Z(\beta)] \Rightarrow$$

$$\lim_{n\to 0} \lim_{N\to\infty} \frac{1}{N} \frac{[Z^n(\beta)] - 1}{n}$$

RS ANSATZ



$$Q_{ab} = \begin{pmatrix} \diagdown & q \\ q & \diagdown \end{pmatrix} = \langle s_a s_b \rangle$$

MARGINAL REPLICON $= 0$

Summary

$$V_J^{(z)}(\{s_i\}) = -\frac{1}{2}\sum_{i\neq j} J_{ij}\, s_i s_j + \frac{z}{2}\left(\sum_i s_i^2 - N\right)$$

- Pot energy landscape

$$z^* = \lambda_\mu$$

- Free-energy landscape

$$f_{TAP}(\{m_i\}, q) = -\frac{1}{2N}\sum_{i\neq j} J_{ij}\, m_i m_j \quad \text{energy}$$

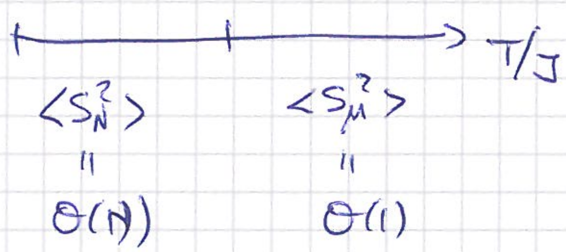
$$-\frac{T}{2}\ln(1-q) - \frac{1}{4T}(1-q)^2$$

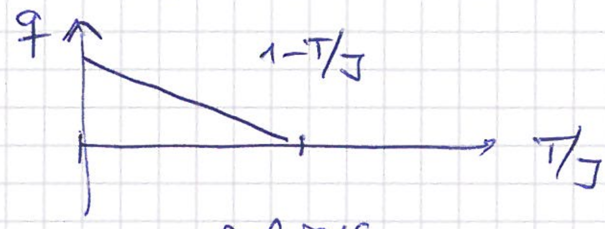
entropy   reaction

$$q \equiv \frac{1}{N}\sum_i m_i^2$$

Pot energy extreme $\longrightarrow$ free energy ones

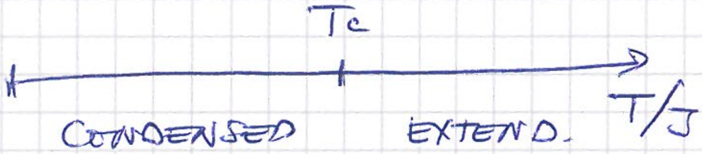
- Equilibrium $\qquad \langle s_\mu^2 \rangle = \dfrac{T}{z-\lambda_\mu} \qquad z = 2J = \lambda_N + \Theta(1/N)$



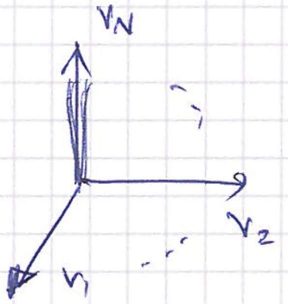


Replicas

Recall that initial conditions can be drawn from the two phases



$\langle S_N^2 \rangle \propto N$ $\qquad$ $\langle S_\mu^2 \rangle = \mathcal{O}(1) \quad \forall \mu.$



$T < T_c$ $\quad$ Something special about $\langle S_N^2 \rangle$ below T_c

It's more gral than just using the ground state.

- One can also use "metast states" as initial conditions - working at fixed J_{ij}.
- One can use $\quad p_{ic}(\{S_\mu\}) \propto e^{-\beta V_J^{(z)}(\{S_\mu\}, z)}$

Boltzmann form

STOCHASTIC DYNAMICS

LFC & DEAN '95

$$\dot{s}_i = J_{ij} s_j - z s_i + h_i + \xi_i$$

$$\dot{s}_\mu = \lambda_\mu s_\mu - z s_\mu + h_\mu + \xi_\mu$$

NB $h_i \Rightarrow$ ERGODICITY BREAKING

FOCUS ON $T = 0$

DIRECT SOLUTION OF EQ EASY FOR 1st TIME DERIV

$$s_\mu(t) = s_\mu(0)\, e^{\lambda_\mu t - \int_0^t dt' z(t')} \left(\xi_\mu(t') + h_\mu(t') \right)$$

CALL $\Gamma(t) \equiv e^{2\int_0^t dt' z(t')}$

$$s_\mu(t) = \frac{s_\mu(0)}{\sqrt{\Gamma(t)}} e^{\lambda_\mu t} + \int_0^t dt'\, e^{\lambda_\mu (t-t')} \sqrt{\frac{\Gamma(t')}{\Gamma(t)}} \left(\xi_\mu(t') + h_\mu(t') \right)$$

CONSTRAINT $\frac{1}{N} \sum_\nu \langle s_\nu^2(t) \rangle = 1$ FOR $h_\mu(t) = 0 \Rightarrow$

I. C.

$$\Rightarrow \Gamma(t) = \langle\langle \underline{s_\mu^2(0)}\, e^{2\lambda_\mu t} \rangle\rangle + 2T \int_0^t dt' \langle\langle e^{2\lambda_\mu (t-t')} \rangle\rangle\, \Gamma(t')$$

$\boxed{\text{INITIAL COND DEP}}$

WHERE $\langle\langle f(\lambda_\mu) \rangle\rangle = \int d\lambda\, \rho(\lambda)\, f(\lambda)$

* WHAT IS $z(t)$ IN $t \to \infty$ LIMIT?

$$\Gamma(t) = e^{2\int_0^t dt'\, z(t')} \Rightarrow \frac{d}{dt}\left(\frac{1}{2}\ln\Gamma(t)\right) = z(t)$$

$$\Gamma(t) \to \frac{e^{4t}}{(2t)^{3/2}} \Rightarrow \frac{d}{dt}\,\frac{1}{2}\left(4t - \underbrace{\ln(2t)^{+3/2}}_{}\right)$$

CORRECTION $\to 0$ LARGE t

$$\boxed{z(t) \longrightarrow 2J}$$ THE EQUIL VALUE!

* $e_{pot} = \langle V_J^{(2)} \rangle$

$$\dot{s}_i = J_{ij} s_j - z s_i$$

EVALUATE AT T, MULT BY $s_i(t')$, TAKE $t' \to t^-$

AND SUM OVER N

$$\underbrace{\frac{1}{N}\sum_i \frac{d}{dt}\langle s_i(t) s_i(t^-)\rangle}_{} = -2V_J(\{s_i\}) - z$$

ARGUE $\to -T$

$$\boxed{z = T - 2e_{pot}}$$

$$e_{pot} = \frac{T - z}{2} \longrightarrow \frac{T - 2J}{2} \Rightarrow -J \text{ AT } T=0.$$

$$\boxed{e_{pot} = -J}$$

ONE TIME MACROSC QUANTITIES APPROACH EQUIL. VALUES

INITIAL CONDITIONS

1 - UNCORRELATED FROM LOW T EQUIL $\Rightarrow$

$$S_N^2(0) = \mathcal{O}(1)$$

2 - CORRELATED W/ LOW T EQUIL $\Rightarrow$

$$S_N^2(0) = \mathcal{O}(N)$$

EASY INITIAL COND.

$$\boxed{S_N^2(0) = 1 \quad \forall \mu .}$$

QUENCH FROM HIGH T TO T=0

THE INTEGRAL EQ OF VOLTERRA TYPE $\Rightarrow$ SOL AT $T=0$ $(J=1)$

$$\boxed{\Gamma_{T=0}(t) = \frac{I_1(4t)}{2t} \xrightarrow[t \gg 1]{} \frac{e^{4t}}{(2t)^{3/2}}}$$

1 - ONE TIME QUANTITIES (GLOBAL)

RESCALING $t \to Jt$

FROM HERE NO SPACE BUT TWO TIME.

3 - TWO TIME QUANTITIES (GLOBAL)

$$C(t,t') = \frac{1}{N} \sum_i \langle S_i(t)\, S_i(t') \rangle_{\xi, i.c.}$$

$$R(t,t') = \frac{1}{N} \sum_i \left. \frac{\delta \langle S_i(t) \rangle}{\delta h_i(t')} \right|_{h=0}$$

$$C(t,t') \underset{T=0}{=} f(t'/t) \qquad R(t,t') = t^{-3/2}\, g(t'/t)$$

AGING $\qquad$ (NNO) MEMORY

$$\langle S_\mu(t)\rangle_\xi = \frac{S_\mu(0)\, e^{\lambda_\mu t}}{\sqrt{\Gamma(t)}} \sim S_\mu(0)\, e^{\lambda_\mu t}$$

$\downarrow$

EVEN W/ TEMP.

$$\sim \frac{S_\mu(0)\, e^{\lambda_\mu t}}{\sqrt{\dfrac{e^{2(2J)t}}{(2Jt)^{3/2}}}}$$

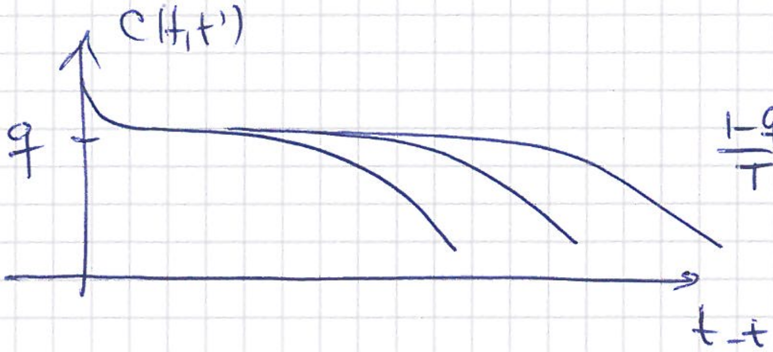
2_ MODES BEHAVIOUR

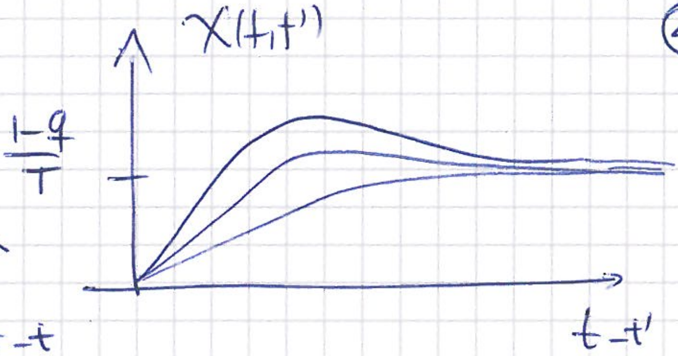
$$\sim S_\mu(0)\, \underbrace{e^{(\lambda_\mu - 2J)t}} \quad (2Jt)^{3/4}$$

ALL MODES DECAY EXPONENTIALLY BUT FOR THE N-th ONE $\lambda_N = 2J$ THAT GROWS AS A POWER LAW

$$\boxed{S_N(0) \sim S_N(0)\,(2Jt)^{3/4}}$$

SIMILARITY W/ COARSENING.

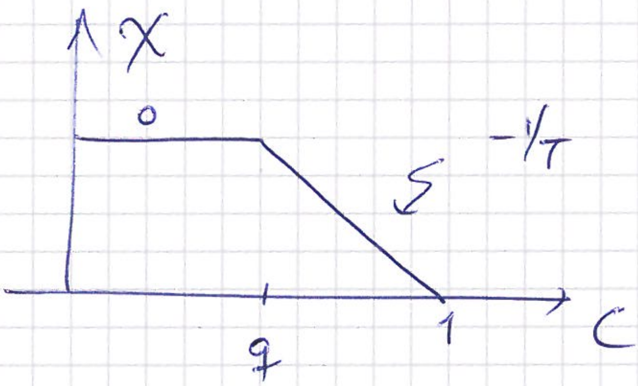




FDT → BEN'S

$$\underbrace{\frac{\partial}{\partial \beta_i} \langle \cdot \rangle}_{\text{LINEAR RESPONSE}} = \underbrace{\langle \cdot \; Q_i \rangle}_{\text{CORR}}$$

SPECIFIC HEAT FLUCT OF ENERGY



Lim
$t' \to \infty$
t/t' FIXED

MARTIN - SIGGIA - ROSE $\Rightarrow$ SCHWINGER DYSON EQS.

GENERATING FCT

EQS ON $\begin{cases} C(t,t') \\ R(t,t') \end{cases}$

DIRECTLY.

ONE COULD DO PERTURBATION & THE RE-SUM "À LA RUSSIAN" BUT FCT FORM IS BETTER

SYMMETRIES, etc.

$$Z_{dyn}[\{\eta, \hat{\eta}\}] = \int \underbrace{\mathcal{D}\xi \, e^{-\frac{\xi^2}{4T}}}_{\substack{\text{INT OVER} \\ \text{NOISE \&} \\ \text{MEASURE}}} + \eta \, \theta(\{s_\xi\})$$

SOL. LANG EQ.

$\langle \theta(\{s_\xi\}) \rangle_\xi$ FROM $\dfrac{\delta Z_{dyn}[\eta]}{\delta \eta(t)}$

NB: $Z_{dyn}[\eta = 0, \hat{\eta} = 0] = 1$ NORM OF NOISE

(NO NEED OF REPLICAS TO AVERAGE)

$$Z_{dyn}[\{\eta, \hat{\eta}\}] = \int \mathcal{D}\xi \, \mathcal{D}s \, \mathcal{D}\hat{s} \, \exp\left[i\hat{s} \, \text{Eq}^{\text{LANG}} \, (= 0) \right] P[\xi]$$

(2 SOURCES NOW)

THE DELTA IMPOSING S TO BE SOL OF LANG EQ

LFC + LECOMTE 17 ALL SUBTLETIES

NOW TRICKS

1) INT OVER J_{ij}'s w/ $P(J_{ij})$

2) DECOUPLE SITES BY INTRODUCING

$$\tilde{C}(t,t') = \frac{1}{N} \sum_i s_i(t) s_i(t')$$

$$\tilde{R}(t,t') = \frac{1}{N} \sum_i s_i(t)\, i\hat{s}_i(t')$$

+ 2 OTHERS

LIKE THE $y = \frac{1}{N} \sum_i s_i$ IN GINZB LANDAU.

3) SP ON $\tilde{C}, \tilde{R}$

$\Rightarrow$ EQS ON

$$\langle \tilde{C}(t,t') \rangle = C(t,t')$$

$$\langle \tilde{R}(t,t') \rangle = R(t,t')$$

JUST THEM IF CAUSALITY

PLAN IV LECTURES

D. BARBIER, LFC, G. LOZANO & N. NESSI

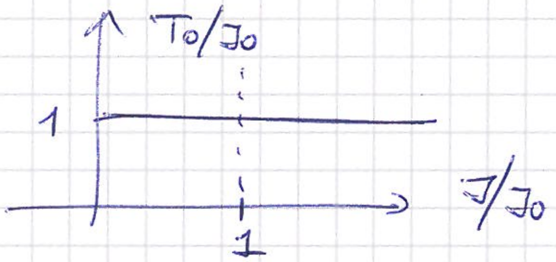
18-20.

1 – NEUMANN MODEL DEF & CONST OF MOTION

2 – THE QUENCH $J^0_{ij} \to J_{ij}$ W/ A SCALING FACTOR

P_{eq} À LA BOLTZM FOR INIT COND.

3 – AVERAGED CONST OF MOTION $\langle I_\mu \rangle_{IC}$ KNOWN



$\uparrow$ IC

$\langle I_\mu \rangle_{IC} = \mathcal{O}(1)$

$\langle I_N \rangle_{IC} = \mathcal{O}(N)$

$\longrightarrow$ ENERGY CHANGE $\quad \Delta e_{TOT} \propto (1 - J/J_0)$

4 – SCHWINGER DYSON : DISCUSS INITIAL COND. TERMS

Σ, D SIMPLE FOR $p=2$

QUENCH (J_0, J, β_0) PARAM.

$\Rightarrow$ HINTS ON PHASE DIAGRAM.

5 – GGE "CALCULATION" & RESULTS

$\bar{c}$ SKETCHED

Hamiltonian Dynamics →

Neumann Model

$$H = \frac{p^2}{2m} + V(x)$$

$$\Rightarrow \quad H_J[\{p_\mu, s_\nu\}] = \frac{1}{2}\sum_\nu p_\nu^2 + V_J[\{s_\nu\}]$$

AND SPH CONST $\sum_\mu s_\mu^2 = N$ PRIMARY $\qquad \sum_\mu p_\mu \cdot s_\mu = 0$ SECONDARY

$I_\mu[\{p_\nu, s_\nu\}]$: N CONST OF MOTION SUCH THAT

$$\frac{dI_\mu}{dt} = \{H, I_\mu\} = 0 \qquad \{I_\mu, I_\nu\} = 0$$

$$\sum_\mu \lambda_\mu I_\mu = -2H \qquad \sum_\mu I_\mu = \sum_\mu s_\mu^2 = N$$

INTEGRABLE MODEL.

$\boxed{\text{QUESTION}}$ $\qquad \boxed{\lim_{t\to\infty} \quad \lim_{N\to\infty}}$ HAMILTONIAN DYNAMICS ? AFTER A QUENCH →

VS A GGE $\quad Z_{GGE}[\{\beta_\mu\}] = \int e^{-\sum_\mu \beta_\mu I_\mu}$

$$I_\mu = s_\mu^2 + \frac{1}{mN}\sum_{\nu(\neq\mu)} \frac{s_\mu^2 p_\nu^2 + s_\nu^2 p_\mu^2 - 2 s_\mu p_\mu s_\nu p_\nu}{\lambda_\nu - \lambda_\mu}$$

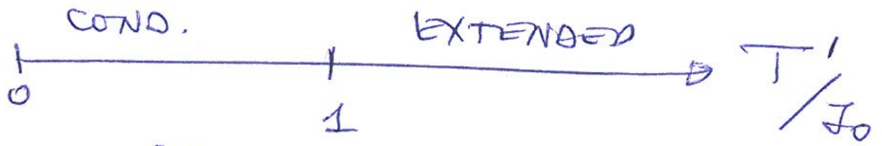
$L_{\mu\nu}^2/(\lambda_\nu - \lambda_\mu)$

↳ NORM. DUE TO $\sum_\mu s_\mu^2 = N$.

THE QUENCH

IN QUANTUM PROBLEMS SYST PREPARED IN GROUND STATE $|\Psi_0\rangle_{\hat{H}_0}$ AND EVOLVED WITH $\hat{H} \neq \hat{H}_0$.

HERE, WE KNOW FULL EQ. PHASE ($T=0$ BUT ALSO $T>0$) $\Longrightarrow$



INITIAL COND.

$\sim$ FM

$\langle S_N^2 \rangle = \Theta(N)$

$\sim$ PM

$\langle S_N \rangle = 0$
$\langle S_N^2 \rangle = \Theta(1)$

ALL DONE ON AVER. OVER IC.

CHOOSE INITIAL COND FROM BOTH PHASES

$J_{ij}^0 \to J_{ij}$
$J_0 \to J$

INJECT / EXTRACT ENERGY BY

ENERGY INJ / EXT.

$$J_0 \longrightarrow J$$

$$[(J_{ij}^{(0)})^2] = \frac{J_0^2}{N} \longrightarrow [J_{ij}^2] = \frac{J^2}{N}$$

$$\boxed{J_{ij}^{(0)} \longrightarrow \frac{J}{J_0} J_{ij}^{(0)} = J_{ij}}$$

"GLOBAL RESCALING

CONSTS OF MOTION ON AVER OVER INITIAL CONDIT.

$$I_\mu = s_\mu^2 + \frac{1}{mN} \sum_{\nu(\neq\mu)} \frac{(s_\mu p_\nu - s_\nu p_\mu)^2}{\lambda_\nu - \lambda_\mu}$$ UHLENBECK 80s

$\langle I_\mu \rangle_{ic}$ DEPENDS ON PHASE

$T_0 > T_c$: $\langle I_\mu \rangle_{ic} = O(1) \quad \forall \mu$

$T_0 < T_c$: $\langle I_N \rangle_{ic} = O(N)$ DUE TO $\langle s_N^2 \rangle$ AND IT WILL IMPOSE STRONG CONDITIONS ON THE DYNAMICS.

ONE CAN DRAW $\{s_\mu, p_\mu\}$ WITH $\frac{e^{-\beta_0 H}}{Z(\beta_0)}$ AND SEE HOW $P(\{I_\mu\})$ BEHAVES.

FROM STRICT SPH. MODEL TO SPH ON AVER OVER I.C.

$\sum_\mu I_\mu = N \quad \Rightarrow \quad \sum_\mu \langle I_\mu \rangle_{ic} = N$

$\sum_\mu \lambda_\mu I_\mu = -2H \quad \Rightarrow \quad \sum_\mu \lambda_\mu \langle I_\mu \rangle_{ic} = -2 \langle H \rangle_{ic}$

- HOW DO WE COMPUTE $\langle I_\mu(0^+) \rangle_{ic}$?

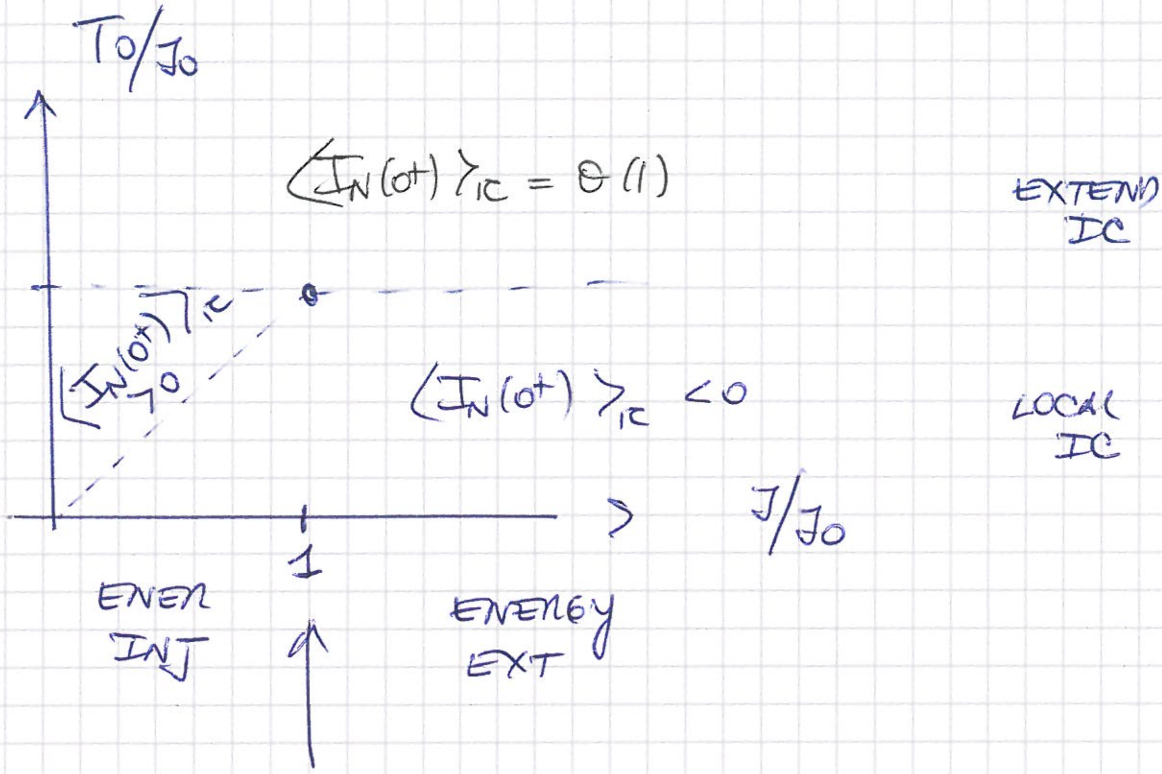
INSTANTANEOUS QUENCH ASSUMPTION

$$\langle s_\mu^2(0^+) \rangle_{ic} = \frac{T_0}{z_0 - \lambda_\mu^{(0)}} \qquad \langle p_\mu^2(0^+) \rangle_{ic} = m T_0$$

BUT THE $\{\lambda_\mu\}$s IN $\{I_\mu\}$'s EXPRESSION ARE THE POST QUENCH ONES. $\longrightarrow$ EXACT EXPRESSIONS.

$$
\langle I_\mu(0^+) \rangle_{IC} =
\begin{cases}
\dfrac{T_0^2}{J_0 J} \, \dfrac{J(J_0+J)/T_0 - \lambda_\mu}{J(J_0+T_0^2/J_0)/T_0 - \lambda_\nu} & T_0 \geq J_0 \\[2ex]
\dfrac{T_0^2}{J_0 J} \, \dfrac{J(J_0+J)/T_0 - \lambda_\mu}{2J - \lambda_\mu} & \mu \neq N \\[2ex]
\left(1 - \dfrac{T_0}{J_0}\right)\left(1 - \dfrac{T_0}{J}\right) N + O(1) & \mu = N
\end{cases}
$$

$$T_0 < J_0$$



NO QUENCH.

WRITE JUST $\quad \langle I_{\mu=N}(0^+)\rangle_{IC} = \left(1-\frac{T_0}{J_0}\right)\left(1-\frac{T_0}{J}\right)N$

FOR $T_0 < J_0$ $\quad + \underbrace{\Theta(N)}$ SMALLER THAN N

$\Rightarrow$ CHANGE of SIGN : AT $T_0 = J_0 \rightarrow 0$

AT $T_0 = J \rightarrow$ TOO.

- MENTION HERE

$$\Delta e_{TOT} = \Delta k + \Delta v \;\propto \left(1 - \frac{J}{J_0}\right) \Rightarrow$$

EXT for $J > J_0$

INJ for $J < J_0$.

IDEA STUDY (DYN & GEE BOTH) W/ SPH CONST IMPOSED ON AVERAGE, W/ LAGRANGE MULTIPLIER

How to study Dynamics?

Two Methods → $N \gg 1$ (DMFT-like) Schwinger-Keldysh Martin-Siggia-Rose ①

& LARGE N LIMIT ⇒ SADDLE-POINT EQS TO DERIVE COUPLED EQS ON $\{C, R, z\}$.

→ Sotiriadis-Cardy VARIATIONS ON PARAM. OSC. SOLUTIONS. $\Theta(N)$ ②

① Very well known in Disordered Systems studies
HANDY TO AVERAGE OVER "DISORDER"

② A BIT SPECIFIC BUT ALLOWS US TO FIND

$\langle S_\mu^2(t) \rangle$; $\langle p_\mu^2(t) \rangle$; $z(t)$

IT'S THE EXTENSION OF WHAT I DID FOR LANGEVIN ONLY THAT $\frac{\partial^2}{\partial t^2}$ IS HARDER TO TREAT.

ALREADY EXPLAINED FOR DISSIPATIVE / OVERDAMPED DYN.

SCHWINGER - DYSON EQS. FOR QUENCH FROM P_{eq} (BOLTZMANN) OF GENERIC MODEL.

$$G_0^{-1}(t)\, C(t,t') = \int_0^t dt''\, \Sigma(t,t'')\, C(t'',t') + \int_0^{t'} dt''\, D(t,t'')\, R(t',t'') + \underbrace{\frac{\beta_0 J_0}{J} \sum_{a=1}^{n} D_a(t,0)\, C_a(t',0)}_{\text{INITIAL COND. } n\to 0}$$

$$G_0^{-1}(t)\, R(t,t') = \int_{t'}^t dt''\, \Sigma(t,t'')\, R(t'',t')$$

$$G_0^{-1}(t)\, C_a(t,0) = \int_0^t dt''\, \Sigma(t,t'')\, C_a(t'',0) + \underbrace{\frac{\beta_0 J_0^2}{J} \sum_{b=1}^{n} D_b(t,0)\, Q_{ab}}_{\text{INIT. COND. } n\to 0}$$

$$G_0^{-1}(t) = m\frac{\partial^2}{\partial t^2} + z(t)$$ FOR NEWTON

" INVERSE FREE PROP "

$$z(t) = -m\frac{\partial^2}{\partial t^2} C(t,t')\Big|_{t'\to t^-} + \int_0^t dt''\, \Sigma(t,t'')\, C(t'',t') + \int_0^t dt''\, D(t,t'')\, R(t,t'') + \underbrace{\frac{J J_0}{T_0} \sum_{a=1}^{n} C_a(t,0)\, D_a(t,0)}_{\text{INIT. COND. } n\to 0}$$

ORIGIN OF "REPLICA" TERMS

$$Z_{dyn}[\eta] = \int \mathcal{D}\xi \, e^{-\frac{\xi^2}{4T}} \, e^{\int \eta \cdot \theta}$$

BUT WHERE IS $P_{INIT}[\{s_i\}, \{p_i\}]$?

HAVE TO ADD IT! IF $P^{(J)}_{INT}$ ← RANDOMNESS

$\Rightarrow$ AVERAGE OVER $P(\{J_{ij}\})$ NEEDS TO BE DONE USING REPLICAS. $\Rightarrow$ NO TRIVIAL DEP OF $Z_{dyn}[\eta=0]$ ON $\{J_{ij}\}$.

HOUGHTON, JAIN, YOUNG 80s
DE DOMINICIS 80s

IN EQS. IN PREV PAGE

$P_{INIT}[\{S_i(0)\}]$ DEPENDS ON $J_{ij}^{(0)}$

$$\left[\left(J_{ij}^{(0)}\right)^2\right] = \frac{J_0^2}{N}$$

AND SAMPLED AT β_0.

NB IN THIS CALC $Z(t)$ IS $\langle Z(t) \rangle_{IC}$

SELF-ENERGY $\Sigma(t,t') = J^2 R(t,t')$

VERTEX $D(t,t') = J^2 C(t,t')$

FOR MODEL W/ TWO-BODY INTERACTIONS.

COMMENT SD EQS CLOSE EXACTLY ON THESE TWO TWO-TIME OBJECTS 'CAUSE "MEAN-FIELD" MODEL.
"FULLY CONNECTED" MODEL.

$\boxed{Z(t) = 2\,(K(t) - V(t))}$
"ACTION"

NB EQ. ON $R(t,t')$ "DEPENDS ONLY" ON $R(t,t')$ (AND $Z(t)$) AND CAN BE STUDIED DIRECTLY INDEP. OF "INIT. COND" INFORM US ABOUT $Z(t) \to Z$
$t \gg t_0$

EASY TO SEE THAT

$$\Delta e = e_{TOT}(0^+) - e_{TOT}(0^-) \propto (J_0 - J) = J_0\left(1 - \frac{J}{J_0}\right)$$

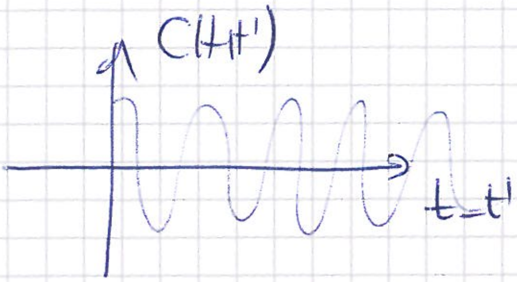
$J > J_0 \Rightarrow$ EXT

$J < J_0 \Rightarrow$ INJ

QUENCH $\Rightarrow$ Δe_{TOT}

SERIES OF PAPERS W/ D. BARBIER, G. LOZANO & N. NESSI 18-20.

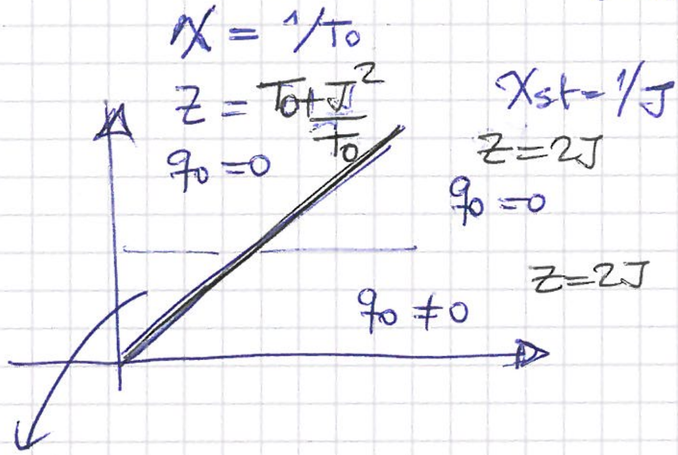
SD EQS → SOLVABLE NUMERICALLY & ANALYTICALLY.



$\lim\limits_{\substack{t'\to\infty \\ t-t' \\ \text{FIXED}}} C(t,t')$ CHECK STATION.

etc.

$\chi(t,t')$ SAME. ← $\lim\limits_{t\to\infty} C(t,0) = q_0$



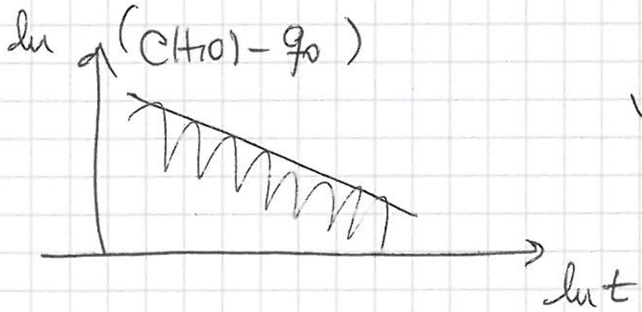
HINTS ON PHASE DIAGRAM

POWER LAW APPROACHES TO ASYMPT. VALUES.

INTERESTING POWERS

LOTS OF OSCILLATIONS

NOT THAT CLEAR SEEMS $q_0 = 0$



w/ osc.

VIOLATIONS OF FDT

⇓

NO BOLTZMANN EQ EXPECTED.

Mode Dynamics

- Not as easy as for Langevin → 2nd time deriv.
- Parametric oscillators $z(t) - \lambda_\mu$ time-dep. freq. $\Omega_\mu(t)$ w/ coupling via $z(t)$

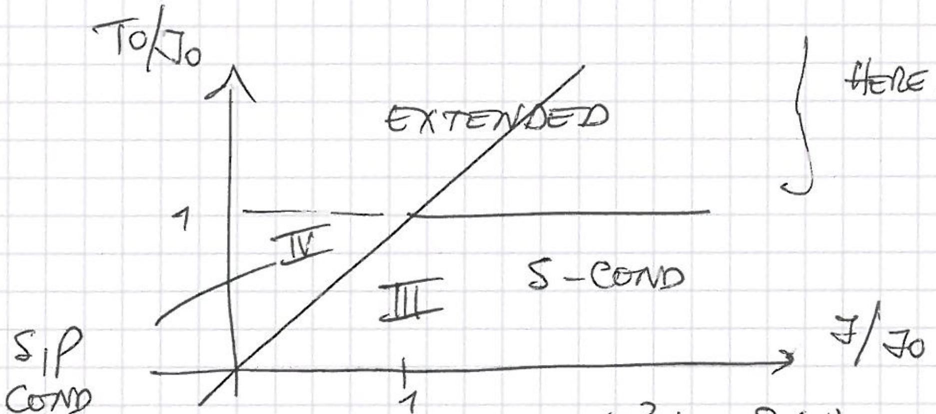
Sotiriadis & Cardy '10
Ermakov 60s

$s_\mu(t)$ → harm. osc solution w/ t-dep freq $\Omega_\mu(t)$
fixed by a 2nd order t-deriv eq.

+ constraint to fix $z(t)$

$$\boxed{\overline{\langle s_\mu^2(t)\rangle}_{ic}, \quad \overline{\langle p_\mu^2(t)\rangle}_{ic}}$$

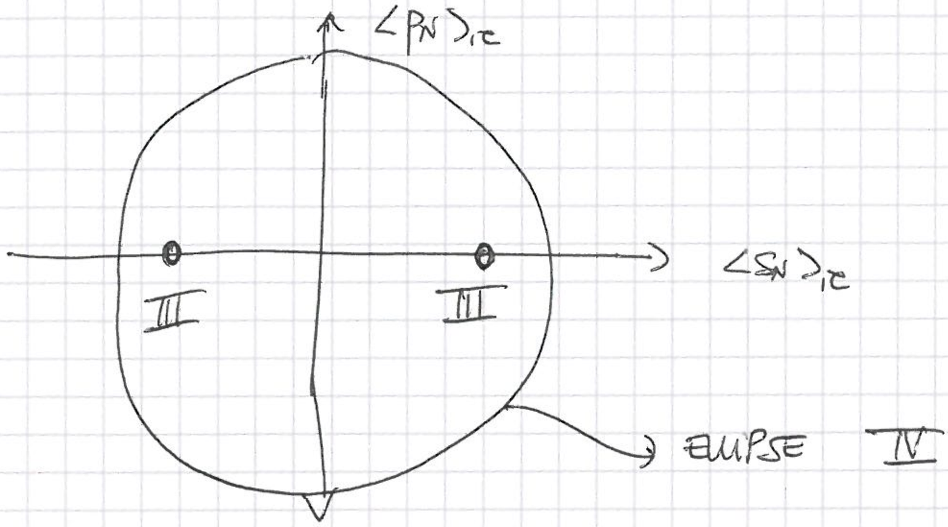
knowing $\langle s_\mu^2(0^+)\rangle_{ic}$, $\langle p_\mu^2(0^+)\rangle_{ic}$



Here no particular projection on $\vec{v}_N$

$\langle s_N^2\rangle = \mathcal{O}(N)$
$\langle p_N^2\rangle = \mathcal{O}(N)$

$\langle s_N^2\rangle = \mathcal{O}(N)$
$\langle p_N^2\rangle = \mathcal{O}(1)$



$T_0 < J_0$

INTERESTING PHASES.

GGE

BOLTZMANN $(J = J_0)$ $\xrightarrow{}$ FOR $\gamma_\mu = -\dfrac{\lambda_\mu \beta_0}{2}$

$$e^{-\sum_\mu \gamma_\mu I_\mu} \quad ?$$

USING

$$\sum_\mu \lambda_\mu I_\mu = -2H$$

SCALING W/N :
$$\begin{cases} \langle I_\mu \rangle_{IC} = O(1) & \text{IN } I, II \ \ \forall \mu . \\ \langle I_{\mu=N} \rangle_{IC} = O(N) & \text{IN } III, IV \end{cases}$$

[IN BOTH CASES I MAY EXPECT A FULLY $O(N)$ OBJECT IN THE EXPON. $\Rightarrow$ OK. FOR $N \to \infty$.

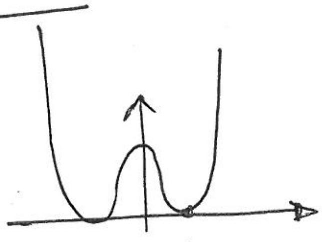
HOW TO COMPUTE

$$\langle s_\mu^2 \rangle_{GGE} \qquad \langle p_\mu^2 \rangle_{GGE} \quad ?$$

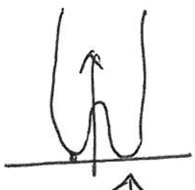
$$\langle I_\mu(0^+) \rangle_{IC} = \langle I_\mu \rangle_{GGE}$$ EXPECTED TO FIX $\{\gamma_\mu\}$

$J > T_0$ III $v(S_N)$

EXTRACTION OF ENERGY



QUENCH



OSCILL

WELL W/ FLAT BOTTOM

INITIAL COND.
" SYMM BROKEN

$\langle S_N^2 \rangle$

$O(N)$

BEFORE QUENCH

STILL MOVING THERE

$\langle S_N^2 \rangle$

$O(N)$

AFTER QUENCH.

$J < T_0$ IV INJECTION OF ENERGY

$\Rightarrow$ THE OSC MOVES AS CRAZY.

RECALL $$I_\mu = s_\mu^2 + \frac{1}{mN} \sum_{\mu (\neq \nu)} \frac{s_\mu^2 p_\nu^2 + s_\nu^2 p_\mu^2 - 2 s_\mu p_\mu s_\nu p_\nu}{\lambda_\nu - \lambda_\mu}$$

$$\frac{1}{N} \sum_\mu \circ \longrightarrow \int d\lambda \, \rho(\lambda) \, \circ$$

BUT IF $\displaystyle\sum_{\mu(\neq\nu)} \frac{\circ}{\lambda_\nu - \lambda_\mu} \longrightarrow ⨍ \frac{\circ}{\lambda - \lambda'}$ CAUCHY PRINCIPAL VALUE

RECALL THE CALCULATION OF $Z(\beta)$ $\Longrightarrow$ $\langle s_\mu^2 \rangle_{eq} = \dfrac{T}{z_{GGE} - \lambda_\mu}$

$\langle p_\mu^2 \rangle_{eq} = Tm$.

GGE $T \rightarrow T(\lambda)$

- HARMONIC ANSATZ NOW (PROVEN TO BE CONSISTENT AT SADDLE-POINT LEVEL)

$$\langle s^2(\lambda) \rangle_{GGE} = \frac{T(\lambda)}{z_{GGE} - \lambda} \qquad \langle p^2(\lambda) \rangle_{GGE} = m T(\lambda)$$

$$\langle s(\lambda) p(\lambda') \rangle_{GGE} = 0 \quad \forall \lambda, \lambda'$$

- EQS FIXING $T(\lambda)$

$$\begin{cases} \langle I(\lambda) \rangle_{IC} = \dfrac{2T(\lambda)}{z_{GGE} - \lambda} \left[1 - ⨍ d\lambda' \dfrac{T(\lambda') \rho(\lambda')}{\lambda - \lambda'} \right] \\[2em] \dfrac{\langle I_N \rangle_{IC}}{2 \langle s_N^2 \rangle_{GGE}} = 1 - ⨍ d\lambda' \dfrac{\rho(\lambda') T(\lambda')}{\lambda_N - \lambda'} - \dfrac{\langle s_N^2 \rangle_{GGE}}{N} \end{cases}$$

$\dfrac{T_0}{J_0} < 1$

HINTS ON SADDLE POINT CALCULATION

INTROD

$$\begin{cases} A_\mu^{(p^2)} = \frac{1}{N} \sum_{\nu(\neq\mu)} \frac{\gamma_\mu - \gamma_\nu}{\lambda_\mu - \lambda_\nu} s_\nu^2 \\ A_\mu^{(s^2)} = \frac{1}{N} \sum_{\nu(\neq\mu)} \frac{\gamma_\mu - \gamma_\nu}{\lambda_\mu - \lambda_\nu} p_\nu^2 \end{cases}$$

SAME TRICKS AS INTRO $\begin{cases} \{\hat{C}, \hat{R}\} & \text{IN DYN} \\ \{Q_{ab}\} & \text{IN REPLICAS} \end{cases}$

$\Rightarrow$ CONTIN. LIMIT w/ APPROP $\int$ WHEN NEEDED

OR CONT CONTINUATIONS

$$e^{-N[\quad]}$$ S.P ON (A's)

DISENTANGLE TO GET $\langle s_\mu^2 \rangle_{GGE}$, $\langle p_\mu^2 \rangle_{GGE}$